Stability and Sensitivity Analysis and Optimization Control of the Hydro-turbine Generator Unit

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1 Stability and sensitivity analysis and multi-objective optimization control of the hydro-turbine generator unit

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4 **Abstract**

5 The hydro-turbine governing system (HTGS) and shafting system are mutually coupled. However, the interaction
6 between them has always been neglected. This paper aims to explore the stability and sensitivity of the governor control
7 parameters to the HTGS and shafting system and make the optimal control of the stable operation for the hydro-turbine
8 generator unit(HTGU). First, a novel HTGU motion equation is proposed, which can make connections between the
9 HTGS and the shafting system of the HTGU. And on this basis, the nonlinear coupling mathematical model of the
10 HTGS and the shafting system is established. According to the nonlinear mathematical model, the sensitivity of the
11 governor control parameters on the operating stability of the HTGU is obtained. Then, a multi-objective governor
12 control parameters optimization strategy is proposed. Furthermore, the chaotic-dominated sorting genetic algorithm
13 II(NSGA-II) and multi-objective evolutionary algorithm based on decomposition(MOEA/D) were introduced to obtain
14 the optimal control parameter and mutually verify the effectiveness of the optimization effect. Finally, the nonlinear
15 dynamic characteristics of HTGU under optimal control were revealed. The simulation results show that the rotation
16 speed deviation and shafting system vibrations are sensitive on the PID parameters in some ranges and the stable region
17 will be decreased when considering the shafting system vibrations. The multi-objective PID parameter optimization
18 strategy shows good control performance on the nonlinear dynamic characteristics of the HTGU. The shafting system
19 vibrations excited by the coupled vibration sources are quasi-period in 3D space. In addition to this, these results and
20 the optimization strategy can provide some bases for the design and stable operation of the HTGU.

1 **keywords**

2 Sensitivity, Stability, Hydro-turbine governing system, Shafting system, Multi-objective optimization control, Nonlinear
3 dynamic.

4 **List of symbols**

Symbol	Symbolic meaning
h	Hydro-turbine working head (H)
q_t	Flows of the penstock (Q)
h_{r0}	Head loss of penstock
T_{wt}	Penstock flows inertia time constant
x	The HTGU speed(n)relative deviation
y	Guide vane relative opening (Y)
m_t	The relative dynamic torque(M_t)
m_g	The relative resistance torque(M_s)
e_h, e_x, e_y	Hydro-turbine torque transfer coefficient
e_{qx}, e_{qy}, e_{qh}	Hydro-turbine flow transfer coefficient
T_{ab}	The inertia time constant of the HTGU
k_p	The proportional gain
k_i	The integral gain
k_d	The differential gain
T_y	Main servomotor time constant
u	Governor regulating output
e_g	Load self-regulation coefficient
a	Center distance between upper guide bearing and generator
b	Center distance between lower guide bearing and generator
c	Center distance between lower guide bearing and hydro-turbine guide bearing
d	Distance between hydro-turbine guide bearing and center of the hydro-turbine runner
l	Length of spindle
L	Generator rotor height
D	Generator rotor diameter
B	The magnetic density
β	General coefficient
e_f	The dynamic eccentricity distance of the generator
δ_1	The normal clearance of generator rotor and stator
K	The equivalent stiffness of the sealing forces
D_s	The equivalent damping coefficient
τ_f	The nonlinear function of displacement perturbation
e_t	The relative eccentricity of the hydro-turbine runner
δ_2	The sealing clearance of hydro-turbine runner
K_z	The undetermined coefficient of axial hydrodynamic thrust
D_1	The hydro-turbine runner diameter
m_{11}	The equivalent mass of the generator spindle and generator
m_{22}	The equivalent mass of the hydro-turbine spindle and hydro-turbine
g	The gravitational acceleration
r_1	Radial displacement of the generator

r_2	Radial displacement of the hydro-turbine runner
r_3	Radial displacement of the upper guide bearing
r_4	Radial displacement of the lower guide bearing
r_5	Radial displacement of the hydro-turbine guide bearing
m_2	The mass of the hydro-turbine runner
m_1	The mass of the generator rotor
e_1	The generator mass eccentricity distance
e_2	The hydro-turbine runner mass eccentricity distance
j_1	Rotary inertia of the generator
j_2	Rotary inertia of the hydro-turbine runner
ϕ_1	Phase position of the generator
ϕ_2	Phase position of the hydro-turbine runner
(x_1, y_1, z_1)	The central coordinates of the generator
(x_2, y_2, z_2)	The central coordinates of the hydro-turbine runner
K_1	The stiffness of the upper guide bearing
K_2	The stiffness of the lower guide bearing
K_3	The stiffness of the hydro-turbine guide bearing
K_y	The elastic coefficient of the spindle
k_z	The thrust guide bearing stiffness
k_l	The axial stiffness
d_H	The inner diameter of the spindle
d_B	The external diameter of the spindle
l	The total length of the generator lower spindle and the hydro-turbine spindle
E	Elastic modulus of the spindle
c_1	The damping coefficient of the generator
c_2	The damping coefficient of the hydro-turbine runner
c_t	The torsional damping coefficient of the spindle
F_{xi}	The excitation force in the horizontal X direction
F_{yi}	The excitation force in the horizontal Y direction
F_{zi}	The excitation force in the horizontal Z direction
T	The total kinetic energy of the HTGU
U	The elastic potential energy of the HTGU
Q_i	The corresponding generalized force
M_g	The generator magnetic torque
M_t	The hydro-turbine torque
φ_1	The initial phase of the generator
φ_2	The initial phase of the hydro-turbine runner
α	Torsion angle of the spindle
ω	The angular speed of the HTGU
PID-MOO	PID parameters multi-objective optimized
PID-Stein	PID parameters obtained by Stein's formula

1. Introduction

Water energy is a renewable energy source that can be efficiently converted into electricity [1]. At present, there

are conventional hydropower generation, pumped storage power generation, tidal power generation, ocean current power generation. They mainly play the role of peak and frequency regulation in the power grid, especially conventional hydropower generation and pumped storage power generation [2-3]. Therefore, the efficient, safe, and stable operation of the HTGU has always been the hotspot and difficulty in the research, which mainly involves the stability, the opening and stopping strategy, the equipment fault diagnosis, and the status evaluation, etc. Of course, stability is the most important one that is concerned with the safety and stability of the power grid and the HTGU.

In recent years, the research on the stability of HTGU has been divided into two main directions: the coupling shafting system of the hydro-turbine governing system and the coupling power network of the hydro-turbine governing system [4-5]. They are all complex nonlinear systems, and the study of their complex nonlinear dynamic behavior is of great significance for the realization of optimal economic operation control [6]. In previous works, the stability and dynamic characteristics of the HTGS were studied using the Routh-Hurwitz criterion [7], root locus method [8], state-space method [9], and Hopf bifurcation theory [10-12]. In [13], Hopf bifurcation theory is applied to analyze the influence of the structural parameters of the sloping ceiling tailrace tunnel on the stability domain of the PID control parameters. In [11,14], the simulation results show that the delay and saturation of the servo-system have a certain degree of impact on the stability of the HTGS. At present, in the practical application of engineering, PID is the most important controller. Some scholars have also proposed fractional order PID controller, sliding mode controller, state feedback controller, and so on. In[15], a novel mathematical model of Francis HTGS with a straight-tube surge tank is proposed, where the unstable oscillations of this model are studied extensively and presented in the forms of the bifurcation diagram, time waveform plot, phase trajectories, and power spectrum. To eliminate these undesirable behaviors, a specified fuzzy sliding mode controller is designed. In[16], the state equation of HTGS with the super long headrace tunnel(SLHT) was derived, and the robust H_∞ control strategy is designed based on the exact linearization of HTGS with SLHT and the construction of

nominal output function (NOF). In[17], it was focused on the design of the fractional-order PID (FOPID) controller for the HTGS and the optimal control parameters were obtained using NSGA-II. In[18], the fuzzy fractional-order PID (FFOPID) controller is designed to improve the stability of the nonlinear pumped storage unit(PSU), and the control parameters of the FOPID are optimized based on the multi-objective gravitational search algorithm (MOGSA).

As far as shafting system vibration is concerned, the research on vibration characteristics gradually develops from single vibration under a single excitation to multi-directional vibration under the influence of the coupled vibration sources. Such as the differential equation of lateral vibration of the shafting is established, and the bifurcated characteristics of the vibration caused by rotor mass eccentricity and rotor speed of the HTGU are revealed [19, 20]. The torsional vibration characteristics of the HTGU are analyzed, meanwhile, the influences of the UMP [21], the fluid seal excitation [22], the rotors Rub-impact [23], the hydraulic unbalance [24], the guide bearing loose [25], the gyroscopic effect [26], the shaft misalignment [27], and pressure pulsation of the draft tube [28] effect on shafting system vibrations are investigated. With the further development of the research, the hydraulic-mechanical-electrical coupling factors are considered in the shafting system to explore the nonlinear vibration dynamic behavior. A mathematical model considering the coupling relationship between hydraulic system, electrical system, and guide vane mechanical system was established, the sensitivity of various parameters was revealed [29-31]. In [32], the generalized Hamiltonian model of the shafting system is established which provides a new idea for exploring shafting vibration. In [33], the fractional-order dynamic model of the shafting system and the HTGS is established to investigate the shafting vibration nonlinear dynamic behavior. Besides, The 3D FEA model for the HTGU coupling foundation of hydropower station is established by taking the guide bearing as the bridge connecting the shafting and the structure of the hydropower house, and then the influence of the shafting system vibration on hydropower house vibration and its response mechanism is studied numerically [34, 35].

To sum up, the stability and dynamic characteristics of HTGS are obtained by different methods, and some controllers are proposed to achieve effective control. On the other hand, the shafting system vibrations dynamic

characteristics under different excitation sources are verified, which lays a theoretical foundation for vibration control and fault diagnosis of the HTGU. However, the stability and sensitivity of the PID parameters to the HTGS and the shafting system have not been revealed. The nonlinear dynamic characteristics of the HTGS and shafting system vibration under the coupling effects of the mass eccentricity, the UMP, the arcuate whirl of the rotor, and the seal excitation under the optimal PID control haven't been explored which seriously restricted the secure and stable operation of the HTGU, and brought huge economic losses. Therefore, it is urgent to explore the stability and sensitivity of the PID parameter under the action of hydraulic-mechanical-electrical coupling vibration sources and realize optimal control to ensure the safe and stable operation of the HTGU.

As summarized above, the novelty and contributions of this study are: (1) A novel and practical nonlinear mathematical model coupling HTGS and shafting system is established. (2) The stability and sensitivity of ration speed deviation and shafting system vibrations to the PID parameters are revealed. (3) The parameters chosen of the PID governor are formulated as a multi-objective optimization problem that considers the HTGS and shafting system vibrations simultaneously for the first time, in which the objective functions are composed by the ITSE. (4)The influence of the self-regulating coefficient(e_g) on the nonlinear dynamic characteristics of the HTGU is revealed. (5) The nonlinear dynamic characteristics of the shafting system vibrations in the 3D space, which are generated by the mass eccentricity, the UMP, the arcuate whirl of the rotor, and the seal excitation coupled vibration sources under the optimal control are revealed. Of course, there are still some shortcomings in this paper, the gyroscopic effect for shafting vibration is ignored. Meanwhile, the idealized models are adopted for external excitation sources of hydraulic, mechanical, and electrical systems, which cannot fully reflect the external excitation action on the HTGU.

The remainder of this paper is organized as follows. In section 2, the nonlinear mathematical modeling. In section 3, the stability and sensitivity of the governor PID parameters to the HTGU. In section 4, the governor PID parameter multi-objective parameter optimization control. In section 5, the nonlinear dynamic characteristics of the HTGU. Finally, in section 6, the conclusions are presented.

2. Nonlinear mathematical modeling

The layout of a conventional hydroelectric power plant and a pumping and storage station is shown in Fig. 1. In this paper, we mainly involve the water diversion system, the speed regulation system, and the shafting system.

Fig. 1 Structure of a hydropower station

2.1 Hydro-turbine governing system equation

Ignoring the complex layout of the hydraulic water diversion system, only consider the upstream reservoir, the penstock, the governor, the hydro-turbine, the downstream reservoir, and the generator. The universal mathematical model of the HTGS is established as follows:

(1) Dynamic equation of linear penstock [13]:

$$h = -T_{wt} \frac{dq_t}{dt} - \frac{2h_{t0}}{H_0} q_t \quad (1)$$

in Eq (1), $h = (H - H_0) / H_0$, $q_t = (Q - Q_0) / Q_0$ are the working head (H), and the flows of the penstock (Q) of the HTGU, respectively. The subscript "0" represents the initial value, h_{t0} is the head loss of penstock, T_{wt} is the inertia time constant of flows in the penstock.

1 (2) Hydro-turbine torque and flows equation [13]:

$$\begin{aligned} m_t &= e_h h + e_x x + e_y y \\ q_t &= e_{qh} h + e_{qx} x + e_{qy} y \end{aligned} \quad (2)$$

2 (3) Generator motion equation [13]:

$$\frac{dx}{dt} = \frac{1}{T_{ab}} (m_t - m_g - e_g x) \quad (3)$$

3 (4) Electro-hydraulic servo system differential equation [36]:

$$\frac{dy}{dt} = \frac{u - y}{T_y} \quad (4)$$

4 (5) PID controller equation[36]:

$$\frac{du}{dt} = -k_p \frac{dx}{dt} - k_i x - k_d \frac{d^2 x}{dt^2} \quad (5)$$

5 in Eq (2)–(5), $x = (n - n_0) / n_0$, $y = (Y - Y_0) / Y_0$, $m_t = (M_t - M_{t0}) / M_{t0}$, $m_g = (M_g - M_{g0}) / M_{g0}$ are the
6 relative values of the deviation of HTGU speed (n), guide vane opening (Y), dynamic torque (M_t), and resistance
7 torque (M_g), respectively. The subscript "0" represents the initial value. e_h , e_x , and e_y are the torque transfer
8 coefficient of the hydro-turbine. e_{qx} , e_{qy} , and e_{qh} are the flow transfer coefficient of the hydro-turbine. T_{ab} is the
9 inertia time constant of the HTGU. k_p , k_i , and k_d are the proportional gain, the integral gain, and the differential
10 gain, respectively. T_y is the time constant of the main servomotor. u is the regulating output for the governor. e_g is
11 load self-regulation coefficient

12 Combining the Eq(1)–Eq(5), the HTGS equation can be written as follows:

$$\begin{cases} \dot{\phi}_t = \left(-\frac{q_t - e_{qx}x - e_{qy}y}{e_{qh}} - \frac{2h_{t0}}{H_0} q_t \right) / T_{wt} \\ \dot{x} = \frac{1}{T_{ab}} \left(\frac{e_h q_t}{e_{qh}} + \left(e_x - e_g - \frac{e_h e_{qx}}{e_{qh}} \right) x + \left(e_y - \frac{e_h e_{qy}}{e_{qh}} \right) y - m_g \right) \\ \dot{y} = \frac{u - y}{T_y} \\ \dot{u} = \frac{-k_p}{T_{ab}} \left(e_h \frac{q_t - e_{qx}x - e_{qy}y}{e_{qh}} + e_x x + e_y y - m_g - e_g x \right) - k_i x - \frac{k_d}{T_{ab}} \left(\frac{e_h}{e_{qh}} \dot{\phi}_t + \left(e_x - e_g - \frac{e_h e_{qx}}{e_{qh}} \right) \dot{x} + \left(e_y - \frac{e_h e_{qy}}{e_{qh}} \right) \dot{y} \right) \end{cases} \quad (6)$$

13 2.2 Shafting Transverse, longitudinal, and torsional coupled vibration equations

As can be seen from Fig. 2, $B_i (i=1,2,3)$ represents the upper, lower, and hydro-turbine guide bearings. $O_i (i=1,2,3,4,5)$ and $r_i (i=1,2,3,4,5)$ represent the geometric centers and radial radii of the generator, hydro-turbine runner, and upper, lower, and hydro-turbine guide bearings, respectively. We have $r_i = \sqrt{(x_i^2 + y_i^2)}$, and we denote $|O_1 O_3| = a$, $|O_1 O_4| = b$, $|O_4 O_5| = c$, $|O_2 O_5| = d$. From the geometrical relation in Fig. 2, the following equations can be obtained [34]:

$$r_3 = \frac{[(a+b)(b+c+d) - a(c+d)]r_1 + abr_2}{b(b+c+d)} \quad (7)$$

$$r_4 = \frac{(c+d)r_1 + br_2}{b+c+d} \quad (8)$$

$$r_5 = \frac{dr_1 + (b+c)r_2}{b+c+d} \quad (9)$$

According to, Eq (7), Eq (8), and Eq (9), the radial vibration radius of the generator, the hydro-turbine runner, and the guide bearings are mutually mapped.

Fig. 2 Shafting structure diagram

As we know the shafting system vibration is mainly caused by hydraulic-mechanical-electrical coupled factors. The mechanical vibration sources are mainly divided into mechanical unbalance forces and rotary unbalance forces, which

are mainly distributed on the generator and the hydro-turbine runner, the calculation formula can be expressed as[23]:

$$F = me\omega^2 \quad (10)$$

The electrical vibration source is the UMP, which is calculated as follows [19]:

$$F_{ump} = \beta\pi DL \left(\frac{B}{0.5} \right)^2 \frac{e_f}{\delta_1} \quad (11)$$

in Eq (11), L , and D are the generator rotor height and diameter, B is the magnetic density, β is a general coefficient ($\beta = 0.2 \sim 0.5$), e_f is the dynamic eccentricity distance of generator ($e_f = \sqrt{x_1^2 + y_1^2}$), and δ_1 is the normal clearance of generator rotor and stator.

The water flowing to the flange seals forms circumference flow causing the sealing forces, which can be written as[37]:

$$\begin{cases} F_{x-ff} = -(K - m_f \tau_f^2 \omega^2) x_{02} - \tau_f \omega D_s y_{02} - D_s x_{02} - 2\tau_f m_f \omega x_{02} - m_f x_{02} \\ F_{y-ff} = -(K - m_f \tau_f^2 \omega^2) y_{02} + \tau_f \omega D_s x_{02} - D_s y_{02} + 2\tau_f m_f \omega y_{02} - m_f y_{02} \end{cases} \quad (12)$$

where K , D_s , and τ_f are the equivalent stiffness of the sealing forces, the equivalent damping coefficient, and the nonlinear function of displacement perturbation, respectively. Their corresponding expressions are written as follows:

$$\begin{cases} K = K_0 (1 - e_t^2)^{-j} \\ D_s = D_0 (1 - e_t^2)^{-j} \\ \tau_f = \tau_0 (1 - e_t)^{b_0} \end{cases} \quad (13)$$

in Eq(13), we have $j = \frac{1}{2} \sim 3$, $0 < b_0 < 1$. The e_t is the relative eccentricity of the runner ($e_t = \sqrt{x_2^2 + y_2^2} / \delta_2$, δ_2 is sealing clearance). Where j , b_0 , and τ_0 were used to describe the specific sealing, generally, we have $\tau_0 < 0.5$.

The sealing force was calculated as follows:

$$K_0 = \mu_3 \mu_0, \quad D_0 = \mu_1 \mu_3 T, \quad m_f = \mu_2 \mu_3 T^2 \quad (14)$$

The operation is accompanied by arcuate whirl centrifugal force of the generator and hydro-turbine, which were calculated as follows[38]:

$$F_{G-G} = \frac{(j_1 + 2m_1 e_1^2)(r_3 + r_4)\omega^2}{2g} \quad (15)$$

$$F_{T-G} = \frac{(j_2 + 2m_2 e_2^2)\omega^2 r_5}{g} \quad (16)$$

The HTGU is always accompanied by axial hydrodynamic thrust during operation, which is determined by the empirical formula as follow[39]:

$$F_z = K_z \frac{\pi}{4} D_1^2 \frac{n\sqrt{Q}}{H^{0.25}} \quad (17)$$

where K_z is the undetermined coefficient (generally $K_z = 0.7 \sim 0.8$). n is the speed of the runner under specific working conditions. Q is the overflow rate of the runner under specific working conditions. H is the working head under specific working conditions. D_1 is the hydro-turbine runner diameter.

Assuming that the hydro-turbine and generator spindles are connected rigidly[32], and when axial vibration is considered, the mass of the generator spindle is concentrated on the rotor equivalently, which was written as m_{11} . The mass of the turbine spindle is concentrated on the runner equivalently, which was written as m_{22} .

The axial force at the generator is mainly provided by the thrust bearing, which is approximately expressed as the combined force of the unit's gravity and axial hydraulic thrust, so we have:

$$F_G = (m_{11} + m_{22})g + F_z \quad (18)$$

Based on the above supposes, the kinetic energy of the SCV can be written as:

$$T = \frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2 + e_1^2 \dot{\phi}_1^2 + 2e_1 \dot{\phi}_1 \dot{x}_1 \cos \phi_1 - 2e_1 \dot{\phi}_1 \dot{y}_1 \sin \phi_1) + \frac{1}{2} (J_1 + m_1 e_1^2) \dot{\phi}_1^2 + \frac{1}{2} m_{11} \dot{\phi}_1^2 + \frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2 + e_2^2 \dot{\phi}_2^2 + 2e_2 \dot{\phi}_2 \dot{x}_2 \cos \phi_2 - 2e_2 \dot{\phi}_2 \dot{y}_2 \sin \phi_2) + \frac{1}{2} (J_2 + m_2 e_2^2) \dot{\phi}_2^2 + \frac{1}{2} m_{22} \dot{\phi}_2^2 \quad (19)$$

in Eq (19), (x_i, y_i, z_i) , J_i , m_i , e_i , ϕ_i , and $\dot{\phi}_i = \omega_i$ ($i=1,2$) are the centroid coordinates, the rotational inertia, the equivalent concentrated masses, the eccentricity distance, the rotation angle, and the rotational angular velocity of the generator rotor and the hydro-turbine runner, respectively.

Considering transverse-longitudinal and flexural-torsional coupled vibrations, the potential energy of the shafting for

1 the HTGU can be expressed as follows:

$$U = (x_1^2 + y_1^2)K_{11} + (x_2^2 + y_2^2)K_{22} + \sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}K_{12} + \frac{1}{2}K_y(\phi_1 - \phi_2)^2 + \frac{1}{2}(k_z + k_l)z_1^2 + m_{11}gz_1 + \frac{1}{2}k_l z_2^2 + m_{22}gz_2 \quad (20)$$

2 where K_{11} , K_{22} , and K_{12} is the equivalent stiffness of the shafting system of the HTGU, K_y is the elastic coefficient
 3 of the spindle, k_l is the axial stiffness, k_z is the thrust guide bearing stiffness. ϕ_1 and ϕ_2 are the rotation angles of the
 4 generator and the hydro-turbine runner, respectively [19].

$$\begin{cases} K_{11} = \frac{A_1^2}{B^2}k_1 + \frac{(c+d)^2}{(b+c+d)^2}k_2 + \frac{d^2}{(b+c+d)^2}k_3 & (a) \\ K_{22} = \frac{A_2^2}{B^2}k_1 + \frac{b^2}{(b+c+d)^2}k_2 + \frac{(b+c)^2}{(b+c+d)^2}k_3 & (b) \\ K_{12} = \frac{-A_1A_2}{B^2}k_1 + \frac{b(c+d)}{(b+c+d)^2}k_2 + \frac{d(b+c)}{(b+c+d)^2}k_3 & (c) \end{cases} \quad (21)$$

5 in Eq (21), $A_1 = (a+b)(b+c+d)$, $A_2 = ab$, $B = b(b+c+d)$ k_1 , k_2 , and k_3 are the supporting stiffness of
 6 the upper guide bearing, the lower guide bearing, and the hydro-turbine guide bearing, respectively.

$$K_y = \frac{E\pi(d_H^2 - d_B^2)}{32l} \quad (22)$$

$$k_l = 32K_y \quad (23)$$

7 in Eq(22), E is the elastic modulus of the spindle. d_H and d_B are the internal and external diameters of the spindle.
 8 l is the total length of the generator lower spindle and the hydro-turbine spindle.

9 Assuming that the shafting damping is simplified as linear damping applied to the generator and hydro-turbine runner,
 10 and the external excitation force is also considered, then the generalized force of the HTGU can be formulated as
 11 follows [32]:

$$\begin{cases} Q_{xi} = -c_i x_i + F_{xi} \\ Q_{yi} = -c_i y_i + F_{yi} \\ Q_{zi} = -c_i z_i + F_{zi} \end{cases} \quad (24)$$

in Eq (24), c_i is the damping coefficient, F_{xi} is the excitation force in the horizontal x direction, F_{yi} is the excitation force in the horizontal y direction, and F_{zi} is the excitation force in the horizontal z direction.

Lagrange equation can be expressed as[32]:

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \left(\frac{\partial T}{\partial q_i} \right) + \left(\frac{\partial U}{\partial q_i} \right) = Q_i \quad (25)$$

where T is the kinetic energy expressed by various generalized coordinates and generalized velocities of the system.

U is the potential energy represented by the generalized coordinates. Q_i is the corresponding generalized force.

Since the quality of the guide bearing is much lower than those of the generator and the hydro-turbine runner, the vibration of the guide bearing can be ignored. Substituting Eq (19), Eq (20), and Eq (24) into Eq (25), the vibration motion equations of the shafting can be written as:

$$\begin{cases} m_1 \ddot{x}_1 + c_1 \dot{x}_1 - m_1 e_1 \ddot{\phi}_1 \sin \phi_1 + 2x_1 K_{11} + x_1 \frac{\sqrt{x_2^2 + y_2^2}}{\sqrt{x_1^2 + y_1^2}} K_{12} - m_1 e_1 \ddot{\phi}_1 \cos \phi_1 = (F_{G-G} + F_{ump}) \cos \phi_1 & (a) \\ m_1 \ddot{y}_1 + c_1 \dot{y}_1 - m_1 e_1 \ddot{\phi}_1 \cos \phi_1 + 2y_1 K_{11} + y_1 \frac{\sqrt{x_2^2 + y_2^2}}{\sqrt{x_1^2 + y_1^2}} K_{12} - m_1 e_1 \ddot{\phi}_1 \sin \phi_1 = (F_{G-G} + F_{ump}) \sin \phi_1 & (b) \\ m_{11} \ddot{z}_1 + c_{z1} \dot{z}_1 + (k_z + k_l) z_1 + m_{11} g = F_G & (c) \\ m_2 \ddot{x}_2 + c_2 \dot{x}_2 - m_2 e_2 \ddot{\phi}_2 \sin \phi_2 + 2x_2 K_{22} + x_2 \frac{\sqrt{x_1^2 + y_1^2}}{\sqrt{x_2^2 + y_2^2}} K_{12} - m_2 e_2 \ddot{\phi}_2 \cos \phi_2 = F_{x-lf} + F_{T-G} \cos \phi_2 & (d) \\ m_2 \ddot{y}_2 + c_2 \dot{y}_2 - m_2 e_2 \ddot{\phi}_2 \cos \phi_2 + 2y_2 K_{22} + y_2 \frac{\sqrt{x_1^2 + y_1^2}}{\sqrt{x_2^2 + y_2^2}} K_{12} - m_2 e_2 \ddot{\phi}_2 \sin \phi_2 = F_{y-lf} + F_{T-G} \sin \phi_2 & (e) \\ m_{22} \ddot{z}_2 + c_{z2} \dot{z}_2 + k_l z_2 + m_{22} g = F_z & (f) \\ (J_1 + 2m_1 e_1^2) \ddot{\phi}_1 + m_1 e_1 \ddot{x}_1 \cos \phi_1 - m_1 e_1 \ddot{y}_1 \sin \phi_1 + K_y \alpha = -M_g & (g) \\ (J_2 + 2m_2 e_2^2) \ddot{\phi}_2 + m_2 e_2 \ddot{x}_2 \cos \phi_2 - m_2 e_2 \ddot{y}_2 \sin \phi_2 - K_y \alpha = M_t & (h) \end{cases} \quad (26)$$

in Eq (26), $M_t = m_t M_{gB}$, $M_g = m_g M_{gB}$ are the hydro-turbine torque, and the generator torque of the HTGU,

respectively. $\dot{\phi}_1 = \dot{\phi}_2 = \omega_1 = \omega_2 = \omega$ is the inherent coaxial rotation, ϕ_1 is the rotational phase of the generator rotor, ϕ_2 is

the rotational phase of the hydro-turbine runner. we have $\phi_1 = \int \omega dt + \varphi_1$ and $\phi_2 = \int \omega dt + \varphi_2$, it can be further obtained that the phase difference between the generator and the hydro-turbine runner is $\alpha = \phi_1 - \phi_2 = \varphi_1 - \varphi_2$.

2.3 Nonlinear equation coupling the HTGS and the shafting system

In Eq(26), the shafting system we have $(J_2 + 2m_2e_2^2) \times (g) - (J_1 + 2m_1e_1^2) \times (h)$ and $(g) + (h)$ which can be obtained:

$$\begin{aligned} \omega = & -\frac{m_1e_1\cos\phi_1 - m_1e_1\sin\phi_1}{(J_1 + 2m_1e_1^2)} + \frac{m_2e_2\cos\phi_2 - m_2e_2\sin\phi_2}{(J_2 + 2m_2e_2^2)} - \\ & \frac{JK_y\alpha}{(J_2 + 2m_2e_2^2)(J_1 + 2m_1e_1^2)} - \frac{S_{gB}(m_g(J_2 + 2m_2e_2^2) + m_t(J_1 + 2m_1e_1^2))}{\omega_{mB}(J_1 + 2m_1e_1^2)(J_2 + 2m_2e_2^2)} \end{aligned} \quad (27)$$

$$J \frac{d\omega}{dt} = M_t - M_g - m_1e_1(\cos\phi_1 - \sin\phi_1) - m_2e_2(\cos\phi_2 - \sin\phi_2) \quad (28)$$

where $J = J_2 + 2m_2e_2^2 + J_1 + 2m_1e_1^2$. Meanwhile, considering the change of angular velocity(ω) in the transient process of perturbation factor, the Eq (28) can be written as:

$$J \frac{d\omega}{dt} = (M_t - M_g - m_1e_1(\cos\phi_1 - \sin\phi_1) - m_2e_2(\cos\phi_2 - \sin\phi_2)) / \omega_{mB} \quad (29)$$

Due to the $x = (n - n_0) / n_0$, and $\omega = (n\pi / 30 - n_0\pi / 30) / n_0\pi / 30$, so we have $\frac{d\omega}{dt} = \frac{dx}{dt}$. M_{gB} is the basic value of the generator torque, ω_{mB} is the basic value of rotation speed (Where $M_{gB} = S_{gB} / \omega_{mB}$, $\omega_{mB} = \frac{n_r\pi}{30}$), $M_t = m_t M_{gB} = m_t S_{gB} / \omega_{mB}$, and $M_g = m_g M_{gB} = m_g S_{gB} / \omega_{mB}$.

The inertia time constant of the HTGU is $T_{ab} = \frac{(J_2 + 2m_2e_2^2 + J_1 + 2m_1e_1^2)\omega_{mB}^2}{S_{gB}}$. Further, Substituting J , T_{ab}

into Eq (29), and rewritten M_t and M_g in Eq(29), the motion equation of the HTGU can be written as:

$$\frac{dx}{dt} = \frac{m_t - m_g}{T_{ab}} + \frac{\omega_{mB}(m_1e_1(\sin\phi_1 - \cos\phi_1) + m_2e_2(\sin\phi_2 - \cos\phi_2))}{T_{ab}S_{gB}} \quad (30)$$

When the load self-regulation factor is considered, and Substituting Eq (2) into Eq (30) the HTGU rotation equation can be obtained as follows:

$$\frac{dx}{dt} = \frac{e_h \frac{q_t - e_{qx}x - e_{qy}y}{e_{qh}} + e_x x + e_y y - m_g - e_g x}{T_{ab}} + \frac{\omega_{mB} (m_1 e_1 (\ddot{\phi}_1 \sin \phi_1 - \dot{\phi}_1^2 \cos \phi_1) + m_2 e_2 (\ddot{\phi}_2 \sin \phi_2 - \dot{\phi}_2^2 \cos \phi_2))}{T_{ab} S_{gB}} \quad (31)$$

1 in Eq (31), $\phi_1 = (1+x)\omega_{mB}t$, $\phi_2 = (1+x)\omega_{mB}t + \alpha$.

2 According to Eq(31), the HTGS and shafting system were connected as a new nonlinear complex coupled system,

3 substituted Eq (31) into Eq (6) replace $\ddot{x}$, and substituted Eq (27) into Eq (26) to replace Eq(g) and Eq(h). the novel

4 nonlinear equation is written as follows:

$$\begin{cases} \ddot{\phi}_t = \left(-\frac{q_t - e_{qx}x - e_{qy}y}{e_{qh}} - \frac{2h_{t0}}{H_0} q_t \right) / T_{wt} \\ \ddot{x} = \frac{e_h \frac{q_t - e_{qx}x - e_{qy}y}{e_{qh}} + e_x x + e_y y - m_g - e_g x}{T_{ab}} + \frac{\omega_{mB} (m_1 e_1 (\ddot{\phi}_1 \sin \phi_1 - \dot{\phi}_1^2 \cos \phi_1) + m_2 e_2 (\ddot{\phi}_2 \sin \phi_2 - \dot{\phi}_2^2 \cos \phi_2))}{T_{ab} S_{gB}} \\ \ddot{y} = \frac{u - y}{T_y} \\ \ddot{x} = \frac{-k_p}{T_{ab}} \left(e_h \frac{q_t - e_{qx}x - e_{qy}y}{e_{qh}} + e_x x + e_y y - m_g - e_g x \right) - k_i x - \frac{k_d}{T_{ab}} \left(\frac{e_h}{e_{qh}} \ddot{\phi}_t + \left(e_x - e_g - \frac{e_h e_{qx}}{e_{qh}} \right) \ddot{x} + \left(e_y - \frac{e_h e_{qy}}{e_{qh}} \right) \ddot{y} \right) \\ \ddot{\phi}_1 = \left((F_{ump} + F_{G-G}) \cos \phi_1 + m_1 e_1 \ddot{\phi}_1 \sin \phi_1 - c_1 \ddot{\phi}_1 - 2x_1 K_{11} - x_1 \frac{\sqrt{x_2^2 + y_2^2}}{\sqrt{x_1^2 + y_1^2}} K_{12} + m_1 e_1 ((1+x)\omega_{mB})^2 \cos \phi_1 \right) / m_1 \\ \ddot{\phi}_1 = \left((F_{ump} + F_{G-G}) \sin \phi_1 + m_1 e_1 \ddot{\phi}_1 \cos \phi_1 - c_1 \ddot{\phi}_1 - 2y_1 K_{11} - y_1 \frac{\sqrt{x_2^2 + y_2^2}}{\sqrt{x_1^2 + y_1^2}} K_{12} + m_1 e_1 ((1+x)\omega_{mB})^2 \sin \phi_1 \right) / m_1 \\ \ddot{\phi}_1 = (F_G - c_{z1} \ddot{\phi}_1 - (k_z + k_l) z_1 - m_{11} g) / m_{11} \\ \ddot{\phi}_2 = \left(F_{x-ff} + F_{T-G} \cos \phi_2 + m_2 e_2 \ddot{\phi}_2 \sin \phi_2 - c_2 \ddot{\phi}_2 - 2x_2 K_{22} - x_2 \frac{\sqrt{x_1^2 + y_1^2}}{\sqrt{x_2^2 + y_2^2}} K_{12} + m_2 e_2 ((1+x)\omega_{mB})^2 \cos \phi_2 \right) / m_2 \\ \ddot{\phi}_2 = \left(F_{y-ff} + F_{T-G} \sin \phi_2 + m_2 e_2 \ddot{\phi}_2 \cos \phi_2 - c_2 \ddot{\phi}_2 - 2y_2 K_{22} - y_2 \frac{\sqrt{x_1^2 + y_1^2}}{\sqrt{x_2^2 + y_2^2}} K_{12} + m_2 e_2 ((1+x)\omega_{mB})^2 \sin \phi_2 \right) / m_2 \\ \ddot{\phi}_2 = (F_z - c_{z2} \ddot{\phi}_2 - k_l z_2 - m_{22} g) / m_{22} \\ \ddot{\phi} = -c_t \ddot{\phi} - \frac{m_1 e_1 \ddot{\phi}_1 \cos \phi_1 - m_1 e_1 \ddot{\phi}_1 \sin \phi_1}{(J_1 + 2m_1 e_1^2)} + \frac{m_2 e_2 \ddot{\phi}_2 \cos \phi_2 - m_2 e_2 \ddot{\phi}_2 \sin \phi_2}{(J_2 + 2m_2 e_2^2)} - \\ K_y \alpha \frac{(J_2 + 2m_2 e_2^2 + J_1 + 2m_1 e_1^2)}{(J_2 + 2m_2 e_2^2)(J_1 + 2m_1 e_1^2)} - \frac{S_{gB} (m_g (J_2 + 2m_2 e_2^2) + m_t (J_1 + 2m_1 e_1^2))}{\omega_{mB} (J_1 + 2m_1 e_1^2)(J_2 + 2m_2 e_2^2)} \end{cases} \quad (32)$$

5 in Eq (32), $c_t \ddot{\phi}$ is the linear torsional damping force.

6 3. Stability and sensitivity of the governor PID parameters to the hydro-turbine generator unit

The governor is one of the most important parts of the HTGU. The control parameters of the governor have a very important influence on the regulation performance and stability of the unit. At the same time, PID governor is widely used in engineering practice. Therefore, we focus on the analysis of PID parameters on the stability and sensitivity of the unit. A hydropower station is selected for analyzing the stability and sensitivity of the governor parameter. The classical fourth-order Runge-Kutta method is adopted to solve the nonlinear equation of the coupling HTGS with the shafting system in MATLAB. The actual parameters of a hydropower station are shown in Table 1.

Table 1 Actual parameters of a hydropower station

parameter	value	unit	parameter	value	unit
m_1	4.91×10^5	kg	a	2.54	m
m_2	5.6×10^4	kg	b	2.04	m
m_{11}	5.65432×10^5	kg	c	8.19	m
m_{22}	1.17×10^5	kg	d	1.765	m
c_1	4.1×10^6	$N \cdot s / m$	e_1	0.0005	m
c_2	2.52×10^6	$N \cdot s / m$	e_2	0.0005	m
c_{z1}	3.6×10^6	$N \cdot s / m$	d_H	1.15	m
c_{z2}	6.31×10^6	$N \cdot s / m$	d_B	0.3	m
c_t	1000	$N \cdot s / rad$	d_1	7.44	m
k_1	1.85×10^8	N / m	L	2.927	m
k_2	1.45×10^8	N / m	D	7.44	m
k_3	1.15×10^7	N / m	δ_1	0.0004	m
k_z	1.27×10^7	N / m	δ_2	0.0021	m
j_1	4.625×10^6	$kg \cdot m^2$	l	10.3	m
j_2	2×10^5	$kg \cdot m^2$	n_r	250	r / min
E	206×10^9	P_a	S_{gB}	306	MW
H_0	195	$m \cdot H_2O$	h_{t0}	3.2723	m
Q	176.1	m^3 / s	T_y	0.02	s
e_x	-1		T_{ab}	10.8072	s
e_{qh}	0.5		T_{wt}	2.74	s
e_{qy}	1		τ_0	0.23	
e_h	1.5		D_1	5.34	m
e_y	1		B	1	T
g	9.81	m / s^{-2}	β	0.5	
μ_2	0.11		n_0	0.066	
μ_0	0.6		m_0	-0.25	
Δp	0.5×10^6	P_a	b_0	3.5	
K_z	0.8		T_0	0.0657	
Ra_0	2703		r_a	1.86	m

In this section, Stein's formula[40] was adopted to obtain the combination of PID parameters. While the relative value

of load disturbance is 0.1 or -0.1, the stability and sensitivity of the PID parameters to the HTGS and shafting system vibrations are obtained using the bifurcation diagram. Since the deviation of the characteristic quantity of the HTGS is consistent, the deviation of the rotation speed is selected as the typical characteristic variable of the HTGS.

3.1 The stability and sensitivity of k_p to the rotation speed deviation and shafting system vibrations

The stability and sensitivity of the rotation speed deviation and shafting system vibrations to k_p are shown in Fig.3. From Fig.3(a), it is seen that when k_p is $[0.1, 1.65]$, the rotation speed deviation is bifurcated which shows a strong sensitivity. When k_p is $[1.65, 5.4]$, the rotation speed deviation is stable. When k_p is $(5.4, 5.6]$, the rotation speed deviation is chaotic that shows a strong sensitivity. As can be seen from Fig.3(b), When k_p is $[0.1, 5.6]$, the negative torsion vibration angles are stable. From Fig.3(c), it is seen that when k_p is $[0.1, 2]$, the generator vibration of x direction is chaotic that shows obvious vibration and strong sensitivity. When k_p is $[2, 4.05]$, the generator vibration of x direction is quasi-periodic which shows slight vibration, and when k_p is $[4.05, 5.35]$, the generator vibration of x direction still shows quasi-periodic but vibration amplitude increase obviously. When k_p is $(5.35, 5.6]$, the generator vibration of x direction is chaotic that shows obvious vibration and strong sensitivity. As can be seen from Fig.3(d), when k_p is $[0.1, 2.15]$, the generator vibration of y direction is chaotic that shows obvious vibration and strong sensitivity. When k_p is $[2.15, 3.2]$, the generator vibration of y direction is quasi-periodic which shows slight vibration, and When k_p is $[3.2, 5.3]$, the generator vibration of y direction still shows quasi-periodic but vibration amplitude increase obviously. When k_p is $(5.3, 5.6]$, the generator vibration of y direction is chaotic that shows obvious vibration and strong sensitivity. As can be seen from Fig.3(e), when k_p is $[0.1, 1.35]$, the generator vibration of z direction is bifurcated which shows slight vibration and slight sensitivity. When k_p is $[1.35, 5.35]$, the generator vibration of z direction is periodic which shows slight vibration. When k_p is $(5.35, 5.6]$, the generator vibration of z direction is chaotic that shows obvious vibration and strong sensitivity. As can be seen from Fig.3(f) and (g), when k_p is $[0.1, 1]$, the hydro-turbine vibration of x and y direction is chaotic that shows obvious vibration and strong sensitivity. When k_p is $[1, 5.4]$, the hydro-turbine vibration of x and y direction is quasi-periodic which

shows slight vibration. When k_p is $(5.4, 5.6]$, the hydro-turbine vibration of x and y direction is chaotic that shows obvious vibration and strong sensitivity. From Fig.3(h), it is seen that when k_p is $[0.1, 1.35)$, the hydro-turbine vibration of z direction is bifurcated which shows obvious vibration and strong sensitivity. When k_p is $[1.35, 5.3]$, the hydro-turbine vibration of y direction is periodic which shows slight vibration. When k_p is $(5.3, 5.6]$, the hydro-turbine vibration of y direction is chaotic that shows obvious vibration and strong sensitivity. Meanwhile, under the different load disturbances, the shafting radial vibration shows almost identical stability, However, the axial vibration shows the same stability of k_p but the larger amplitude was generated by smaller the load.

(a) Rotation speed deviation bifurcation diagram of k_p

(b) Torsion angle bifurcation diagram of k_p

(c) Generator vibration (x_1) bifurcation diagram of k_p

(d) Generator vibration (y_1) bifurcation diagram of k_p

(e) Generator vibration (z_1) bifurcation diagram of k_p

(f) Hydro-turbine vibration (x_2) bifurcation diagram of k_p

(g) Hydro-turbine vibration (y_2) bifurcation diagram of k_p

(h) Hydro-turbine vibration (z_2) bifurcation diagram of k_p

Fig.3 The rotation speed deviation and SCV dynamic characteristics to k_p

3.2 The Stability and sensitivity of k_i to the rotation speed deviation and shafting systems vibrations

The stability and sensitivity of k_i to the rotation speed deviation and shafting system vibrations are shown in Fig.4.

From Fig.4(a), it is seen that when k_i is $[0.1, 0.77]$, the rotation speed deviation is stable. When k_i is $[0.77, 1.15]$, the rotation speed deviation is bifurcated which shows strong sensitivity. As can be seen from Fig.4(b), When k_i is $[0.1, 1.15]$, the negative torsion vibration of HTGU is stable. From Fig.4(c), it is seen that when k_i is $[0.1, 0.21]$, the generator vibration of x direction is chaotic that shows obvious vibration and strong sensitivity. When k_i is $[0.21, 0.72]$, the generator vibration of x direction is quasi-periodic which shows slight vibration. When k_i is $(0.72, 1.15]$, the generator vibration of x direction is chaotic that shows obvious vibration and strong sensitivity. As can be seen from Fig.4(d), when k_i is $[0.1, 0.26]$, the generator vibration of y direction is chaotic that shows obvious vibration and strong sensitivity. When k_i is $[0.26, 0.59]$, the generator vibration of y direction is quasi-periodic which shows slight vibration. When k_i is $(0.59, 1.15]$, the generator vibration of y direction is chaotic

1 that shows obvious vibration and strong sensitivity. As can be seen from Fig.4(e), when k_i is $[0.1, 0.83)$, the
2 generator vibration of z direction is periodic which shows slight vibration. When k_i is $(0.83, 1.15]$, the generator
3 vibration of z direction is bifurcated which shows obvious vibration and strong sensitivity. As can be seen from
4 Fig.4(f) and (g), when k_i is $[0.1, 0.89)$, the hydro-turbine vibration of x and y direction is quasi-periodic which shows
5 slight vibration. When k_i is $[0.89, 1.15]$, the hydro-turbine vibration of x and y direction is chaotic that shows
6 obvious vibration and strong sensitivity. From Fig.4(h), it is seen that when k_i is $[0.1, 0.86)$, the hydro-turbine
7 vibration of y direction is periodic which shows slight vibration. When k_i is $[0.86, 1.15]$, the hydro-turbine vibration
8 of y direction is chaotic that shows obvious vibration and strong sensitivity. Obviously, under the different load
9 disturbances, the shafting radial vibration shows almost identical stability, However, the axial vibration shows the same
10 bifurcated characteristics of k_i but the smaller amplitude was generated by a larger load.

(a) Rotation speed deviation bifurcation diagram of k_i

(b) Torsion angle bifurcation diagram of k_i

(c) Generator vibration (x_1) bifurcation diagram of k_i

(d) Generator vibration (y_1) bifurcation diagram of k_i

(e) Generator vibration (z_1) bifurcation diagram of k_i

(f) Hydro-turbine vibration (x_2) bifurcation diagram of k_i

(g) Hydro-turbine vibration (y_2) bifurcation diagram of k_i

(h) Hydro-turbine vibration (z_2) bifurcation diagram of k_i

Fig. 4 The rotation speed deviation and SCV dynamic characteristics of k_i

3.3 The stability and sensitivity of k_d to the rotation speed deviation and shafting system vibrations

The stability and sensitivity of k_d to the rotation speed deviation and shafting system vibrations are shown in Fig.5.

From Fig.5(a), it is seen that when k_d is $[0.1, 5.48)$, the rotation speed deviation is stable. When k_d is $(5.48, 5.5]$, the rotation speed deviation is chaotic that shows a strong sensitivity. As can be seen from Fig.5(b), When k_d is $[0.1, 5.5]$, the negative torsion vibration of HTGU is stable. From Fig.5(c), it is seen that when k_d is $[0.1, 5.46)$, the generator vibration of x direction is quasi-periodic that shows slight vibration. When k_d is $(5.46, 5.5]$, the generator vibration of x direction is chaotic that shows obvious vibration and strong sensitivity. As can be seen from Fig.5(d), when k_d is $[0.1, 5.46)$, the generator vibration of y direction is quasi-periodic which shows slight vibration, but when k_d is $[0.1, 1.78)$ and $[3.62, 5.46)$, it shows the slight sensitivity. When k_d is $(5.46, 5.5]$, the generator vibration of y direction is chaotic that shows obvious vibration and strong sensitivity. As can be seen from Fig.5(e), when k_d is $[0.1, 5.44)$, the generator vibration of z direction is periodic which shows slight vibration. When k_d

is $[5.44, 5.5]$, the generator vibration of z direction is chaotic that shows obvious vibration and strong sensitivity. As can be seen from Fig.5(f) and (g), when k_d is $[0.1, 0.5.46)$, the hydro-turbine vibration of x and y direction is quasi-periodic that shows slight vibration. When k_d is $[5.46, 5.5]$, the hydro-turbine vibration of x and y direction is chaotic that shows obvious vibration and strong sensitivity. From Fig.5(h), it is seen that when k_d is $[0.1, 5.48)$, the hydro-turbine vibration of y direction is periodic which shows slight vibration. When k_d is $(5.48, 5.5]$, the hydro-turbine vibration of y direction is chaotic that shows obvious vibration and strong sensitivity. Obviously, under the different load disturbances, the shafting vibration shows almost identical stability of k_d , However, the axial vibration's larger amplitude was generated by a smaller load.

(a) Rotation speed deviation bifurcation diagram of k_d

(b) Torsion angle bifurcation diagram of k_d

(c) Generator vibration (x_1) bifurcation diagram of k_d

(d) Generator vibration (y_1) bifurcation diagram of k_d

(e) Generator vibration (z_1) bifurcation diagram of k_d

(f) Hydro-turbine vibration (x_2) bifurcation diagram of k_d

(g) Hydro-turbine vibration (y_2) bifurcation diagram of k_d

(h) Hydro-turbine vibration (z_2) bifurcation diagram of k_d

Fig. 5 The rotation speed deviation and SCV dynamic characteristics to k_d

4. The governor PID parameter multi-objective optimization control

Based on the coupling of HTGS with shafting system nonlinear mathematical model, a multi-objective optimization strategy of PID parameters is proposed with the following innovations: (1) The objective functions consider the rotation speed deviation, the opening of the guide vane, and the shafting system vibrations simultaneously. (2) The objective function is composed of the ITSE. (3) The NSGA-II[41] and MOEAD[42] which have shown high efficiency and competitive performance in solving the multi-objective optimization problems compared with other evolutionary algorithms were introduced.

4.1 Objective functions

In the process of load regulation, the rotation speed, guide vane opening, and shafting system vibration will be volatile. Under small load disturbance, to make the complex coupled system to obtain the response output overshoot

minimize and the shortest regulation time. The ITSE ($ITSE = \int_0^\infty te^2(t)dt$) of rotation speed deviation, guide vane opening deviation, vibration radius deviation of the generator and hydro-turbine in 3D space are selected to establishes the objective functions, which was written as follows:

$$\begin{cases} J_1 = W_1 \cdot ITSE_x + ITSE_{r11} + ITSE_{r22} \\ J_2 = W_2 \cdot ITSE_y + ITSE_{r11} + ITSE_{r22} \end{cases} \quad (33)$$

In Eq(33), $W_1 = W_2 = 2$, $r_{11} = \sqrt{(x_1^2 + y_1^2 + z_1^2)}$, and $r_{22} = \sqrt{(x_2^2 + y_2^2 + z_2^2)}$.

4.2 Decision variables

When load disturbance occurs to the HTGU, improper PID parameter combination may lead to the whole system slow stable response, or even goes into an unstable or chaotic state, which will lead to control failure of the whole system and lead to power grid oscillation and abnormal vibration of the HTGU. Therefore, we select the parallel PID parameters k_p , k_i , and k_d as the decision variables. In section 3, the rotation speed deviation and shafting system vibration stability and sensitivity of k_p , k_i , and k_d are given, which can be concluded that the system has different stability domains for different characteristic variables of k_p , k_i , and k_d . There, we define the periodic and quasi-periodic vibration regions of shafting system vibrations and the stable region of HTGS to form the relative stability region. Therefore, the intersection principle was used to obtain the common relative stable domain consider the shafting system vibration of the parameters of k_p , k_i , and k_d , and then the boundary of the common stability domain is taken as the boundary of the decision variables. so we have:

$$\begin{cases} k_{ps} = k_{pxs} \cap k_{p\alpha s} \cap k_{px_1s} \cap k_{py_1s} \cap k_{pz_1s} \cap k_{px_2s} \cap k_{py_2s} \cap k_{pz_2s} \\ k_{is} = k_{ixs} \cap k_{i\alpha s} \cap k_{ix_1s} \cap k_{iy_1s} \cap k_{iz_1s} \cap k_{ix_2s} \cap k_{iy_2s} \cap k_{iz_2s} \\ k_{ds} = k_{dxs} \cap k_{d\alpha s} \cap k_{dx_1s} \cap k_{dy_1s} \cap k_{dz_1s} \cap k_{dx_2s} \cap k_{dy_2s} \cap k_{dz_2s} \end{cases} \quad (34)$$

Thus, the relative stable domain that considers the HTGS and the shafting system of the decision variables are $k_{ps} = [2.15, 3.2]$, $k_{is} = [0.26, 0.59]$, and $k_{ds} = [1.78, 3.62]$, respectively.

4.3 Multi-objective optimization

According to the above determined objective functions and decision variable boundary, the NSGA-II and MOEAD

multi-objective optimization algorithms were used to establish the optimization process as shown in Fig.6, and the optimization results of the dual-objective and parameter were obtained as shown in Fig.7. When the load disturbance occurs to the HTGU, since the current focus is on the overregulation amount and regulation time of the rotation speed deviation of the unit, so the smaller J_1 is selected after optimized. The detailed results are shown in Table 2.

Fig 6 Optimization process diagram

Fig 7. Pareto optimal solution obtained by different algorithms

Table 2 the optimal results

algorithms	Load disturbance	J_1	J_2	k_p	k_i	k_d
NSGA-II	$m_g = \pm 0.1$	34.9	3585.5	3.2	0.49	2.69
MOEAD	$m_g = \pm 0.1$					

5 Nonlinear dynamic characteristics of the HTGU

In this section, the nonlinear dynamic characteristics of the HTGS, generator vibration, hydro-turbine vibration, and torsion vibration were revealed using the time domain feature, phase trajectory, Poincare map, and frequency spectrum while the small load disturbance happened with the optimal PID control parameters.

5.1 Nonlinear dynamic characteristics of the HTGU to the self-regulating coefficient

The nonlinear dynamic characteristics of the HTGS are shown in Fig.8. As can be seen from Fig.8(a) and (b), while the load disturbance is $m_g = \pm 0.1$, the optimized PID parameters make the adjustment time of q_t , x , and y were significantly shortened, whereas the overshoot of q_t , x and y was increased obviously. Meanwhile, it is seen that the overshoot of q_t , x , and y was decreased gradually with the self-regulation coefficient increased.

Fig.8 The nonlinear dynamic characteristics of the HTGS

The dynamic characteristics of generator vibration radii in 3D space are shown in Fig.9. As can be seen from Fig.9(a) and (b), while the load disturbance is $m_g = \pm 0.1$ the optimized PID parameters make the adjustment time was significantly shortened and the vibration amplitude decreased slightly. Meanwhile, it is seen that the dithering amplitude of r_{11} was decreased gradually with the self-regulation coefficient increased, however, the dithering time is increased slightly.

(a) $m_g = -0.1$

(b) $m_g = 0.1$

Fig 9 Robustness of optimized PID controller for r_{11} to the load-self regulation coefficient

The dynamic characteristics of hydro-turbine vibration radii in 3D space are shown in Fig.10. As can be seen from Fig.10(a) and (b), while the load disturbance is $m_g = \pm 0.1$ the optimized PID parameters make the adjustment time was significantly shortened and the vibration amplitude decreased slightly. Meanwhile, it is seen that the dithering amplitude of r_{22} was decreased gradually with the self-regulation coefficient increased, however, the dithering time is increased slightly.

$m_g = -0.1$

$m_g = 0.1$

Fig 10 Robustness of optimized PID controller for r_{22} to the load-self regulation coefficient

The dynamic characteristics of spindle torsion vibration are shown in Fig.11. As can be seen from Fig.11(a) and (b), while the load disturbance is $m_g = \pm 0.1$ the shafting torsion vibration is stable. Meanwhile, it is seen that the torsion vibration remains same with the self-regulation coefficient increased.

Fig.11 Robustness of optimized PID controller for torsion angle to the load-self regulation coefficient

5.2 Nonlinear dynamic characteristics of shafting system vibrations

According to the above research, the nonlinear dynamic characteristics of the shafting system vibrations excited by the mass eccentricity, the UMP, the arcuate whirl of the rotor, and the seal excitation coupled vibration sources, controlled with the optimal PID parameters, and self-regulation coefficient is 1.05 are analyzed.

(1) The nonlinear dynamic characteristics of generator vibration

The generator vibration nonlinear dynamic characteristics are shown in Fig.12. From Fig.12(a), the generator vibration axis locus is multiple closed curves in 3D space, which shows quasi-periodic vibration. As can be seen from Fig.12 (b), (c), (d), (e), (f), and (g), the generator vibration axis locus are multiple closed curves in $x - y$, $x - z$, $y - z$ 2D space, and the Poincare section show the attractors are a finite number, which can qualitatively analyze that the hydro-turbine vibration of $x - y$, $x - z$, $y - z$ 2D space are quasi-periodic. As can be seen from Fig.12 (h) and (l), during load disturbance, the generation vibration of x and y direction go through three fluctuations: first, it decreases rapidly and then increases rapidly; second, it decreases slightly and then increases; finally, it decreases rapidly and then increases rapidly and stable in a certain area. As can be seen from Fig.12 (i), (j), (m), and (n) the generator vibration of x and y direction velocity displacement phase plane are multiple closed curves and the velocity displacement Poincare section show the attractors are a finite number, which can qualitatively analyze that the generator vibration of x and y direction are quasi-periodic. As can be seen from Fig.12(k) and (o), the generator vibration of x and y directions vibration is accompanied by three main vibration frequency components, namely,

0.08 octaves, 0.18 octaves, and 0.24 octaves. As can be seen from Fig.12(p), during load disturbance, the generator vibration of z direction changes with the corresponding change of load and stable in a certain area, which can be analyzed qualitatively the vibration and the load of the HTGU stay the positive correlation. As can be seen from Fig.12 (q) and (r) the generator vibration of z direction velocity displacement phase plan are multiple closed curves and the velocity displacement Poincare section shows the attractors are a finite number, which can qualitatively analyze that the generator vibration of z direction is quasi-periodic. As can be seen from Fig.12(s), the generator vibration of z direction is accompanied by 0.1 octaves. To sum up the above, it can be seen that the vibration of the generator of x and y directions has almost the same vibration characteristics.

(a) Three-dimensional axis locus of generator vibration

(b) The axis locus of x - y direction

(c) The Poincare section of x - y direction

(d) The axis locus of x - z direction

(e) The Poincare section of x - z direction

(f) The axis locus of y - z direction

(g) The Poincare section of y - z direction

(h) The time-domain diagram of the x direction

(i) The velocity displacement phase plan of x direction

(j) The velocity displacement Poincare section of x direction

(k) The frequency spectrum of x direction

(l) The time-domain diagram of y direction

(m) The velocity displacement phase plan of y direction

(n) The velocity displacement Poincare section of y direction

(o) The frequency spectrum of y direction

(p) The time-domain diagram of z direction

(q) The velocity displacement phase plan of z direction

(r) The velocity displacement Poincare section of z direction

(s) The frequency spectrum of the z direction

Fig. 12 The nonlinear dynamic characteristics of generator vibration

(2) The nonlinear dynamic characteristics of hydro-turbine vibration

The hydro-turbine vibration nonlinear dynamic characteristics are shown in Fig.13. From Fig.13(a), the hydro-turbine vibration axis locus is multiple closed curves in 3D space, which shows quasi-periodic vibration. As can be seen from Fig.13 (b), (c), (d), (e), (f), and (g), the hydro-turbine vibration axis locus are multiple closed curves in $x-y$, $x-z$, $y-z$ 2D space, and the Poincare section show the attractors are a finite number, which can qualitatively analyze that the hydro-turbine vibration of $x-y$, $x-z$, $y-z$ 2D space are quasi-periodic. As can be seen from Fig.13 (h) and (l), during load disturbance, the hydro-turbine vibration of x and y direction go through small fluctuations and steadily in a certain area. As can be seen from Fig.13 (i), (j), (m), and (n), the hydro-turbine vibration of x and

1 y direction velocity displacement phase plane are multiple closed curves and the velocity displacement Poincare
 2 section show the attractors are a finite number, which can qualitatively analyze that the hydro-turbine vibration of x and
 3 y direction are quasi-periodic. As can be seen from Fig.13 (k) and (o), the hydro-turbine vibration of x and
 4 y directions vibration is accompanied by three main vibration frequency components, namely, 0.1 octaves,
 5 0.34octaves, 0.56 octaves, 0.78 octaves, and 0.96 octaves. As can be seen from Fig.13 (p), during load disturbance
 6 the hydro-turbine vibration of z direction changes with the corresponding change of load and stable in a certain area,
 7 which can be analyzed qualitatively the vibration and the load of the HTGU stay the positive correlation. As can
 8 be seen from Fig.13 (q) and (r), the hydro-turbine vibration of z direction velocity displacement phase plan are
 9 multiple closed curves, and the velocity displacement Poincare section shows the attractors are a finite number,
 10 which can qualitatively analyze that the hydro-turbine vibration of z direction is quasi-periodic. As can be seen
 11 from Fig.13 (s), the hydro-turbine vibration of z direction is accompanied by 0.04 octaves.

(a) Three-dimensional axis locus of hydro-turbine vibration

(b) The axis locus of $x - y$ direction

(c) The Poincare section of $x - y$ direction

(d) The axis locus of $x - z$ direction

(e) The Poincare section of $x - z$ direction

(f) The axis locus of $y - z$ direction

(g) The Poincare section of $y - z$ direction

(h) The time-domain diagram of x direction

(i) The velocity displacement phase plan of x direction

(j) The velocity displacement Poincare section of x direction

(k) The frequency spectrum of x direction

(l) The time-domain diagram of y direction

(m) The velocity displacement phase plan of y direction

(n) The velocity displacement Poincare section of y direction

(o) The frequency spectrum of y direction

(p) The time-domain diagram of z direction

(q) The velocity displacement phase plan of z direction

(r) The velocity displacement Poincare section of z direction

(s) The frequency spectrum of z direction

Fig.13 The nonlinear dynamic characteristics of hydro-turbine vibration

1 (3) The nonlinear dynamic characteristics of torsion vibration

According to Fig. 14(a), it can be seen that the torsional vibration angle quickly stabilizes at 0.013 rad. From Fig.14(b) and (c), the Poincare section is approximately linear, which can qualitatively analyze that the torsional vibration is stable. It can be seen from Fig.14(d) that the spindle has smaller torsional vibration spectrum components.

(a) Torsion vibration time-domain diagram

(b) Torsion vibration velocity displacement phase plan

(c) Torsion vibration velocity displacement Poincare section

(d) Torsion vibration frequency spectrum

Fig.14 The nonlinear dynamic characteristics of torsion vibration

6. Conclusions

In this study, first, we establish a nonlinear mathematical model that considers the HTGS and the shafting system. Meanwhile, the stability and sensitivity of the rotation speed deviation and shafting system vibrations to the PID parameters were revealed by the bifurcation diagram. second, an innovational optimized strategy that considers the ITSE of the HTGS and shafting system vibrations was proposed, and then the NSGA-II and MOEAD were introduced to obtain the optimal PID control parameter for the multi-objective function. Finally, the nonlinear dynamic

characteristics of the HTGS and the shafting system vibrations which are controlled by the optimized PID parameter were analyzed. The major conclusions are summarized as follows:

(1) The simulation results show that the nonlinear mathematical model coupling the HTGS and the shafting system is effective. It can reveal the interaction characteristics between the shafting system and the HTGS and reveal the stability and sensitivity of some key parameters to the HTGS and shafting system vibrations, and these results will provide some bases for the design and stable operation of the HTGU.

(2) The multi-objective PID parameter optimization strategy shows good performance, it makes the adjusting-time shorten 12 seconds of the HTGS and the shafting system vibrations. The self-regulating coefficient can reduce the overshoot of the HTGS effectively, and suppressed the fluctuation amplitude.

(3) The rotation speed deviation and shafting system vibrations have different stability and sensitivity to the PID parameters. The relative stable region of k_p , k_i , and k_d will decrease when considering the shafting system vibrations, the specific region are $k_{ps} = [2.1, 3.85]$, $k_{is} = [0.26, 0.59]$, and $k_{ds} = [0.1, 5.4]$, respectively.

(4) The generator and hydro-turbine vibrations in the 3D space that excited by the mass eccentricity, the UMP, the arcuate whirl of the rotor, and the seal excitation coupled vibration sources are quasi-periodic composed of several frequency components, whereas the torsion vibration is stable with a tiny frequency component,

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Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Figures

Figure 1

Structure of a hydropower station

Figure 2

Shafting structure diagram

Figure 3

The rotation speed deviation and SCV dynamic characteristics to K_p

Figure 4

The rotation speed deviation and SCV dynamic characteristics of K_I

Figure 5

The rotation speed deviation and SCV dynamic characteristics to K_d

Figure 6

Optimization process diagram

(a) $Mg=-0.1$

(b) $Mg=0.1$

Figure 7

Pareto optimal solution obtained by different algorithms

(a) $m_g = -0.1$

(b) $m_g = 0.1$

Figure 8

The nonlinear dynamic characteristics of the HTGS

(a) $m_g = -0.1$

(b) $m_g = 0.1$

Figure 9

Robustness of optimized PID controller for to the load-self regulation coefficient

$$m_g = -0.1$$

$$m_g = 0.1$$

Figure 10

Robustness of optimized PID controller for to the load-self regulation coefficient

$$m_g = -0.1$$

$$m_g = 0.1$$

Figure 11

Robustness of optimized PID controller for torsion angle to the load-self regulation coefficient

Figure 12

The nonlinear dynamic characteristics of generator vibration

Figure 13

The nonlinear dynamic characteristics of hydro-turbine vibration

(a) Torsion vibration time-domain diagram

(b) Torsion vibration velocity displacement phase plan

(c) Torsion vibration velocity displacement Poincare section

(d) Torsion vibration frequency spectrum

Figure 14

The nonlinear dynamic characteristics of torsion vibration