

Do Equidistant Energy Levels Necessitate a Harmonic Potential?

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Research Article

Keywords: equidistant spectrum , anharmonic oscillator , shift operator , quantum well state , semiconductor physics , angle-resolved photoemission spectroscopy (ARPES)

Posted Date: March 30th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-345038/v1>

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Version of Record: A version of this preprint was published at Optical and Quantum Electronics on July 14th, 2021. See the published version at <https://doi.org/10.1007/s11082-021-03015-6>.

Abstract

Experimental results from literature show equidistant energy levels in thin Bi films on surfaces, suggesting a harmonic oscillator description. Yet this conclusion is by no means imperative, especially considering that any measurement only yields energy levels in a finite range and with a nonzero uncertainty. Within this study we review isospectral potentials from the literature and investigate the applicability of the harmonic oscillator hypothesis to recent measurements. First, we describe experimental results from literature by a harmonic oscillator model, obtaining a realistic size and depth of the resulting quantum well. Second, we use the shift-operator approach to calculate anharmonic non-polynomial potentials producing (partly) equidistant spectra. We discuss different potential types and interpret the possible modeling applications. Finally, by applying n th order perturbation theory we show that exactly equidistant eigenenergies cannot be achieved by polynomial potentials, except by the harmonic oscillator potential. In summary, we aim to give an overview over which conclusions may be drawn from the experimental determination of energy levels and which may not.

Full Text

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Figures

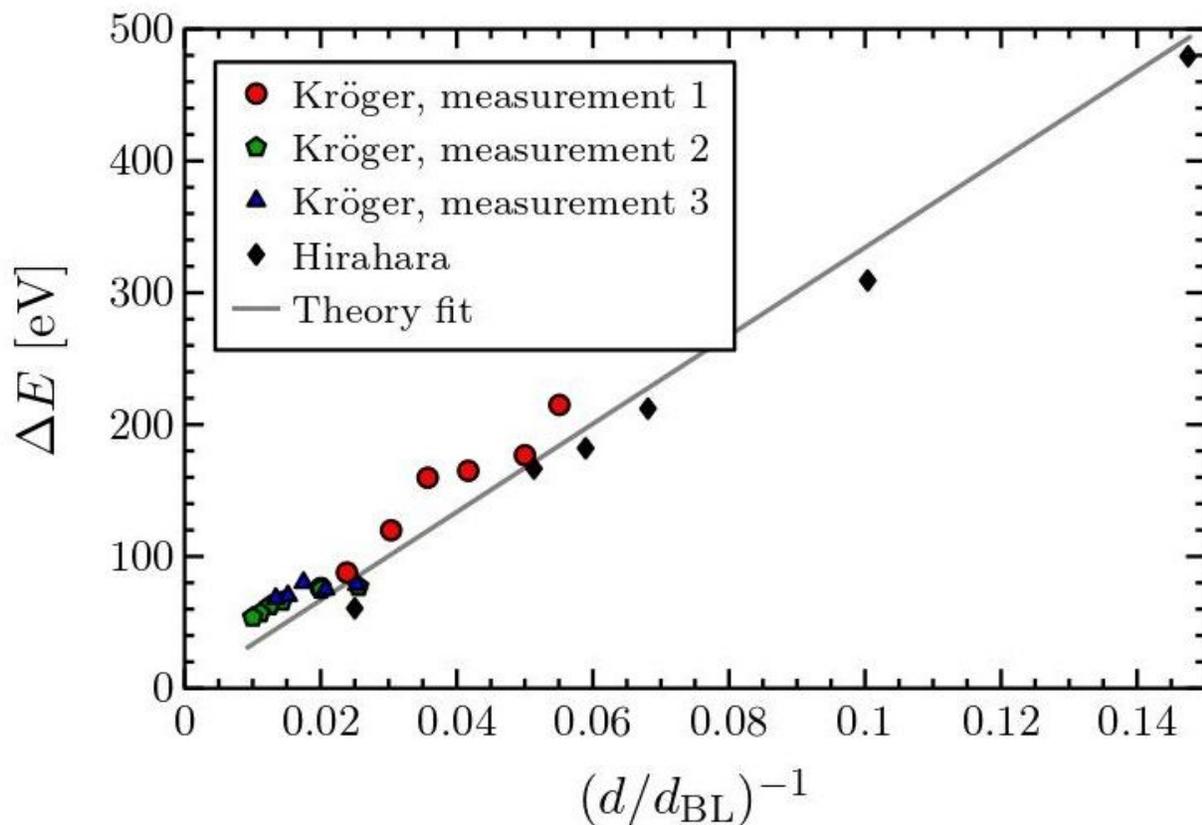


Figure 1

Experimental data of Kroger et al. [4] (3 different Bi films) and Hirahara et al. [5] in comparison with a fit to the harmonic oscillator.

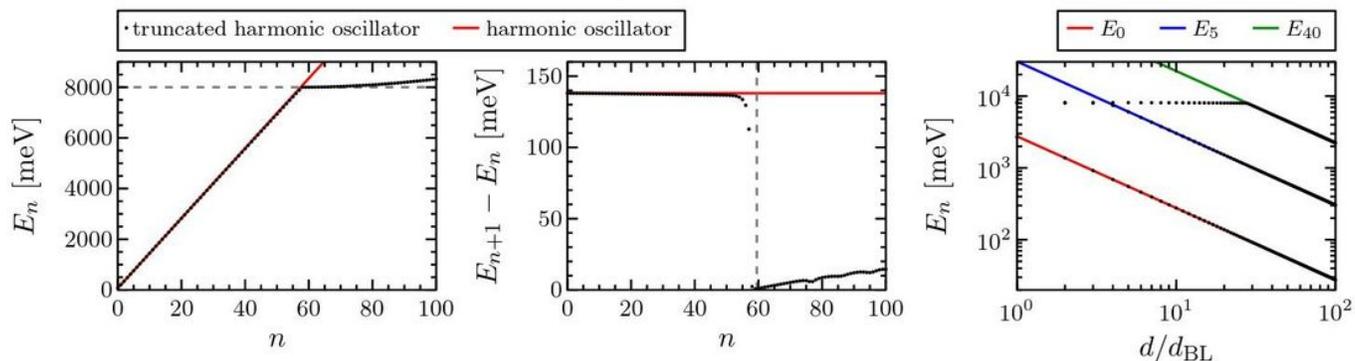


Figure 2

Left: energy levels. Center: energy level spacings. Right: thickness-dependent energy level. Colored lines denote the 1D harmonic oscillator. Black dots denote the truncated 1D harmonic oscillator of 16 nm thickness.

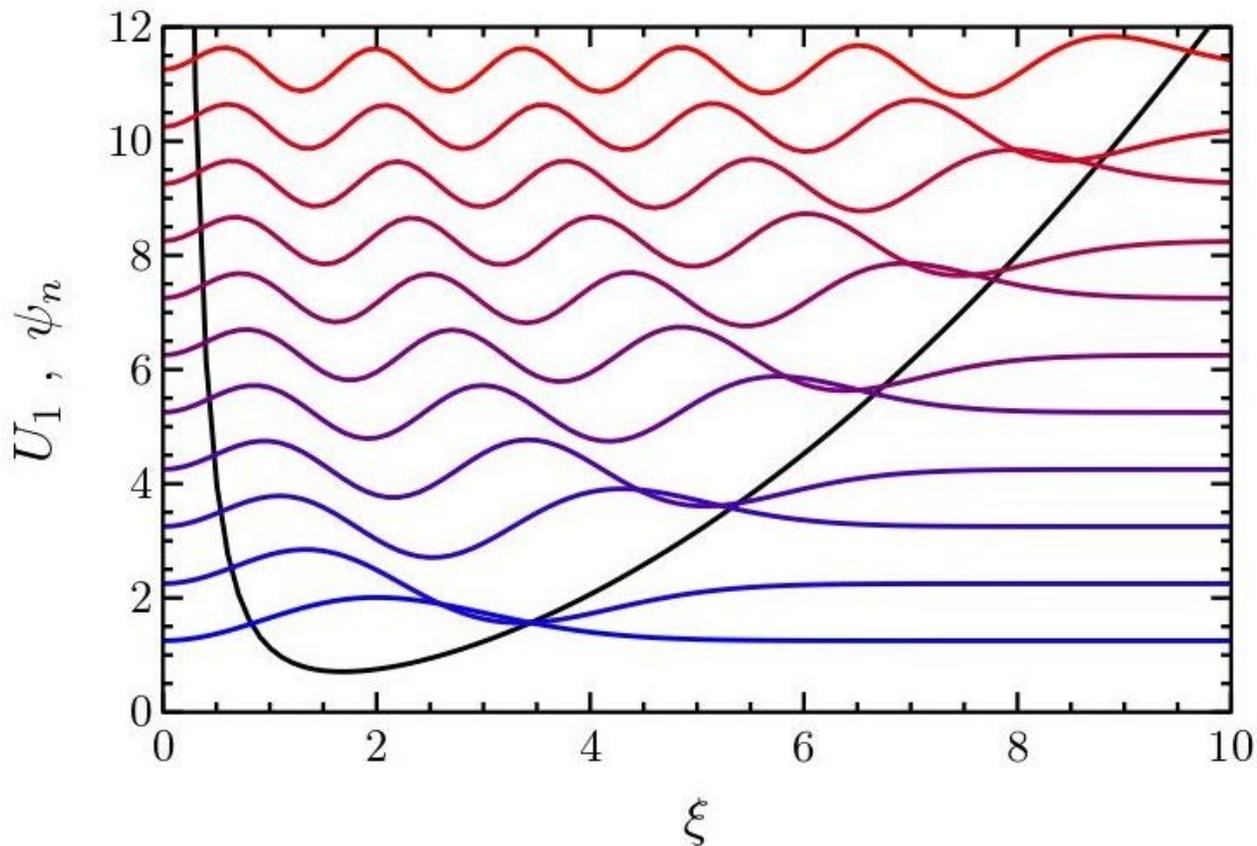


Figure 3

Potential and lowest states of the second-order shift operator for $A = 1$. The offset of the states are the corresponding eigenenergies.

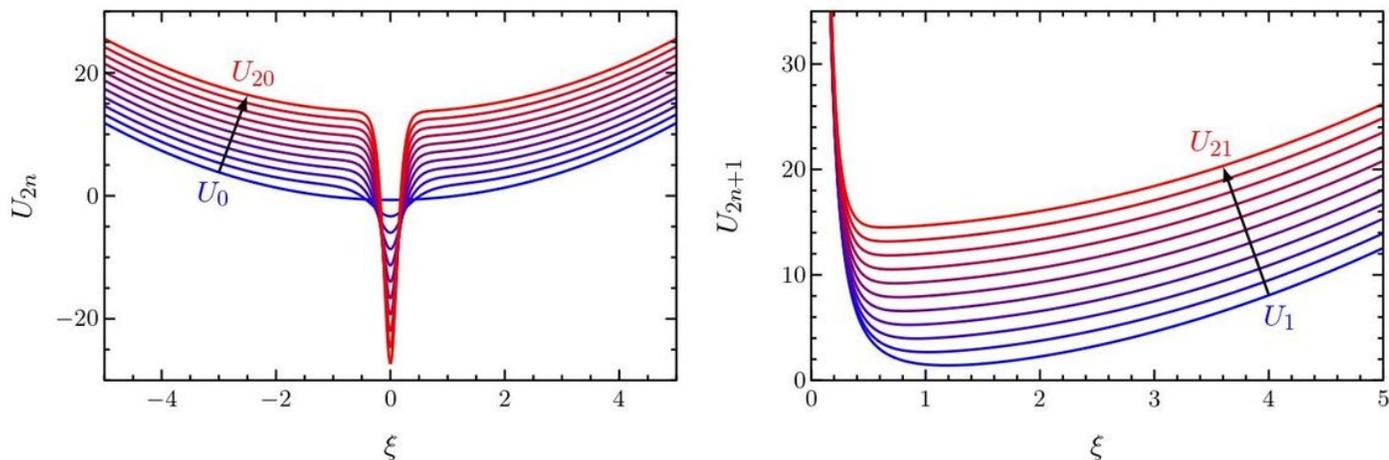


Figure 4

Potentials of the third-order shift operator $U_0 \dots U_{21}$.

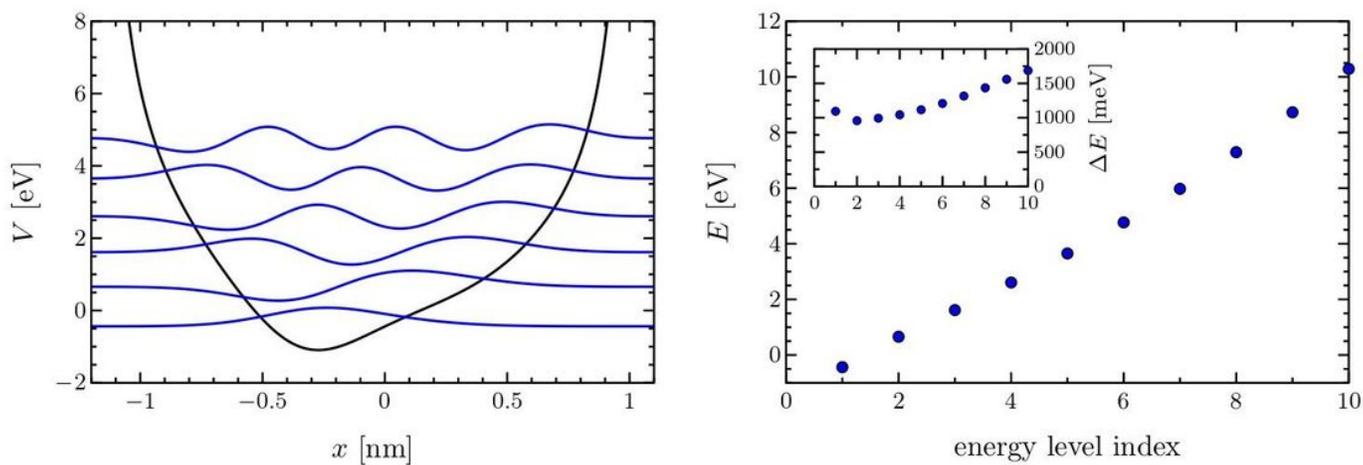


Figure 5

Numerical solution of Eq. (13) with $A = -0.4$ and $W(1) = W_0(1) = W_{00}(1) = 0$. Left: type-1 potential and lowest states. The offset of the states are the corresponding eigenenergies. Right: energy levels. The inset shows the energy level spacings.

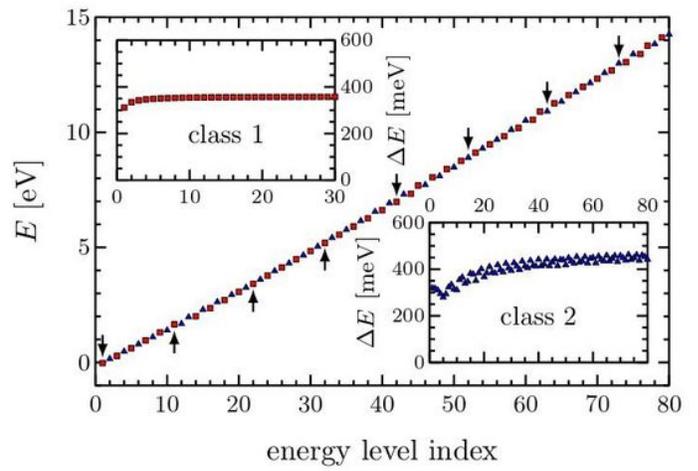
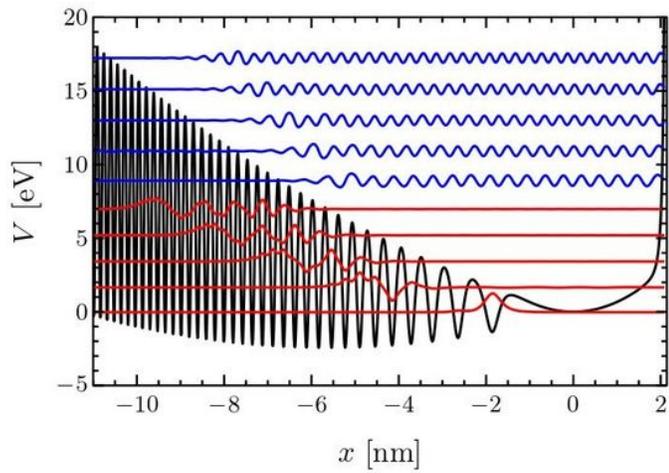


Figure 6

Please see the Manuscript PDF file for the complete figure caption

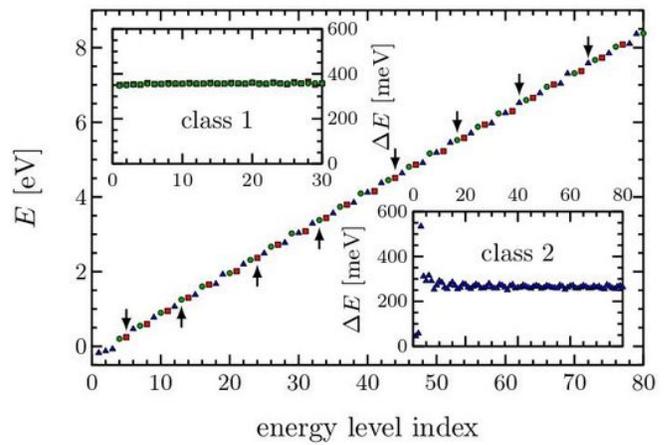
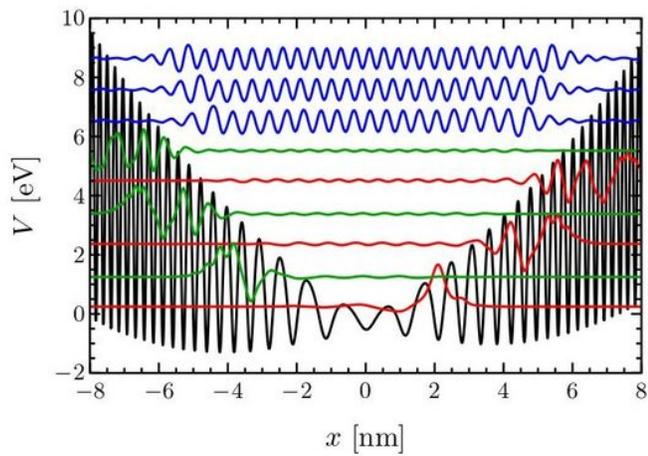


Figure 7

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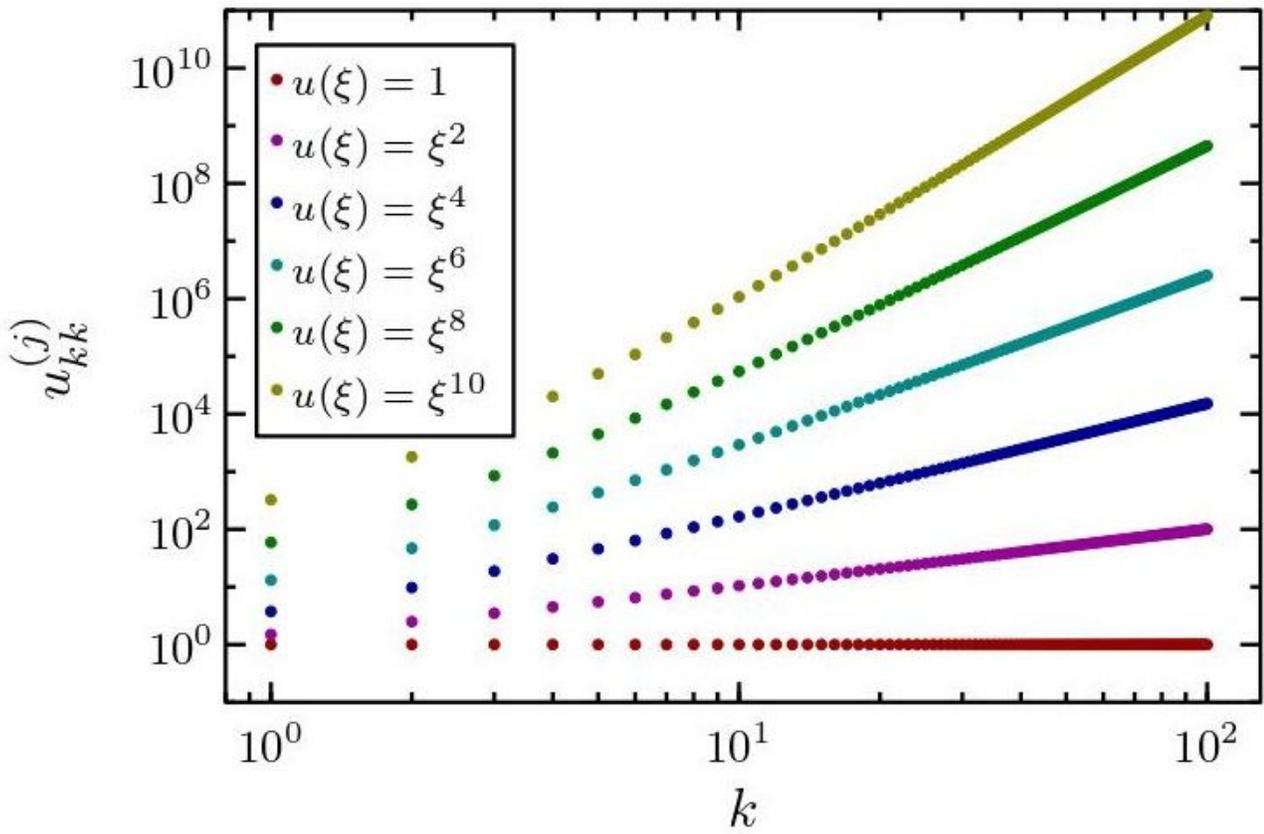


Figure 8

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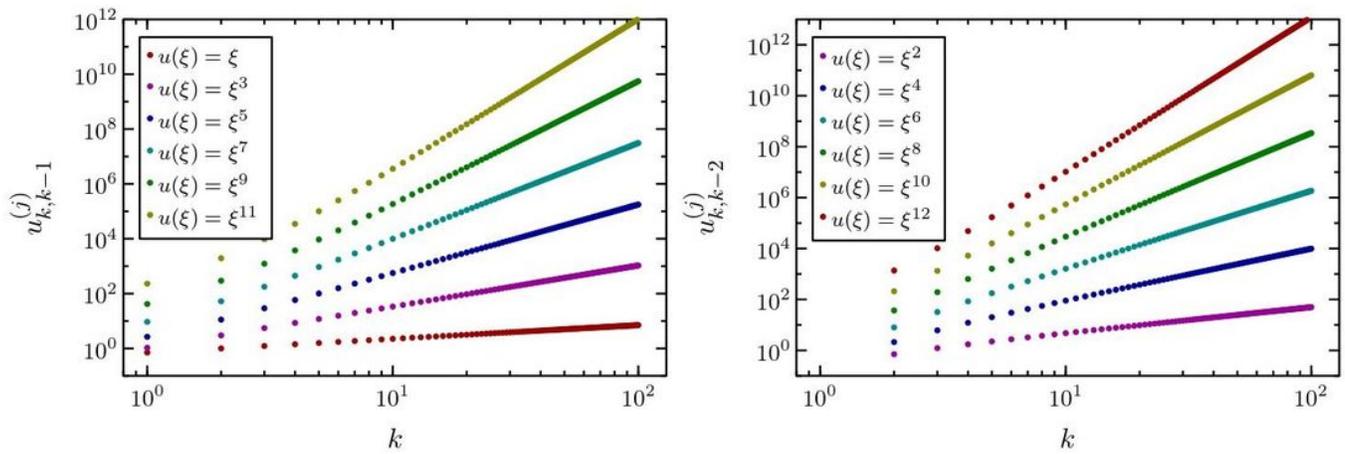


Figure 9

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