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Machining accuracy reliability optimization of three-axis CNC machine tools using doubly-weighted vector projection response surface method

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Research Article

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Posted Date: October 26th, 2023

DOI: https://doi.org/10.21203/rs.3.rs-3467413/v1

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Version of Record: A version of this preprint was published at The International Journal of Advanced Manufacturing Technology on March 16th, 2024. See the published version at https://doi.org/10.1007/s00170-024-13426-w.

Machining accuracy reliability optimization of three-axis CNC machine tools using doubly-weighted vector projection response surface method

Zhiming Wang[1]*; **Wenbin Lu**[1]

Abstract The reasonable allocation of geometric errors of NC machine tools can improve the machining accuracy reliability. However, due to the complexity and high nonlinearity of limit state function (LSF) of machining accuracy reliability, the fitting accuracy is usually low when the traditional method is used to approximate LSF. To solve this problem, a doubly-weighted vector projection response surface (DWVPRS) method, which considers not only the approximation results of the test sample point to LSF but the distance between the test sample point and the most probable failure point (MPFP), is proposed. Using the reliability sensitivity analysis method, the key geometric errors were identified and optimized to meet the design requirements. Finally, taking a large gantry guideway grinding machine as an example to verifies the correctness and effectiveness of the DWVPRS method proposed in this paper, the results show that after the optimization of geometric accuracy, the minimum and average reliability value of the grinding machine meet the design requirements.

Keywords Doubly-weighted vector projection response surface method · Machining accuracy reliability · Sensitivity analysis · Geometric error · Accuracy optimization

List of symbols

δ_{xx} Working table linear error in X-direction

δ_{yx} Working table linear error in Y-direction

δ_{zx} Working table linear error in Z-direction

δ_{xy} Vertical slide linear error in X-direction

δ_{yy} Vertical slide linear error in Y-direction

δ_{zy} Vertical slide linear error in Z-direction

δ_{xz} Movable beam linear error in X-direction

δ_{yz} Movable beam linear error in Y-direction

δ_{zz} Movable beam linear error in Z-direction

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ε_{xx} Roll error of working table

ε_{yx} Pitch error of working table

ε_{zx} Yaw error of working table

ε_{xy} Roll error of vertical slide

ε_{yy} Pitch error of vertical slide

ε_{zy} Yaw error of vertical slide

ε_{xz} Roll error of movable beam

ε_{yz} Pitch error of movable beam

ε_{zz} Yaw error of movable beam

S_{xy} Perpendicularity error between working table and movable beam

S_{xz} Perpendicularity error between vertical slide and movable beam

S_{yz} Perpendicularity error between working table and vertical slide

1 Introduction

Machining accuracy reliability of CNC machine tools refers to the ability of machine tools to perform a machining accuracy requirement under particular circumstances for a given specific time interval [1]. It is a crucial reliability index in evaluating the machining performance of CNC machine tools, which significantly impacted by force errors, thermal errors, and particularly geometric errors [2,3]. This is because that geometric errors and thermal errors are the main influencing factors, accounting for 45%-65% of the total errors. If the temperature of machine tools changes to a stable state, the impact of geometric errors is the largest, accounting for about 40% of the total errors [4]. Therefore, it is imperative to address practical engineering challenges by rationally optimizing the geometric errors of CNC machine tools. This optimization process aims to enhance the machining accuracy reliability of CNC machine tools, ensuring that it aligns with the design requirements [5].

At recent years, in literature many studies have focused on error modeling techniques for CNC machine tools. The three main methods are used including the differential matrix method [6], homogeneous transformation matrix (HTM) method based on multi-body system theory [7-9], and error modeling method based on the product-of-exponential theory [10,11]. Among these methods, the HTM method has gained the most popularity [12]. However, its drawbacks are also obvious. Using the HTM method to model the error of machine tools, four kinds of matrixes, which including stationary characteristic matrix, motion characteristic matrix, stationary error matrix and motion error matrix, are all should be considered simultaneously, this makes the HTM method is complex, time-consuming and error-prone. The product-of-exponential modeling approach is commonly employed in the robotics industry because of its simplicity and clear physical significance [13,14], while it has not yet been paid enough attention in the field of machine tools. The product-of-exponential theory was utilized by Li et al [15]. to model the kinematics of the rotating axis of a five-axis motion platform and obtain the geometric errors of each position. This approach offers advantages such as solving the singularity problem of error matrix and providing a clear physical significance for angular error. Moreover, the motion spinor index can easily describe the move of a rigid structure, thereby the product-of-exponential modeling approach can simplify the kinematic analysis of the series mechanism [10,11]. Therefore, the product-of-exponential theory is also adopted in this study to model the geometric error of CNC machine tools.

In the field of machining accuracy reliability analysis for machine tools, several methods including the importance sampling (IS) method, first-order second-moment (FOSM) method, advanced first-order second-moment (AFOSM) method, response surface (RS) method and Monte Carlo simulation (MCS) method [9,16-20] are commonly used. Among these methods, the RS method [21-23] is widely employed due to its ability to effectively approximate limit state function (LSF) and its simplicity in calculation. However, the accuracy of the RS method is greatly influenced by the shape of the response surface function and the selection of test sampling points. With the development of reliability analysis theory, Kim and Na [24] proposed an improved sequential response surface method. Nevertheless, this method does not take into account the distance between the test sample points and the actual LSF, and it treats all test sample points as having the same effect. Kaymaz and Mcmahon [25] proposed the idea of weighted regression response surface method, which reduced the amount of calculation and improved the fitting accuracy to a certain extent. However, the test sample points obtained by this method are not representative, and the selection strategy of test sample points needs to be improved. Fan et al. [26] proposed an adaptive response surface method considering cross-terms. However, unreasonable interpolation coefficients may cause large errors, and even lead to wrong calculation results. Zhang et al. [27] considered the distance between the test sample point and the LSF, and proposed an improved weighted response surface method based on vector projection sampling. This method addresses the limitations of previous approaches and aims to enhance the accuracy of reliability analysis. The region near the actual most probable failure point (MPFP) is a critical area with a high probability of failure, making it crucial in reliability calculations. Therefore, it is necessary to accurately fit the LSF near the MPFP. Inspired by the research of Zhang et al. [27], this paper further tries to improve the fitting accuracy by considering not only the approximation results of the test sample point to the LSF but the distance between the test sample point and the MPFP, which ensure the test sample points are accurately approaching the limit state surface and the LSF has also been fitted well at the MPFP. Based on these two factors, taking into account the approximation results, the test sample points are doubly weighted, a doubly-weighted vector projection response surface (DWVPRS) method is proposed in this paper. Consequently, based on the proposed method, the machining accuracy reliability of CNC grinding machine is optimized. Compared with the conditional method, the proposed method enhances the computational accuracy to a certain extent.

2 Machining accuracy reliability modeling of CNC machine tools

Any rigid body in three-dimensional space possesses six degrees of freedom, and each motion axis associated with six fundamental errors: three linear displacement errors and three angular displacement errors. Consequently, a three-axis CNC machine tool exhibits a total of 21 geometric errors, comprising 18 geometric errors and 3 perpendicularity errors. Specifically, the linear displacement errors of the X-axis are represented as δ_{xx}, δ_{yx} and δ_{zx}, while the angular displacement errors are ε_{xx}, ε_{yx} and ε_{zx}. Similarly, the linear displacement errors of the Y-axis are denoted as δ_{xy}, δ_{yy} and δ_{zy}, and the angular displacement errors are ε_{xy}, ε_{yy} and ε_{zy}, respectively. The linear displacement errors of the Z-axis are denoted as δ_{xz}, δ_{yz} and δ_{zz}, and the angular displacement errors are ε_{xz}, ε_{yz} and ε_{zz}, respectively. Furthermore, the manufacturing and assembly errors of machine tools introduce perpendicularity errors between the axes. The perpendicularity error between the X and Y axes is denoted as S_{xy}, and the perpendicularity errors between the X and Z axes and the Y and Z axes are represented by S_{xz} and S_{yz}, respectively.

Figure 1 is the structural diagram of a CNC MKW5230A/3×160 gantry guide grinding machine of the XFZY type. The open-loop motion chain from the workbench to the grinding head is as follows: the workbench - X axis - bed - Z axis - Y axis - the grinding head. Using the product-of-exponential theory to model the geometric errors of the grinding machine, ignoring the influence of higher-order terms and using the assumption of small angle approximation, some of the 21 geometric errors would be canceled each other, and the 17 geometric errors are left at last. Therefore, the position error $\Delta\boldsymbol{E}$ of the grinding head relative to the workbench can be given as:

$$\Delta\boldsymbol{E}=\begin{Bmatrix} e_x \\ e_y \\ e_z \end{Bmatrix}=\begin{Bmatrix} \delta_{xz}-\delta_{xx}+\delta_{xy}+z\varepsilon_{yx}+y(\varepsilon_{zz}-\varepsilon_{zx})-zS_{xz} \\ \delta_{yz}-\delta_{yx}+\delta_{yy}+x\varepsilon_{zx}+z\varepsilon_{xx}+xS_{xy}-zS_{yz} \\ \delta_{zz}-\delta_{zx}+\delta_{zy}-x\varepsilon_{yx}+y(\varepsilon_{xz}+\varepsilon_{xx}) \end{Bmatrix} \tag{1}$$

where e_x, e_y and e_z are the position errors of grinding head in X, Y and Z directions, x, y, and z denote the displacements along the X, Y, and Z axes, respectively.

In order to assess the machining accuracy reliability of grinding machines, the model of the machining accuracy reliability, which is often expressed by LSF, is needed. Using Eq. (1), the LSF of machining accuracy reliability can be obtained by

$$G=I-\sqrt{e_x^{\ 2}+e_y^{\ 2}+e_z^{\ 2}} \tag{2}$$

where I = 0.03 mm is the allowable machining error. When Eq. (2) is greater than zero, it means that the machining accuracy of grinding machines is reliable; otherwise, it is unreliable. From Eq. (2), it can be seen that this function is a complex, high-dimensional nonlinear function, making it challenging to calculate the reliability of grinding machine using the conventional reliability methods.

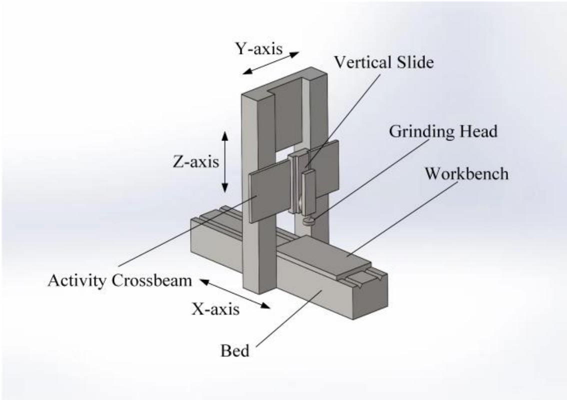

Fig. 1 Structure diagram of grinding machine

3 Doubly weighted vector projection response surface method

To fit the LSF of machining accuracy reliability of grinding machine, a DWVPRS method is proposed in this study. As mentioned in introduction, response surface function is widely used to approximate LSF because of its simplicity. The general expression for quadratic polynomial response surface function, without considering cross terms, can be given as follows:

$$Z = h(Y) = a + \sum_{i=1}^{n} b_i Y_i + \sum_{i=1}^{n} c_i Y_i^2 \tag{3}$$

where $h(Y)$ is the response surface function, Y_i is the design variable, and a， b_i and c_i denote unknown coefficients whose total number is $2n + 1$.

Ignoring the influence of quadratic terms and utilizing linear response surface function to fit the true LSF, the computing burden would be reduced, the simplified expression of the linear polynomial response surface function is given as follows:

$$Z = h(Y) = a + \sum_{i=1}^{n} b_i Y_i = a + \boldsymbol{b}^{\mathrm{T}} \boldsymbol{Y} \tag{4}$$

3.1 The basic principle of vector projection sampling

Assuming that a probable failure point of the previous iteration is y^*, the unit column vector of the response surface function at y^* is calculated according to Eq. (4), which can be obtained by the definition of the unit vector:

$$\boldsymbol{\tau}_r = -\frac{\nabla h(y^*)}{\left\| \nabla h(y^*) \right\|} = -\frac{\boldsymbol{b}}{\left\| \boldsymbol{b} \right\|} \tag{5}$$

where $\boldsymbol{\tau_r}$ represents the unit vector which is perpendicular to the tangent line of the LSF and pointing towards the direction of the LSF, $\boldsymbol{\nabla} h(y^*)$ is the gradient vector of $h(y^*)$.

The unit projection vector $\boldsymbol{u}_i$ of the test sample point for the i-th random variable can be given by [28]:

$$\boldsymbol{u}_i=\frac{\boldsymbol{e}_i-\boldsymbol{\tau}_r^{\mathrm{T}}\boldsymbol{e}_i\boldsymbol{\tau}_r}{\left\|\boldsymbol{e}_i-\boldsymbol{\tau}_r^{\mathrm{T}}\boldsymbol{e}_i\boldsymbol{\tau}_r\right\|}\quad(i=1,2,\cdots,n) \tag{6}$$

where $\boldsymbol{e}_i=(\delta_{1i},\delta_{2i},\cdots,\delta_{ni})^{\mathrm{T}}$ represents the unit basis vector along the coordinate axis X_i, δ_{ij} is the Kronecker symbol, and $\delta_{ij}=1$ $(i=j)$, $\delta_{ij}\neq 0$ $(i\neq j)$. If $\boldsymbol{e}_i-\boldsymbol{\tau}_r^{\mathrm{T}}\boldsymbol{e}_i\boldsymbol{\tau}_r=\mathbf{0}$, then $\boldsymbol{u}_i$ is specified as $\mathbf{0}$.

In order to make the test sample points reasonably distributed near the LSF, the MPFP obtained from each iteration is projected onto the previous iteration RS, and the unit projection vector $\boldsymbol{p}_i$ can be obtained by [28]:

$$\boldsymbol{p}_i=\frac{\lambda\boldsymbol{u}_i-\varepsilon_q\boldsymbol{\tau}_r+(1-\lambda)\boldsymbol{e}_i}{\left\|\lambda\boldsymbol{u}_i-\varepsilon_q\boldsymbol{\tau}_r+(1-\lambda)\boldsymbol{e}_i\right\|}\quad(i=1,2,\cdots,n) \tag{7}$$

where ε_q is an extremely small number, λ is a value between 0 and 1, and when λ is 0 or 1, it corresponds to traditional central composite design sampling and full vector projection sampling, respectively.

Using the projection sampling method defined in Eq. (7), the coordinates of the test sample points are calculated as follows:

$$y_i=y_i^*\pm f\sigma_{yi}\boldsymbol{p}_i^{\mathrm{T}}\boldsymbol{e}_i\quad(i=1,2,\cdots,n) \tag{8}$$

where y_i^* is the MPFP, f is the step size, σ_{yi} is the standard deviation of the test sample point, and $f\sigma_{yi}\boldsymbol{p}_i^{\mathrm{T}}\boldsymbol{e}_i$ represents the distance between the test sample point and the MPFP.

By using the above $2n+1$ test sample points, an overdetermined linear Eq. system can be obtained as follows:

$$\boldsymbol{A\gamma}=\boldsymbol{h}^* \tag{9}$$

where $\boldsymbol{\gamma}=(a,b_1,\cdots,b_n)^T$, $\boldsymbol{h}^*=(h_1^*,h_2^*,\cdots,h_{2n+1}^*)^{\mathrm{T}}$; $\boldsymbol{A}$ is the regression coefficient matrix composed of test sample points, which can be written by:

$$\boldsymbol{A}=\begin{bmatrix}1 & y^{\mathrm{T}}\\ 1 & y^{\mathrm{T}}\\ \vdots & \vdots\\ 1 & y^{\mathrm{T}}\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & -f\mathrm{diag}\left[\sigma_{yi}\boldsymbol{p}_i^{\mathrm{T}}\boldsymbol{e}_i\right]\\ \vdots & \vdots\\ 0 & f\mathrm{diag}\left[\sigma_{yi}\boldsymbol{p}_i^{\mathrm{T}}\boldsymbol{e}_i\right]\end{bmatrix} \tag{10}$$

From Eq. (9):

$$\boldsymbol{\gamma}=(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{T}}h^* \tag{11}$$

The selection of step size, denoted as f, has been identified as a critical factor affecting the convergence speed and calculation accuracy of the RS method [29]. When the step size is large, due to the low fitting accuracy of the polynomial to the LSF, the RS method cannot converge to the actual MPFP, but converges locally around it. Conversely, when the step size is very small, although the response surface function can be well fitted to the real LSF at the actual MPFP. However, if the initial value of step f is small for the LSF with a high nonlinearity, periodic oscillation or even non-convergence will occur in the iterative process. To avoid this situation, a large initial value is given in the iterative process, and f is replaced with $\sqrt{f}$, so as to ensure that the step size f gradually converges to a smaller and reasonable value. By employing this approach, the problem of the convergence failure and computational errors of the RS method can be effectively resolved, then the convergence speed and calculation accuracy of the RS method would be improved.

3.2 Construction of double weighting coefficient

There are two type of weighted coefficients which affect the fitting accuracy of the LSF. The Type I weighted coefficient considers the distance between the test sample point and the limit state surface (also known as the failure surface, that is $h(y) = 0$). In order to make the response surface function approach the failure surface better, a larger weighted coefficient should be given to the test sample points which are closer to the failure surface. Because the test sample points with larger weighted coefficient play a more important role in fitting the response surface function, thus it can improve the fitting accuracy of the LSF. The expression for Type I weighted coefficient is given as:

$$
\begin{gathered}
t = \min\{|h(y_1)|, |h(y_2)|, \ldots, |h(y_n)|\} \\
w_{1i} = \begin{cases} 1 & h(y_i) = 0 \\ \dfrac{t}{|h(y_i)|} & h(y_i) \neq 0 \end{cases}
\end{gathered} \tag{12}
$$

where w_{1i} is the Type I weighted coefficient at each test sample point, $|h(y_i)|$ denotes the distance between each test sample point and the LSF, and t is the optimal value of the distance between the two, that is, the smallest non-zero value.

The Type II weighted coefficient measures the distance between the test sample point and the MPFP. As the region near the MPFP has a high failure probability, this region also plays an important role in reliability calculation. A larger weight for the test sample points which are close to the MPFP can make the response surface function accurately approximate the real LSF near the actual MPFP. The weighted coefficient is defined as follows [30]:

$$w_{2i} = \begin{cases} 1-6d_i^2+8d_i^3-3d_i^4 & 0 \le d_i \le 1 \\ 0 & d_i > 1 \end{cases} \tag{13}$$

and

$$d_i = \frac{l_i}{l_{\max}} = \frac{\left\| y^{*(k)} - y_i \right\|}{\max l_i} \tag{14}$$

where l_i is the distance between each test sample point and the MPFP, $l_{\max}$ is the maximum distance between the test sample point and the MPFP among all the test sample points, and $y^{*(k)}$ is the MPFP generated by the k-th iteration.

Taking the average of the two weights, the weight coefficient of each test sample point can be obtained, and then the weight matrix is constructed with the weight coefficient of each test sample point as the diagonal element, and respectively given by:

$$\begin{cases} w_i = \dfrac{w_{1i}+w_{2i}}{2} \\ \boldsymbol{W} = \operatorname{diag}(w_1, w_2, \cdots w_n) \end{cases} \tag{15}$$

At this stage, the coefficient matrix $\boldsymbol{\gamma}$ in Eq. (11) is updated using the weighted matrix to obtain the new coefficient matrix $\boldsymbol{B}$ by

$$\boldsymbol{B} = (\boldsymbol{A}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{A})^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{W}h^{*} \tag{16}$$

Therefore, the details for calculating the machining accuracy reliability of a grinding machine using the DWVPRS method are given as follows:

Step 1: Construct the initial iteration point $y = (y_1, y_2, \cdots, y_n)^{\mathrm{T}}$ by taking the mean value μ_y of the random variable.

Step 2: Set the initial iteration point y as the sampling center and choose an appropriate step size f (at first it can be selected from 1 to 3). Generate $2n + 1$ test sample points using the central composite design method, the coordinates of each test sample point can be expressed as $y_i = y \pm f\sigma_{y_i}$.

Step 3: Substitute the test sample points into Eq. (10) to obtain the coefficient matrix $\boldsymbol{A}(X)$. According to the test sample points, the corresponding response values $h_i (i = 1, 2, \cdots, 2n+1)$ are obtained by numerical analyses or test, and the column vector $\boldsymbol{m} = (h_1, h_2, \cdots, h_{2n+1})$ is given.

Step 4: Construct the weight matrix $\boldsymbol{W}$ using Eqs. (12) to (15), and determine the new coefficient matrix $\boldsymbol{B}$ using the least squares method. Note that the weight matrix is a unit matrix when using the method of central composite design to generate the test sample point.

Step 5: Calculate the MPFP y^* of the LSF and the reliability index $\beta = \mu_Z/\sigma_Z$ using the AFOSM method. Here, μ_Z is the mean of the response function and σ_Z is the standard deviation of the response function.

Step 6: Take the MPFP y^* as the expansion point, carry out projection sampling using Eqs. (5) to (8), generate the new test sample points y_i. Replace f with $\sqrt{f}$ in the iteration process to ensure accuracy and convergence of the calculation results.

Step 7: Repeat Steps 3 to 6 until the difference between the last two times value of $\|y\|$ is less than the allowed error ε (typically set between 10^{-6} and 10^{-4}), output the reliability index β. Finally, compute the machining accuracy reliability of CNC machine tools using $R = \Phi(\beta)$, where $\Phi(\cdot)$ is a standard normal distribution function.

3.3 Reliability sensitivity analysis of machining accuracy of CNC machine tools

To analyze the influence of error variable on the machining accuracy reliability of a grinding machine, based on the doubly weighted vector projection response surface model of LSF, the sensitivity of CNC machining accuracy reliability is obtained by the partial derivation of the geometric error parameters.

From Eq. (4), the mean and variance of the response surface function for CNC machine tools' machining accuracy can be given as follows:

$$\begin{cases} \mu_Z = E(Z) = a + \sum_{i=1}^{n} b_i \mu_{Y_i} \\ \sigma_Z^2 = D(Z) = \sum_{i=1}^{n} b_i^2 \sigma_{Y_i}^2 \end{cases} \tag{17}$$

where E is the mean of response surface function, and D represents the variance. Therefore, one can derive the partial derivative of geometric error parameter as:

$$\begin{cases} \partial \mu_Z / \partial \mu_{Y_i} = b_i \\ \partial \mu_Z / \partial \sigma_{Y_i} = \partial \sigma_Z / \partial \mu_{Y_i} = 0 \\ \partial \sigma_Z / \partial \sigma_{Y_i} = b_i^2 \sigma_{Y_i} / \sigma_Z \end{cases} \tag{18}$$

The sensitivity of the geometric error parameter to the machining accuracy reliability can be expressed as:

$$\begin{cases} \dfrac{\partial R}{\partial \mu_{Y_i}} = \dfrac{\partial R}{\partial \beta}\left(\dfrac{\partial \beta}{\partial \mu_Z}\dfrac{\partial \mu_Z}{\partial \mu_{Y_i}} + \dfrac{\partial \beta}{\partial \sigma_Z}\dfrac{\partial \sigma_Z}{\partial \mu_{Y_i}} \right) \\ \dfrac{\partial R}{\partial \sigma_{Y_i}} = \dfrac{\partial R}{\partial \beta}\left(\dfrac{\partial \beta}{\partial \mu_Z}\dfrac{\partial \mu_Z}{\partial \sigma_{Y_i}} + \dfrac{\partial \beta}{\partial \sigma_Z}\dfrac{\partial \sigma_Z}{\partial \sigma_{Y_i}} \right) \end{cases} \tag{19}$$

where R is the reliability, and $\dfrac{\partial R}{\partial \beta} = \dfrac{1}{\sqrt{2\pi}}\exp\left(-\dfrac{\beta}{2}\right)$, $\dfrac{\partial \beta}{\partial \mu_Z} = \dfrac{1}{\sigma_Z}$, $\dfrac{\partial \beta}{\partial \sigma_Z} = -\dfrac{\mu_Z}{\sigma_Z^2}$. Using Eq. (19), the sensitivity of geometric error parameter of machining accuracy reliability can be obtained.

The proposed methods consist of three models include geometric error model, machining accuracy reliability model based on the DWVPRS method and reliability sensitivity analysis model. In order to optimize the machining accuracy reliability of grinding machine, the geometric error model of the grinding machine is needed in first step. Subsequently, utilizing the accuracy reliability model and sensitivity analysis model, the machining accuracy reliability was optimized by adjusting the geometric error parameters with high sensitivity, so that the reliability design requirements is satisfied. The details are illustrated in Fig. 2.

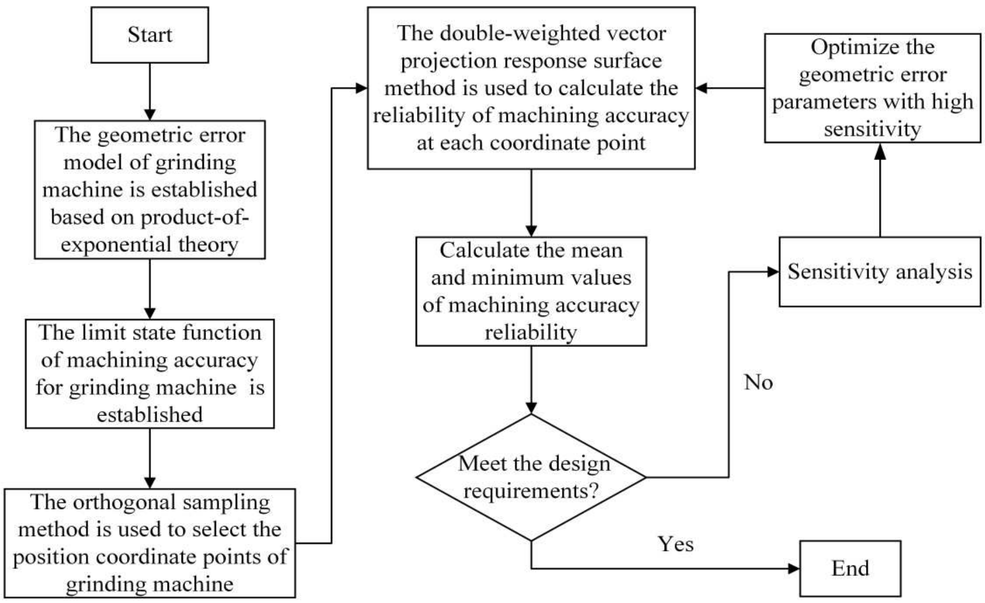

Fig. 2 The flow chart of the DWVPRS method proposed in this paper

4 Case study

The MKW5230A/3×160 guideway grinding machine adopts a gantry layout and consists of double-column, activity crossbeam, vertical slide, and bed. Its dimensions are 22 m × 7.8 m × 5.8

m, making it suitable for machining large parts of heavy machinery, ships, aerospace, and metallurgical equipment. During the machining process, the working table moves longitudinally along the bed guideway, while the activity crossbeam lifts and lowers along the double-column guideway. A vertical sliding plate is installed on the activity crossbeam, which moves transversely. The vertical slide is equipped with a universal vertical grinding head that moves along the guideway on the sliding plate.

The design requirements of the grinding machine are given as follows: the average reliability should exceed 97% when the positional error is below the maximum allowable value of 0.03 mm, and the minimum reliability should be above 95%. The study of reference [31] shows that the component geometric errors of machine tools follow a normal distribution closely. Thus, in this study, all geometric errors of the grinding machine are also assumed normally distributed. The distribution parameters of the geometric errors primarily depend on the assembly tolerance and design tolerance of the grinding machine. The relationship between the standard deviation (σ) of the geometric error and the tolerance (T) is given by $T = 6\sigma$. During the design and manufacturing process, the tolerance is controlled strictly ensures that each component meets the design requirements as closely as possible. Consequently, the mean of geometric error of the grinding machine can be assumed to be close to zero on average. According to the machining accuracy that can be achieved by general CNC equipment and the national standard of the People's Republic of China for precision testing of gantry guideway grinding machines (GB/T5288-2007/ISO4703: 2001), the standard deviations of the 21 selected geometric errors are given in Table 1.

Table 1 The standard deviation of 21 geometric errors of grinding machine

Error variance	δ_{xx}	δ_{yx}	δ_{zx}	ε_{xx}	ε_{yx}	ε_{zx}	δ_{xy}
Standard deviation/mm	0.05/6	0.05/6	0.05/6	0.03/6000	0.06/6000	0.05/6000	0.04/6
Error variance	δ_{yy}	δ_{zy}	ε_{xy}	ε_{yy}	ε_{zy}	δ_{xz}	δ_{yz}
Standard deviation/mm	0.05/6	0.04/6	0.05/6000	0.04/6000	0.04/6000	0.03/6	0.03/6
Error variance	δ_{zz}	ε_{xz}	ε_{yz}	ε_{zz}	S_{xy}	S_{xz}	S_{yz}
Standard deviation/mm	0.05/6	0.03/6000	0.04/6000	0.03/6000	0.03/3000	0.03/3000	0.02/3000

5 Results and discussion

In order to optimize the geometric error of the grinding machine reasonably, the machining accuracy reliability of the grinding machine should be predicted correctly at first. During the operation of the grinding machine, although the bed guide rail may be moved, the theoretical working point on the X-axis remains unchanged. As a result, the position coordinate of X direction, denoted as x, remains constant, while the displacements of the grinding head along the Y and Z directions are represented by y and z, respectively. Adopting the orthogonal sampling method, five equidistant points within the range of -1500 to 1500 mm on the Y-axis and 0 to 2000 mm on the Z-axis of the grinding machine are chosen. Thus a total of 25 sets of the position coordinate points of grinding head are obtained. Taking them as the input variables, the machining accuracy reliability of each position coordinate point is used to reflect the machining accuracy reliability of the grinding machine. Firstly, the LSF for the machining accuracy of the grinding machine is established using Eq. (2). Due to the complexity and high nonlinearity of the LSF, the response surface function described in Eq. (4) is used to approximate the LSF. To ensure the fitting accuracy of the response surface function, the vector gradient projection method is applied to select the test sampling points, as shown in Eqs. (5) to (8). Although the test sample points selected by the vector gradient projection method have higher accuracy than other sampling methods, some test sample points have important influence on the fitting accuracy and reliability calculation of the LSF during the fitting process, and the influence of all test sample points on the fitting accuracy of the LSF cannot be considered equal. Therefore, Eqs. (12) and (13) were used to carry out doubly weighted regression on each test sample point, so that each important sample point could play its actual role, and then Eq. (16) was applied to calculate the unknown coefficients of the response surface function. Finally, the AFOSM method was used to estimate the machining precision reliability at each position coordinate point. Using Matlab programming, the machining accuracy reliability corresponding to 25 groups of position coordinate points is calculated in turn, and it was then determined whether it meets the design requirements of minimum reliability of 95% and average reliability of more than 97%. At the same time, in order to verify the correctness and effectiveness of the DWVPRS method, MCS method was used as the standard method and compared with the RS method and VPRS method, the corresponding residual and mean residual values of machining accuracy reliability could be obtained. The calculation results are shown in Fig. 3. As can be seen from Fig. 3, compared with the MCS method, the errors given by the RS and VPRS methods are large. However, the error of the DWVPRS method is the smallest, its calculative result is closer to the result of MCS method. Among them, the residual value of the RS method ranges from -3.68 to 0.86, and the average residual value is -1.07. The residual value range of VPRS method is -1.66~1.17, and the mean residual value is -0.26. While the residual range of the DWVPRS method is -0.29 to 0.22, and the mean residual is only -0.03. Therefore, it can be said that the proposed method in this paper enhances the calculation accuracy. The lower calculation accuracy observed in the other two methods can be attributed to two factors as follows: the RS method is affected by the shape of response surface function and the selection of test sample points, which results in a large difference between the calculation results and the standard results; in addition, the VPRS method only considers the distance between the test sample points and the LSF, and does not consider the important sample points near the real MPFP point, thereby resulting in a low calculation accuracy.

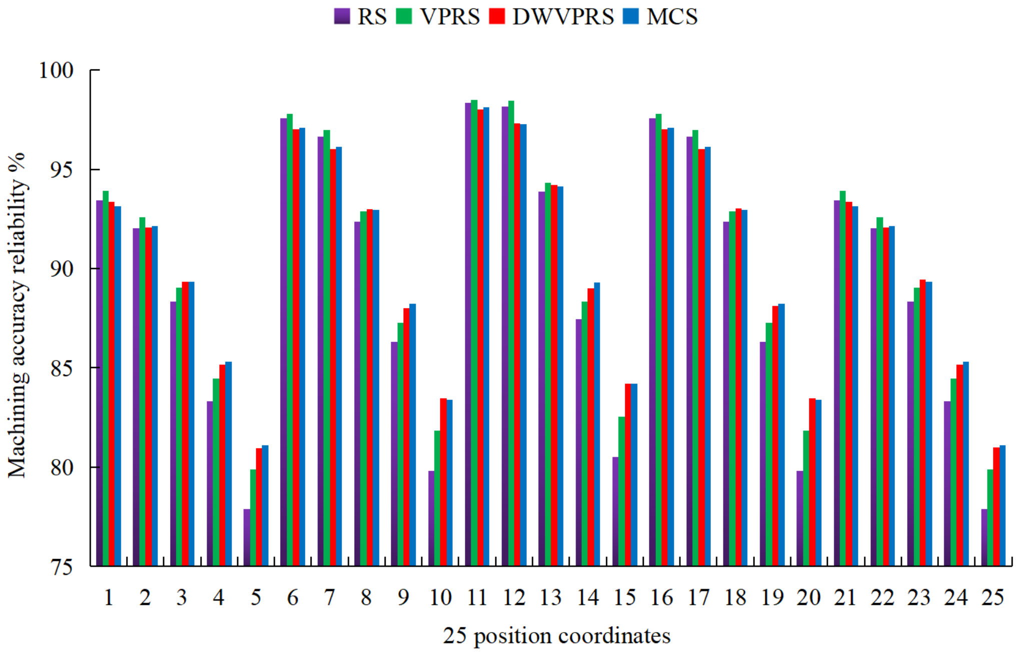

Fig. 3 Machining accuracy reliability of grinding machine

As shown in Fig. 3, the reliability of the grinding machine was evaluated using 25 selected positional coordinate points within its travel range. The minimum reliability value was found to be 80.95%, and the average value was 90.39%. Both values did not meet the design requirements of the grinding machine. Since the geometric error parameters of the grinding machine significantly affect its machining accuracy reliability, it is necessary to optimize these parameters to improve the machining accuracy reliability. Since the mean value of the geometric error is 0, the main parameter influencing the machining accuracy reliability is the standard deviation of the geometric error. Therefore, only the sensitivity of the standard deviation needs to be calculated. Select the coordinate point with the least reliability, namely the 5th coordinate point, and Eq. (19) was used to analyze the sensitivity of the standard deviation of the geometric error. Therefore, the geometric error parameters that have a great influence on the machining accuracy reliability are identified, and the analysis results are shown in Fig. 4.

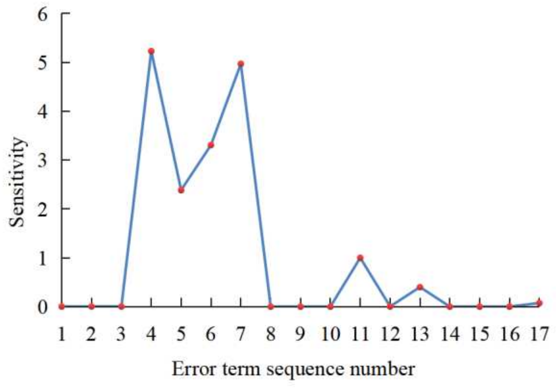

Fig. 4 Machining accuracy sensitivity analysis of grinding machine

From Fig. 4, it can be seen that the 4th, 5th, 6th, 7th, 11th, and 13th geometric errors, denoted as ε_{yx}, ε_{zz}, ε_{zx}, S_{xz}, ε_{xx} and S_{yz}, respectively, have a more significant impact on the machining accuracy reliability of the grinding machine. Therefore, these six geometric errors need to be optimized to meet its design requirements. Considering the inverse relationship between the cost and reliability of machine tools, a precision balance approach is adopted to optimize these geometric errors. The optimization model is expressed as follows:

$$\begin{cases} \text{min} & F = \min\left(-\sum_{i=1}^{n} p_i^2\right) \\ \text{subjet to} & a \le p_i \le b \\ & R_{\text{min}} \ge 95\% \\ & R_{\text{mean}} \ge 97\% \end{cases} \tag{20}$$

where F is the objective function, a is the minimum value of the geometric error term ($a > 0$, determined according to design requirements), p_i represents the geometric error term to be optimized, b is the maximum value of the geometric error term (generally less than the initial value of the geometric error), R_{min} is the minimum value of reliability, and R_{mean} is the mean value of reliability. The optimization results are given in Table 2.

Table 2 The optimization results of accuracy reliability

Geometric error term	ε_{yx}	ε_{zz}	ε_{zx}	S_{xz}	ε_{xx}	S_{yz}
Before optimization/mm	0.06/6000	0.03/6000	0.05/6000	0.03/3000	0.03/6000	0.02/3000
After optimization/mm	0.016/6000	0.016/6000	0.018/6000	0.012/3000	0.02/6000	0.013/3000
Mean reliability	97.17					
Minimum reliability	95.42					

The optimization results show that after reliability optimization, the minimum reliability of the grinder is increased from 80.95% to 95.42%, and the average reliability is increased from 90.39% to 97.17%, both of them meet the design requirements, which shows the correctness of the reliability optimization method proposed in this paper. So far, the correctness and effectiveness of the proposed method has been verified, and the machining accuracy reliability of CNC grinding machine can be improved to meet the design requirements by optimizing the geometric error term.

6 Conclusion

To improve the machining accuracy reliability of CNC machine tools, a DWVPRS method is proposed. Based on the geometric error model and the reliability model of machining accuracy, using the analysis results of reliability sensitivity, the geometric accuracy of the main parts of the grinding machine is optimized to meet design requirement. This method also has a reference significance for error modeling and precision enhancing of other types of machine tools. The main conclusions can be drawn as follows:

The purpose of the DWVPRS method is to obtain important test sample points, and to reduce the influence of unimportant test sample points. Both the approximation results of the test sample point to LSF and the distance between the test sample point and the MPFP are considered simultaneously by the DWVPRS method. Compared with the RS and VPRS methods, the DWVPRS method has a highest fitting accuracy to LSF at the MPFP, which can improve the calculation accuracy of reliability.

Author contribution Conceptualization, Zhiming Wang; methodology, Wenbin Lu; investigation, Wenbin Lu; data curation, Wenbin Lu; writing—original draft preparation, Zhiming Wang;

writing—review and editing, Wenbin Lu; supervision, Zhiming Wang.

Funding The research was sponsored by the National Natural Science Foundation of China (grant no. 52365063).

Data availability All data generated or analyzed during this study are included in this published article.

Code availability Not applicable.

Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent for publication All authors have read and agreed to the published version of the manuscript.

Competing interests The authors declare no competing interests.

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