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Research Article

Keywords: Flutter stability, Functionally graded material, Metastructure, Nonlinear aeroelastic suppression, Nonlinear vibration absorbers

Posted Date: April 8th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-364180/v1>

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Metastructure plate with periodically embedded nonlinear vibration absorbers for nonlinear aeroelastic suppression

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Abstract: A metastructure plate featuring periodically embedded nonlinear vibration absorbers (NVAs) is proposed for the passive suppression of nonlinear aeroelastic responses under supersonic flow conditions. Using the von Karman large deformation theory and supersonic piston theory, the motion equations of a supersonic functionally graded material plate coupled with NVAs are derived from the Hamilton principle. Linear flutter analysis shows that the multiple-NVA design can significantly enhance the aerothermoelastic stability of the metastructure plate. Subsequently, the nonlinear aeroelastic behaviors of the plate and the energy transfer mechanism between it and the NVAs are examined using an energy-based analysis approach. The comparison of bifurcation diagrams indicates that the attachment of periodic NVAs realizes a superior suppression of vibration absorption than a single NVA. Numerical results show that the nonlinear dynamic responses of the plate can be substantially reduced via the targeted energy transfer of NVAs in the post-flutter regime. In particular, the passive control performance of the periodic NVAs does not degrade under an increase in the dynamic pressure. Furthermore, a significant reduction of more than 95% in the response amplitude of the plate can be realized by properly tuning the NVA parameters. The present work demonstrates that

the metastructure design, based on periodically distributed NVAs, is effective at enhancing the flutter stability and mitigating nonlinear aeroelastic responses.

Keywords: *Flutter stability; Functionally graded material; Metastructure; Nonlinear aeroelastic suppression; Nonlinear vibration absorbers.*

1 Introduction

In aerospace science and engineering, the aeroelasticity of the plates and shells of high-speed aircraft is a focal research topic [1-3]. Because of their structural properties, the plate/shell structures exhibit nonlinear motions under supersonic/hypersonic flow; furthermore, the lightweight structure, high speed, and high manoeuvrability of high-speed aircraft puts them at risk of severe aerodynamic loads. Though they can improve flight performance, such design concepts may induce flutter instability and complex nonlinear dynamic phenomena. Hence, panel flutter and nonlinear aeroelastic suppression have attracted considerable attention amongst researchers, and an increasing numbers of control approaches have been proposed to mitigate or eliminate the adverse impacts of nonlinear phenomena.

Passive control techniques, excellent robustness without external energy input, have been applied for vibration suppression. A nonlinear energy sink (NES) is a passive vibration-control device that can capture and dissipate broadband vibration energy through targeted energy transfer (TET) from primary structures [4-7]. Compared with the linear tuned mass damper (which is only applied for narrow frequency bands), a well-designed NES can improve the vibration absorption and realize highly efficient energy transmission. NESs have been widely developed and applied to various fields, including architecture [8], machinery [9-11], aerospace [12-13], and engineering structures. To improve the control performance of NESs, different NES variants have also been designed and assessed, including bistable [14-15], vibro-impact [16-17], lever-enhanced [18-20], and energy-harvester-enhanced [21-22] NESs. Several studies on NES-based panel-flutter suppression have

provided valuable insights into the passive-flutter control mechanisms of NESs [23-25]. For example, Zhang et al. [23] analyzed the vibration mitigation of a supersonic composite laminated plate, and they obtained superior attenuation under pre-flutter conditions. Pacheco et al. [24] used an energy analysis approach to evaluate the panel-flutter suppression performance of an NES, and they discussed the influence of the NES parameters on its control performance. Subsequently, we applied an NES to the nonlinear aeroelastic suppression of a three-dimensional (3D) hypersonic wing by constructing a 3D wing with an NES-based structure design. The obtained results demonstrated excellent aeroelastic behaviors [25]. Moreover, existing studies have shown that the installation position of the NES plays an important role in its behavior and efficiency, and its performance is also sensitive to parameter variations [7,25-26]. However, the NES has a limited influence on the dynamic behavior of the system, i.e., the vibration reduction and flutter enhancement.

Metastructures, a metamaterial-inspired concept, feature distributed periodic vibration absorbers [27-29]. These structures exhibit interesting dynamic behaviors (e.g., bandgaps and pass bands) that have attracted significant research interest in recent years. The metastructures can behave unique dynamic properties because of the periodic arrangement and parametric design of their local resonators. Although classified as a passive control approach, the ability to suppress multiple modes simultaneously is a key feature of metastructures. The application of local resonators can reduce the structure vibrations by allowing waves of a certain frequency to propagate along the periodic substructure. Furthermore, waves in other frequency bands are unable to pass.

Owing to these superior dynamic behaviors, a number of metastructure studies have focused upon the vibration suppression of systems. Reichl and Inman [30] proposed a conserved-mass 1D metastructure model to suppress the axial vibrations of a host structure. The numerical results indicated that an excellent attenuation was achieved by varying the natural frequency of the local resonators. Ji [31] designed a nonlinear vibration absorber using three-to-one internal resonances, to suppress nonlinear

vibrations and eliminate saddle-node bifurcations of the primary structure. Using a distributed array of local vibration absorbers, Casalotti et al. [32] evaluated the multi-mode vibration absorptions of a nonlinear metamaterial beam. The vibration-suppression performance was improved by tuning the parameters of the local absorbers. A type of graded metamaterial beam with attached local resonators was also applied by Hu et al. [33] for broadband-vibration suppression. Basta et al. [34] proposed a mechanical rotating metamaterial cantilever beam coupled to a periodic array of spring-mass-damper subsystems, to realize vibration control. In terms of the designs of the metastructure plates, Sheng et al. [35] analyzed the vibration behaviors of a sandwich plate containing viscoelastic periodic cores; they found that these cores could provide a better attenuation performance than uniform viscoelastic ones. Subsequently, Sheng et al. [36] optimized a lightweight nonlinear acoustic metamaterial beam to realize low-frequency, broadband, high-efficiency vibration reduction. Furthermore, Li et al. [37] designed a sandwich-like plate containing mass-beam resonators for vibration attenuation and isolation, and the performance of this structure was validated theoretically and experimentally. Later that year, Song et al. [38] proposed metamaterial sandwich panels comprising a host sandwich panel and periodically attached resonators, to analyze the vibration and sound properties of the system. The use of the periodic structure design may represent an effective approach for suppressing vibrations, sound radiation, and sound transmission over a wide frequency range.

As demonstrated by the aforementioned studies, various metastructure designs can be used to effectively tune bandgaps and suppress vibrations. Casadei and Bertoldi [39-40] proposed periodic arrays of airfoil-type vibration absorbers to allow wave propagation along a beam, and their design achieved wide band gaps by harnessing fluid–structure interactions and varying the airfoil properties. However, to the best of our knowledge, most studies of metastructures have focused on the vibration mitigation of linear metastructures, particularly for wave propagation and vibro-acoustic suppression. In addition, the design of metastructures with local

resonators or periodic substructures has seldom been applied to suppress the vibrations of nonlinear aeroelastic systems. In supersonic/hypersonic aircraft, the skins of wings and engines are exposed to thermal and aerodynamic loads during high-speed flight, which may produce rich nonlinear dynamic phenomena. Therefore, it is essential to investigate how the metastructure design can be applied to the nonlinear aeroelastic suppression of a 3D supersonic plate.

The objective of the present study is to extend the metastructure concept to the nonlinear aeroelastic suppression of a plate. We focus on assessing the effects of a metastructure embedded with periodically distributed nonlinear vibration absorbers (NVAs) on the passive control of flutter stability and the nonlinear aeroelastic response of a functionally graded material (FGM) plate. The remainder of this paper is organized as follows: in Section 2, we formulate the nonlinear aeroelastic equations of the metastructure FGM plate coupled with multiple NVAs; in Section 3.1, the convergence and accuracy of the proposed analytical model are validated; in Sections 3.2 and 3.3, we evaluate the effects of the NVA parameters on the linear flutter and nonlinear aeroelastic behaviors of the metastructure plate, respectively; the main conclusions are drawn in Section 4.

2 Theoretical analyses

We consider a metastructure that consists of an FGM plate coupled with a distributed array of $N_a (= n_a \times n_a)$ nonlinear absorbers, as shown in Fig. 1. The local NVAs are modelled as small masses m_j attached to certain positions (x_j, y_j) of the plate by a linear damper c_j and a spring with linear stiffness coefficient $k_{l,j}$ and nonlinear stiffness coefficient $k_{nl,j}$. The multiple NVAs are periodically and evenly distributed across the plate, with the spaces between the NVAs and plate assumed sufficiently large to prevent the NVAs from colliding with the plate during motion. Thus, new displacements p_j ($j = 1, 2, \dots, N_a$) must be introduced as additional degrees of freedom (DOFs).

Figure 1. Schematic of the metastructure plate with periodic NVAs under supersonic flow conditions.

2.1 Material properties

Because of their excellent thermo-mechanical performances, FGMs have potential applications in aerospace engineering structures [41-42]. Therefore, the nonlinear aeroelastic behaviors of a supersonic FGM plate are here examined. In general, an FGM plate is composed of a mixture of metals and ceramics. The material properties of the FGM plate vary continuously along the thickness direction of the plate in accordance with a power-law distribution of volume fractions, expressed as

$$\begin{aligned}
 E(z) &= (E^u - E^b) \left(\frac{2z+h}{2h} \right)^p + E^b, \\
 \nu(z) &= (\nu^u - \nu^b) \left(\frac{2z+h}{2h} \right)^p + \nu^b, \\
 \rho(z) &= (\rho^u - \rho^b) \left(\frac{2z+h}{2h} \right)^p + \rho^b, \\
 \alpha(z) &= (\alpha^u - \alpha^b) \left(\frac{2z+h}{2h} \right)^p + \alpha^b, \\
 \kappa(z) &= (\kappa^u - \kappa^b) \left(\frac{2z+h}{2h} \right)^p + \kappa^b,
 \end{aligned} \tag{1}$$

where E , ν , ρ , α , and κ denote the elastic modulus, Poisson's ratio, density, thermal expansion coefficient, and thermal conductivity, respectively. The symbol p denotes the volume-fraction index. The superscripts u and b denote the upper and bottom surfaces of the FGM plate, respectively.

The temperature-dependent material properties can be written as

$$P = P_0 \left(P_{-1} T^{-1} + 1 + P_1 T^1 + P_2 T^2 + P_3 T^3 \right), \quad (2)$$

where P_0 , P_{-1} , P_1 , P_2 , and P_3 are coefficients of temperature $T(K)$.

To consider the thermal effects, we assume that the temperature varies only in the thickness direction and that the in-plane temperature field remains constant. Thus, the temperature-field distribution throughout the plate can be obtained using the 1D Fourier heat-conduction equation, expressed as

$$-\frac{d}{dz} \left[\kappa(z) \frac{dT}{dz} \right] = 0, \quad (3)$$

where $\kappa(z)$ is the thermal conductivity of the FGM plate. The upper and bottom temperature boundary conditions are $T(h/2) = T^u$ and $T(-h/2) = T^b$, respectively.

The solution of Eq. (3) can be obtained as

$$T(z) = T^b + (T^u - T^b) \eta(z), \quad (4)$$

where

$$\eta(z) = \frac{\int_{-h/2}^z \frac{1}{\kappa(z)} dz}{\int_{-h/2}^{h/2} \frac{1}{\kappa(z)} dz}.$$

The temperature change along the thickness is defined as $\Delta T(z) = T(z) - T^b$. To simulate aircraft structures under the thermal load conditions that occur during supersonic flight, the temperature T^b exerted on the bottom surface is defined as the reference temperature T_0 , and the upper surface is exposed to a high uniform temperature field.

2.2 Energy expressions

Using the first-order shear deformation theory, the displacement components for the FGM plate in the x -, y -, and z -directions can be written as

$$\begin{aligned} U(x, y, z, t) &= u(x, y, t) + z\varphi_x, \\ V(x, y, z, t) &= v(x, y, t) + z\varphi_y, \\ W(x, y, z, t) &= w(x, y, t), \end{aligned} \quad (5)$$

where u , v , and w denote the displacements on the middle surface of the FGM plate in the x , y , and z directions, respectively. φ_x and φ_y denote the rotations of the transverse normal to the middle surface about the x - and y -axes, respectively.

Based on the von Karman large deformation assumption and the first-order shear deformation theory, the nonlinear strain–displacement relations can be expressed as

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} + z \frac{\partial \varphi_x}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2, \\ \varepsilon_y &= \frac{\partial v}{\partial y} + z \frac{\partial \varphi_y}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2, \\ \gamma_{xy} &= \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + z \left(\frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x} \right) + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}, \\ \gamma_{yz} &= \varphi_x + \frac{\partial w}{\partial x}, \gamma_{xz} = \varphi_y + \frac{\partial w}{\partial y}, \end{aligned} \quad (6)$$

In Eq. (6), the strain vector can be divided into linear and nonlinear components, as

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}_L + \boldsymbol{\varepsilon}_{NL}, \quad (7)$$

where $\boldsymbol{\varepsilon} = [\varepsilon_x \quad \varepsilon_y \quad \gamma_{yz} \quad \gamma_{xz} \quad \gamma_{xy}]^T$ and the subscripts L and NL indicate the linear and nonlinear parts, respectively. The nonlinear component of the strain vector is

expressed as

$$\boldsymbol{\varepsilon}_{NL} = \left[\frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \quad \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \quad \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \quad 0 \quad 0 \right]^T. \quad (8)$$

The constitutive relation of the FGM plate considering thermal effects can be written as

$$\boldsymbol{\sigma} = \mathbf{C}(\boldsymbol{\varepsilon} - \boldsymbol{\alpha}\Delta T), \quad (9)$$

where $\boldsymbol{\sigma}$ and $\boldsymbol{\alpha}$ denote the stress and thermal expansion coefficient vectors, respectively. Here, $\mathbf{C}$ denotes the elastic constant matrix. The vectors and matrices are given as follows:

$$\boldsymbol{\sigma} = [\sigma_x \quad \sigma_y \quad \tau_{yz} \quad \tau_{xz} \quad \tau_{xy}]^T, \boldsymbol{\alpha} = [\alpha(z, T) \quad \alpha(z, T) \quad 0 \quad 0 \quad 0]^T, \quad (10)$$

$$\mathbf{C} = \begin{bmatrix} Q_{11}(z, T) & Q_{12}(z, T) & 0 & 0 & 0 \\ Q_{12}(z, T) & Q_{22}(z, T) & 0 & 0 & 0 \\ 0 & 0 & \kappa_s Q_{66}(z, T) & 0 & 0 \\ 0 & 0 & 0 & \kappa_s Q_{66}(z, T) & 0 \\ 0 & 0 & 0 & 0 & Q_{66}(z, T) \end{bmatrix}.$$

where Q_{ij} ($i, j = 1, 2$, and 6) are the elastic constants of the FGM, which can be obtained using

$$Q_{11}(z, T) = \frac{E(z, T)}{1 - \nu^2(z)}, \quad Q_{12}(z, T) = \frac{\nu(z)E(z, T)}{1 - \nu^2(z)}, \quad Q_{66}(z, T) = \frac{E(z, T)}{2[1 + \nu(z)]}. \quad (11)$$

The kinetic energy of the metastructure plate is expressed as follows:

$$T_k = \frac{1}{2} \int_{-h/2}^{h/2} \int_0^b \int_0^a \rho(z) \left[\left(\frac{\partial U}{\partial t} \right)^2 + \left(\frac{\partial V}{\partial t} \right)^2 + \left(\frac{\partial W}{\partial t} \right)^2 \right] dx dy dz + \frac{1}{2} \sum_j^{N_a} m_j \dot{p}_j^2. \quad (12)$$

Substituting Eq. (4) into Eq. (12), the kinetic energy can be written as

$$T_k = \frac{1}{2} \int_0^a \int_0^b \left\{ I_0 \left[\left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial v}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right] + 2I_1 \left(\frac{\partial u}{\partial t} \frac{\partial \varphi_x}{\partial t} + \frac{\partial v}{\partial t} \frac{\partial \varphi_y}{\partial t} \right) + I_2 \left[\left(\frac{\partial \varphi_x}{\partial t} \frac{\partial \varphi_x}{\partial t} \right)^2 + \left(\frac{\partial \varphi_y}{\partial t} \right)^2 \right] \right\} dx dy + \frac{1}{2} \sum_j^{N_a} m_j \dot{p}_j^2, \quad (13)$$

where $(I_0, I_1, I_2) = \int_{-h/2}^{h/2} \rho(z)(1, z, z^2) dz$.

The potential energy U_p of the metastructure plate under thermal loads is given as

$$\begin{aligned}
U_p = & \int_{-h/2}^{h/2} \int_0^b \int_0^a \left(\frac{1}{2} \boldsymbol{\varepsilon}_L^T \mathbf{C} \boldsymbol{\varepsilon}_L + \frac{1}{2} \boldsymbol{\varepsilon}_{NL}^T \mathbf{C} \boldsymbol{\varepsilon}_L + \frac{1}{2} \boldsymbol{\varepsilon}_L^T \mathbf{C} \boldsymbol{\varepsilon}_{NL} + \frac{1}{2} \boldsymbol{\varepsilon}_{NL}^T \mathbf{C} \boldsymbol{\varepsilon}_{NL} \right. \\
& \left. - \boldsymbol{\varepsilon}_L^T \mathbf{C} \boldsymbol{\alpha} \Delta T - \boldsymbol{\varepsilon}_{NL}^T \mathbf{C} \boldsymbol{\alpha} \Delta T \right) dx dy dz + \frac{1}{2} \sum_j^{N_a} k_{l,j} \left[w(x_j, y_j, t) - p_j(t) \right]^2 \quad (14) \\
& + \frac{1}{4} \sum_j^{N_a} k_{nl,j} \left[w(x_j, y_j, t) - p_j(t) \right]^4
\end{aligned}$$

The Rayleigh's dissipation function of the nonlinear resonators is

$$\mathcal{G} = \frac{1}{2} \sum_j^{N_a} c_j \left[\dot{w}(x_j, y_j, t) - \dot{p}_j(t) \right]^2. \quad (15)$$

The external work performed by the aerodynamic pressure can be obtained using

$$W = \int_0^b \int_0^a \Delta p w dx dy, \quad (16)$$

and the aerodynamic pressure Δp is modelled using first-order piston aerodynamics,

expressed as

$$\Delta p = \frac{2q_\infty}{\beta} \left[\frac{\partial w}{\partial x} + \frac{Ma^2 - 2}{Ma^2 - 1} \frac{1}{V_\infty} \frac{\partial w}{\partial t} \right], \quad (17)$$

where $q_\infty (= \rho_\infty V_\infty^2 / 2)$, V_∞ , and Ma represent the dynamic pressure, flow velocity, and

Mach number, respectively. $\beta (= (Ma^2 - 1)^{1/2})$ represents the compressibility

correction factor. The non-dimensional dynamic pressure $\lambda = 2q_\infty a^3 / \beta D_{m0}$ and

non-dimensional mass ratio $\mu = \rho_\infty a / \rho_{m0} h$ are introduced in the following linear

flutter analysis, in which $D_{m0} (= E_{m0} h^3 / 12(1 - \nu^2))$ is defined as the plate stiffness.

ρ_{m0} and E_{m0} denote the density and elastic modulus of the metallic material,

respectively.

The total energy function of the metastructure plate can be obtained as

$$\Pi = T_k - U_p + \mathcal{G} + W. \quad (18)$$

2.3 Governing equations in state-space form

The displacement components of the FGM plate are expressed as

$$\begin{aligned}
 u(x, y, t) &= \sum_{m=0}^M \sum_{n=0}^N A_{mn}(t) \xi_{mn}(x, y) = \boldsymbol{\xi}^T(x, y) \mathbf{A}(t), \\
 v(x, y, t) &= \sum_{m=0}^M \sum_{n=0}^N B_{mn}(t) \varsigma_{mn}(x, y) = \boldsymbol{\varsigma}^T(x, y) \mathbf{B}(t), \\
 w(x, y, t) &= \sum_{m=0}^M \sum_{n=0}^N C_{mn}(t) \varphi_{mn}(x, y) = \boldsymbol{\varphi}^T(x, y) \mathbf{C}(t), \\
 \varphi_x(x, y, t) &= \sum_{m=0}^M \sum_{n=0}^N D_{mn}(t) \chi_{mn}(x, y) = \boldsymbol{\chi}^T(x, y) \mathbf{D}(t), \\
 \varphi_y(x, y, t) &= \sum_{m=0}^M \sum_{n=0}^N E_{mn}(t) \eta_{mn}(x, y) = \boldsymbol{\eta}^T(x, y) \mathbf{E}(t),
 \end{aligned} \tag{19}$$

where $\mathbf{A}(t) = [A_{11}(t), \dots, A_{mn}(t)]^T$, $\mathbf{B}(t) = [B_{11}(t), \dots, B_{mn}(t)]^T$, $\mathbf{C}(t) = [C_{11}(t), \dots, C_{mn}(t)]^T$, $\mathbf{D}(t) = [D_{11}(t), \dots, D_{mn}(t)]^T$, and $\mathbf{E}(t) = [E_{11}(t), \dots, E_{mn}(t)]^T$ are the generalized coordinate vectors.

For a simply supported FGM plate, the shape functions can be expressed as

$$\begin{aligned}
 \xi_{mn} &= \chi_{mn} = \cos \frac{m\pi x}{a} \sin \frac{n\pi y}{b}, \quad (m = 1, 2, \dots, M; n = 1, 2, \dots, N), \\
 \varsigma_{mn} &= \eta_{mn} = \sin \frac{m\pi x}{a} \cos \frac{n\pi y}{b}, \quad (m = 1, 2, \dots, M; n = 1, 2, \dots, N), \\
 \varphi_{mn} &= \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}, \quad (m = 1, 2, \dots, M; n = 1, 2, \dots, N),
 \end{aligned} \tag{20}$$

According to the Hamilton principle, the governing equations for the complete system can be obtained as

$$\begin{bmatrix} \mathbf{M}_{ss} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_A \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}}_s \\ \ddot{\mathbf{q}}_a \end{Bmatrix} + \begin{bmatrix} \mathbf{D}_s & \mathbf{D}_C \\ \mathbf{D}_C^T & \mathbf{D}_A \end{bmatrix} \begin{Bmatrix} \dot{\mathbf{q}}_s \\ \dot{\mathbf{q}}_a \end{Bmatrix} + \begin{bmatrix} \mathbf{K}_{ss} + \mathbf{K}_s & \mathbf{K}_C \\ \mathbf{K}_C^T & \mathbf{K}_A \end{bmatrix} \begin{Bmatrix} \mathbf{q}_s \\ \mathbf{q}_a \end{Bmatrix} + \begin{bmatrix} \mathbf{F}_{NL} + \Delta \mathbf{F} \\ \mathbf{F}_A \end{bmatrix} + \begin{bmatrix} \mathbf{Q} \\ \mathbf{0} \end{bmatrix} = \mathbf{0} \tag{21}$$

or, in short form,

$$\hat{\mathbf{M}} \begin{Bmatrix} \ddot{\mathbf{q}}_s \\ \ddot{\mathbf{q}}_a \end{Bmatrix} + \hat{\mathbf{C}} \begin{Bmatrix} \dot{\mathbf{q}}_s \\ \dot{\mathbf{q}}_a \end{Bmatrix} + \hat{\mathbf{K}} \begin{Bmatrix} \mathbf{q}_s \\ \mathbf{q}_a \end{Bmatrix} + \hat{\mathbf{F}}_{NL} + \hat{\mathbf{Q}} = \mathbf{0}, \tag{22}$$

where $\mathbf{q}_s = [\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}, \mathbf{E}]^T$ and $\mathbf{q}_a = [p_1, p_2, \dots, p_{N_a}]^T$ denote state vectors, $\mathbf{Q}$ is the generalized aerodynamic force of the plate, $\mathbf{F}_{NL}$ is the nonlinear term of the plate,

and $\Delta \mathbf{F}$ is an incremental nonlinear term generated by the NVAs. $\mathbf{F}_A$ is the nonlinear term of the NVAs. The components of $\mathbf{M}_{ss}$, $\mathbf{K}_{ss}$, $\mathbf{M}_A$, $\mathbf{D}_S$, $\mathbf{D}_A$, $\mathbf{D}_C$, $\mathbf{K}_S$, $\mathbf{K}_A$, $\mathbf{K}_C$, $\mathbf{Q}$, $\mathbf{F}_{NL}$, $\Delta \mathbf{F}$, and $\mathbf{F}_A$ are provided in the **Appendix**.

2.4 Energy evaluation

To explore the energy transfer of the proposed metastructure plate and its passive control performance, several quantities (e.g., the total mechanical energy contained in the plate, the energy input by the airflow, and the energy dissipated by the NVAs) are estimated using an energy-based analysis approach.

The total instantaneous energy of the plate is expressed as

$$E_s(t) = \frac{1}{2} \dot{\mathbf{q}}_s^T \mathbf{M}_{ss} \dot{\mathbf{q}}_s + \frac{1}{2} \dot{\mathbf{q}}_s^T \mathbf{K}_{ss} \dot{\mathbf{q}}_s + \dot{\mathbf{q}}_s^T \mathbf{F}_{NL}. \quad (23)$$

The total instantaneous energy of the NVAs is expressed as

$$E_{NVA}(t) = \frac{1}{2} \sum_j^{N_a} m_j \dot{p}_j^2 + \frac{1}{2} \sum_j^{N_a} k_{l,j} [p_j(t) - w(x_j, y_j, t)]^2 + \frac{1}{4} \sum_j^{N_a} k_{nl,j} [p_j(t) - w(x_j, y_j, t)]^4. \quad (24)$$

The energy input by the airflow and dissipated by the NVAs for $t \in [0, T]$ are obtained as

$$E_a(t) = \int_0^T \left[\iint \Delta p \frac{\partial w}{\partial t} dx dy \right] dt = \int_0^T \left[-\dot{\mathbf{q}}_s^T (\mathbf{K}_a \mathbf{q}_s + \mathbf{C}_a \dot{\mathbf{q}}_s) \right] dt, \quad (25)$$

$$E_{dis}(t) = \sum_j^{N_a} \int_0^T c_j [\dot{p}_j(t) - \dot{w}(x_j, y_j, t)]^2 dt, \quad (26)$$

respectively.

3 Results and discussions

In the present work, the passive flutter control and nonlinear aeroelastic behaviors of a supersonic FGM plate featuring periodically attached NVAs are assessed. The FGM plate was composed of Si₃N₄ and SUS304, and its temperature-dependent material properties are listed in Table 1. The mass density and thermal conductivity of Si₃N₄ are $\rho^u = 2370 \text{ kg/m}^3$ and $\kappa^u = 9.19 \text{ W/(mK)}$, respectively; those of SUS304 are $\rho^b = 8166 \text{ kg/m}^3$ and $\kappa^b = 12.04 \text{ W/(mK)}$, respectively. The Poisson's ratio is defined

as $\nu^u = \nu^b = 0.28$, and the shear correction factor is defined as $\kappa_s = 0.91$ [43]. For all case studies, the parameters were set as $a = b = 1.0$ m, $h = 0.02$ m, and $\mu/Ma = 0.01$. In numerical simulations, all figures were plotted with respect to a typical point $(x, y) = (0.75a, 0.5b)$ of the plate.

Table 1. Temperature-dependent material properties of Si_3N_4 and SUS304.

Materials	Properties	P_0	P_{-1}	P_1	P_2	P_3	P ($T_0 = 300$ K)
Si_3N_4	E^u (Pa)	348.430e9	0.0	-3.070e-4	2.160e-7	-8.946e-11	322.2715e9
	α^u (1/K)	5.8723e-6	0.0	9.095e-4	0	0.0	7.4746e-6
SUS304	E^b (Pa)	201.04e9	0.0	3.079e-4	-6.534e-7	0.0	207.7877e9
	α^b (1/K)	12.330e-6	0.0	8.086e-4	0	0.0	15.321e-6

3.1 Convergence study and validation

The convergence and accuracy of the proposed formulation were validated for the proposed metastructure plate, using the numerical examples described in this subsection. First, the effects of mode truncation on the nonlinear aeroelastic response of the system were evaluated, to determine the appropriate number of modes. We take the metastructure plate ($p = 1.0$) with 4×4 NVAs as an example; Fig. 2 shows the limit cycle oscillation (LCO) amplitudes of the plate and selected 10th NVA with respect to the dynamic pressure for different numbers of retained modes along the airflow and span directions. Here, the parameters of the NVAs were selected as $m_j = 0.01$, $c_j = 0.1$, $k_{l,j} = 20$, and $k_{nl,j} = 2000$, and the reference temperature T_0 was set equal to 300 K. The results show that the LCO amplitudes obtained for $M = 6$ and $N = 2$ agree well with those obtained for $M = 7$ and $N = 2$. Furthermore, Fig. 3 shows the corresponding nonlinear aeroelastic responses of the plate and 10th NVA at $\lambda = 710$, thereby verifying that $M = 6$ and $N = 2$ are suitable settings to achieve a good compromise between computational accuracy and efficiency in the present configuration.

Figure 2. LCO amplitude versus dynamic pressure for different numbers of retained modes along the airflow and span directions in the case of a simply supported FGM plate ($p = 1.0$) with 4×4 NVAs.

Figure 3. Nonlinear aeroelastic responses of the simply supported FGM plate ($p = 1.0$) with 4×4 NVAs at $\lambda = 710$ for different numbers of retained modes: (a) plate and (b) 10th NVA.

Next, to further validate the feasibility and accuracy of the present analytical model, the metastructure plate was degenerated into an aluminium rectangular plate with a simply supported boundary, for which the relevant parameters were $E = 7.17 \times 10^{10}$ Pa, $\rho = 2700$ kg/m³, $\nu = 0.3$, and $\mu/Ma = 0.1$. The LCO behaviors obtained by the present formulation for this rectangular plate were compared with those reported in Xie et al. [44]. The comparison is shown in Fig. 4 and indicates strong consistency. Consequently, the aforementioned results demonstrate that the proposed analytical

model predicts the aeroelastic stability and nonlinear dynamic responses of the metastructure plate with sufficient accuracy.

Figure 4. Comparison of LCO behaviors obtained under the present formulation for an aluminium rectangular plate with a simply supported boundary.

3.2 Linear flutter analysis

We conducted a linear flutter analysis to elucidate the aeroelastic stability of the metastructure plate embedded with NVAs. The flutter stability can be obtained by neglecting the nonlinear force $\hat{\mathbf{F}}_{\text{NL}}$ in Eq. (22). To examine the effects of the distributed NVAs on the flutter behavior of the metastructure, the eigenvalue solutions of the first five modes (under non-dimensional dynamic pressure λ) for the pure plate are shown in Fig. 5, which indicates that the flutter of the pure plate is produced by the coupling between the 1st and 2nd modes. The critical flutter aerodynamic pressure and flutter frequency were $\lambda_{cr} = 656.79$ and $f_{cr} = 7.55$, respectively; this agrees well with the results ($\lambda_{cr} = 648.71$ and $f_{cr} = 7.533$) reported in Ref. [43].

Figure 5. Eigenvalue solutions of the first five modes (under non-dimensional dynamic pressure λ) for the FGM plate ($p = 1.0$) without NVAs.

When multiple NVAs are attached to the FGM plate, the flutter behavior of the metastructure plate can be analyzed. Fig. 6 shows the variations in the critical flutter aerodynamic pressure λ_{cr} under NVA linear stiffness coefficient k_l for different numbers of NVAs. The change in the NVA linear stiffness coefficient can be seen to have a marked influence on the flutter boundary of the system. In the small- k_l regime, the flutter boundary of the metastructure changes drastically in all cases; however, in the large- k_l regime, the flutter boundary begins to stabilize under an increase in k_l . Moreover, the variation trends of flutter stability also differ significantly for different numbers of NVAs. For $N_a = 1 \times 1$, the flutter boundary of the metastructure was lower than that of the pure plate when $k_l < 55$. Under a further increase in k_l , the linear flutter behavior improved. Fig. 7 shows the flutter behavior of the plate with 1×1 NVAs for $k_l = 40$ and $k_l = 200$. For $k_l = 40$, the flutter was generated by the coupling between the NVA mode and 2nd plate mode, and the coupled 2nd mode of the plate became unstable at $\lambda_{cr} = 493.05$. Here, the NVA mode represents the mode of the derived linear NVA. Thus, for the plate with 1×1 NVA, when the frequency of the NVA mode was between the 1st and 2nd plate modes, the aeroelastic behavior of the system was degraded. However, when the frequency of the NVA mode far exceeded that of the 2nd plate mode (as shown in Fig. 7 for the case of $k_l = 200$), the attached

NVA can be treated as a lumped mass; this reduces the 1st mode frequency of the plate and can delay the flutter onset; as a result, the corresponding critical flutter aerodynamic pressure was $\lambda_{cr} = 688.62$.

Figure 6. Critical flutter aerodynamic pressure versus NVA linear stiffness k_l for different number of NVAs.

Figure 7. Flutter behaviors of the FGM plate with 1×1 NVAs for $k_l = 40$ and $k_l = 200$.

For $N_a = 2 \times 2$, the flutter behaviors of the plate with 2×2 NVAs for $k_l = 20$ and $k_l = 400$ are presented in Fig. 8. The frequencies of the NVA modes were located

between those of the 1st and 2nd modes of the plate, which increased the 2nd mode frequency. The system flutter finally occurred at $\lambda_{cr} = 685.83$ via the coupling of the NVA mode and the 2nd plate mode. Under the increase in the k_l of the NVAs, the NVA mode frequencies gradually approached the 2nd mode frequency of the plate, reducing the flutter boundary of the metastructure. These variation trends of the flutter boundary are displayed in Fig. 7. As shown in Fig. 8(b), when the NVA mode frequency exceeded the 2nd mode frequency, the system's flutter boundary was $\lambda_{cr} = 609.33$ for $k_l = 40$. Moreover, the 2nd mode frequency was remarkably decreased compared with the pure plate case. Under these conditions, the system flutter produced by the coupling between the 1st and 2nd modes of the plate occurred at a lower aerodynamic pressure. Under a further increase in k_l , the flutter boundary remained almost unchanged.

Figure 8. Flutter behaviors of the FGM plate with 2×2 NVAs for different NVA linear stiffness coefficients: (a) $k_l = 20$ and (b) $k_l = 40$.

For larger numbers of NVAs, a significant improvement in the aeroelastic behavior of the metastructure plate was observed. As shown in Fig. 6, the plates coupled with 3×3 , 4×4 , 5×5 , and 6×6 NVAs exhibited an enhanced aeroelastic stability over the entire parameter domain. Considering the case of 6×6 NVAs for $k_l = 20$ as an example (Fig. 9), we see that the flutter stability of the metastructure was enhanced by 16.46%, and the NVA mode became unstable at $\lambda_{cr} = 764.87$. As k_l was increased

further, the flutter boundary of the system finally stabilized near $\lambda = 655.3$.

Furthermore, a similar trend was observed for the cases of 3×3 , 4×4 , and 5×5 NVAs. Therefore, multiple attached NVAs can enhance the aeroelastic stability of the supersonic FGM plate.

Figure 9. Flutter behaviors of the metastructure plate with 6×6 NVAs for NVA linear stiffness coefficients of $k_l = 20$.

However, the question of whether the flutter stability of the metastructure plate under different volume fraction indices could be improved using distributed NVAs is not yet resolved. Hence, we evaluated the effect of the volume fraction indices on the flutter behavior of the metastructure plate for the case of 6×6 NVAs. Fig. 10 depicts the flutter behaviors of the system for different volume fraction indices, including the critical flutter aerodynamic pressure and corresponding flutter frequency. When the volume fraction index p was increased, the flutter boundary decreased for an identical NVA linear stiffness coefficient. The variation of the flutter boundary with respect to k_l was similar for different volume fraction indices; that is, λ_{cr} increased and then declined. As shown in Fig. 10(b), a sudden change occurred at the cutting point of the flutter frequency variation. Thus, two regions can be distinguished in the subsequent discussion. Figs. 11 and 12 depict the flutter behaviors of the metastructure plate ($p = 0.2$) with 6×6 NVAs for $k_l = 10$ and $k_l = 40$, respectively; these are considered typical cases for Region I and Region II, respectively. When $k_l = 10$, the coupling of

the NVA mode and 2nd mode of the plate generated flutter instability of the metastructure at $\lambda_{cr} = 764.87$, in which the NVA mode was destabilized under the increase in λ ; hence, it was the flutter dominant DOF, and the corresponding flutter frequency was $\omega_{cr} = 10.92$. For $k_l = 40$, the flutter instability of the system was produced by the coupling between the 1st mode of the plate and the NVA mode, and the coupled NVA mode destabilized at $\lambda_{cr} = 759.44$. The results demonstrate that the change in k_l affects the coupled modes of the system flutter, and the flutter frequency of the unstable NVA mode is also markedly altered.

Figure 10. Flutter behaviors of the metastructure plate with 6×6 NVAs for different volume fraction indices: (a) critical flutter aerodynamic pressure and (b) flutter frequency.

Figure 11. Flutter behaviors of the metastructure plate with 6×6 NVAs ($p = 0.2$, $k_l = 10$) in Region I.

Figure 12. Flutter behaviors of the metastructure plate with 6×6 NVAs ($p = 0.2, k_l = 40$) in Region II.

Next, the effects of the temperature rise on the aeroelastic stability of the metastructure plate were also examined. Figs. 13(a) and 13(b) depict the aerothermoelastic behaviors of the metastructure plate for $p = 0.5$, and $p = 2.0$, respectively. From the results, the aerothermoelastic stability was seen to decrease under the temperature increase occurring on the upper surface. For temperature increase ΔT , the value of k_l corresponding to the peak of the critical flutter aerodynamic pressure λ_{cr} was gradually decreased. Using the periodically distributed NVAs, the maximum increment of λ_{cr} in the FGM plate ($p = 0.5$) was reduced from 16.71% to 14.64% under an increase in ΔT . However, for the FGM plate ($p = 2.0$), the maximum increment in λ_{cr} was reduced from 17.33% to 6.87%. Furthermore, to clearly determine the aerothermoelastic stability of the metastructure plate, Fig. 14 depicts the stability boundaries of the system in terms of the flutter and critical temperature boundaries. The attachment of NVAs on the FGM plates for both $p = 0.5$ and $p = 2.0$ produced wider flat-plate regions; this indicates a more stable FGM plate, especially in the range of low temperature rise. However, the NVAs had very little effect on the flutter and thermal buckling behavior of the plate at high temperatures. Therefore, we conclude that the designed NVAs can improve the aeroelastic behaviors of metastructure plates for various volume fraction indices, and the

appropriate NVA linear stiffness should be selected to effectively enhance the aerothermoelastic stability of the FGM plate for different thermal environments.

Figure 13. Effects of temperature rise on the flutter stability of the metastructure plate with 6×6 NVAs: (a) $p = 0.5$ and (b) $p = 2.0$.

Figure 14. Stability boundaries of the metastructure plates with 6×6 NVAs under the effects of aerodynamic and thermal loads: (a) $p = 0.5$ and (b) $p = 2.0$.

3.3 Nonlinear aeroelastic analysis

In this section, the geometrical nonlinearity of the plate and the nonlinear stiffness of the NVAs are considered. The effects of the distributed NVAs on the nonlinear aeroelastic behaviors of the metastructure plate were carefully analyzed. Meanwhile,

an energy-based approach was used to evaluate the energy transfer of the present metastructure. Fig. 15 shows the bifurcation diagrams of the local amplitude extremes for the pure and metastructure plates. Here, the parameters of the NVAs were set as $m_j = 0.01$, $c_j = 0.1$, $k_{lj} = 10$, and $k_{nlj} = 2000$. As shown in Fig. 15, the attachment of the distributed NVAs resulted in a significant difference in complex dynamic bifurcation characteristics compared to the pure plate. A slight reduction in the flutter boundary was found to occur when 1×1 NVA was used. In addition, the amplitude of the plate did not decrease; instead, the plate–NVA system exhibited complex nonlinear dynamic responses in the region of $740 < \lambda < 960$. However, the use of 6×6 NVAs notably improved the nonlinear aeroelastic suppression performance for the metastructure plate. In the following, the nonlinear aeroelastic behaviors and energy transfers of the present metastructure plate under typical dynamic pressures are discussed in detail.

Figure 15. Bifurcation diagram in terms of local amplitude extreme for the pure and metastructure plates.

When $742 < \lambda < 798$, the pure plate exhibited a stable LCO; however, because of the attachment of 6×6 NVAs, the motion of the metastructure plate converged to a

static equilibrium. Fig. 16(a) compares the nonlinear aeroelastic responses at $\lambda = 780$ for the pure and metastructure plates, and it confirms that the addition of periodic NVAs can improve the aeroelastic stability of the system. In addition, the position of the 28th NVA ($x_{28} = 3/4, y_{28} = 7/12$) was close to the typical point; hence, its response was compared with that of the plate, as shown in Fig. 16(b). The vibration amplitude of this NVA was much smaller than that of the plate, which indicates that no targeted energy transfer occurred. The system's own stability in the pre-flutter regime means that the vibration energy of the plate is insufficiently strong to meet the energy threshold of the TET. Thus, the absorption energy of NVAs is small. To better understand the energy transfer mechanism of NVAs, the time histories of the energy for the present system are depicted in Fig. 16(c). As time elapsed, the input and dissipated energy gradually increased and eventually converged. During this process, the input energy was entirely dissipated by the NVAs. Accordingly, the remained energy in the system was reduced to zero.

(a)

(b)

(c)

Figure 16. Nonlinear aeroelastic responses at $\lambda = 780$: (a) pure and metastructure plates, (b) the plate and 28th NVA, and (c) the energy time histories for the metastructure plate.

For the post-flutter regime, Fig. 17(a) depicts the nonlinear aeroelastic responses of the pure and metastructure plates at $\lambda = 806$. The numerical results show that the LCO amplitude of the plate was significantly (i.e., up to 88%) reduced. Furthermore, the LCO amplitude did not increase under increasing aerodynamic pressure in the region $802 < \lambda < 850$. As shown in Fig. 17(b), the vibration amplitude of the NVA was still smaller than that of the plate. The corresponding energy analysis is shown in Fig.

17(c). The energy dissipation and energy input are clearly seen to reach a balance, and the remained energy in the system maintains the stable LCO of the plate.

Furthermore, the time histories of the plate energy and the total energy of the NVAs are shown in Fig. 17(d), where we can see that the total NVA energy is far below that of the plate and the TET phenomenon does not occur. Under a further increase in λ , the plate produces a random response with a small amplitude. The case of $\lambda < 900$ is shown in Fig. 18. Similarly, the plate's response was reduced by ~85%. Nevertheless, the metastructure plate exhibited a recurrent motion with an undulating amplitude, and the amplitude of the 28th NVA exceeded that of the plate. The energy time histories shown in Fig. 18(c) indicate a dynamic balance of energy input and dissipation. To better understand the energy transfer mechanism, Fig. 18(d) presents the time histories of the plate energy and the total energy of the NVAs for $t \in [0, 5]$.

When the vibration energy of the plate reached the critical energy threshold, the NVA energy suddenly increased, a large quantity of energy was transferred from the plate to the NVAs, and the response amplitude of the plate began to decrease. Thus, the vibration energy of the plate gradually weakened, and the NVAs lost their ability to capture energy. Subsequently, owing to the continuous energy input of the airflow, the energy of the plate increased rapidly again. The aforementioned process was repeated to illustrate the phenomenon shown in Fig. 18(b). Furthermore, this process indicates that a TET was accomplished via recurrent transient resonance capture. In addition, Fig. 19 compares the power spectrum diagrams of the responses for the plate and 28th NVA. The two peak frequencies of the responses were remarkably consistent, and this NVA had a wider frequency band than the plate. Based on the above analysis, we observe from Fig. 15 that the passive control performance does not degrade under the dynamic pressure increase for the present metastructure plate. In contrast, the NVAs consistently provided excellent suppression of nonlinear dynamic responses of the plate in the entire post-flutter regime. This differs from systems featuring a single nonlinear absorber, for which the suppression capability of the absorber can be reduced by a higher aerodynamic load or more prominent nonlinear effect [22, 25].

(c)

(d)

Figure 17. Nonlinear aeroelastic responses at $\lambda = 806$: (a) pure and metastructure plates, (b) the plate and 28th NVA, (c) energy time histories for the metastructure plate, and (d) time histories of the plate energy and total energy of NVAs.

(a)

(b)

(c)

(d)

Figure 18. Nonlinear aeroelastic responses at $\lambda = 900$: (a) pure and metastructure plates, (b) the plate and 28th NVA, (c) energy time histories for the metastructure plate and (d) time histories of the plate energy and total energy of NVAs for $t \in [0, 5]$.

Figure 19. Power spectrum diagrams of the responses for the plate and 28th NVA at $\lambda = 900$.

Furthermore, another difference between this and the single nonlinear resonator case is that the installation positions of the periodic NVAs can be neglected, because they are evenly distributed across the plate. To further improve the suppression performance of the nonlinear aeroelastic responses for the metastructure plate, a parametric NVA design was performed. For ease of comparison, the parameters m_j , c_j , and $k_{l,j}$ remained unchanged, to obtain the linear flutter behavior; we investigated the effects of the nonlinear stiffness coefficient $k_{nl,j}$ on the nonlinear aeroelastic behaviors of the system. Here, $\lambda = 1000$ was selected as the design aerodynamic pressure. Fig. 20 shows the bifurcation diagrams of the local amplitude extremes for different nonlinear stiffness coefficients $k_{nl,j}$. Under an increase in $k_{nl,j}$, the response amplitude of the plate was significantly reduced in the region of $k_{nl,j} < 5000$; however, when $k_{nl,j}$ increased further, the amplitude change became unclear. Hence, we selected a larger value for the nonlinear stiffness coefficient, to improve the passive control performance of the NVAs. Taking the case of $k_{nl,j} = 40000$ as an example, the bifurcation diagram of the improved metastructure plate is illustrated in Fig. 21. The

comparison indicates that a proper parameter design for the NVAs can potentially improve the suppression of nonlinear aeroelastic responses. Notably, this 95% reduction of the plate response was achieved across the entire post-flutter region, by applying the improved parameters.

Figure 20. Bifurcation diagram of the metastructure plate with different NVA nonlinear stiffness coefficients $k_{nl,j}$ at $\lambda = 1000$.

Figure 21. Bifurcation diagram of local amplitude extreme for the improved metastructure plate ($k_{nl,j} = 40000$).

4 Conclusions

In this study, the concept of a metastructure design featuring periodically distributed NVAs was proposed and explored for the nonlinear aeroelastic suppression of supersonic FGM plates. An analytical model of the metastructure FGM plate coupled with multiple NVAs was formulated using the von Karman large deformation theory and supersonic piston theory. Convergence and comparison studies validated the effectiveness and accuracy of the proposed model in predicting nonlinear aeroelastic responses. The effects of the NVA parameters on the flutter and nonlinear dynamic behaviors of the metastructure plate were evaluated. The numerical results suggest that the distributed-NVAs-based metastructure design can significantly improve the aerothermoelastic stability and mitigate nonlinear dynamic responses. The following conclusions were drawn from the numerical examples:

1) Increasing the number of distributed NVAs can notably improve the aeroelastic behavior of the metastructure plate. The change in the linear stiffness coefficient of the NVAs can determine the coupled modes of the system flutter, which can effectively enhance the aerothermoelastic stability of the FGM plate for an appropriate stiffness value.

2) Energy-based analysis results reveal that the nonlinear aeroelastic responses of the metastructure plate can be substantially suppressed through the TET of NVAs in the post-flutter regime. Notably, the passive control performance does not degrade under an increase in dynamic pressure for the design of periodically distributed NVAs.

3) A significant suppression performance enhancement can be achieved by appropriately tuning the NVA nonlinear parameter, and a reduction of up to 95% was achieved for the response amplitude of the metasurface plate. To summarise, the present study demonstrates the excellent potential of the proposed metastructure for realising performance enhancements in term of both flutter stability and nonlinear dynamic behaviors.

Acknowledgments This work was supported by the National Natural Science Foundation of China (Grant No.11972296) and the Overseas Expertise Introduction Project for Discipline Innovation (111 Project, BP0719007).

Conflict of interest The authors declare that they have no conflict of interest.

Appendix

The detailed expressions in the matrices $\mathbf{M}_{ss}$ and $\mathbf{K}_{ss}$ are given as:

$$\mathbf{M}_{ss} = \begin{bmatrix} \mathbf{M}_{uu} & \mathbf{0} & \mathbf{0} & \mathbf{M}_{ux} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{vv} & \mathbf{0} & \mathbf{0} & \mathbf{M}_{vy} \\ \mathbf{0} & \mathbf{0} & \mathbf{M}_{ww} & \mathbf{0} & \mathbf{0} \\ \mathbf{M}_{ux}^T & \mathbf{0} & \mathbf{0} & \mathbf{M}_{xx} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{vy}^T & \mathbf{0} & \mathbf{0} & \mathbf{M}_{yy} \end{bmatrix}, \quad (\text{A-1})$$

$$\mathbf{K}_{ss} = \begin{bmatrix} \mathbf{K}_{uu} & \mathbf{K}_{uv} & \mathbf{0} & \mathbf{K}_{ux} & \mathbf{K}_{uy} \\ \mathbf{K}_{uv}^T & \mathbf{K}_{vv} & \mathbf{0} & \mathbf{K}_{vx} & \mathbf{K}_{vy} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{ww} & \mathbf{K}_{wx} & \mathbf{K}_{wy} \\ \mathbf{K}_{ux}^T & \mathbf{K}_{vx}^T & \mathbf{K}_{wx}^T & \mathbf{K}_{xx} & \mathbf{K}_{xy} \\ \mathbf{K}_{uy}^T & \mathbf{K}_{vy}^T & \mathbf{K}_{wy}^T & \mathbf{K}_{xy}^T & \mathbf{K}_{yy} \end{bmatrix}, \quad (\text{A-2})$$

where

$$\mathbf{M}_{uu} = \int_0^b \int_0^a I_0 \boldsymbol{\xi}^T \boldsymbol{\xi} dx dy \quad (\text{A-3})$$

$$\mathbf{M}_{ux} = \int_0^b \int_0^a I_1 \boldsymbol{\xi}^T \boldsymbol{\chi} dx dy \quad (\text{A-4})$$

$$\mathbf{M}_{vv} = \int_0^b \int_0^a I_0 \boldsymbol{\varsigma}^T \boldsymbol{\varsigma} dx dy \quad (\text{A-5})$$

$$\mathbf{M}_{vy} = \int_0^b \int_0^a I_1 \boldsymbol{\varsigma}^T \boldsymbol{\eta} dx dy \quad (\text{A-6})$$

$$\mathbf{M}_{ww} = \int_0^b \int_0^a I_0 \boldsymbol{\phi}^T \boldsymbol{\phi} dx dy \quad (\text{A-7})$$

$$\mathbf{M}_{xx} = \int_0^b \int_0^a I_2 \boldsymbol{\chi}^T \boldsymbol{\chi} dx dy \quad (\text{A-8})$$

$$\mathbf{M}_{yy} = \int_0^b \int_0^a I_2 \boldsymbol{\eta}^T \boldsymbol{\eta} dx dy \quad (\text{A-9})$$

$$\mathbf{K}_{uu} = \int_0^b \int_0^a \left[A_{11} \left(\frac{\partial \boldsymbol{\xi}}{\partial x} \right)^T \left(\frac{\partial \boldsymbol{\xi}}{\partial x} \right) + A_{66} \left(\frac{\partial \boldsymbol{\xi}}{\partial y} \right)^T \left(\frac{\partial \boldsymbol{\xi}}{\partial y} \right) \right] dx dy \quad (\text{A-10})$$

$$\mathbf{K}_{uv} = \int_0^b \int_0^a \left[A_{12} \left(\frac{\partial \xi}{\partial x} \right)^T \left(\frac{\partial \zeta}{\partial y} \right) + A_{66} \left(\frac{\partial \xi}{\partial y} \right)^T \left(\frac{\partial \zeta}{\partial x} \right) \right] dx dy \quad (\text{A-11})$$

$$\mathbf{K}_{ux} = \int_0^b \int_0^a \left[B_{11} \left(\frac{\partial \xi}{\partial x} \right)^T \left(\frac{\partial \chi}{\partial x} \right) + B_{66} \left(\frac{\partial \xi}{\partial y} \right)^T \left(\frac{\partial \chi}{\partial y} \right) \right] dx dy \quad (\text{A-12})$$

$$\mathbf{K}_{uy} = \int_0^b \int_0^a \left[B_{12} \left(\frac{\partial \xi}{\partial x} \right)^T \left(\frac{\partial \eta}{\partial y} \right) + B_{66} \left(\frac{\partial \xi}{\partial y} \right)^T \left(\frac{\partial \eta}{\partial x} \right) \right] dx dy \quad (\text{A-13})$$

$$\mathbf{K}_{vv} = \int_0^b \int_0^a \left[A_{22} \left(\frac{\partial \zeta}{\partial y} \right)^T \left(\frac{\partial \zeta}{\partial y} \right) + A_{66} \left(\frac{\partial \zeta}{\partial x} \right)^T \left(\frac{\partial \zeta}{\partial x} \right) \right] dx dy \quad (\text{A-14})$$

$$\mathbf{K}_{vx} = \int_0^b \int_0^a \left[B_{12} \left(\frac{\partial \zeta}{\partial y} \right)^T \left(\frac{\partial \chi}{\partial x} \right) + B_{66} \left(\frac{\partial \zeta}{\partial x} \right)^T \left(\frac{\partial \chi}{\partial y} \right) \right] dx dy \quad (\text{A-15})$$

$$\mathbf{K}_{vy} = \int_0^b \int_0^a \left[B_{22} \left(\frac{\partial \zeta}{\partial y} \right)^T \left(\frac{\partial \eta}{\partial y} \right) + B_{66} \left(\frac{\partial \zeta}{\partial x} \right)^T \left(\frac{\partial \eta}{\partial x} \right) \right] dx dy \quad (\text{A-16})$$

$$\mathbf{K}_{ww} = \int_0^b \int_0^a \left[(\kappa_s A_{66} + N_x^T) \left(\frac{\partial \phi}{\partial x} \right)^T \left(\frac{\partial \phi}{\partial x} \right) + (\kappa_s A_{66} + N_y^T) \left(\frac{\partial \phi}{\partial y} \right)^T \left(\frac{\partial \phi}{\partial y} \right) \right] dx dy \quad (\text{A-17})$$

$$\mathbf{K}_{wx} = \int_0^b \int_0^a \left[\kappa_s A_{66} \left(\frac{\partial \phi}{\partial x} \right)^T \chi \right] dx dy \quad (\text{A-18})$$

$$\mathbf{K}_{wy} = \int_0^b \int_0^a \left[\kappa_s A_{66} \left(\frac{\partial \phi}{\partial y} \right)^T \eta \right] dx dy \quad (\text{A-19})$$

$$\mathbf{K}_{xx} = \int_0^b \int_0^a \left[\kappa_s A_{66} \chi^T \chi + D_{11} \left(\frac{\partial \chi}{\partial x} \right)^T \left(\frac{\partial \chi}{\partial x} \right) + D_{66} \left(\frac{\partial \chi}{\partial y} \right)^T \left(\frac{\partial \chi}{\partial y} \right) \right] dx dy \quad (\text{A-20})$$

$$\mathbf{K}_{xy} = \int_0^b \int_0^a \left[D_{12} \left(\frac{\partial \chi}{\partial x} \right)^T \left(\frac{\partial \eta}{\partial y} \right) + D_{66} \left(\frac{\partial \chi}{\partial y} \right)^T \left(\frac{\partial \eta}{\partial x} \right) \right] dx dy \quad (\text{A-21})$$

$$\mathbf{K}_{yy} = \int_0^b \int_0^a \left[\kappa_s A_{66} \eta^T \eta + D_{22} \left(\frac{\partial \eta}{\partial y} \right)^T \left(\frac{\partial \eta}{\partial y} \right) + D_{66} \left(\frac{\partial \eta}{\partial x} \right)^T \left(\frac{\partial \eta}{\partial x} \right) \right] dx dy \quad (\text{A-22})$$

where $(A_{ij}, B_{ij}, D_{ij}) = \int_{-h/2}^{h/2} Q_{ij}(1, z, z^2) dz$, $(N_x^T, N_y^T) = -\int_{-h/2}^{h/2} \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{12} & Q_{22} \end{bmatrix} \begin{bmatrix} \alpha \Delta T \\ \alpha \Delta T \end{bmatrix} dz$.

The expressions in the matrices $\mathbf{M}_A$, $\mathbf{D}_S$, $\mathbf{D}_A$, $\mathbf{D}_C$, $\mathbf{K}_S$, $\mathbf{K}_A$ and $\mathbf{K}_C$ are expressed as:

$$\mathbf{M}_A = \begin{bmatrix} m_1 & & & \\ & m_2 & & \\ & & \ddots & \\ & & & m_{N_a} \end{bmatrix} \quad (\text{A-23})$$

$$\mathbf{K}_S = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{Sw} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \mathbf{K}_C = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{K}_{Cw} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \mathbf{K}_A = \begin{bmatrix} k_{l,1} & & & & \\ & k_{l,2} & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & k_{l,N_a} \end{bmatrix} \quad (\text{A-24})$$

$$\mathbf{K}_{Swmn}^{rs} = \sum_j^{N_a} k_{l,j} \varphi_{mn}(x_j, y_j) \varphi_{rs}(x_j, y_j), \quad \mathbf{K}_{Cwj}^{rs} = -k_{l,j} \varphi_{rs}(x_j, y_j) \quad (\text{A-25})$$

$$\mathbf{D}_S = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{D}_{Sw} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \mathbf{D}_C = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{D}_{Cw} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \mathbf{D}_A = \begin{bmatrix} c_1 & & & & \\ & c_2 & & & \\ & & \ddots & & \\ & & & \ddots & \\ & & & & c_{N_a} \end{bmatrix} \quad (\text{A-26})$$

$$\mathbf{D}_{Swmn}^{rs} = \sum_j^{N_a} c_j \varphi_{mn}(x_j, y_j) \varphi_{rs}(x_j, y_j), \quad \mathbf{D}_{Cwj}^{rs} = -c_j \varphi_{rs}(x_j, y_j) \quad (\text{A-27})$$

The expression of the generalized aerodynamic force $\mathbf{Q}$ is given as

$$\mathbf{Q} = \mathbf{K}_a \mathbf{q}_s + \mathbf{C}_a \dot{\mathbf{q}}_s \quad (\text{A-28})$$

where

$$\mathbf{K}_a = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{K}_{ww} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad \mathbf{C}_a = \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_{ww} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix}, \quad (\text{A-29})$$

$$\mathbf{K}_{ww} = \frac{\lambda D_{m0}}{a^3} \int_0^b \int_0^a \boldsymbol{\varphi}^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial x} \right) dx dy, \quad \mathbf{C}_{ww} = \frac{Ma^2 - 2}{Ma^2 - 1} \sqrt{\frac{\mu \lambda D_{m0} \rho_m h}{\beta a^4}} \int_0^b \int_0^a \boldsymbol{\varphi}^T \boldsymbol{\varphi} dx dy \quad (\text{A-30})$$

The nonlinear terms $\mathbf{F}_{NL}$, $\Delta \mathbf{F}$ and $\mathbf{F}_A$ are given as

$$\mathbf{F}_{NL} = [\mathbf{F}_u \quad \mathbf{F}_v \quad \mathbf{F}_w \quad \mathbf{F}_x \quad \mathbf{F}_y]^T, \quad \Delta \mathbf{F} = [\mathbf{0} \quad \mathbf{0} \quad \Delta \mathbf{F}_w \quad \mathbf{0} \quad \mathbf{0}]^T \quad (\text{A-31})$$

where

$$\mathbf{F}_u = \frac{1}{2} \int_0^b \int_0^a \left[\begin{aligned} &A_{11} \left(\frac{\partial \xi}{\partial x} \right)^T \mathbf{C}^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial x} \right)^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial x} \right) \mathbf{C} + A_{12} \left(\frac{\partial \xi}{\partial x} \right)^T \mathbf{C}^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial y} \right)^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial y} \right) \mathbf{C} \\ &+ 2A_{66} \left(\frac{\partial \xi}{\partial y} \right)^T \mathbf{C}^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial x} \right)^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial y} \right) \mathbf{C} \end{aligned} \right] dx dy \quad (\text{A-32})$$

$$\mathbf{F}_v = \frac{1}{2} \int_0^b \int_0^a \left[\begin{aligned} &A_{12} \left(\frac{\partial \xi}{\partial y} \right)^T \mathbf{C}^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial x} \right)^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial x} \right) \mathbf{C} + A_{22} \left(\frac{\partial \xi}{\partial y} \right)^T \mathbf{C}^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial y} \right)^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial y} \right) \mathbf{C} \\ &+ 2A_{66} \left(\frac{\partial \xi}{\partial x} \right)^T \mathbf{C}^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial x} \right)^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial y} \right) \mathbf{C} \end{aligned} \right] dx dy \quad (\text{A-33})$$

[illegible]

$$\mathbf{F}_x = \frac{1}{2} \int_0^b \int_0^a \left[B_{11} \left(\frac{\partial \chi}{\partial x} \right)^T \mathbf{C}^T \left(\frac{\partial \Phi}{\partial x} \right)^T \left(\frac{\partial \Phi}{\partial x} \right) \mathbf{C} + B_{12} \left(\frac{\partial \chi}{\partial x} \right)^T \mathbf{C}^T \left(\frac{\partial \Phi}{\partial y} \right)^T \left(\frac{\partial \Phi}{\partial y} \right) \mathbf{C} \right. \\ \left. + 2B_{66} \left(\frac{\partial \chi}{\partial y} \right)^T \mathbf{C}^T \left(\frac{\partial \Phi}{\partial x} \right)^T \left(\frac{\partial \Phi}{\partial y} \right) \mathbf{C} \right] dx dy \quad (\text{A-35})$$

$$\mathbf{F}_y = \frac{1}{2} \int_0^b \int_0^a \left[B_{12} \left(\frac{\partial \boldsymbol{\eta}}{\partial y} \right)^T \mathbf{C}^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial x} \right)^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial x} \right) \mathbf{C} + B_{22} \left(\frac{\partial \boldsymbol{\eta}}{\partial y} \right)^T \mathbf{C}^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial y} \right)^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial y} \right) \mathbf{C} \right. \\ \left. + 2B_{66} \left(\frac{\partial \boldsymbol{\eta}}{\partial x} \right)^T \mathbf{C}^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial x} \right)^T \left(\frac{\partial \boldsymbol{\varphi}}{\partial y} \right) \mathbf{C} \right] dx dy \quad (\text{A-36})$$

$$\Delta \mathbf{F}_w^{rs} = \sum_j^{N_a} k_{nl,j} \left\{ \sum_{kl}^{MN} \sum_{mn}^{MN} \sum_{pq}^{MN} C_{kl} C_{mn} C_{pq} \varphi_{kl}(x_j, y_j) \varphi_{mn}(x_j, y_j) \varphi_{pq}(x_j, y_j) \varphi_{rs}(x_j, y_j) \right. \\ - 3p_j \sum_{kl}^{MN} \sum_{mn}^{MN} C_{kl} C_{mn} \varphi_{kl}(x_j, y_j) \varphi_{mn}(x_j, y_j) \varphi_{rs}(x_j, y_j) \\ \left. + 3p_j^2 \sum_{kl}^{MN} C_{kl} \varphi_{kl}(x_j, y_j) \varphi_{rs}(x_j, y_j) - p_j^3 \varphi_{rs}(x_j, y_j) \right\} \quad (\text{A-37})$$

$$\mathbf{F}_A^j = k_{nl,j} \left\{ p_j^3 - 3p_j^2 \sum_{kl}^{MN} C_{kl} \varphi_{kl}(x_j, y_j) \right. \\ + 3p_j \sum_{kl}^{MN} \sum_{mn}^{MN} C_{kl} C_{mn} \varphi_{kl}(x_j, y_j) \varphi_{mn}(x_j, y_j) \\ \left. - \sum_{kl}^{MN} \sum_{mn}^{MN} \sum_{pq}^{MN} C_{kl} C_{mn} C_{pq} \varphi_{kl}(x_j, y_j) \varphi_{mn}(x_j, y_j) \varphi_{pq}(x_j, y_j) \right\} \quad (\text{A-38})$$

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Figures

Figure 1

Schematic of the metastructure plate with periodic NVAs under supersonic flow conditions.

Figure 2

LCO amplitude versus dynamic pressure for different numbers of retained modes along the airflow and span directions in the case of a simply supported FGM plate ($p = 1.0$) with 4×4 NVAs.

Figure 3

Nonlinear aeroelastic responses of the simply supported FGM plate ($p = 1.0$) with 4×4 NVAs at $\lambda = 710$ for different numbers of retained modes: (a) plate and (b) 10th NVA.

Figure 4

Comparison of LCO behaviors obtained under the present formulation for an aluminium rectangular plate with a simply supported boundary.

Figure 5

Eigenvalue solutions of the first five modes (under non-dimensional dynamic pressure λ) for the FGM plate ($p = 1.0$) without NVAs.

Figure 6

Critical flutter aerodynamic pressure versus NVA linear stiffness kl for different number of NVAs.

Figure 7

Flutter behaviors of the FGM plate with 1×1 NVAs for $kl = 40$ and $kl = 200$.

Figure 8

Flutter behaviors of the FGM plate with 2×2 NVAs for different NVA linear stiffness coefficients: (a) $k_l = 20$ and (b) $k_l = 40$.

Figure 9

Flutter behaviors of the metastructure plate with 6×6 NVAs for NVA linear stiffness coefficients of $k_l = 20$.

Figure 10

Flutter behaviors of the metastructure plate with 6×6 NVAs for different volume fraction indices: (a) critical flutter aerodynamic pressure and (b) flutter frequency.

Figure 11

Flutter behaviors of the metastructure plate with 6×6 NVAs ($p = 0.2, k_l = 10$) in Region I.

Figure 12

Flutter behaviors of the metastructure plate with 6×6 NVAs ($p = 0.2, k_l = 40$) in Region II.

Figure 13

Effects of temperature rise on the flutter stability of the metastructure plate with 6×6 NVAs: (a) $p = 0.5$ and (b) $p = 2.0$.

Figure 14

Stability boundaries of the metastructure plates with 6×6 NVAs under the effects of aerodynamic and thermal loads: (a) $p = 0.5$ and (b) $p = 2.0$.

Figure 15

Bifurcation diagram in terms of local amplitude extreme for the pure and metastructure plates.

(a)

(b)

(c)

Figure 16

Nonlinear aeroelastic responses at $\lambda = 780$: (a) pure and metastructure plates, (b) the plate and 28th NVA, and (c) the energy time histories for the metastructure plate.

(a)

(b)

(c)

(d)

Figure 17

Nonlinear aeroelastic responses at $\lambda = 806$: (a) pure and metastructure plates, (b) the plate and 28th NVA, (c) energy time histories for the metastructure plate, and (d) time histories of the plate energy and total energy of NVAs.

(a)

(b)

(c)

(d)

Figure 18

Nonlinear aeroelastic responses at $\lambda = 900$: (a) pure and metastructure plates, (b) the plate and 28th NVA, (c) energy time histories for the metastructure plate and (d) time histories of the plate energy and total energy of NVAs for $t \in [0, 5]$.

Figure 19

Power spectrum diagrams of the responses for the plate and 28th NVA at $\lambda = 900$.

Figure 20

Bifurcation diagram of the metastructure plate with different NVA nonlinear stiffness coefficients $k_{nl,j}$ at $\lambda = 1000$.

Figure 21

Bifurcation diagram of local amplitude extreme for the improved metastructure plate ($k_{nl,j} = 40000$).