

Fault Compensation Control of MIMO Nonlinear Systems Subject to Unknown Control Directions

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Fault compensation control of MIMO nonlinear systems subject to unknown control directions

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Abstract This study is primarily focused on the issue of low-complexity prescribed performance fault compensation for multi-input and multi-output (MIMO) uncertain nonlinear systems with actuator failures, coupling states, and unknown control directions. First of all, proper logarithm-type error conversion functions and smooth orientation functions are linked to design the continuous control signals in the state feedback controller. Then, based on the idea of proof by contradiction, it is shown that the state errors converge to a predictable compact aggregate at the definite rate. Meanwhile, the boundedness of any closed-loop signals might be guaranteed. The simulation example results are delivered to demonstrate the effectiveness of the developed control strategy at last.

Keywords MIMO nonlinear systems · Unknown control directions · Fault-tolerant control (FTC) · Prescribed performance control (PPC)

1 Introduction

With the rapid development of modern industrial technology, composite engineering systems with uncertain nonlinearities, strong state couplings, actuator failures, and external disturbances are more and more widely used. Because of these difficulties, numerous research findings are committed to the single-input and single-output (SISO) nonlinear systems like [1,2]. It is worth noting that lots of actual industrial systems contain multiple inputs and outputs, such as flexible multi-link manipulators [3,4], satellite [5,6], and permanent magnet synchronous motor [7,8]. Hence, for the past few years, a large number of scholars have been working on the robust controller design of MIMO uncertain nonlinear systems. In [9,10], regarding some systems with unknown control directions, the Nussbaum gain technique and control laws were merged into the controller model. However, as Oliveira et al. pointed out in [11–13], the practical value of this method was controversial owing to the inherent lack of robustness, large fluctuation peaks, and poor transients. To avoid these matters, adaptive control mechanisms were applied to observe the unknown parameters by referring to [14–18]. Their shortcoming was that the real-time updating of online learning parameters greatly increased the computational complication of the control algorithm, which made the control machine difficult to be implemented in engineering. Furthermore, see [19–23] and reference therein, the implementation of backstepping method needs to calculate the derivatives of virtual con-

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trol rate, it is quite complicated and easy to cause relatively large errors.

On the one hand, in order to dodge the trouble of virtual control mechanisms, more approximation techniques were adopted. In [24], a neural network adaptive robust control (NNARC) method was proposed. It exploited the multi-layer neural network to approximate the unknown nonlinear term. Further, better model compensation and improved performance were obtained. With the help of fuzzy adaptive observer making up for unmeasured states, the authors of [25] transformed the tracking errors into new virtual error variables, a fuzzy adaptive output-feedback control method was offered. On the other hand, the computational complexity has been reduced by adopting dynamic surface control (DSC) technology. In [26], a first-order filter was introduced which overcame the phenomenon of parameter expansion caused by traditional backstepping. Besides, the assumption of the fault boundary was eliminated. From this foundation, reference [27] considered the actuator failures of MIMO uncertain systems. In addition, DSC was applied to FTC, then a fuzzy fault-tolerant control (FFTC) mechanism was raised. In [28], the authors derived a distributed FTC scheme by combining neural network, backstepping, DSC, and algebraic graph theory. It was usually used to settle the puzzle of output tracking control based on pure feedback multi-agents. The intelligent approximate technique and DSC method could effectively handle with tracking control problem of uncertain nonlinear systems during the backstepping process. However, there still exists a relatively complex recursive design with adaptation mechanism or parameter updated laws. Also, the corresponding stability analysis results are only confined in the local compact sets.

Although there have been a lot of improvements, none of the above approaches are able to specify transient performance, only the convergence of tracking error to residual set is established. Exactly, the involvement of PPC in [29–31] compensated for this deficiency. Because it can not only cause the tracking errors converge to a predictable compact set, but also ensure that the convergence rate is not less than a predetermined value. Correspondingly, just as Theodorakopoulos analyzed in [32], the simplification in structure and calculation still made PPC more widely used. For example, [33] focused on solving PPC of SISO strict-feedback nonlinear systems. The error transformation functions were associated with the specific Nussbaum-type functions. Additionally, a trigonometric type error transformation function was come up in [34, 35].

Inspired by the above discussions, this paper studies the FTC and prescribed performance control problems

for the type of MIMO uncertain nonlinear systems with actuator failures and unknown control directions. For the research results given, the major contributions of this paper are summarized as below:

1) Here we consider the more general uncertain MIMO nonlinear systems with actuator failures, it is worth mentioning that [35] and [36] are special cases of this paper.

2) Different from [33–35], by introducing the idea of inverse hyperbolic tangent function, an appropriate logarithm-type of error conversion function cooperated with directional function is proposed to realize PPC.

3) By the programming of the control form, neither the approximation strategies in [24, 25] nor the higher-order derivatives of reference signals in [36–38] are used. As a result, the controller is simpler than comparable results in structure and calculation. Besides, the adaptive parameters are not any more needed to be updated compared with [15, 16], which both reduce the computational cost, and ensure that the state error converges to an arbitrarily small residual set at a definite rate not less than the predetermined value.

The remaining part of this paper is organized as follows: In Section 2, the problem statement and several related assumptions are introduced. Meanwhile, a design procedure of the controller is given. Section 3 presents the control process and stability analysis. The numerical simulation results are provided in Section 4. In the end, Section 5 draws the conclusions.

2 Problem Formulation and Preliminaries

Take the following class of MIMO uncertain nonlinear systems with actuator failures and unknown control directions into account:

$$\begin{aligned} \dot{x}_{i,1} &= f_{i,1}(\underline{x}_{i,1}) + g_{i,1}(\underline{x}_{i,1})x_{i,2} + d_{i,1}(t) \\ \dot{x}_{i,2} &= f_{i,2}(\underline{x}_{i,2}) + g_{i,2}(\underline{x}_{i,2})x_{i,3} + d_{i,2}(t) \\ &\vdots \\ \dot{x}_{i,n_i-1} &= f_{i,n_i-1}(\underline{x}_{i,n_i-1}) + g_{i,n_i-1}(\underline{x}_{i,n_i-1})x_{i,n_i} \\ &\quad + d_{i,n_i-1}(t) \\ \dot{x}_{i,n_i} &= f_{i,n_i}(x) + g_{i,n_i}(x)u_{i,f} + d_{i,n_i}(t) \\ y_i &= x_{i,1} \end{aligned} \quad (1)$$

where $\underline{x}_{i,j} = [x_{i,1}, x_{i,2}, \dots, x_{i,j}]^T \in \mathbb{R}^j$, $j = 1, 2, \dots, n_i$, $i = 1, 2, \dots, m$ and $x = [x_{1,n_1}^T, x_{2,n_2}^T, \dots, x_{m,n_m}^T]^T \in \mathbb{R}^{\sum_{i=1}^m n_i}$ are the state variables of the systems; $u_{i,f} \in \mathbb{R}$ and $y_i \in \mathbb{R}$ denote respectively the system control input and output; both $f_{i,j}(\cdot) : \mathbb{R}^i \rightarrow \mathbb{R}$ and $g_{i,j}(\cdot) : \mathbb{R}^i \rightarrow \mathbb{R}$ are the unknown smooth nonlinear functions;

furthermore, the perturbation term $d_{i,j}(t) \in \mathbb{R}$ is piecewise continuous in t .

The sign of $g_{i,j}(\cdot)$ plays the role of control direction. The actual control input $u_{i,f} \in \mathbb{R}$ describes as follow:

$$u_{i,f} = \lambda_i(t)u_i(t) + \delta_i(t) \quad (2)$$

where $u_i(t)$ is a control input to be designed, $\lambda_i(t)$ and $\delta_i(t)$ are the time-varying partial loss of effectiveness and the float faults.

Assumption 1(See [33], [34]): There exist unknown positive constants $\underline{\lambda}_i$, $\bar{\delta}_i$, and \bar{d}_i , such that

$$\underline{\lambda}_i \leq \lambda_i(t) \leq 1, \quad |\delta_i(t)| \leq \bar{\delta}_i, \quad |d_{i,j}(t)| < \bar{d}_i \quad (3)$$

for $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n_i$.

Assumption 2(See [35]): There exists the unknown constant $g_i > 0$, satisfies that $g_i \leq |g_{i,j}(\cdot)|$, for $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n_i$.

Assumption 3(See [39], [40]): Both reference signal $r_i(t)$ and its first-order derivative $\dot{r}_i(t)$ keep bounded, for $i = 1, 2, \dots, m$.

Remark 1 For the systems with unknown control direction in [41] and [42], it is assumed that $g_{i,1}(\underline{x}_{i,1}) = g_{i,2}(\underline{x}_{i,2}) = \dots = g_{i,n_i-1}(\underline{x}_{i,n_i-1}) = 1$, $i = 1, 2, \dots, m$ and $g_{i,n_i}(x) = g_{n_i}$ with $g_{n_i} \neq 0$ being a constant, whereas all the control direction functions $g_{i,j}(\cdot)$, $j = 1, 2, \dots, n_i$ are allowed to be unknown in this paper. In addition, more universalities on given system comparing with [34] and [36]: 1) just constrain $|g_{i,j}(\cdot)|$ to have a lower bound; 2) the unknown nonlinear function $f_{i,j}(\cdot)$ can be any smooth function, it does not require boundaries.

Remark 2 As mentioned in Assumption 3, the target trajectory and its first-order derivative are bounded that mean fewer restrictions than [36] and [37]. Because the controller design strategy combines error transformation and smooth orientation functions, it is no longer needed to obtain higher derivatives of $r_i(t)$, $i = 1, 2, \dots, m$.

The ultimate control objectives are to design the continuous virtual control signals and control law to achieve:

1) the closed-loop system (1) is globally stable in this sense that each reference signal is uniformly ultimately bounded;

2) state errors are limited within prescribe performance functions;

3) control outputs of the system can track the given reference signals $r_i(t)$, $i = 1, 2, \dots, m$ as close as possible.

2.1 Controller Design

In this subsection, the state feedback controller is contrived to make the above-mentioned control objectives come true. First of all, define the state errors as

$$\begin{aligned} e_{i,1} &= y_i - r_i(t), \quad i = 1, 2, \dots, m \\ e_{i,j+1} &= x_{i,j+1} - \alpha_{i,j}, \quad j = 1, \dots, n_i - 1 \end{aligned} \quad (4)$$

where $r_i(t)$ is the user-defined tracking trajectory and $\alpha_{i,j}$ expresses the continuous virtual control signal which will be defined later.

The prescribed performance functions $q_{i,j}(t)$ were selected to restrict state errors, so that

$$q_{i,j}(t) = (q_{i,j}^{(0)} - q_{i,j}^{(\infty)})e^{-\mu_{i,j}t} + q_{i,j}^{(\infty)} \quad (5)$$

where set up $0 < q_{i,j}^{(\infty)} \leq q_{i,j}(t) \leq q_{i,j}^{(0)}$, with $q_{i,j}^{(0)} > q_{i,j}^{(\infty)} > 0$ and $\mu_{i,j} > 0$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n_i$.

For $\forall t \geq 0$, define error conversion function as

$$\xi_{i,j} = \ln \left(\frac{q_{i,j} + e_{i,j}}{q_{i,j} - e_{i,j}} \right) \quad (6)$$

where $e_{i,j}$ and $q_{i,j}$ meet the above definitions (4) and (5).

Remark 3 The design inspiration of the error conversion function (6) comes from the reverse hyperbolic tangent function $\tanh^{-1}(x) = \frac{1}{2} \ln \left(\frac{1+x}{1-x} \right)$ by referring to [39] and [40]. Owing to the logarithmic function limits the domain, there will be $|x| < 1 - \varepsilon$, $0 < \varepsilon < 1$ hold. For the same reason, there exists a constant $0 < c_{i,j} < q_{i,j}^{(\infty)}$ that guarantees $|e_{i,j}| < q_{i,j} - c_{i,j}$.

Next, combine the error conversion function $\xi_{i,j}$ with the smooth orientation function $N_{i,j}(\cdot)$, the continuous virtual control messages and control law are described as

$$\begin{aligned} \alpha_{i,j} &= \gamma_{i,j} N_{i,j}(\xi_{i,j}) \xi_{i,j}, \quad j = 1, 2, \dots, n_i \\ u_i(t) &= \alpha_{i,n_i}, \quad i = 1, 2, \dots, m, \end{aligned} \quad (7)$$

where $\gamma_{i,j}$ represents the positive control gain. Meanwhile, the function $N_{i,j}(\xi_{i,j})$ satisfies that

$$\limsup_{\xi_{i,j} \rightarrow \infty} N_{i,j}(\xi_{i,j}) = a_{i,j}, \quad \liminf_{\xi_{i,j} \rightarrow \infty} N_{i,j}(\xi_{i,j}) = -b_{i,j} \quad (8)$$

with $a_{i,j}$ and $b_{i,j}$ being two positive constants. For example, $N_{i,j}(\xi_{i,j}) = \arctan(\xi_{i,j})$.

Remark 4 As we all know, for the logarithmic functions, the function value approaches minus infinity as the independent variable verges to the original point; the function value approaches positive infinity as the independent variable verges to the positive infinity. Therefore, from equations (6)-(8), thinking about the state

error $e_{i,j}$ could be positive or negative, an important conclusion is drawn as

$$\begin{aligned} \liminf_{(e_{i,j}+q_{i,j}) \rightarrow 0^+} \alpha_{i,j} &= -\infty, & \limsup_{(e_{i,j}+q_{i,j}) \rightarrow 0^+} \alpha_{i,j} &= +\infty \\ \liminf_{(e_{i,j}-q_{i,j}) \rightarrow 0^-} \alpha_{i,j} &= -\infty, & \limsup_{(e_{i,j}-q_{i,j}) \rightarrow 0^-} \alpha_{i,j} &= +\infty \end{aligned} \quad (9)$$

where $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n_i$.

3 Stability Analysis

In this section, a sufficient cause for the derivative bounded of the continuous virtual control signal is given by proving Lemma 1. Furthermore, employ the usual proof by contradiction in mathematics: 1) demonstrate that all the closed-loop signals maintain bounded; 2) the control output can track the target trajectory by means of the proof process of Theorem 1.

Lemma 1 For each $j \in \{1, \dots, n_i - 1\}$, the derivative of the continuous virtual control signals $\dot{\alpha}_{i,j}$, $i = 1, 2, \dots, m$ are bounded when $e_{i,j}$, $\dot{e}_{i,j}$ and $\xi_{i,j}$ remain bounded.

Proof: From equation (7), the derivative of the continuous virtual control signal can be obtained

$$\begin{aligned} \dot{\alpha}_{i,j} &= \gamma_{i,j} \frac{\partial N_{i,j}}{\partial \xi_{i,j}} \dot{\xi}_{i,j} \xi_{i,j} + \gamma_{i,j} N_{i,j}(\xi_{i,j}) \dot{\xi}_{i,j} \\ &= \gamma_{i,j} \dot{\xi}_{i,j} \left(\frac{\partial N_{i,j}}{\partial \xi_{i,j}} \xi_{i,j} + N_{i,j}(\xi_{i,j}) \right). \end{aligned}$$

Differentiating (6) leads to

$$\dot{\xi}_{i,j} = \frac{2}{q_{i,j}^2} \frac{\dot{e}_{i,j} q_{i,j} - e_{i,j} \dot{q}_{i,j}}{1 - \left(\frac{e_{i,j}}{q_{i,j}}\right)^2}.$$

By hypothesis $\xi_{i,j} = \ln \left(\frac{q_{i,j} + e_{i,j}}{q_{i,j} - e_{i,j}} \right) \in L^\infty$ that $\left| \frac{e_{i,j}}{q_{i,j}} \right| \in L^\infty$. According to (5), we have

$$\frac{1}{q_{i,j}^0} \leq \frac{1}{q_{i,j}(t)} \leq \frac{1}{q_{i,j}^\infty}, \quad \mu_{i,j} (q_{i,j}^\infty - q_{i,j}^0) \leq \dot{q}_{i,j} \leq 0, \quad (10)$$

$$i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n_i.$$

To sum up, $q_{i,j}$, $\dot{q}_{i,j}$ and $1/q_{i,j}$ are bounded, then $\dot{\xi}_{i,j} \in L^\infty$. Every component of $\dot{\alpha}_{i,j}$ is bounded, Lemma 1 holds. A complete proof of Lemma 1.

To limit the state errors to the prescribed performance bounds in the form of

$$|e_{i,j}(t)| < q_{i,j}(t), \quad t \geq 0, \quad (11)$$

$$i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n_i.$$

By looking for the opposite condition, there exists at least one state error variable $e_{i,j}^{(k)}(t_l)$, it follows that

$$|e_{i,j}^{(k)}(t_l)| \geq q_{i,j}^{(k)}(t_l), \quad k \in \{1, 2, \dots, n_i\}, \quad l \in \mathbb{Z}^+ \quad (12)$$

where $t_l < t_{l+1}$. Suppose that t_1 represents the first moment that (11) is violated.

Therefore, when $t \in [0, t_1)$, one obtains

$$|e_{i,j}(t)| < q_{i,j}(t), \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n_i. \quad (13)$$

It means that $e_{i,1}, e_{i,2}, \dots, e_{i,n_i}$, $i = 1, 2, \dots, m$ are bounded over $[0, t_1)$. In view of this, a state error variable $e_{i,j}^{(k)}(t)$ satisfying

$$\lim_{t \rightarrow t_1^-} |e_{i,j}^{(k)}(t)| = q_{i,j}^{(k)}(t_1), \quad k \in \{1, 2, \dots, n_i\} \quad (14)$$

where t_1^- is the left limit of t_1 .

As the state error $e_{i,j}$ close to its prescribed performance limits $q_{i,j}$ in two directions, it naturally holds that

$$\lim_{(e_{i,j}-q_{i,j}) \rightarrow 0^-} \dot{e}_{i,j} \geq \dot{q}_{i,j}, \quad \lim_{(e_{i,j}+q_{i,j}) \rightarrow 0^+} \dot{e}_{i,j} \leq -\dot{q}_{i,j}. \quad (15)$$

In the meantime, the derivative of $q_{i,j}$ can be found by (5). Rewrite (15) as follows

$$\lim_{(e_{i,j}-q_{i,j}) \rightarrow 0^-} \dot{e}_{i,j} \geq h_{i,j}, \quad \lim_{(e_{i,j}+q_{i,j}) \rightarrow 0^+} \dot{e}_{i,j} \leq -h_{i,j} \quad (16)$$

where $h_{i,j} = \mu_{i,j} (q_{i,j}^{(\infty)} - q_{i,j}^{(0)})$ with $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n_i$.

Theorem 1 Discuss one type of MIMO uncertain nonlinear systems (1) with actuator failures and unknown control directions, compatible with Assumptions 1 and 2. Given initial condition $|e_{i,j}(0)| < q_{i,j}(0)$, and the target track obeys aforementioned Assumption 3, the advanced state-feedback control strategy (7) fulfills the following conditions:

- 1) the system control output could track the reference signal with the prescribed performance;
- 2) the whole signals of the closed-loop system remain bounded for $t \geq 0$.

Proof: Substituting (1), (2), and (7) into (4) yields

$$\begin{aligned} \dot{e}_{i,j} &= \phi_{i,j} + g_{i,j}(\underline{x}_{i,j}) \alpha_{i,j}, \quad j = 1, 2, \dots, n_i - 1 \\ \dot{e}_{i,n_i} &= \phi_{i,n_i} + g_{i,n_i}(x) \lambda_i(t) \alpha_{i,n_i}, \quad i = 1, 2, \dots, m \end{aligned} \quad (17)$$

where $\phi_{i,1} = f_{i,1}(\underline{x}_{i,1}) + g_{i,1}(\underline{x}_{i,1}) e_{i,2} + d_{i,1}(t) - \dot{r}_i(t)$, $\phi_{i,j} = f_{i,j}(\underline{x}_{i,j}) + g_{i,j}(\underline{x}_{i,j}) e_{i,j+1} + d_{i,j}(t) - \dot{\alpha}_{i,j-1}$ and $\phi_{i,n_i} = f_{i,n_i}(x) + g_{i,n_i}(x) \delta_i(t) + d_{i,n_i}(t) - \dot{\alpha}_{i,n_i-1}$.

Step 1: Firstly, analyze the boundedness of $\phi_{i,1}$ when $t < t_1$:

1) According to Assumptions 1 and 3, $d_{i,1}(t) \in L^\infty$ and $\dot{r}_i(t) \in L^\infty$, $i = 1, 2, \dots, m$ hold.

2) From (13), $e_{i,2}$, $i = 1, 2, \dots, m$ is bounded.

3) Notice that $\underline{x}_{i,1} = y_i \in L^\infty$, in consideration the smoothness of $f_{i,1}(\underline{x}_{i,1})$ and $g_{i,1}(\underline{x}_{i,1})$ in $\underline{x}_{i,1}$, $f_{i,1}(\underline{x}_{i,1})$ and $g_{i,1}(\underline{x}_{i,1})$, $i = 1, 2, \dots, m$ keep bounded.

Hence variable $\phi_{i,1} \in L^\infty$, $i = 1, 2, \dots, m$ when $t < t_1$. By (9) and Assumption 2, it is easy to observe that

$$\begin{aligned} \liminf_{(e_{i,j}-q_{i,j}) \rightarrow 0^-} g_{i,j}(\underline{x}_{i,j})\alpha_{i,j} &= -\infty, \\ \limsup_{(e_{i,j}+q_{i,j}) \rightarrow 0^+} g_{i,j}(\underline{x}_{i,j})\alpha_{i,j} &= +\infty. \end{aligned} \quad (18)$$

In the case of $j = 1$, based on (17), (18) and $\phi_{i,1} \in L^\infty$, we have

$$\begin{aligned} \liminf_{(e_{i,1}-q_{i,1}) \rightarrow 0^-} \dot{e}_{i,1} &= -\infty, & \limsup_{(e_{i,1}+q_{i,1}) \rightarrow 0^+} \dot{e}_{i,1} &= +\infty, \\ & & i &= 1, 2, \dots, m. \end{aligned} \quad (19)$$

By the contradiction between (19) and (16), it can be deduced that the state error $|e_{i,1}|$ cannot arrive the prescribed performance $q_{i,1}$, such that

$$|e_{i,1}| \leq q_{i,1} - c_{i,1}, \quad i = 1, 2, \dots, m, \quad t < t_1. \quad (20)$$

with a positive constant $c_{i,1} < q_{i,1}^{(\infty)}$.

Based on (20), error transformation function $\xi_{i,1}$, $i = 1, 2, \dots, m$ defined in (6) is bounded as $t \in [0, t_1]$.

Step 2: Similar to the first step, judge the boundedness of $\phi_{i,2}$ when $t < t_1$:

1) Under the Assumption 1, the disturbance of the system $d_{i,2}(t) \in L^\infty$, $i = 1, 2, \dots, m$ holds.

2) In the light of Lemma 1, $\dot{\alpha}_{i,1}$ is bounded.

3) According to Step 1, $\xi_{i,1} \in L^\infty$ and $\phi_{i,1} \in L^\infty$. Then by equations (7) and (17), $\alpha_{i,1}$ and $\dot{e}_{i,1}$, $i = 1, 2, \dots, m$ keep bounded.

4) Since (4) and (13) hold, the boundary of $x_{i,2}$ and $e_{i,3}$ are further ensured.

5) Owing to the continuity of $f_{i,2}(\underline{x}_{i,2})$ and $g_{i,2}(\underline{x}_{i,2})$ in $\underline{x}_{i,2}$, $f_{i,2}(\underline{x}_{i,2})$ and $g_{i,2}(\underline{x}_{i,2})$ are bounded.

One can easily get the variable $\phi_{i,2}$ is bounded in the interval $[0, t_1]$. Further, combine with (17) and (18), there are conclusions that

$$\begin{aligned} \liminf_{(e_{i,2}-q_{i,2}) \rightarrow 0^-} \dot{e}_{i,2} &= -\infty, & \limsup_{(e_{i,2}+q_{i,2}) \rightarrow 0^+} \dot{e}_{i,2} &= +\infty, \\ & & i &= 1, 2, \dots, m. \end{aligned} \quad (21)$$

The contradiction between (16) and (21) implies

$$|e_{i,2}| \leq q_{i,2} - c_{i,2}, \quad i = 1, 2, \dots, m, \quad t < t_1 \quad (22)$$

where $0 < c_{i,2} < q_{i,2}^{(\infty)}$, thereby the function $\xi_{i,2} \in L^\infty$.

Step $j(3 \leq j \leq n_i - 1)$: The same procedure as **Step 2**, $\phi_{i,j} \in L^\infty$ can be ensured. At the same time, there exists a positive constant $c_{i,j} < q_{i,j}^{(\infty)}$, leading to

$$|e_{i,j}| \leq q_{i,j} - c_{i,j}, \quad t < t_1 \quad (23)$$

$$j = 3, 4, \dots, n_i - 1, \quad i = 1, 2, \dots, m.$$

It is clear that the error transformation function $\xi_{i,j}$ is bounded.

Step n_i : For the boundedness of ϕ_{i,n_i} as $t < t_1$:

1) According to Assumptions 1 and 3, $d_{i,n_i}(t) \in L^\infty$ and $\delta_i(t) \in L^\infty$, $i = 1, 2, \dots, m$ hold.

2) In the light of Lemma 1, $\dot{\alpha}_{i,n_i-1}$, $i = 1, 2, \dots, m$ is bounded.

3) From Step $j(3 \leq j \leq n_i - 1)$, ξ_{i,n_i-1} and ϕ_{i,n_i-1} are bounded. By (7) and (17), $\alpha_{i,n_i-1} \in L^\infty$ and $\dot{e}_{i,n_i-1} \in L^\infty$, $i = 1, 2, \dots, m$.

4) Since (4) and (13) hold, the boundary of x_{i,n_i} , $i = 1, 2, \dots, m$ is further ensured.

5) In consideration the smoothness of $f_{i,n_i}(x)$ and $g_{i,n_i}(x)$ in x , $f_{i,n_i}(x)$ and $g_{i,n_i}(x)$ keep bounded.

Therefore, ϕ_{i,n_i} , $i = 1, 2, \dots, m$ is bounded over $[0, t_1]$. Incorporating Assumption 1, (17) and (18) yields

$$\begin{aligned} \liminf_{(e_{i,n_i}-q_{i,n_i}) \rightarrow 0^-} \dot{e}_{i,n_i} &= -\infty, & \limsup_{(e_{i,n_i}+q_{i,n_i}) \rightarrow 0^+} \dot{e}_{i,n_i} &= +\infty, \\ & & i &= 1, 2, \dots, m. \end{aligned} \quad (24)$$

Because it contradicts (16) when $j = n_i$, one has

$$|e_{i,n_i}| \leq q_{i,n_i} - c_{i,n_i}, \quad i = 1, 2, \dots, m, \quad t < t_1. \quad (25)$$

with $0 < c_{i,n_i} < q_{i,n_i}^{(\infty)}$ being a constant.

This indicates the function ξ_{i,n_i} , $i = 1, 2, \dots, m$ is bounded over $[0, t_1]$. Combining (20), (22), (23), and (25), it follows that

$$|e_{i,j}| \leq q_{i,j} - c_{i,j}, \quad j = 1, 2, \dots, n_i \quad (26)$$

which contradicts (14). So the hypothesis (12) is not true, as a result (11) is true.

Now let's testify the closed-loop signals are bounded when $t \in [0, \infty)$. Redo **Step 1** – n_i by substituting (11) into (13), the function $\phi_{i,j}$ maintain bounded. Also, it's given by

$$\liminf_{(e_{i,j}-q_{i,j}) \rightarrow 0^-} \dot{e}_{i,j} = -\infty, \quad \limsup_{(e_{i,j}+q_{i,j}) \rightarrow 0^+} \dot{e}_{i,j} = +\infty. \quad (27)$$

Similarly, state error $|e_{i,j}|$ cannot get to its prescribed performance boundary $q_{i,j}$ for any $t \geq 0$, or in other words, there exists a constant $0 < c_{i,j} < q_{i,j}^{(\infty)}$, such that

$$|e_{i,j}| \leq q_{i,j} - c_{i,j}, \quad i = 1, 2, \dots, m, \quad j = 1, 2, \dots, n_i. \quad (28)$$

In summary, the whole signals of the closed-loop system are bounded. This completes the proof.

Remark 5 The reason that proof by contradiction sets up because when $e_{i,j} - q_{i,j}$ reaches the origin from left, the limit of $\dot{e}_{i,j}$ is greater than a constant in (16). Thus, it is impossible that the lower limit of $\dot{e}_{i,j}$ equal to minus infinity in (19). The same reason for the $e_{i,j} + q_{i,j}$ when it gets to the origin from right. Therefore, (27) conflicts with (16), the assumption (11) is invalid, i. e., the state errors can be limited in the prescribed performance functions.

Remark 6 It shows that all closed-loop signals keep bounded by proving Theorem 1. For clarity, the specific processes are summarized as follows: 1) in the case of $i = 1$, due to (28) holds, so the tracking error $e_{i,1}$ is bounded, and then control output y_i is bounded by (4); 2) because the continuous virtual control signals and control rate defined by (7) are bounded, so the control input is bounded; 3) due to (4) and (28), the states of the system are bounded.

4 Simulation Studies

In this section, one numerical simulation example is listed to verify the serviceability of the proposed control scheme. Specifically, a third-order unknown MIMO nonlinear system including actuator failures and unknown control directions is given by

$$\begin{aligned}\dot{x}_{1,1} &= f_{1,1}(x_{1,1}) + g_{1,1}(x_{1,1})x_{1,2} + d_{1,1}(t) \\ \dot{x}_{1,2} &= f_{1,2}(x) + g_{1,2}(x)u_{1,f} + d_{1,2}(t) \\ y_1 &= x_{1,1}\end{aligned}\quad (29)$$

$$\begin{aligned}\dot{x}_{2,1} &= f_{2,1}(x_{2,1}) + g_{2,1}(x_{2,1})x_{2,2} + d_{2,1}(t) \\ \dot{x}_{2,2} &= f_{2,2}(x) + g_{2,2}(x)u_{2,f} + d_{2,2}(t) \\ y_2 &= x_{2,1}\end{aligned}\quad (30)$$

and

$$\begin{aligned}\dot{x}_{3,1} &= f_{3,1}(x_{3,1}) + g_{3,1}(x_{3,1})x_{3,2} + d_{3,1}(t) \\ \dot{x}_{3,2} &= f_{3,2}(x) + g_{3,2}(x)u_{3,f} + d_{3,2}(t) \\ y_3 &= x_{3,1}\end{aligned}\quad (31)$$

where $x = [x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}, x_{3,1}, x_{3,2}]^T$, $u_{i,f}$ and $d_{i,j}$ denote the system state variables, control inputs and external disturbances. Next, choose the unknown smooth nonlinear functions $f_{1,1}(x_{1,1}) = x_{1,1}$, $f_{1,2}(x) = 0$, $g_{1,1}(x_{1,1}) = -e^{x_{1,1}^2}$, $g_{1,2}(x) = 1 + x_{1,1}^2$, $f_{2,1}(x_{2,1}) = -\cos(x_{2,1})x_{2,1}^2$, $f_{2,2}(x) = 0$, $g_{2,1}(x_{2,1}) = -0.8$, $g_{2,2}(x) = 2 - \cos(x_{3,2})$, $f_{3,1}(x_{3,1}) = -x_{3,1}^3/2$, $f_{3,2}(x) = x_{1,2}x_{2,2}x_{3,2}$, $g_{3,1}(x_{3,1}) = -1$, $g_{3,2}(x) = 0.4$, the control inputs $u_{1,f} = 0.8u_1 + 0.2$, $u_{2,f} = 0.6u_2 + 0.4$, $u_{3,f} = 0.7u_3 + 0.3$, the external disturbances $d_{1,1}(t) = \sin(t)$, $d_{1,2}(t) = -0.4\cos(t)$, $d_{2,1}(t) = 0.2\cos(t)$, $d_{2,2}(t) = -0.6$, $d_{3,1}(t) =$

$\sin(t)$, $d_{3,2}(t) = 0.2\sin(t)$. This system certainly satisfies our previous Assumptions 1-3.

In addition, the tracking signals $r_i(t)$ are taken as $r_1(t) = 0.2\cos(t)$, $r_2(t) = 0.2\cos(t) + 0.2\sin(t)$ and $r_3(t) = 0.4\sin(t)$. By the prescribed performance bounds of the state errors (5), the simulation parameters are selected as:

$$\begin{aligned}q_{1,1} &= (2 - 0.1)e^{-t} + 0.1 \\ q_{1,2} &= (4 - 0.3)e^{-0.3t} + 0.3\end{aligned}\quad (32)$$

$$\begin{aligned}q_{2,1} &= (2.5 - 0.1)e^{-0.6t} + 0.1 \\ q_{2,2} &= (3 - 0.25)e^{-0.25t} + 0.25\end{aligned}\quad (33)$$

and

$$\begin{aligned}q_{3,1} &= (1 - 0.05)e^{-0.4t} + 0.05 \\ q_{3,2} &= (5 - 0.3)e^{-0.25t} + 0.3.\end{aligned}\quad (34)$$

At the same time, set the appropriate initial values $x(0) = [-0.2, -0.4, 0.1, -0.3, -0.2, 0.1]^T$ of the system. In view of the continuous virtual control signals and control law defined in (7), the control gain are given as $\gamma_{1,1} = 10$, $\gamma_{2,1} = 12$, $\gamma_{3,1} = 15$, $\gamma_{1,2} = 8$, $\gamma_{2,2} = 20$, $\gamma_{3,2} = 20$, respectively.

Figs. 1-6 are the simulation end results of above MIMO system (29)-(31), which realized by the state feedback controller designed in Section 2. Although the system has uncertainties and disturbances, it can be obtained by Theorem 1:

- 1) from Figs. 1-3, the state errors $e_{i,j}$, $j = 1, 2$ of subsystem i , $i = 1, 2, 3$ are strictly limited to the prescribed performance bounds $\pm q_{i,j}$;
- 2) the output signals y_i can track the reference signals r_i , $i = 1, 2, 3$ very well in Fig. 4;
- 3) system states $x_{i,2}$ of subsystem i , $i = 1, 2, 3$ are bounded (see Fig. 5);
- 4) system input signals u_i and $u_{i,f}$, $i = 1, 2, 3$ maintain bounded (see Fig. 6).

From the above, it is clear that the prescribed transient and steady-state tracking capability of the uncertain MIMO unknown nonlinear system could be guaranteed.

5 Conclusion

In this paper, a lower complexity prescribed performance and FTC approach for MIMO uncertain nonlinear systems accompanied by coupling states, actuator failures, and unknown control directions are proposed. Utilizing state feedback control, the intermediate control signals which combining error transform functions and smooth orientation functions are designed to make the state deviations converge to a predictable compact

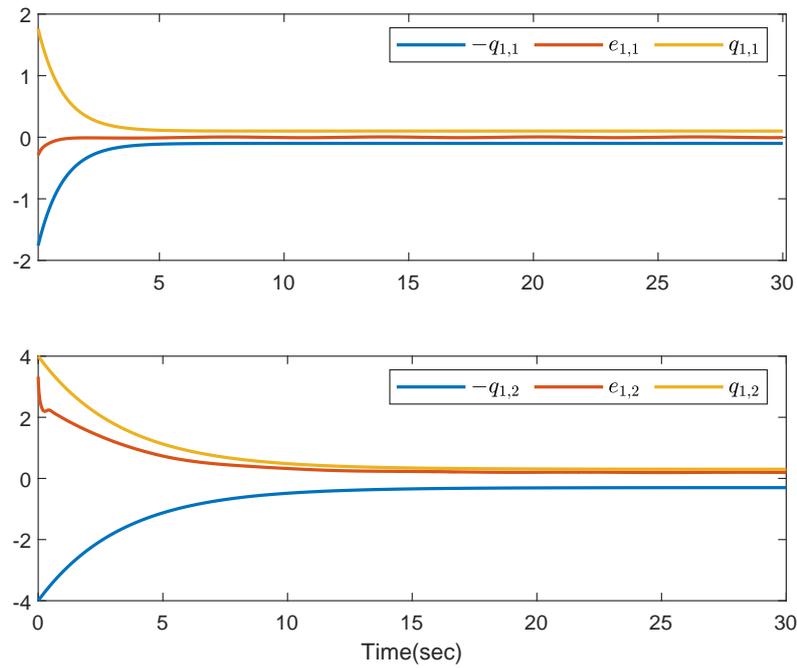


Fig. 1 Tracking error $e_{1,j}$ of the subsystem 1 under the prescribed performance limits $q_{1,j}$, $j = 1, 2$

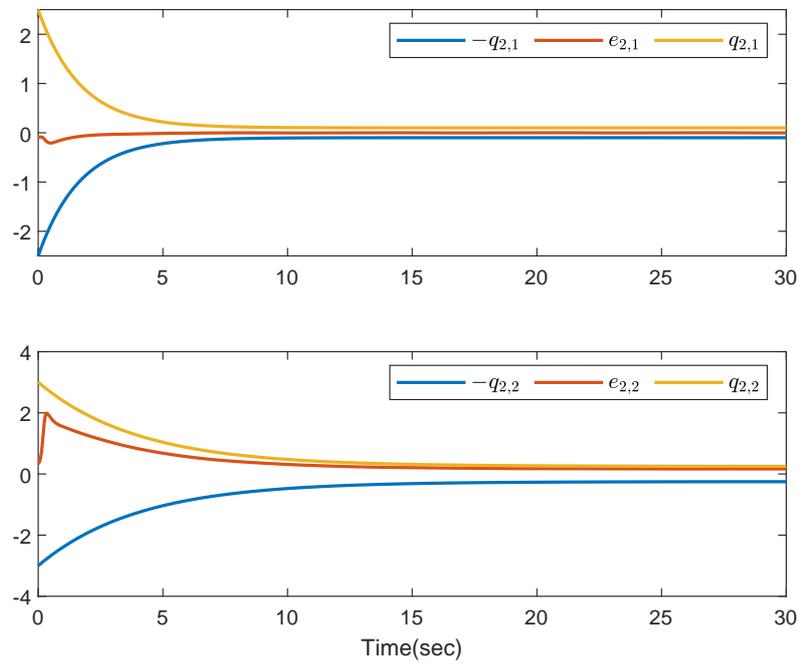


Fig. 2 Tracking error $e_{2,j}$ of the subsystem 2 under the prescribed performance limits $q_{2,j}$, $j = 1, 2$

aggregate and guarantee the boundedness of possessive closed-loop signals. Ultimately, a simulation example turns out to show the effectiveness of the proposed control scheme.

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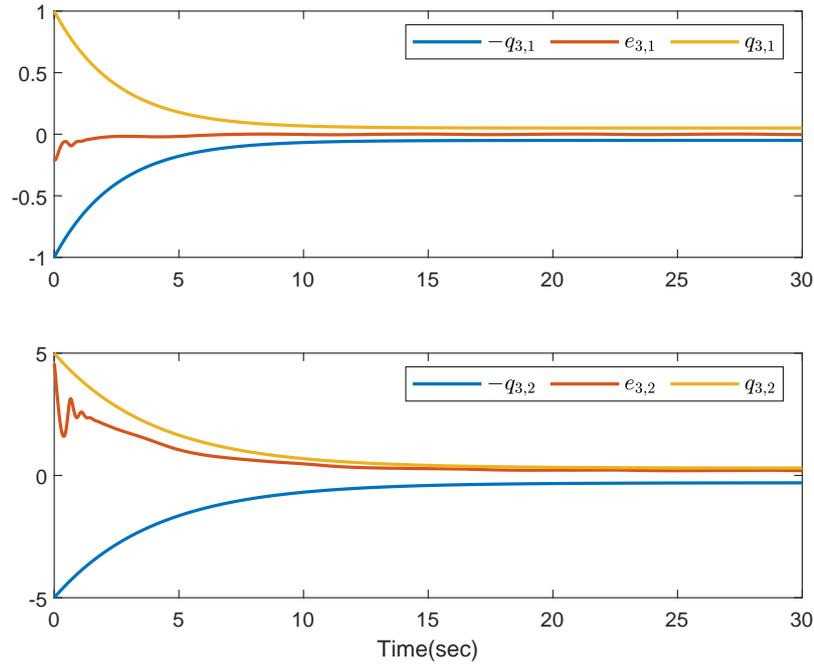


Fig. 3 Tracking error $e_{3,j}$ of the subsystem 3 under the prescribed performance limits $q_{3,j}$, $j = 1, 2$

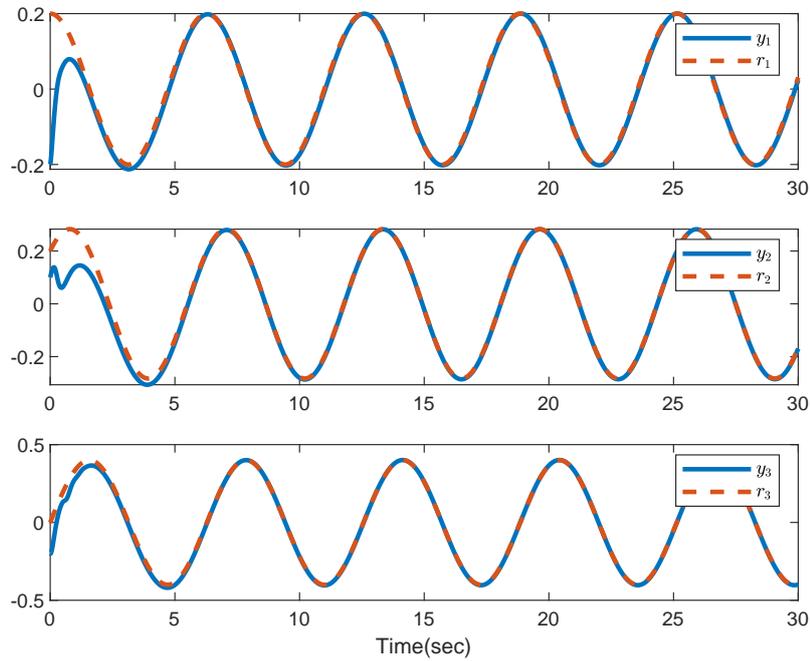


Fig. 4 Output signal y_i and target trajectory r_i of subsystem i , $i = 1, 2, 3$

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Conflict of interest

The authors declare that they have no conflict of interest.

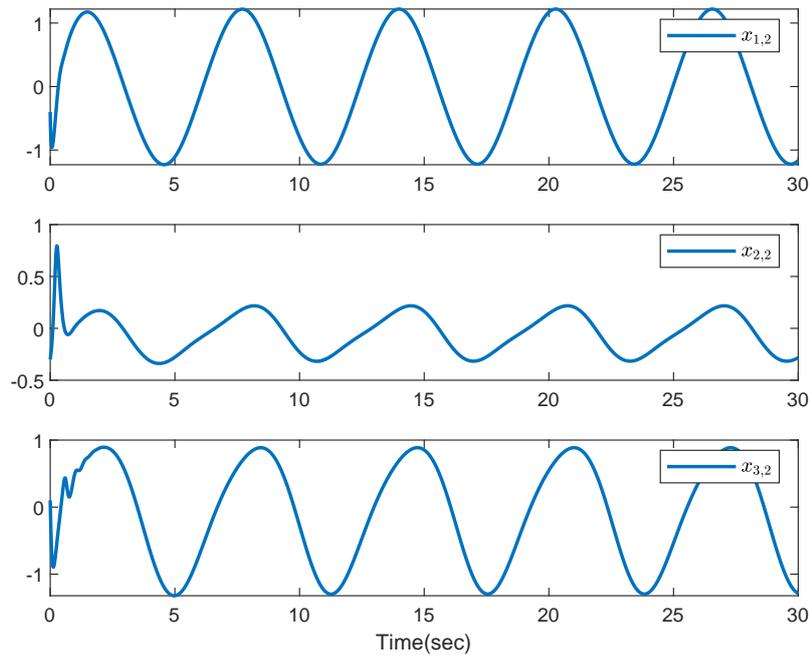


Fig. 5 System state $x_{i,2}$ of subsystem $i, i = 1, 2, 3$

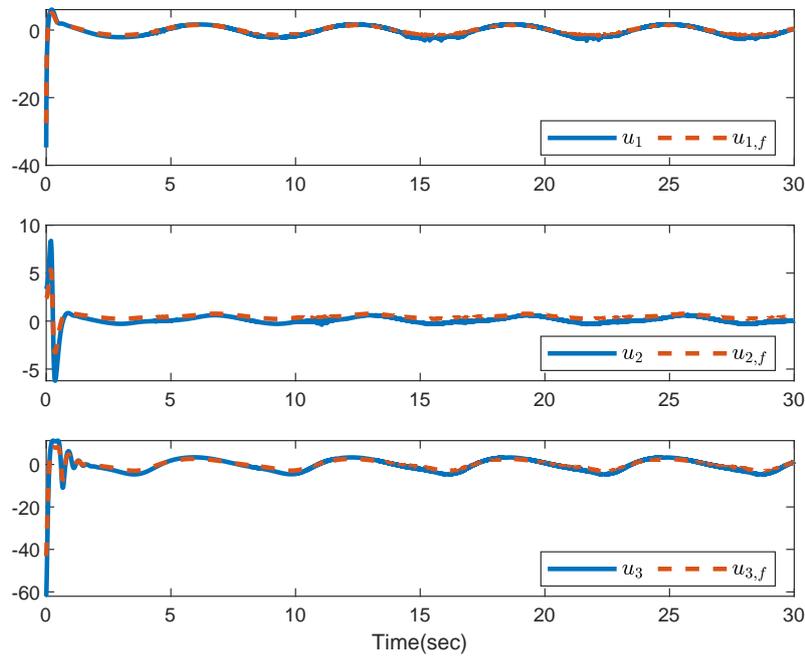


Fig. 6 Control input signal u_i and $u_{i,f}$ of subsystem $i, i = 1, 2, 3$

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Figures

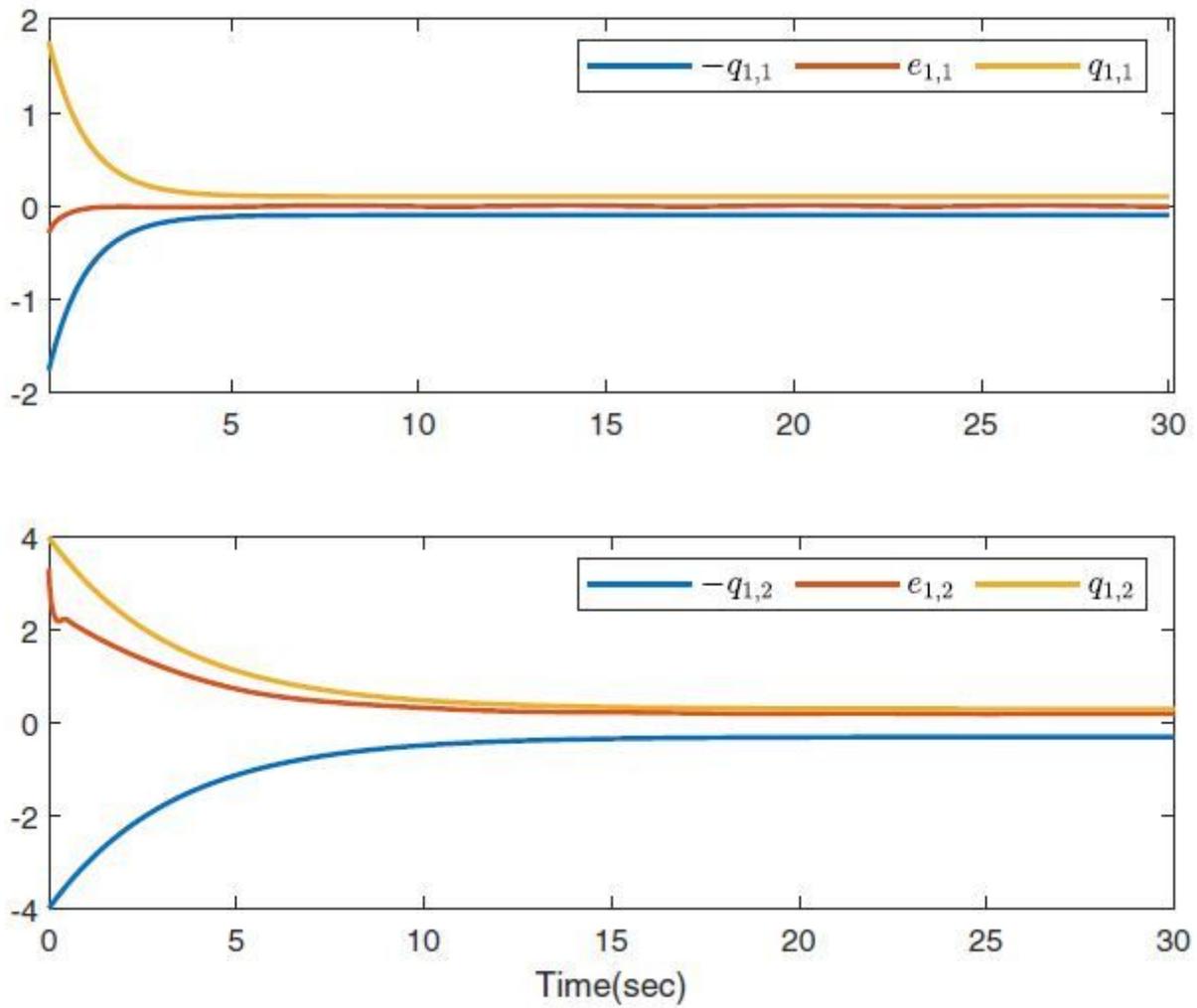


Figure 1

Tracking error $e_{1;j}$ of the subsystem 1 under the prescribed performance limits $q_{1;j}$, $j = 1; 2$

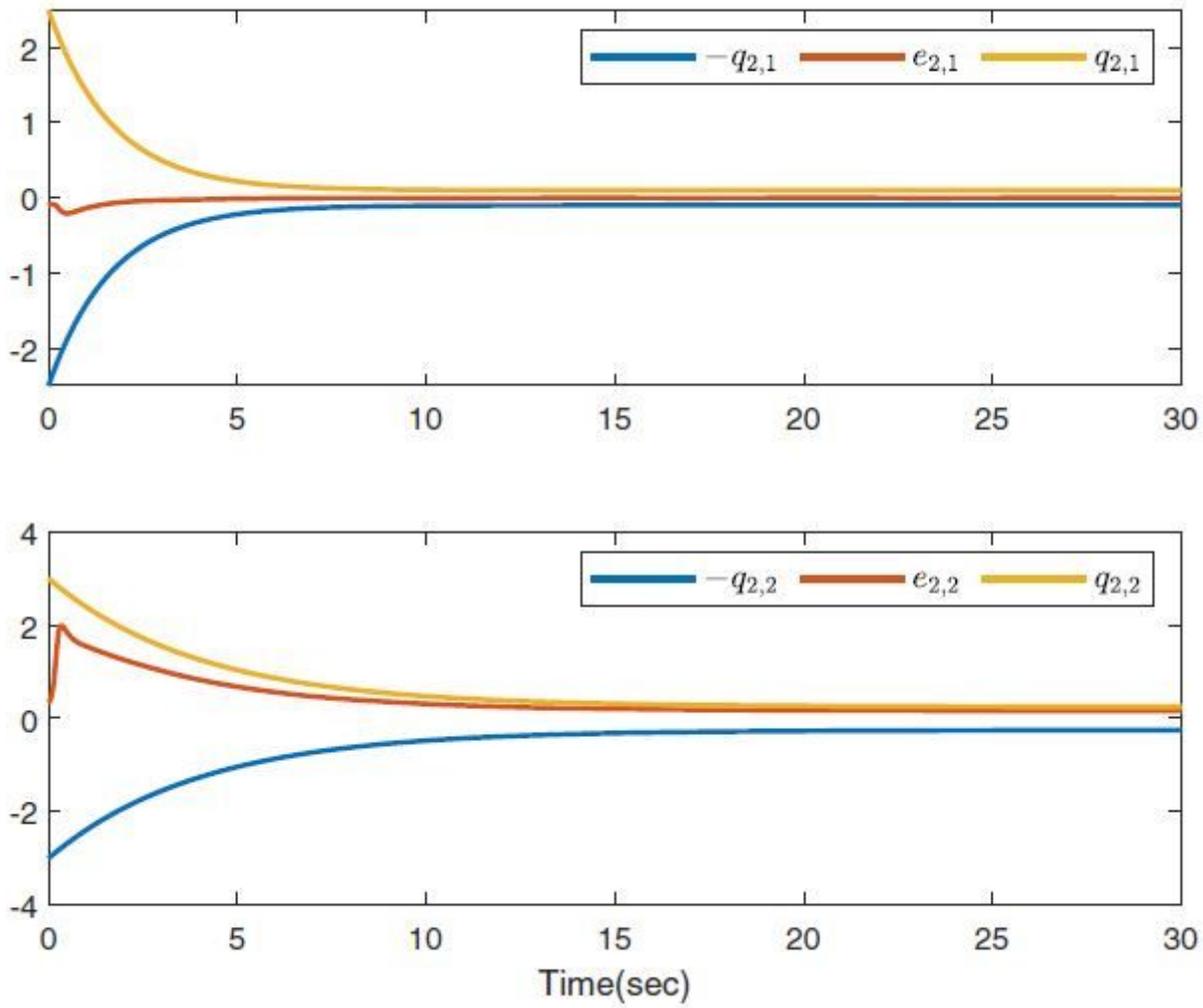


Figure 2

Tracking error $e_{2;j}$ of the subsystem 2 under the prescribed performance limits $q_{2;j}$, $j = 1; 2$

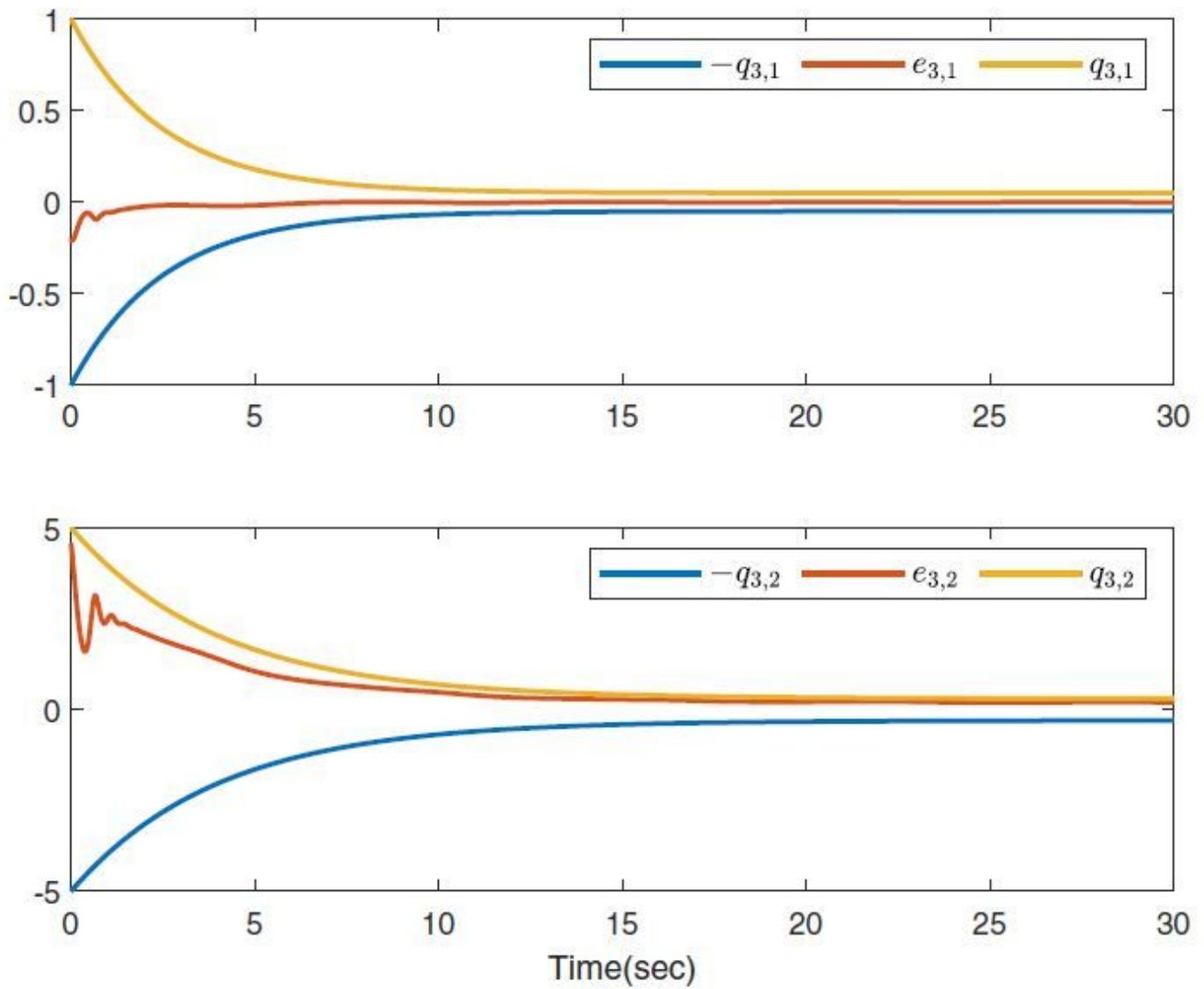


Figure 3

Tracking error $e_{3;j}$ of the subsystem 3 under the prescribed performance limits $q_{3;j}$, $j = 1; 2$

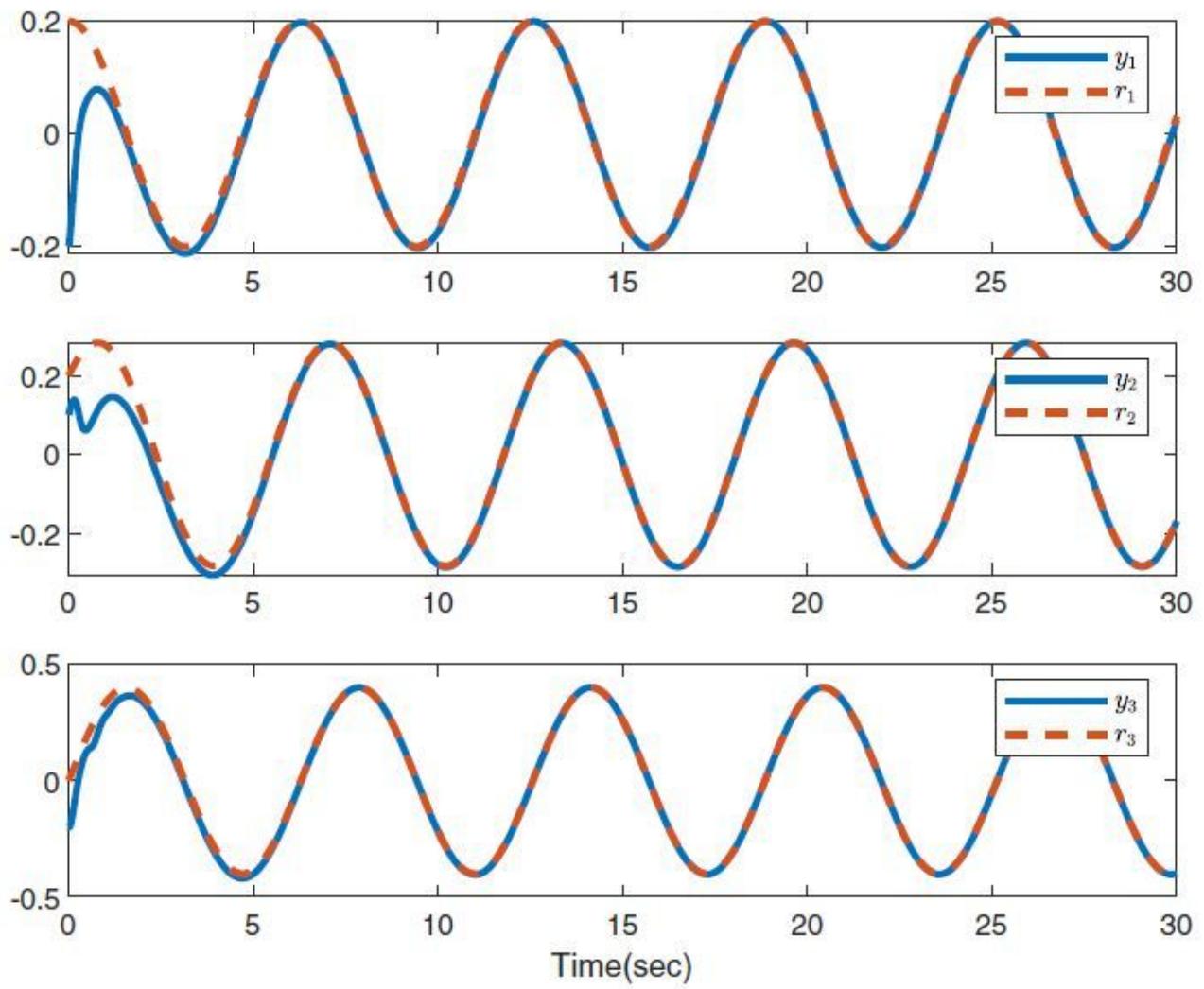


Figure 4

Output signal y_i and target trajectory r_i of subsystem i ; $i = 1; 2; 3$

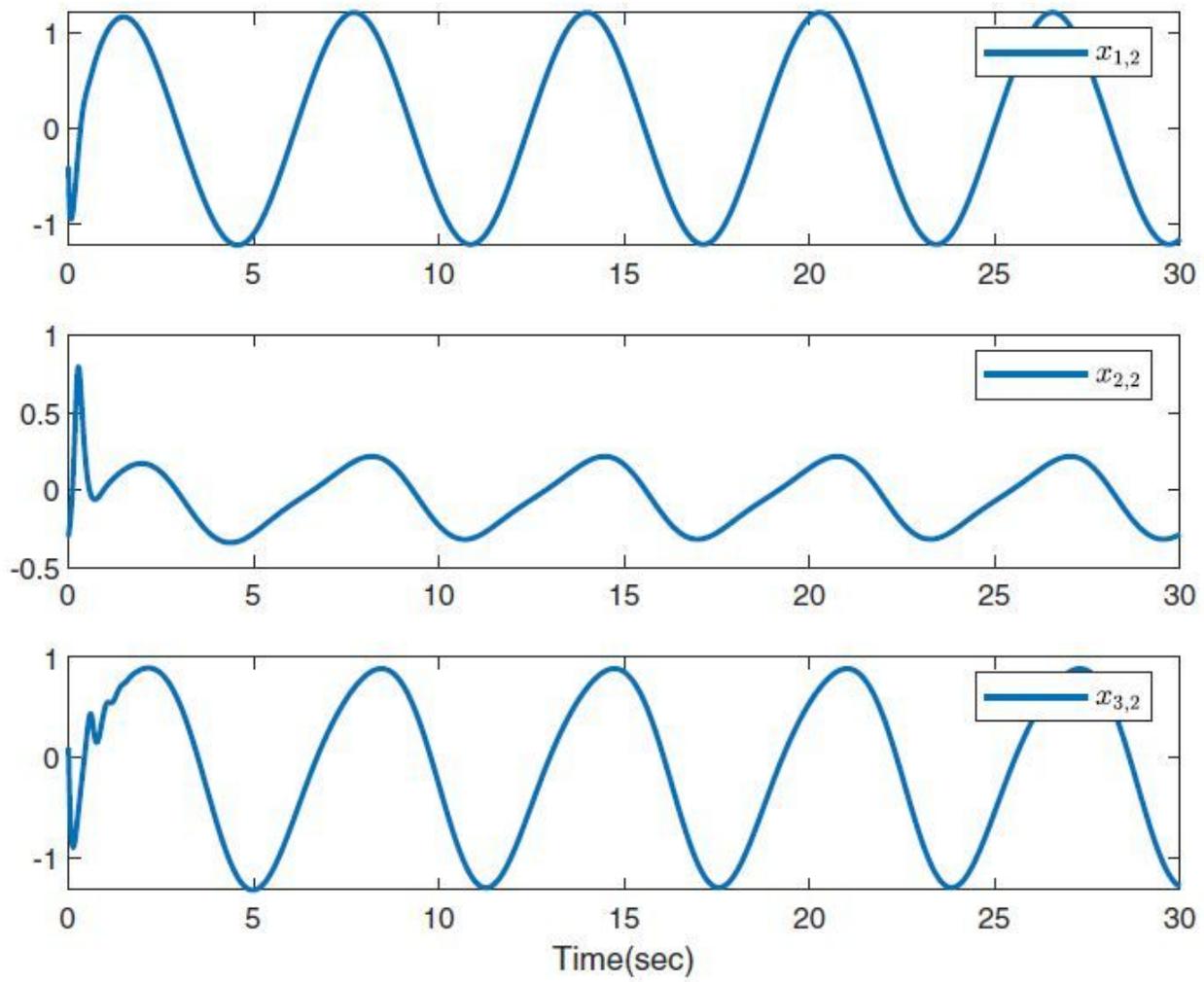


Figure 5

System state $x_{i,2}$ of subsystem i ; $i = 1; 2; 3$

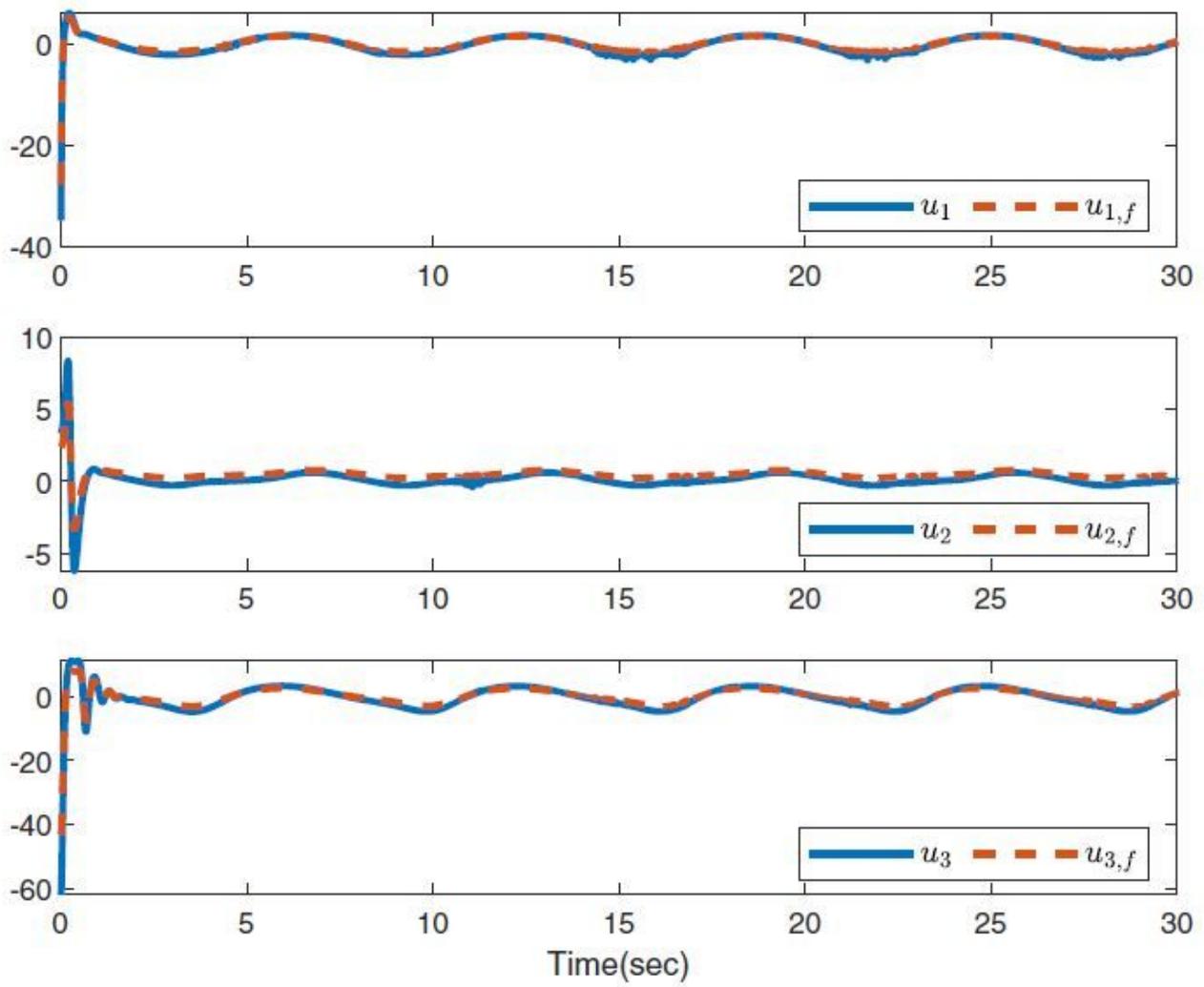


Figure 6

Control input signal u_i and $u_{i,f}$ of subsystem i ; $i = 1; 2; 3$