The κ-statistics approach to epidemiology

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ABSTRACT

A great variety of complex physical, natural and artificial systems are governed by statistical distributions, which often follow a standard exponential function in the bulk, while their tail obeys the Pareto power law. The recently introduced κ-statistics framework predicts distribution functions with this feature. A growing number of applications in different fields of investigation are beginning to prove the relevance and effectiveness of κ-statistics in fitting empirical data. In this paper, we use κ-statistics to formulate a statistical approach for epidemiological analysis. We validate the theoretical results by fitting the derived κ-Weibull distributions with data from the plague pandemic of 1417 in Florence as well as partial data (until April 16, 2020) from the COVID-19 pandemic in China. The fact that both the approximate dataset of the Florence plague and the partial data of the Covid-19 pandemic in China are well described by means of the proposed model suggests that the κ-deformed Weibull model is relevant and that both datasets faithfully represent the spreading of the epidemics.

Introduction

Many phenomena in a large variety of disciplines present apparent regularities described by well-known statistical distributions[1]. Some examples include the populations of cities, the frequencies of words in a text, the energy distribution in solids, the behaviour of financial markets, the statistical distribution of the kinetic energy in a gas, etc.[2,3]. Moreover, statistical approaches provide powerful tools for medical and epidemiological applications, since they allow predicting the behaviour of certain diseases[4,5]. Thus, such approaches are very useful for informing health policy and decision making, particularly regarding control and mitigation measures in response to the societal impacts of epidemics and pandemics.

While an epidemic is defined as “the occurrence in a community or region of cases of an illness clearly in excess of normal expectancy”[6,7], a pandemic is defined as “an epidemic occurring over a very wide area, crossing international boundaries, and usually affecting a large number of people”[6,7]. Pandemics are large-scale outbreaks of infectious diseases which cause a growth in mortality over a wide geographic area.

In addition to the obvious health consequences, both epidemics and pandemics also cause economic stress and social hardship. It has been emphasized that the probability of occurrence of pandemics is increasing due to the high interconnectedness of the modern world, which is facilitated by the ease of travel and a continuous increase of urbanisation[6,8,9]. Consequently, the international community is increasing its efforts toward the mitigation of the impacts of pandemics[6]. Some recent examples of pandemics are the 2003 SARS (Severe Acute Respiratory Syndrome), the 2014 West Africa Ebola epidemic, and the present COVID-19 (COronaVIrus Disease) pandemic caused by the SARS-CoV-2 virus.

In this paper we formulate a statistical thermodynamic approach to epidemiology, which demonstrates the utility of κ-statistics for the analysis of epidemics and pandemics. First, we must consider certain facts which form the biomedical base of these phenomena. Below we summarise the principle causes of pandemics[6]:

- Pathogens such as influenza viruses that are capable of efficient transmission among humans, with a high potential to cause global and severe pandemics. They are characterized by relatively long and asymptomatic infectious period, during which we are unable to detect the infected persons and their movements.
- Pathogens such as Nipah virus which are capable of generating a moderate global threat. They are transmitted efficiently as a result of mutations and adaptation;.
- Pathogens such as Ebola that are capable of causing local epidemics, with a non-negligible risk of evolving into a global pandemic.

Recently, pandemics have also found their origin in zoonotic transmissions of pathogens from animals to humans[10].

The risk of pandemic spread is conditioned by the following factors[11]:

- pathogen factors such as genetic adaptation and mode of transmission.
- human factors such as population density, susceptibility to infection, travel, migration, poverty, malnutrition, and caloric deficits;
- factors related to public policy such as public health surveillance and measurements.

In order to better predict and mitigate the societal impact, the ability to quantify the morbidity and mortality associated with pandemics is very important. Consequently, statistical approaches based on available data that can lead to accurate models for predicting the behaviour of future pandemics are very useful. Historical data from past pandemics play a fundamental role, because they enable comparisons with theoretical models and can also be used to assess the performance of model-based forecasts. Historical records are often sparse and incomplete. Nonetheless, in all fields of research, scientists and the engineers always search for "optimal" statistical distributions that can reliably predict the behaviour of different natural, engineered, and social systems[12–16].

In this paper we introduce κ-statistics as a useful quantitative tool for analysing epidemiological data and providing probabilistic predictions. We validate the proposed approach by means of mortality data from the Florence plague that occurred in the XV century and the COVID-19 pandemic based on the data available until April 16, 2020. We demonstrate the *physical* basis of the statistics obtained from these outbreaks and briefly describe how the relevant equations are derived.

Analysing data from the plague in Florence in 1417 allows us to test the ability of the proposed approach to forecast infectious outbreaks. The analysis of the plague is of particular interest because the mortality data were carefully recorded. The Chinese data (up to April 16, 2020) are of current interest, because of the ongoing coronavirus outbreak. The Chinese data are partial because China declared a correction on April 17, 2020; however, this first set of data (without correction) yields fundamental information about the universal law of diffusion obeyed by such pandemics (diffusion rate, etc.).

The structure of this paper is as follows. In Section 2 we briefly present several mathematical models used in epidemiology, starting from a statistical mechanics background. Section 3 describes in detail the κ-deformed statistical model. This section introduces the κ-deformed survival function S_κ and other related quantities such as the hazard function λ_κ, the cumulative hazard function Λ_κ and the quantile function Q_κ. In Section 4 we analyse the number of deaths caused by the 1417 plague pandemic in Florence and those caused by the recent COVID-19 outbreak in China until April 16, 2020. In both cases we find good agreement between the available data and the proposed κ-deformed statistical model. Finally, Section 5 presents our conclusions.

Results

An intriguing application of the κ-deformed model described in the next section is the in analysis of the number of deaths during the plague pandemic that ravaged the city of Florence in the XV century.

The Bubonic Plague or Black Death arrived in Europe in 1348 and in Italy in the spring of the same year[17,18]. It should be noted that the plague inspired Boccaccio to write his famous nouvelle *Decameron* only a couple of years after the end of the pandemic. Its consequence was the death of around 25-50% of Europe's population in 1351. The pandemic is believed to have started in China and came to Europe via the trading routes though Asia and the Black Sea. In 1417, Florence had a population of 60,000 inhabitants and the Italian city's Grain Office kept a series of Books of Dead which recorded the number of human deaths caused by the bubonic plague. Florentines were infected after coming in contact with infected black rats and the fleas that they carried[18]. From a thermodynamic point of view, deaths which are due to the spread of an epidemic can be considered as an irreversible process inside an open system. Herein we show how the evolution of the plague in relation to the time and the number of deceased people can be explained using the κ-deformed statistics and its variation in relation to time. In Table 1 the statistical data of the 1417 plague in Florence are summarised as a function of time[19].

We consider the *death occurrence probability* or the pdf f_i (where the index i counts months) as the ratio between the monthly number of deaths (as given by the second column of the Table 1) and the cumulative number of deaths. We assume that the latter is equal to the total number of deaths between May and December plus 100 deaths which we hypothesize that they occurred in the months following December, raising the total number of deaths to $N = 12000$.

Figure 1 depicts the quantile function Q_κ versus the time t for the Florence epidemic. The theoretical quantile function of the κ-deformed model, defined in Eq. (34), is just the bisectrix of the plane represented by the continuous straight line. The empirical quantile function is represented by the set of the dots. The optimal fit parameters have the values: $\kappa = 0.612$, $\alpha = 3.460$ and $\beta = 0.012$. It is clear that the model describes the empirical data very well. Deviations of the empirical quantiles from the straight line are likely due to detection errors, since as it is evidenced in Table 1, the number of monthly deaths is approximated with an error margin of the order of 100 units. This statistical error is responsible for the dispersion of empirical quantiles around the bisectrix.

Figure 2 shows the cumulative distribution function $F_\kappa(t)$ versus time t. The continuous sigmoid curve represents the theoretical prediction of the model as given by Eq. (25), while the dots represent the empirical cdf as deduced from the data in Table 1. The optimal fit of the model parameters are the same as those shown in Figure 1. The sigmoid curve fits accurately the empirical data. We recall that in general $0 < \kappa < 1$ with $\kappa = 0$ corresponding to the standard Weibull model. Hence, the optimal parameter estimate $\kappa = 0.612$ clearly denotes the difference (supported by the data) between the optimal κ-deformed model and the standard Weibull model.

As a second application of the κ-deformed statistical model, we consider the analysis of the number of deaths caused by COVID-19 in China until April 16, 2020. The statistical data are taken from the site https://www.worldometers.info/coronavirus/

Figure 3 reports the quantile function Q_κ versus the time t (measured in days) for the COVID-19 pandemic in China. The best fit parameters have the values: $\kappa = 0.720$, $\alpha = 3.827$ and $\beta = 7.6 \cdot 10^{-7}$. This figure confirms the goodness of the model as there is no systematic deviation of the empirical quantiles from the theoretical linear trend while the statistical nature of data leads to their dispersion around the bisectrix.

Figure 4 displays the cumulative distribution function $F_\kappa(t)$ versus the time t for the Covid-19 death data. The best fit values of the free parameters of the κ-deformed model are the same as in Figure 3. The sigmoid curve of the κ-deformed model is a κ-generalized Weibull curve with $\kappa = 0.720$. Therefore it differs considerably from the standard Weibull curve ($\kappa = 0$) especially in the right tail where $t \to \infty$.

The survival function $S_\kappa(t)$ versus time is shown on a log-log plot in Figure 5. The continuous curve is the theoretical function defined in Eq. (21). One of the most important features is its power-law tail i.e. $S_\kappa(t) \propto t^{-p}$ obtained for $t \to \infty$, which in the log-log plot is evidenced in the almost straight line. For the Chinese pandemic data the value of Pareto's index $p = \alpha/\kappa$ is equal to 5.31.

In Figures 6 and 7, we plot versus time the probability density function $f_\kappa(t)$ and the hazard function $\lambda_\kappa(t)$; these functions are given respectively by Eq. (26) and Eq. (29). The dispersion of the empirical data represented by the dots reflects statistical fluctuations.

Finally, Figure 8 shows the cumulative hazard function $\Lambda_\kappa(t)$ versus time. Once again, the continuous curve is the theoretical model defined in Eq. (32) while the dots represent the respective empirical curve based on the mortality data.

For the purpose of a comparison between the data from Florence and those from China, we recall that the parameters κ and α of the κ-deformed model have very similar values in both cases, since they assume the values $\kappa_F = 0.612$, $\alpha_F = 3.460$ and $\kappa_C = 0.720$, $\alpha_C = 3.827$, respectively.

On the other hand, values of β differ significantly between the two cases. Since β is a dimensional parameter, its value depends on the temporal scale used. In the case of Florence, the unit of measurement of time is "months", while for the Chinese data the unit of measurement is "days". If we use days as a unit of measurement for the Florence epidemic data, this parameter assumes the value of $\beta_F = 9.3 \cdot 10^{-7}$ which is very close to that of the Covid-19 data in China, i.e., $\beta_C = 7.6 \cdot 10^{-7}$. Instead of the parameter β, the dimensional time constant τ defined by means of $\tau_F = \beta^{-1/\alpha}$ can be introduced. This assumes the value of $\tau_F = 55$ days for the data from the Florence plague while for the Covid-19 data from China it leads to $\tau_C = 41$ days.

The last parameter of the model, i.e. the parameter N, representing the cumulative asymptotic value of the deaths, assumes the values $N_F = 12000$ and $N_C = 3366$. Summarizing, the vector of the four parameters $\mathbf{v} = (\kappa, \alpha, \tau, N)$ for Florence and China assumes the values $\mathbf{v}_F = (0.61, 3.46, 55, 12000)$ for the plague data and $\mathbf{v}_C = (0.72, 3.80, 41, 3366)$ for the Covid-19 data, respectively.

Furthermore, it is important to note that the cumulative survival function in both cases presents a power-law tail i.e. $S_\kappa(t) \sim t^{-p}$ with Pareto's index $p = \alpha/\kappa$ having similar values in both cases, namely $p_F = 5.65$ and $p_C = 5.31$.

Discussion

Both the approximate plague data from the plague pandemic in Florence around 600 years ago as well as the partial Covid-19 data from China (collected until April 16, 2020) can be used to obtain information about the spreading dynamics of these two deadly disease outbreaks.

It is worth recalling that the Florence plague data are approximate and relatively sparse, and for these reasons they cannot be used to reliably test a statistical model. However, the Florence data are of great historical importance as they represent the first quantitative and regularly collected record of a pandemic. The first two figures in this paper undoubtedly show that at least the cumulative Florence data are compatible with the proposed κ-deformed statistical model. On the other hand, the Chinese data set on SARS-CoV-2 is important, as it is the first statistical record for the entire evolutionary cycle of the COVID-19 pandemic in the world. The Chinese mortality data, albeit partial, can be used to test the proposed κ-deformed Weibull model and to obtain reliable information on the spreading dynamics of the pandemic.

Most notably, both data sets—even though they are separated by more than 600 years—are successfully described by the same theoretical model, the κ-deformed Weibull. In addition, the estimates of the optimal parameters have similar values for both datasets. These results suggest that the κ-deformed statistical model is sufficiently flexible and physically informed to accurately represent the main statistical functions of epidemic datasets. Furthermore, it is surprising that the linear ODE model (27) equipped with the nonlinear clock can accurately capture the evolution of the survival function, while standard epidemic models such as the SIR model involve nonlinear differential equations[20].

In addition, the fact that both the approximate dataset of the Florence plague and the partial data of the Covid-19 epidemic (until April 16, 2020) in China are well described by means of the proposed model suggests that both datasets faithfully represent the spreading of the epidemics.

Methods

The fundamental difficulty faced by mathematical approaches to epidemiology is that all forecasts strongly depend on the model employed, the parameter estimates, as well as the choices of the initial conditions. There are both deterministic and stochastic epidemiological models[5,21]. The model that we propose below has its roots in statistical mechanics and is therefore stochastic by construction.

The κ-deformed exponential function

Let us consider the function $n(t)$ to represent the number density of deaths at any given time $t \in \mathscr{D}$, where $\mathscr{D}$ is the temporal domain of interest. In most cases of interest $\mathscr{D} = [t_0, \infty)$, where $t_0 \geq 0$. The probability of death within a short time interval δt around t is given by $f(t)\delta t$, where $f(t)$ is the *probability density function (pdf)*. Then, the respective number density is given by $n(t) = N f(t)$, where N is the total number of deaths.

In statistical mechanics a general rate equation for $f(t)$ is the first-order linear ordinary differential equation (ODE)

$$\frac{df(t)}{dt} = -r(t)\, f(t), \tag{1}$$

where the function $r(t)$ is the *decay rate*. The solution of the above ODE is the exponential

$$f(t) = c \exp\left(-\int_{t_0}^{t} r(t')\, dt' \right) , \tag{2}$$

with the standard normalisation condition which determines the constant c:

$$\int_{t_0}^{t} f(t')\, dt' = 1 \ . \tag{3}$$

The normalization also enforces a constraint on the number density, i.e., $N = \int_{t_0}^{\infty} n(t) dt$.

In the context of the exponential solution, the following three simple cases must be considered.

- The Exponential Model: this model is fundamental in every branch of science; indeed, it allows us to describe a great variety of phenomena, from elasticity to electricity, from nuclear decay to thermal transitory, to name a few. The exponential model can be obtained for constant decay rate, i.e.,

  $$r(t) = \beta \ , \tag{4}$$

  which leads to the following exponential pdf:

  $$f(t) = \beta \, \exp(-\beta\, t) \ , \tag{5}$$

  with $\mathscr{D} = (0, +\infty)$.

- Power-law Model (Pareto distribution): Zipf[22–25], in his studies on the size distribution of cities, incomes and word frequencies, pointed out the notion of regularity in the distribution of sizes. In a great variety of these cases the distributions follow a power law with an exponent close to -1, also known as Zipf's Law[26–32]. Such power law distributions have been considered with increasing interest in the description of regular distributions. Distributions with exponents different from -1 are known as Pareto's law[3,30,33]. In general, distributions with power-law tails are known as *heavy-tailed, fat-tailed* or *subexponential* distributions, in juxtaposition to distributions whose tails decay exponentially[2].

  Perline introduced the following classification of power laws[34]:

  - A power law is *strong* if the distribution is a power law over the entire domain of definition;
  - A power law is *weak* if only part of the distribution is fitted by a power law;
  - A power law is *false* if only a highly truncated part of the distribution is approximated by a power law (in the scientific literature there are many examples of false power laws[35]).

  Pareto obtained his *Type I model*, named *the Pareto law* by fitting the available data in his time on increasing social inequalities. He concluded that only economic growth can increase the income of the poor and decrease inequality[36]. Today, we know that his conclusions were partially right and partially wrong, because economic growth and equity are strictly related to the social relations of production, to the technological level and institutional structures of the economy, and to the composition, accumulation and distribution of human capital, as well as the quality and accessibility of the educational and financial structures in place[37]. Moreover, since Pareto disseminated his approach in 1897[38], the application of heavy-tailed distributions in economics has been developed[39].

  The Pareto pdf $f(t)$ is obtained from Eq. (1) by inserting the following time-dependent decay rate function

$$r(t) = \frac{p}{t},\ p > 1\ , \tag{6}$$

  which leads to a power-law solution for the pdf, i.e.,

$$f(t) = \frac{p-1}{t_0}\left(\frac{t_0}{t}\right)^p,\ p > 1\ , \tag{7}$$

  in the domain $\mathscr{D} = (t_0, +\infty)$, with $t_0 > 0$:

- κ-Exponential Model: This model, which is based on a fundamental approach derived from relativity[40], has proved useful in many applications. Experimental evidence suggests that probability density functions should resemble the exponential function for $t \to 0$. However, for $t \to 0$ the Pareto pdf diverges. On the other hand, for high values of t many experimental results show a Pareto-like pdf with power law tails instead of exponential decay. Consequently, for $t \to 0$ it follows that $r(t) \sim \beta$ while for $t \to +\infty$ it follows $r(t) \sim p/t$. So, the actual decay rate function $r(t)$ should smoothly interpolate between these two regimes; a good proposal for $r(t)$ has been introduced in the context of special relativity, where the function $r(t)$ is given in terms of the Lorentz factor. We recall the expression of the Lorentz factor $\gamma_\kappa(q) = \sqrt{1+\kappa^2 q^2}$; this expression involves the dimensionless momentum q where the parameter κ is the reciprocal of the dimensionless light speed c, i.e. $\kappa \propto 1/c$. After posing $r(t) = \beta/\gamma_\kappa(\beta t)$ or more explicitly

$$r(t) = \frac{\beta}{\sqrt{1+\kappa^2\beta^2 t^2}}\ , \tag{8}$$

  it follows that for $t \to 0$ the decay rate $r(t)$ approaches the exponential regime, i.e. $r(t) \sim \beta$. On the other hand, for $t \to +\infty$ it follows that $r(t)$ approaches the decay rate of the Pareto model, i.e. $r(t) \sim 1/\kappa t$.

  The solution of the rate equation in this case yields the following pdf

$$f(t) = (1-\kappa^2)\,\beta\,\exp_\kappa(-\beta t)\ , \tag{9}$$

  where the κ-deformed exponential function is given by

$$\exp_\kappa(t) = \left(\sqrt{1+\kappa^2 t^2} + \kappa t\right)^{1/\kappa}\ , \tag{10}$$

with $0 < \kappa < 1$. It is important to note that in the $\kappa \to 0$ limit or alternatively in the $t \to 0$ limit the function $\exp_\kappa(t)$ approaches the ordinary exponential $\exp(t)$, i.e.

$$\exp_\kappa(t) \underset{\kappa\to 0}{\sim} \exp(t) \ , \tag{11}$$

$$\exp_\kappa(t) \underset{t\to 0}{\sim} \exp(t). \tag{12}$$

On the other hand the function $\exp_\kappa(-t)$ for $t \to +\infty$ presents a power-law tail, i.e.

$$\exp_\kappa(-t) \underset{t\to +\infty}{\sim} (2\kappa t)^{-1/\kappa}. \tag{13}$$

Furthermore, the κ-exponential satisfies the following identity

$$\exp_\kappa(t) \exp_\kappa(-t) = 1 \ , \tag{14}$$

in analogy with the standard, non-deformed, exponential.

The κ-exponential represents a very powerful tool which can be used to formulate a generalized statistical theory capable of treating systems described by distribution functions that exhibit power-law tails[40–48]. The mechanism generating the κ-exponential function is based on first principles from special relativity, and therefore the new function appears very promising for physical applications. Generalized statistical mechanics, based on the κ-exponential, preserves the main features of ordinary Boltzmann-Gibbs statistical mechanics which is based on the ordinary exponential through the Boltzmann factor. For this reason, it has attracted the interest of many researchers over the last two decades who have studied its foundations and mathematical aspects[49–58], the underlying thermodynamics[59–65], and specific applications of the theory to various fields. A non-exhaustive list of applications includes, among others, those in quantum statistics[66–69], in quantum entanglement[70,71], in plasma physics[72–78], in nuclear fission[79–82], in astrophysics[83–86], in quantum gravity[87–92], in geomechanics[93,94], in genomics[95,96], in complex networks[97–99], in economy[100–104], in finance[105–108], as well as in reliability analysis and seismology[109–111].

The κ-deformed statistical model

Given a pdf $f(t)$ which represents the death rate, the *cumulative distribution function (cdf)* $F(t) : \mathscr{D} \to [0,1]$ represents the probability of death between the initial time and the current time t. $F(t)$, which is also known as the *lifetime distribution*, is given by means of the following integral $F(t) = \int_{t_0}^{t} f(t')dt'$.

Conversely, the pdf $f(t)$ is given by the derivative of $F(t)$. In the following, we will assume without loss of generality that $t_0 = 0$. The complement of $F(t)$, i.e., $S(t) = 1 - F(t)$ is known as the *survival function*, and it represents the probability of survival at time t.

In the most models of population dynamics the following time-dependent monomial

$$T(t) = \beta t^\alpha \ . \tag{15}$$

The expression (15) for T contains the real-valued parameters $\alpha > 0$ and $\beta > 0$. We can think of $T(t)$ as the time measured by a *nonlinear clock*[112]. $T(t)$ is regularly used in the definition of the survival function $S(t)$ which becomes an implicit function of time based on the dependence of S on $T = T(t)$, i.e. $S = S(T)$.

The survival function of the *Weibull model* is then defined according to

$$S = \exp(-T) \ , \tag{16}$$

and it represents a stretched exponential function in time.

Popular models for empirical data that exhibit power-law tails include the Log-Logistic model[113], with survival function given by

$$S = \frac{1}{1+T} \ , \tag{17}$$

the Burr type XII or Singh-Maddala model[114], with survival function given by

$$S = \frac{1}{(1+T)^p}, \quad p > 0 \ , \tag{18}$$

and the Dagum model[115], with survival function given by

$$S = 1 - \frac{T^p}{(1+T)^p}, \quad p > 0\,. \tag{19}$$

Various other postulates can be used for the analytical expression of the survival function. However, for physical applications it is extremely important to identify physical mechanisms or first principles which lead to such expressions for the survival function.

The κ-deformed statistical model can be viewed as a one-parameter generalization of the Weibull model, obtained by replacing the ordinary exponential $\exp(t)$ in the definition (16) of the survival function by the κ-deformed exponential $\exp_\kappa(t)$. Then, using the Weibull dependence for T given by Eq. (15), we obtain the following expression for the κ-deformed survival function $S_\kappa = S_\kappa(t)$

$$S_\kappa = \exp_\kappa(-T)\,, \tag{20}$$

or more explicitly

$$S_\kappa(t) = \exp_\kappa(-\beta\, t^\alpha),\ \alpha > 0, \beta > 0\,. \tag{21}$$

The κ-deformed survival function $S_\kappa(t)$ reduces to the ordinary survival function of the Weibull model in the $\kappa \to 0$ limit. The rate equation obeyed by S_κ is expressed in terms of T as a linear first-order ODE

$$\frac{dS_\kappa}{dT} = -\frac{1}{\sqrt{1+\kappa^2\, T^2}}\, S_\kappa\,, \tag{22}$$

with initial condition $S_\kappa(0) = 1$. The ODE (22) represents an interpolation between the rate equation of the exponential model and that of the Pareto model; moreover, the functional form in Eq. (22) is dictated by the first principles of special relativity.

The most important feature of the function $S_\kappa(t)$ is that it continuously interpolates between a power-law tail for large $t \gg 1$, i.e.,

$$S_\kappa(t) \sim (2\,\kappa\beta)^{-1/\kappa}\, t^{-\alpha/\kappa}\,, \tag{23}$$

and exponential dependence

$$S_\kappa(t) \sim \exp(-\beta\, t^\alpha)\,, \tag{24}$$

for $t \ll 1$.

The lifetime distribution function $F_\kappa = F_\kappa(t)$ is given by the expression

$$F_\kappa(t) = 1 - \exp_\kappa(-\beta\, t^\alpha)\,, \tag{25}$$

while the pdf $f_\kappa = f_\kappa(t)$, defined $f_\kappa = dF_\kappa/dt$, is given by

$$f_\kappa(t) = \frac{\alpha\,\beta\, t^{\alpha-1}}{\sqrt{1+\kappa^2\,\beta^2\, t^{2\alpha}}}\, \exp_\kappa(-\beta\, t^\alpha)\,. \tag{26}$$

The rate equation for the survival function assumes the form of the first-order linear ODE

$$\frac{dS_\kappa(t)}{dt} = -\lambda_\kappa(t)\, S_\kappa(t)\,, \tag{27}$$

where $S_\kappa(0) = 1$, and $\lambda_\kappa = \lambda_\kappa(t)$ is the *the hazard function (hazard rate)* defined through

$$f_\kappa(t) = \lambda_\kappa(t)\, S_\kappa(t). \tag{28}$$

Hence, the hazard function assumes the expression

$$\lambda_\kappa(t) = \frac{\alpha\,\beta\, t^{\alpha-1}}{\sqrt{1+\kappa^2\,\beta^2\, t^{2\alpha}}}\,. \tag{29}$$

The cumulative hazard function $\Lambda_\kappa = \Lambda_\kappa(t)$ is defined by means of the integral

$$\Lambda_\kappa(t) = \int_0^t \lambda_\kappa(u)\,du \ , \tag{30}$$

and is linked with S_κ through

$$\Lambda_\kappa(t) = -\ln S_\kappa(t) \ . \tag{31}$$

After taking into account that the κ-exponential function (10) can also be expressed in the form

$$\exp_\kappa(x) = \exp\left(\frac{1}{\kappa}(\mathrm{arcsinh}(\kappa x)\right) \ , \tag{32}$$

the cumulative hazard function $\Lambda_\kappa = \Lambda_\kappa(t)$ assumes the following explicit expression

$$\Lambda_\kappa(t) = \frac{1}{\kappa}\,\mathrm{arcsinh}\,(\kappa\beta\,t^\alpha) \ , \tag{33}$$

and in the $\kappa \to 0$ limits reduces to the standard Weibull cumulative hazard function $\Lambda(t) = \beta\,t^\alpha$. It is important to note that in the standard Weibull model the cumulative hazard function coincides with the function $T(t)$.

Finally, the *quantile function* Q_κ is defined as the inverse of the survival function follows $S_\kappa = S_\kappa(t)$. By expressing the quantile function in the form $t = Q_\kappa(S_\kappa)$, one easily obtains

$$Q_\kappa(u) = \left(-\frac{1}{\beta}\,\ln_\kappa(u)\right)^{1/\alpha} \ , \tag{34}$$

where the κ-logarithm $\ln_\kappa(u)$ is the inverse function of $\exp_\kappa(u)$, i.e. $\ln_\kappa(\exp_\kappa(u)) = \exp_\kappa(\ln_\kappa(u)) = u$ and is given by

$$\ln_\kappa(u) = \frac{u^\kappa - u^{-\kappa}}{2\,\kappa} \ . \tag{35}$$

After observing that the function $\ln_\kappa(u)$ approaches the function $\ln(u)$ in the $\kappa \to 0$ limit, it follows that in the same limit $Q_\kappa(u)$ reduces to the quantile function of the standard Weibull model.

The κ-deformed statistical model presented above, which represents a one-parameter continuous deformation of the Weibull model, has been successfully applied in econophysics for the analysis of personal income distribution[100] and in seismology[109].

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Author contributions statement

G.K. developed the statistical model and U.L. conceived the idea to adopt the model in the analysis of pandemic data. All authors provided critical feedback and helped shape the research, analysis and manuscript.

Additional information

Competing interests: The authors declare no competing interests.
The corresponding author is responsible for submitting a competing interests statement on behalf of all authors of the paper.

Month	Dead people	Population of Florence
May	600	59400
June	700	58700
July	2700	56000
August	5000	51000
September	2000	49000
October	600	48400
November	200	48200
December	100	48100

Table 1. Temporal distribution of plague victims and population in Florence during the year 1417[19]

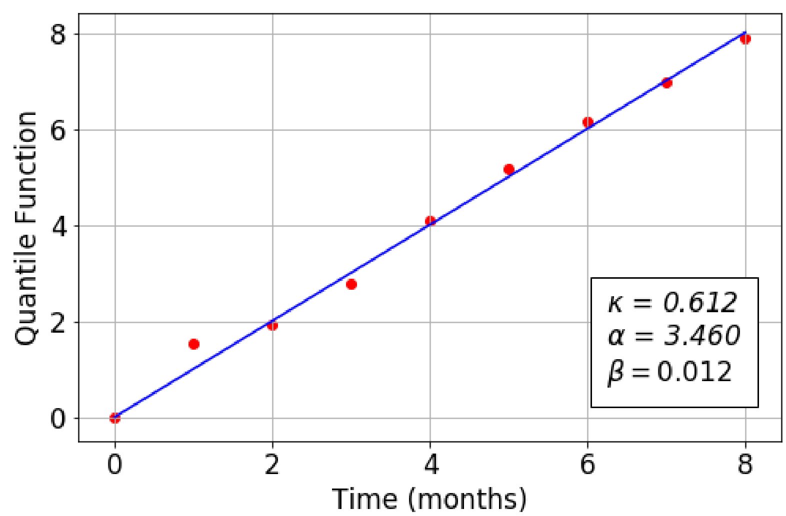

Figure 1. Theoretical (continuous curve) and empirical (dots) of the quantile function versus time for the 1417 Florence plague epidemic. The theoretical curve is based on Eq. (35) for the κ-deformed statistical model.

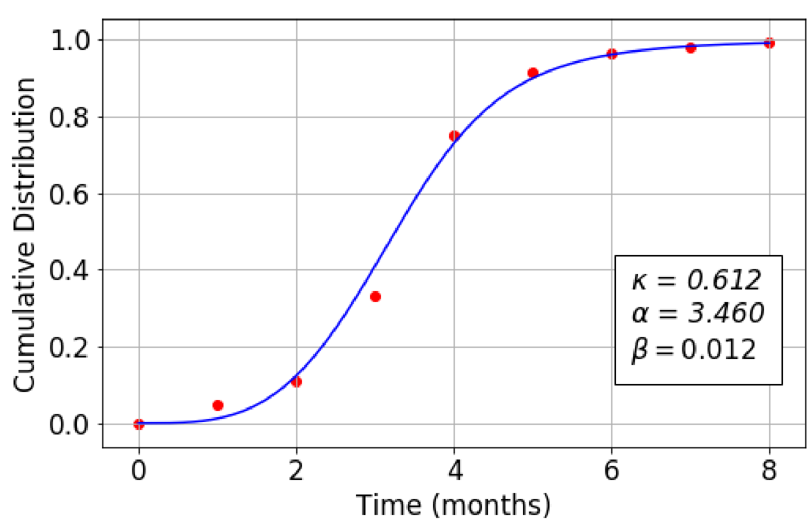

Figure 2. Theoretical (continuous curve) and empirical (dots) cumulative distribution function versus time for the 1417 Florence plague epidemic. The theoretical curve is based on Eq. (25) for the κ-deformed statistical model.

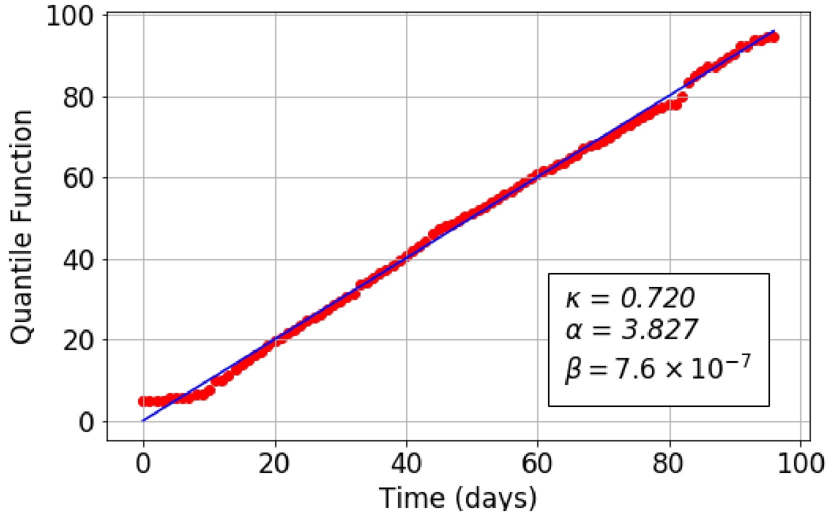

Figure 3. Theoretical (continuous curve) and empirical (dots) quantile function versus time for the Covid-19 mortality data in China. The theoretical curve is based on Eq. (35) for the κ-deformed statistical model.

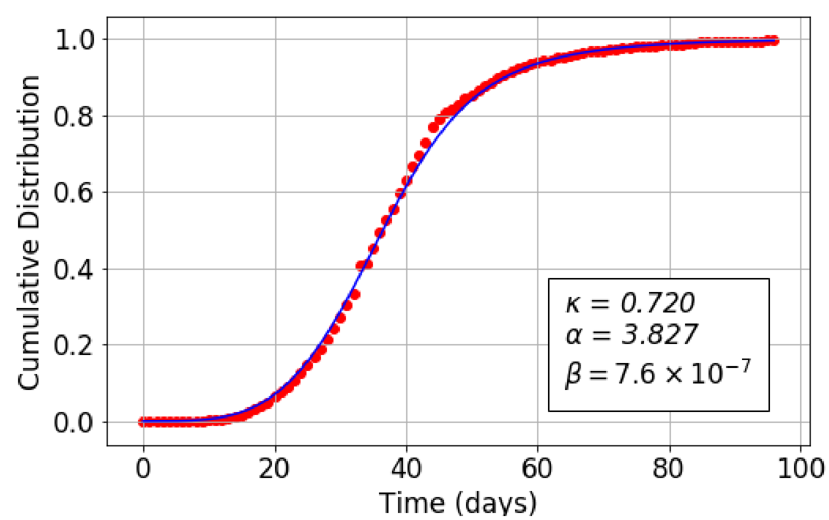

Figure 4. Theoretical (continuous curve) and empirical (dots) cumulative function versus time for the Covid-19 mortality data in China. The theoretical curve is based on Eq. (25) for the κ-deformed statistical model.

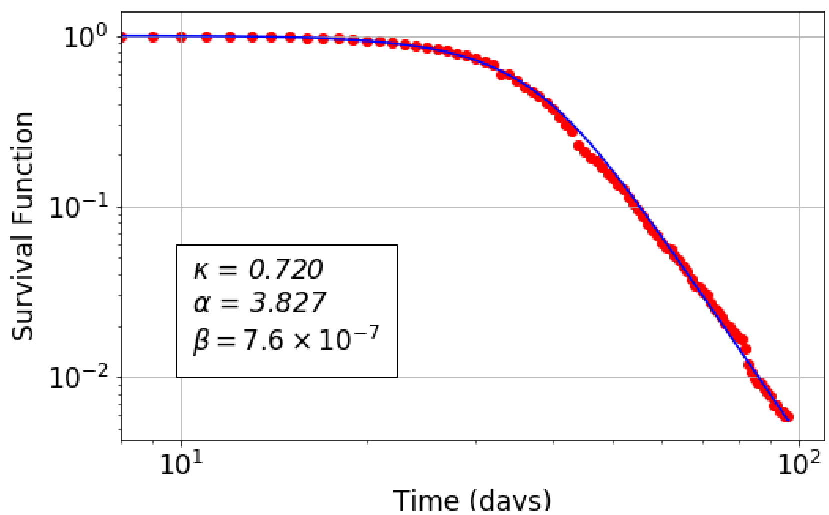

Figure 5. Theoretical (continuous curve) and empirical (dots) survival function of versus time related to the Covid-19 China's pandemic data.

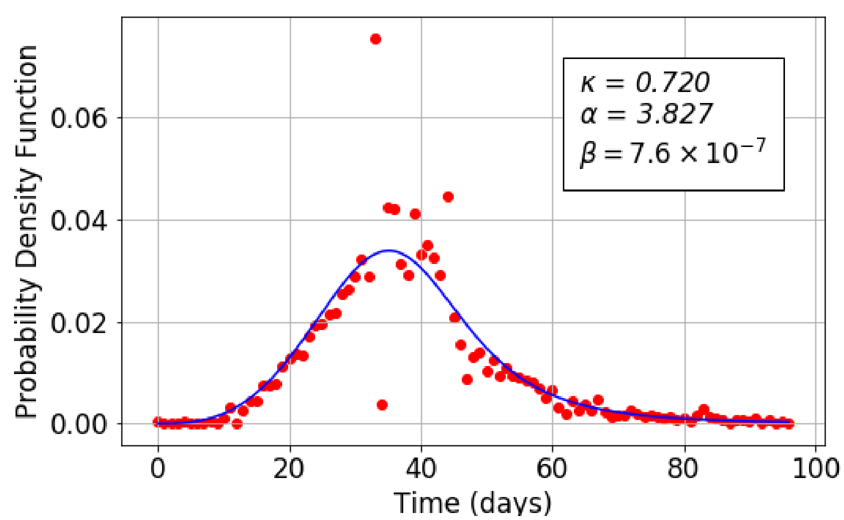

Figure 6. Theoretical (continuous curve) and empirical (dots) probability density function versus time related to the Covid-19 data from China.

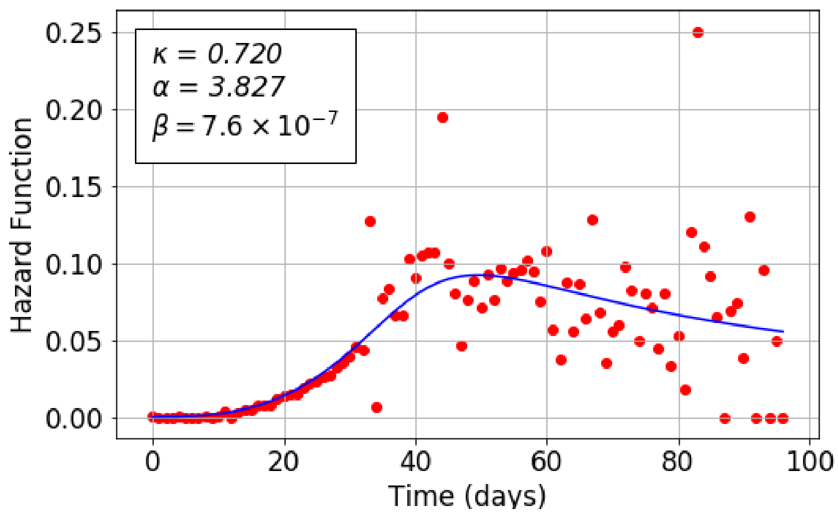

Figure 7. Theoretical (continuous curve) and empirical (dots) hazard function versus time related to the Covid-19 data.

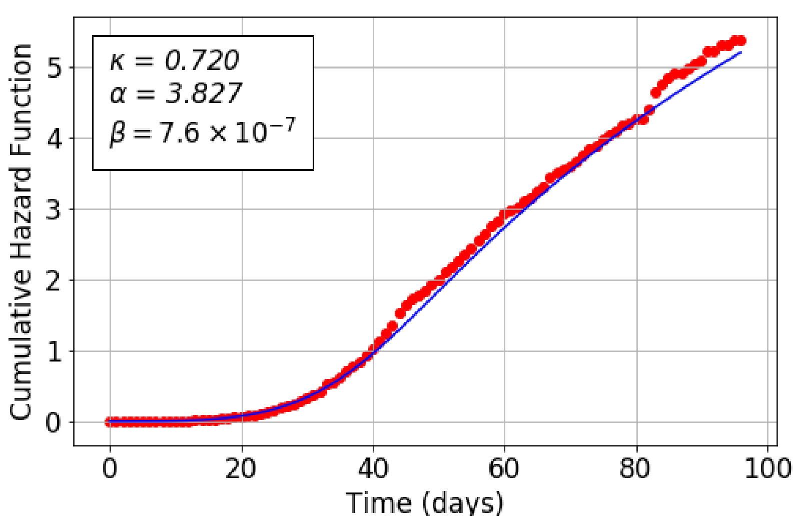

Figure 8. Theoretical (continuous curve) and empirical (dots) cumulative hazard function versus time related to the Covid-19 data from China.