Agile Attitude Maneuvers with Active Vibration-suppression for Flexible-spacecraft

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Agile attitude maneuvers with active vibration-suppression for flexible-spacecraft

Jianqiao Zhang, Xianglong Kong, Chuang Liu, and Qing Deng

Abstract

This paper addresses the agile attitude stabilization maneuver control of flexible-spacecraft using control moment gyros (CMGs) in the absence of modal information. Here, piezoelectric actuators are employed to actively suppress the vibration of flexible appendages. Both the attitude dynamics and the proposed robust controller are globally developed on the Special Orthogonal Group $SO(3)$, avoiding ambiguities and singularities associated with other attitude representations. More specifically, an observer is first designed to estimate the modal information of vibration. A robust control law is developed by synthesizing a proportional derivative (PD) controller, an adaptive sliding mode controller, and an active vibration-suppression controller, which use the information of the estimated structural modes. The stability of the closed-loop system is proved using Lyapunov stability theory. Finally, numerical examples are performed to show the effectiveness of the proposed method.

Index Terms

Agile attitude control; flexible-spacecraft; $SO(3)$; robust control law; active vibration-suppression.

I. INTRODUCTION

Control moment gyros (CMGs) are momentum exchange devices used for generating high torque which can enable fast attitude maneuvers of large space stations or small agile spacecraft [1]. Moreover, single-gimbal CMGs (SGCMGs) require very small power consumption, such that they can apply very large torques for minimal electrical inputs. The ability to perform fast and agile maneuvers for spacecraft can enable a wide range of current and future space mission applications such as spacecraft rendezvous [2] and high-resolution Earth observation [3]. However, most spacecraft have flexible parts such as solar arrays and antennas for power and communication respectively, since they can increase functionality at a reduced launch cost and within mass requirements [4], [5], [6]. This can be problematic for agile spacecraft since fast attitude maneuvers can excite the elastic vibration of the flexible appendages, which may degrade the control performance and even lead to instability of the system. Thus when employing CMGs on a flexible spacecraft it is essential to suppress these vibrations [7]. Moreover, the modal

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information of appendage vibrations is hard to measure for on-orbit spacecraft, making vibration-suppression a difficult task for an attitude control system. Furthermore, the spacecraft is subject to external disturbances, which will degrade control performance and unavoidably lead to additional vibration of the flexible appendages [8]. These challenges compound the control difficulties when one strives for high performance agile attitude maneuver using CMGs.

Attitude stabilization and tracking problems for flexible-spacecraft have been studied extensively [9]. For instance, proportional derivative (PD) control [10] and PD plus feedforward compensation control [6] were proposed to investigate the attitude tracking control problem. The attitude stabilization problem was addressed in [11] and [12], where the requirement on the angular velocity was removed. In addition, other control methods, such as backstepping control [13], sliding mode control (SMC) [14], PD plus SMC [15] and H_∞ control [16], are also utilized in the area of flexible-spacecraft attitude control. However, these methods do not attempt to suppress the vibration itself but provide robustness to the disturbance caused by it. This approach means that the control is over-engineered, requiring unnecessarily large torques. For example, a sign function of the sliding surface was used in [15] to deal with the upper bound of the disturbance caused by vibrations. Though SMC can guarantee the robustness of the system, the controller is conservative and suffers from chattering.

To realize vibration-suppression, Fracchia et al. [17] developed a low-jerk attitude path planning method based on an analytical smoothing of a bang-off-bang maneuver, Hu et al. [18] proposed a path planning method to generate a smooth attitude maneuver trajectory, and in [19], an integral SMC with input shaping method was proposed for a slewing flexible spacecraft equipped with on-off thrusters. These methods can reduce the vibration of flexible structures during attitude maneuvers, however, path planning and input shaping cannot guarantee the convergence of modal coordinates and will deteriorate the steady-state performance of the attitude control system [8]. Therefore, active vibration control must be incorporated in the loop of the attitude control system [20]. Based on an observer to estimate the appendage vibrations, the concept of using piezoelectric actuators was proposed in [21] for active vibration-suppression of the flexible modes, where the advantages of piezoelectric materials include small size, low mass, and high bandwidth control capability [22]. However, the proposed control algorithm in [21], based on PD control, can only guarantee the bounded stability of the perturbed system.

In addition, the previous work on control design for flexible-spacecraft were all based on coordinate representations, such as the modified Rodriguez parameters (MRPs) or quaternions. As stated in [23], the unwinding problem exists in quaternions, and MRPs and other local coordinate representations suffer from singularities. Fortunately, pioneering studies on the attitude control have been conducted recently based on rotation matrices, such as [24], [25], [26], which provide a global and unique attitude representation on the Special Orthogonal Group $SO(3)$. Since $SO(3)$ is a non-Euclidean manifold the classical methods developed for nonlinear control systems defined on Euclidean space cannot be applied. An alternative way to deal with this problem is to construct a smooth positive definite configuration error function on $SO(3)$, and then the attitude controller is designed based on the error function [23], [27], [28]. This method has also been applied for multiple flexible-spacecraft attitude control [29]. However, actuator dynamics and vibration-suppression were not taken into account.

This paper extends the state-of-the-art in control law design on $SO(3)$ to flexible-spacecraft attitude control

using CMGs and active vibration-suppression using piezoelectric actuators. The controllers are designed without knowledge of the modal coordinates, and without using a co-ordinate representation. Based on the developed model, an observer is designed to estimate the modal information of the flexible appendages, and then an adaptive PD plus SMC (PD+) coupled with an active vibration-suppression controller is proposed. The control algorithm uses the observer outputs, where a modified singular direction avoidance (SDA) steering law based on [30] is employed to generate the attitude control signal. The designed controllers guarantee the uniformly asymptotic stability of the perturbed attitude stabilization and vibration-suppression system.

The remainder of this paper is organized as follows. Sec. II develops the mathematical model of the flexible-spacecraft attitude stabilization system and formulates the control objective of this paper. The main results of this paper including the designs of the appendage vibration observer and the controllers, and the stability analysis of the closed-loop system are presented in Sec. III. Numerical examples of a flexible-spacecraft attitude stabilization scenario is undertaken in Sec. IV to illustrate the performance of the proposed method. Finally, some conclusions are obtained in Sec. V.

II. PROBLEM FORMULATION

A. Flexible-Spacecraft Dynamics Based on Rotation Matrix

In this paper, we investigate the rotational motion of a fully-actuated flexible-spacecraft, where the rigid part is controlled by a cluster of angular momentum exchange actuators, i.e., SGCMGs, and vibration-suppression is realized using piezoelectric actuators. In addition, without loss of generality, the spacecraft is assumed to be equipped with two flexible appendages, such as solar arrays, antennas, or other flexible structures. Firstly, a standard Earth centered inertial frame $\mathcal{F}_I$ is introduced to be the reference frame, whose origin is located in the spacecraft mass center. The body-fixed frame $\mathcal{F}_B$ is defined with its origin located in the mass center of the spacecraft, and its three axes coincide with the spacecraft's principal body axes. Then, the spacecraft's configuration, namely its attitude, is the orientation from $\mathcal{F}_B$ to $\mathcal{F}_I$, which can be described by the set of 3×3 rotation matrices $\mathbf{R} \in \text{SO}(3)$. The Special Orthogonal Group $\text{SO}(3)$ is a Lie group with a determinant of one, and can be described by $\text{SO}(3) = \{\mathbf{R} \in \mathbb{R}^{3 \times 3} : \mathbf{R}^T \mathbf{R} = \mathbf{I}_{3 \times 3}, \det(\mathbf{R}) = 1\}$, where $\mathbf{I}_{3 \times 3}$ is an 3×3 identity matrix [31]. Then, the attitude kinematics and dynamics of a flexible-spacecraft can be described as [4], [5], [23]

$$\dot{\mathbf{R}} = \mathbf{R}\boldsymbol{\Omega}^\times \quad (1)$$

$$\mathbf{J}\dot{\boldsymbol{\Omega}} + \boldsymbol{\Omega}^\times(\mathbf{J}\boldsymbol{\Omega} + \boldsymbol{\Xi}^T \dot{\boldsymbol{\eta}}) + \sum_{i=1}^n \boldsymbol{\Omega}^\times \mathbf{h}_i + \boldsymbol{\Xi}^T \ddot{\boldsymbol{\eta}} = \mathbf{u}_c + \mathbf{d} \quad (2)$$

$$\ddot{\boldsymbol{\eta}} + \mathbf{C}\dot{\boldsymbol{\eta}} + \mathbf{K}\boldsymbol{\eta} = -\boldsymbol{\Xi}\dot{\boldsymbol{\Omega}} - \boldsymbol{\Xi}_p \mathbf{u}_p \quad (3)$$

where $\boldsymbol{\Omega} \in \mathbb{R}^{3 \times 1}$ denotes the angular velocity of the spacecraft expressed in $\mathcal{F}_B$, $\mathbf{J} \in \mathbb{R}^{3 \times 3}$ is the total inertia matrix of the flexible-spacecraft, $\boldsymbol{\Xi} \in \mathbb{R}^{N \times 3}$ is the coupling coefficient matrix between the flexible structures and the rigid body, $\boldsymbol{\eta} \in \mathbb{R}^{N \times 1}$ is the modal coordinate vector reflecting the elastic deflection, N is the number of the vibration modes considered in the dynamics, $\mathbf{h}_i$ is the angular momentum of each actuator, n is the number

of the actuators, $\mathbf{u}_c \in \mathbb{R}^{3 \times 1}$ and $\mathbf{d} \in \mathbb{R}^{3 \times 1}$ are the control torque and external disturbance respectively, $\mathbf{C} = \text{diag}([2\xi_1\omega_1, 2\xi_2\omega_2, \dots, 2\xi_N\omega_N]) \in \mathbb{R}^{N \times N}$ represents the modal damping matrix, $\mathbf{K} = \text{diag}([\omega_1^2, \omega_2^2, \dots, \omega_N^2]) \in \mathbb{R}^{N \times N}$ represents the stiffness matrix, where ξ_j and ω_j ($j = 1, 2, \dots, N$) denote the damping ratios and the natural frequencies respectively, $\mathbf{u}_p \in \mathbb{R}^{M \times 1}$ is the M -dimensional piezoelectric input, and $\mathbf{\Xi}_p \in \mathbb{R}^{N \times M}$ is the corresponding coupling matrix.

The symbol $(\cdot)^\times$ is defined as follows: For any vectors $\mathbf{x} \in \mathbb{R}^{3 \times 1}$ and $\mathbf{y} \in \mathbb{R}^{3 \times 1}$, the mapping $(\cdot)^\times : \mathbb{R}^{3 \times 1} \rightarrow \mathfrak{so}(3)$ can convert $\mathbf{x}$ into a 3×3 skew-symmetric matrix, such that $\mathbf{x}^\times \mathbf{y} = \mathbf{x} \times \mathbf{y}$. $\mathfrak{so}(3)$ is the Lie algebra of $\text{SO}(3)$, and can be described by $\mathfrak{so}(3) = \{\mathbf{H} \in \mathbb{R}^{3 \times 3} | \mathbf{H}^T = -\mathbf{H}\}$. The inverse mapping of $(\cdot)^\times$ is defined as $(\cdot)^\vee$, namely $(\mathbf{x}^\times)^\vee = \mathbf{x}$. In addition, some useful properties are summarized as follows [26], [28]

$$\begin{cases} \text{tr}(\mathbf{H}\mathbf{x}^\times) = -\mathbf{x}^T(\mathbf{H} - \mathbf{H}^T)^\vee \\ (\mathbf{x}^\times \mathbf{H} + \mathbf{H}^T \mathbf{x}^\times)^\vee = (\text{tr}(\mathbf{H})\mathbf{I}_{3 \times 3} - \mathbf{H})\mathbf{x} \\ (\mathbf{R}\mathbf{x})^\times = \mathbf{R}\mathbf{x}^\times \mathbf{R}^T \end{cases} \quad (4)$$

where $\text{tr}(\cdot)$ is the trace of a matrix.

B. Actuator Dynamics

Fig. 1: Pyramid mounting arrangement of SGCMGs

In this paper, four ($n=4$) SGCMGs configured on the x - y plane of the spacecraft constructing a pyramid-type is considered, as depicted in Fig. 1. Then $\mathbf{h} = \sum_{i=1}^n \mathbf{h}_i$ in Eq.(2) can be described by

$$\mathbf{h} = h_0 \begin{bmatrix} -c\beta s\delta_1 - c\delta_2 + c\beta s\delta_3 + c\delta_4 \\ c\delta_1 - c\beta s\delta_2 - c\delta_3 + c\beta s\delta_4 \\ s\beta(s\delta_1 + s\delta_2 + s\delta_3 + s\delta_4) \end{bmatrix} \in \mathbb{R}^{3 \times 1} \quad (5)$$

where h_{0i} is the magnitude of angular momentum of each CMG and without loss of generality, they are equal to h_0 . β is the skew angle, $(\delta_i, i = 1, 2, 3, 4)$ is the i th gimbal angle, $c(x) = \cos(x)$, and $s(x) = \sin(x)$. The dynamics of the SGCMGs can be obtained by [2], [5]

$$\mathbf{u}_c = -\dot{\mathbf{h}} = \mathbf{A}\dot{\boldsymbol{\delta}} \quad (6)$$

where $\boldsymbol{\delta} = [\delta_1 \ \delta_2 \ \delta_3 \ \delta_4]^T$ denotes the gimbal angle vector, and $\mathbf{A}$ is the Jacobian matrix with the following expression

$$\mathbf{A} = -h_0 \begin{bmatrix} -c\beta c\delta_1 & s\delta_2 & c\beta c\delta_3 & -s\delta_4 \\ -s\delta_1 & -c\beta c\delta_2 & s\delta_3 & c\beta c\delta_4 \\ s\beta c\delta_1 & s\beta c\delta_2 & s\beta c\delta_3 & s\beta c\delta_4 \end{bmatrix} \in \mathbb{R}^{3 \times 4} \quad (7)$$

The control signal allocation problem can be described by the following constraint

$$\min \|\dot{\boldsymbol{\delta}}\|^2, \quad \text{subject to } \mathbf{A}\dot{\boldsymbol{\delta}} = \mathbf{u}_c \quad (8)$$

The solution of this minimization problem is [2]

$$\dot{\boldsymbol{\delta}} = \mathbf{A}^\dagger \mathbf{u}_c \quad (9)$$

where $\mathbf{A}^\dagger = \mathbf{A}^T(\mathbf{A}\mathbf{A}^T)^{-1}$, which is referred to as the pseudoinverse steering law. If $\text{rank}(\mathbf{A}) < 3$ for certain sets of $\boldsymbol{\delta}$, the system has been caught in a singular state. A singular state occurs when all the momentum vectors $\mathbf{h}_i$ of the SGCMGs system have either maximal or minimal projections in the direction of $\mathbf{u}_c$. One of the principal difficulties in the widely application of SGCMGs to attitude control is the geometric singularity problem. Therefore, a singular-free steering law is required.

The inputs of the piezoelectric actuators are the vibration displacements of the flexible appendages. However, the modal information of appendage vibrations is hard to measure for on-orbit spacecraft. Thus, the inputs of $\mathbf{u}_p$ will be the estimation of the modal information, which can be expressed by the following equation

$$\mathbf{u}_p = \mathbf{f}(\hat{\boldsymbol{\eta}}, \dot{\hat{\boldsymbol{\eta}}}) \quad (10)$$

where $\hat{\boldsymbol{\eta}}, \dot{\hat{\boldsymbol{\eta}}}$ are the estimation of $\boldsymbol{\eta}$ and its derivative.

C. Relative Attitude Dynamics of a Spacecraft

The attitude stabilization control problem investigated in this paper requires the development of a control law, such that the spacecraft can rotate to a desired smooth attitude command $\mathbf{R}_d \in \text{SO}(3)$

$$\dot{\mathbf{R}}_d = \mathbf{R}_d \boldsymbol{\Omega}_d^\times(t), \quad \forall t \geq 0, \quad \boldsymbol{\Omega}_d \in C^1 \cap L_\infty \quad (11)$$

where $\boldsymbol{\Omega}_d$ is the desired angular velocity. Then we can get the attitude error $\mathbf{R}_e = \mathbf{R}_d^T \mathbf{R} \in \text{SO}(3)$. An appropriate smooth positive configuration error function $\Psi : \text{SO}(3) \times \text{SO}(3) \rightarrow \mathbb{R}$ is required to measure the attitude error $\mathbf{R}_e$, and then the derivative of $\Psi(\mathbf{R}, \mathbf{R}_d)$ is used to obtain an attitude error vector and a velocity error vector in the tangent bundle of $\text{SO}(3)$. Following these, the attitude controller can be designed with these error vectors via Lyapunov analysis. In this paper, the so-called modified trace function $\Psi(\mathbf{R}, \mathbf{R}_d)$ defined in [23], [28] is adopted

$$\Psi(\mathbf{R}, \mathbf{R}_d) = \frac{1}{2} \text{tr}[\mathbf{G}(\mathbf{I}_{3 \times 3} - \mathbf{R}_d^T \mathbf{R})] \quad (12)$$

where $\mathbf{G} = \text{diag}([g_1, g_2, g_3])$, $g_i > 0$, $i = 1, 2, 3$. Taking the derivative of Ψ , we have

$$\frac{d}{dt}(\Psi(\mathbf{R}, \mathbf{R}_d)) = \mathbf{e}_R^T \mathbf{e}_\Omega \quad (13)$$

$$\mathbf{e}_R = \frac{1}{2}(\mathbf{G}\mathbf{R}_d^T \mathbf{R} - \mathbf{R}^T \mathbf{R}_d \mathbf{G})^\vee \quad (14)$$

$$\mathbf{e}_\Omega = \Omega - \mathbf{R}^T \mathbf{R}_d \Omega_d \quad (15)$$

where $\mathbf{e}_R \in \mathbb{R}^{3 \times 1}$ is the attitude tracking error, and $\mathbf{e}_\Omega \in \mathbb{R}^{3 \times 1}$ is the angular velocity tracking error. Since the attitude stabilization problem is considered in this paper, the desired angular velocity is equal to zero, which means $\mathbf{e}_\Omega = \Omega$, and $\Omega_d = 0$.

Defining a new variable $\vartheta = [\eta^T (\dot{\eta} + \Xi \Omega)^T]^T \in \mathbb{R}^{2N \times 1}$, taking the derivatives of $\mathbf{e}_R$ and $\mathbf{e}_\Omega$, substituting Eqs. (2) and (3) into the results, and using the relationships in Eq. (4), we can obtain the relative dynamics of the flexible-spacecraft attitude stabilization system studied in this paper

$$\dot{\mathbf{e}}_R = \mathbf{E}(\mathbf{R}, \mathbf{R}_d) \mathbf{e}_\Omega \quad (16)$$

$$\mathbf{J}^* \dot{\mathbf{e}}_\Omega = -\Omega^\times (\mathbf{J}^* \Omega + \mathbf{Z} \vartheta) + \mathbf{L} \vartheta - \mathbf{F} \Omega - \Omega^\times \mathbf{h} + \mathbf{u}_c + \Xi^T \Xi_p \mathbf{u}_p + \mathbf{d} \quad (17)$$

$$\dot{\vartheta} = \mathbf{A}_\vartheta \vartheta + \mathbf{B}_\vartheta \Omega - [0_{N \times M}^T \Xi_p^T]^T \mathbf{u}_p \quad (18)$$

where $\mathbf{E}(\mathbf{R}, \mathbf{R}_d) = \frac{1}{2}(\text{tr}(\mathbf{R}_e^T \mathbf{G}) \mathbf{I}_{3 \times 3} - \mathbf{R}_e^T \mathbf{G}) \in \mathbb{R}^{3 \times 3}$, $\mathbf{J}^* = \mathbf{J} - \Xi^T \Xi$, $\mathbf{Z} = [0, \Xi^T] \in \mathbb{R}^{3 \times 2N}$, $\mathbf{L} = [\Xi^T \mathbf{K}, \Xi^T \mathbf{C}] \in \mathbb{R}^{3 \times 2N}$, $\mathbf{F} = \Xi^T \mathbf{C} \Xi \in \mathbb{R}^{3 \times 3}$, $\mathbf{A}_\vartheta = \begin{bmatrix} \mathbf{0}_{N \times N} & \mathbf{I}_{N \times N} \\ -\mathbf{K} & -\mathbf{C} \end{bmatrix} \in \mathbb{R}^{2N \times 2N}$, and $\mathbf{B}_\vartheta = \begin{bmatrix} -\Xi \\ \mathbf{C} \Xi \end{bmatrix} \in \mathbb{R}^{2N \times 3}$. It is obvious that $\mathbf{A}_\vartheta$ is Hurwitz. For the developed relative attitude dynamics, the following properties are given (the proof is available in [28])

- 1) $\Psi(\mathbf{R}, \mathbf{R}_d)$ is locally positive definite about $\mathbf{R} = \mathbf{R}_d$.
- 2) The critical points of $\Psi = 0$ are $\mathbf{R} = \{\mathbf{R}_d\} \cup \{\mathbf{R}_d \exp(\pi \mathbf{e}^\times) | \mathbf{e} \in \{e_1, e_2, e_3\}\}$, where e_1, e_2, e_3 are the unit column vectors of $\mathbf{R}_d$.
- 3) $\Psi(\mathbf{R}, \mathbf{R}_d)$ is locally quadratic, satisfying

$$b_1 \|\mathbf{e}_R\|^2 \leq \Psi(\mathbf{R}, \mathbf{R}_d) \leq b_2 \|\mathbf{e}_R\|^2 \quad (19)$$

where $b_1 = h_1/h_2 + h_3$, $b_2 = h_1 h_4 / (h_5(h_1 - \psi))$, $\|\mathbf{e}_R\|$ denotes the Euclidean norm, and ψ is a positive constant with $\Psi(\mathbf{R}, \mathbf{R}_d) < \psi < h_1$. The expressions of $h_i (i = 1, \dots, 5)$ are $h_1 = \min\{g_1 + g_2, g_2 + g_3, g_3 + g_1\}$, $h_2 = \max\{(g_1 - g_2)^2, (g_2 - g_3)^2, (g_3 - g_1)^2\}$, $h_3 = \max\{(g_1 + g_2)^2, (g_2 + g_3)^2, (g_3 + g_1)^2\}$, $h_4 = \max\{g_1 + g_2, g_2 + g_3, g_3 + g_1\}$, and $h_5 = \min\{(g_1 + g_2)^2, (g_2 + g_3)^2, (g_3 + g_1)^2\}$.

- 4) The matrix $\mathbf{E}(\mathbf{R}, \mathbf{R}_d)$ in the relative attitude kinematics (16) is bounded by

$$\|\mathbf{E}(\mathbf{R}, \mathbf{R}_d)\| \leq \frac{\sqrt{2}}{2} \text{tr}(\mathbf{G}) \quad (20)$$

In addition, the following assumptions hold

Assumption 1: The disturbance $\mathbf{d}$ that the spacecraft suffers is unknown but bounded, and satisfies $\|\mathbf{d}\| \leq \bar{d}$, where $\bar{d}$ is an unknown constant.

Assumption 2: Among the feedback states used in the control law design, only the modal information $\boldsymbol{\eta}$ and its first and second order derivatives $\dot{\boldsymbol{\eta}}$, $\ddot{\boldsymbol{\eta}}$ cannot be measured by the spacecraft.

Remark 1: The disturbance in the space environment is due to solar radiation pressure, gravity gradient torque, and aerodynamic torque, which is bounded [4]. Thus, Assumption 1 is satisfied in practice. By introducing the attitude error function Ψ , we develop the model in a coordinate free way [23]. In addition, the piezoelectric input $\mathbf{u}_p$ in Eq. (18) is applied to realize active vibration-suppression. Since it also appears in Eq. (17) and has an influence on the convergence of $\mathbf{e}_\Omega$, the designs of $\mathbf{u}_p$ and $\mathbf{u}_c$ should be considered together in the stability analysis of the overall closed-loop system.

D. Control Objective

The attitude stabilization problem of a flexible-spacecraft is studied in this paper, and the control objective herein can be stated as follows. Based on the relative attitude dynamics of the flexible-spacecraft described by Eqs. (16)-(18), by designing $\mathbf{u}_c$ and $\mathbf{u}_p$ properly, we aim to control the spacecraft to rotate to any given desired attitude $\mathbf{R}_d$, and suppress the flexible vibration $\boldsymbol{\eta}$ simultaneously. Meanwhile, the above reasonable assumptions hold, and the controller $\mathbf{u}_c$ should be robust against external disturbance $\mathbf{d}$. Moreover, $\mathbf{u}_p$ is designed based on the modal information, however, $\boldsymbol{\eta}$ and its derivatives cannot be measured. Therefore, an observer should be designed to estimate the modal information. These goals can be described mathematically as $\lim_{t \rightarrow \infty} \mathbf{e}_R = \mathbf{e}_\Omega = 0$, $\lim_{t \rightarrow \infty} \boldsymbol{\eta} = \dot{\boldsymbol{\eta}} = 0$, $\lim_{t \rightarrow \infty} (\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}) = 0$, and $\lim_{t \rightarrow \infty} (\dot{\boldsymbol{\eta}} - \dot{\hat{\boldsymbol{\eta}}}) = 0$, where $\hat{\boldsymbol{\eta}}$ and $\dot{\hat{\boldsymbol{\eta}}}$ are the observer outputs.

III. GEOMETRIC STABILIZATION CONTROL AND ACTIVE VIBRATION-SUPPRESSION ON SO(3)

In this section, based on the relative attitude dynamics described by Eqs. (16)-(18), we solve the flexible-spacecraft attitude stabilization problem, and realize vibration-suppression at the same time. Since the modal information is not available in the controller design, the first step is to design an observer to estimate $\boldsymbol{\eta}$ and $\dot{\boldsymbol{\eta}}$. This is the key point to guarantee the fine performance of the designed control system. Then an adaptive PD plus SMC and an active vibration-suppression controller $\mathbf{u}_p$ are proposed by using the observer outputs. In addition, the stability of the closed-loop system resulted by $\mathbf{u}_c$ and $\mathbf{u}_p$ is proved. Furthermore, since SGCMGs are employed to provide control torque, the method to distribute control signals to actuators without singularity will be discussed. First, the following necessary lemmas are given to provide theoretical support for the stability analysis of the designed controller.

Lemma 1: (Schur complement lemma [4]) For a symmetric matrix $\mathbf{H}$, ($\mathbf{H} = \mathbf{H}^T$) partitioned as:

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{12}^T & \mathbf{H}_{22} \end{bmatrix} \quad (21)$$

The sufficient and necessary conditions for $\mathbf{H} > 0$ are one of the following two cases holds

$$\begin{cases} \mathbf{H}_{11} > 0, \mathbf{H}_{22} - \mathbf{H}_{12}^T \mathbf{H}_{11}^{-1} \mathbf{H}_{12} > 0 \\ \mathbf{H}_{22} > 0, \mathbf{H}_{11} - \mathbf{H}_{12} \mathbf{H}_{22}^{-1} \mathbf{H}_{12}^T > 0 \end{cases} \quad (22)$$

Lemma 2: (Barbalat's lemma [32]) For a differentiable function $V(t)$ and its derivative $\dot{V}(t)$, if $V(t), \dot{V}(t) \in L_\infty$, and $V(t) \in L_p$ for $p \in [1, \infty)$, then $\lim_{t \rightarrow \infty} V(t) = 0$, where $V(t) \in L_\infty$ means $\sup_{t \geq 0} \|V(t)\| < \infty$, and $V(t) \in L_p$ means $(\int_0^\infty \|V(t)\|^p dt)^{1/p} < \infty$, respectively.

A. Controller Design and Stability Analysis

An observer is required to estimate the modal information as a stepping-stone in the design of $\mathbf{u}_c$ and $\mathbf{u}_p$. The piezoelectric input $\mathbf{u}_p$ is designed by employing a feedback method with

$$\mathbf{u}_p = \begin{bmatrix} \mathbf{K}_1 & \mathbf{K}_2 \end{bmatrix} \hat{\boldsymbol{\vartheta}} \quad (23)$$

where $\mathbf{K}_1, \mathbf{K}_2 \in \mathbb{R}^{1 \times N}$ are constant matrices. $\hat{\boldsymbol{\vartheta}}$ is the output of the observer, and the estimation error is denoted by $\tilde{\boldsymbol{\vartheta}} = \boldsymbol{\vartheta} - \hat{\boldsymbol{\vartheta}}$. Substituting the expression of $\mathbf{u}_p$ into Eq. (18), we have

$$\dot{\boldsymbol{\vartheta}} = \mathbf{A}_{\vartheta 1} \boldsymbol{\vartheta} + \mathbf{B}_{\vartheta} \boldsymbol{\Omega} + \mathbf{B} \tilde{\boldsymbol{\vartheta}} \quad (24)$$

where $\mathbf{A}_{\vartheta 1} = \begin{bmatrix} \mathbf{0}_{N \times N} & \mathbf{I}_{N \times N} \\ -\mathbf{K} - \Xi_p \mathbf{K}_1 & -\mathbf{C} - \Xi_p \mathbf{K}_2 \end{bmatrix} \in \mathbb{R}^{2N \times 2N}$, and $\mathbf{B} = \begin{bmatrix} \mathbf{0}_{N \times N} & \mathbf{0}_{N \times N} \\ \Xi_p \mathbf{K}_1 & \Xi_p \mathbf{K}_2 \end{bmatrix}$. Then, the observer is designed with the following form

$$\dot{\hat{\boldsymbol{\vartheta}}} = \mathbf{A}_{\vartheta 1} \hat{\boldsymbol{\vartheta}} + \mathbf{B}_{\vartheta} \boldsymbol{\Omega} + \mathbf{P}^{-1} (-\mathbf{Z}^T (\boldsymbol{\Omega}^\times)^T \mathbf{S} + \mathbf{L}_1^T \mathbf{S}) \quad (25)$$

where $\mathbf{P} \in \mathbb{R}^{2N \times 2N}$ is a symmetric positive definite matrix, satisfying $\mathbf{A}_{\vartheta}^T \mathbf{P} + \mathbf{P} \mathbf{A}_{\vartheta} < 0$ and $\mathbf{A}_{\vartheta 1}^T \mathbf{P} + \mathbf{P} \mathbf{A}_{\vartheta 1} < 0$, $\mathbf{L}_1 = [\Xi^T (\mathbf{K} + \Xi_p \mathbf{K}_1), \Xi^T (\mathbf{C} + \Xi_p \mathbf{K}_2)] \in \mathbb{R}^{3 \times 2N}$, and $\mathbf{S}$ is an auxiliary error vector defined as

$$\mathbf{S} = \mathbf{e}_\Omega + c \mathbf{e}_R \quad (26)$$

where c is a positive constant. It can be observed that $\mathbf{S}$ has the same structure as a sliding mode surface, by which we propose the following adaptive controller

$$\mathbf{u}_c = \mathbf{u}_{eq} - K_p \mathbf{e}_R - K_d \mathbf{e}_\Omega - \frac{k_1 \mathbf{S}}{\|\mathbf{S}\| + \epsilon k_2^2} + \boldsymbol{\Omega}^\times \mathbf{Z} \hat{\boldsymbol{\vartheta}} - \mathbf{L}_1 \hat{\boldsymbol{\vartheta}} \quad (27)$$

$$\mathbf{u}_{eq} = \boldsymbol{\Omega}^\times \mathbf{h} + \mathbf{F} \boldsymbol{\Omega} + \boldsymbol{\Omega}^\times \mathbf{J}^* \boldsymbol{\Omega} \quad (28)$$

$$\dot{k}_2 = -\gamma \frac{\epsilon k_1 k_2 \|\mathbf{S}\|}{\|\mathbf{S}\| + \epsilon k_2^2} \quad (29)$$

where $K_p, K_d, k_1, k_2, \gamma$, and ϵ are all positive constants, and $k_1 > \bar{d}$.

To analyze the stability of the closed-loop system yielded by the controller (27), we consider the following

candidate Lyapunov function

$$V = \frac{1}{2} \mathbf{e}_\Omega^T \mathbf{J}^* \mathbf{e}_\Omega + K_p \Psi(\mathbf{R}, \mathbf{R}_d) + c \mathbf{J}^* \mathbf{e}_\Omega^T \mathbf{e}_R + \frac{1}{2} \tilde{\boldsymbol{\vartheta}}^T \mathbf{P} \tilde{\boldsymbol{\vartheta}} + \frac{1}{2\gamma} k_2^2 \quad (30)$$

Using the property of Ψ in (19), we have

$$V \geq \frac{1}{2} \mathbf{e}_\Omega^T \mathbf{J}^* \mathbf{e}_\Omega + K_p b_1 \|\mathbf{e}_R\|^2 + c \mathbf{J}^* \mathbf{e}_\Omega^T \mathbf{e}_R + \frac{1}{2} \tilde{\boldsymbol{\vartheta}}^T \mathbf{P} \tilde{\boldsymbol{\vartheta}} \quad (31)$$

By defining $\boldsymbol{\xi} = [\|\mathbf{e}_R\|; \|\mathbf{e}_\Omega\|; \|\tilde{\boldsymbol{\vartheta}}\|] \in \mathbb{R}^{3 \times 1}$, Eq. (31) can be written as

$$\boldsymbol{\xi}^T \mathbf{Q}_1 \boldsymbol{\xi} \leq V \quad (32)$$

where

$$\mathbf{Q}_1 = \begin{bmatrix} b_1 K_p & -\frac{1}{2} c \lambda_{\max}(\mathbf{J}^*) & 0 \\ -\frac{1}{2} c \lambda_{\max}(\mathbf{J}^*) & \frac{1}{2} \lambda_{\min}(\mathbf{J}^*) & 0 \\ 0 & 0 & \frac{1}{2} \lambda_{\min}(\mathbf{P}) \end{bmatrix} \quad (33)$$

$\lambda_{\max}$ and $\lambda_{\min}$ represent the maximum and minimum eigenvalues of a matrix, respectively. From Lemma 1, we have that if c satisfies

$$c < \sqrt{\frac{2b_1 K_p \lambda_{\min}(\mathbf{J}^*)}{\lambda_{\max}^2(\mathbf{J}^*)}} \quad (34)$$

then $\mathbf{Q}_1 > 0$. Thus, it is obvious that V is positive and radially unbounded.

Taking the derivative of V with respect to time along the system trajectories leads to

$$\begin{aligned} \dot{V} &= \mathbf{e}_\Omega^T \mathbf{J}^* \dot{\mathbf{e}}_\Omega + K_p \mathbf{e}_R \cdot \dot{\mathbf{e}}_\Omega + c \mathbf{J}^* \dot{\mathbf{e}}_\Omega^T \mathbf{e}_R + c \mathbf{J}^* \mathbf{e}_\Omega^T \dot{\mathbf{e}}_R + \tilde{\boldsymbol{\vartheta}}^T \mathbf{P} \dot{\tilde{\boldsymbol{\vartheta}}} + \frac{1}{\gamma} k_2 \dot{k}_2 \\ &= \mathbf{e}_\Omega^T (-\boldsymbol{\Omega}^\times (\mathbf{J}^* \boldsymbol{\Omega} + \mathbf{Z} \boldsymbol{\vartheta}) + \mathbf{L} \boldsymbol{\vartheta} - \mathbf{F} \boldsymbol{\Omega} - \boldsymbol{\Omega}^\times \mathbf{h} + \mathbf{u}_c + \boldsymbol{\Xi}^T \boldsymbol{\Xi}_p \mathbf{u}_p + \mathbf{d}) \\ &\quad + c \mathbf{e}_R^T (-\boldsymbol{\Omega}^\times (\mathbf{J}^* \boldsymbol{\Omega} + \mathbf{Z} \boldsymbol{\vartheta}) + \mathbf{L} \boldsymbol{\vartheta} - \mathbf{F} \boldsymbol{\Omega} - \boldsymbol{\Omega}^\times \mathbf{h} + \mathbf{u}_c + \boldsymbol{\Xi}^T \boldsymbol{\Xi}_p \mathbf{u}_p + \mathbf{d}) \\ &\quad + K_p \mathbf{e}_R^T \mathbf{e}_\Omega + c \mathbf{J}^* \mathbf{e}_\Omega^T (\mathbf{E}(\mathbf{R}, \mathbf{R}_d) \mathbf{e}_\Omega) + \tilde{\boldsymbol{\vartheta}}^T \mathbf{P} \dot{\tilde{\boldsymbol{\vartheta}}} + \frac{1}{\gamma} k_2 \dot{k}_2 \end{aligned} \quad (35)$$

Substituting the adaptive control law (27) and using (26), yield

$$\begin{aligned} \dot{V} &= \mathbf{S}^T (\mathbf{L} \boldsymbol{\vartheta} - \mathbf{L}_1 \hat{\boldsymbol{\vartheta}} - \boldsymbol{\Omega}^\times \mathbf{Z} \boldsymbol{\vartheta} + \boldsymbol{\Omega}^\times \mathbf{Z} \hat{\boldsymbol{\vartheta}} + \boldsymbol{\Xi}^T \boldsymbol{\Xi}_p \mathbf{u}_p + \mathbf{d} - \frac{k_1 \mathbf{S}}{\|\mathbf{S}\| + \epsilon k_2^2}) \\ &\quad + \mathbf{e}_\Omega^T (-K_p \mathbf{e}_R - K_d \mathbf{e}_\Omega) + c \mathbf{e}_R^T (-K_p \mathbf{e}_R - K_d \mathbf{e}_\Omega) + K_p \mathbf{e}_R^T \mathbf{e}_\Omega \\ &\quad + c \mathbf{J}^* \mathbf{e}_\Omega^T (\mathbf{E}(\mathbf{R}, \mathbf{R}_d) \mathbf{e}_\Omega) + \tilde{\boldsymbol{\vartheta}}^T \mathbf{P} (\dot{\tilde{\boldsymbol{\vartheta}}} - \dot{\boldsymbol{\vartheta}}) + \frac{\epsilon k_1 k_2^2 \|\mathbf{S}\|}{\|\mathbf{S}\| + \epsilon k_2^2} \end{aligned} \quad (36)$$

Substituting the observer (25) and $\mathbf{u}_p$ into (36), and after some algebraic manipulations, one can obtain

$$\begin{aligned}
\dot{V} &= -K_d \|\mathbf{e}_\Omega\|^2 - K_p c \|\mathbf{e}_R\|^2 - cK_d \mathbf{e}_R^T \mathbf{e}_\Omega + c\mathbf{J}^* \mathbf{e}_\Omega^T (\mathbf{E}(\mathbf{R}, \mathbf{R}_d) \mathbf{e}_\Omega) \\
&\quad + \mathbf{S}^T \mathbf{d} - \mathbf{S}^T \frac{k_1 \mathbf{S}}{\|\mathbf{S}\| + \epsilon k_2^2} + \frac{\epsilon k_1 k_2^2 \|\mathbf{S}\|}{\|\mathbf{S}\| + \epsilon k_2^2} + \mathbf{S}^T (-\Omega^\times \mathbf{Z} \tilde{\boldsymbol{\theta}} + \mathbf{L}_1 \tilde{\boldsymbol{\theta}}) \\
&\quad + \tilde{\boldsymbol{\theta}}^T \mathbf{P} (\mathbf{A}_{\vartheta 1} \tilde{\boldsymbol{\theta}} + \mathbf{B}_{\vartheta} \Omega + \mathbf{B} \tilde{\boldsymbol{\theta}} - \mathbf{A}_{\vartheta 1} \hat{\boldsymbol{\theta}} - \mathbf{B}_{\vartheta} \Omega - \mathbf{P}^{-1} (-\mathbf{Z}^T (\Omega^\times)^T \mathbf{S} + \mathbf{L}_1^T \mathbf{S})) \\
&= -K_d \|\mathbf{e}_\Omega\|^2 - K_p c \|\mathbf{e}_R\|^2 - cK_d \mathbf{e}_R^T \mathbf{e}_\Omega + c\mathbf{J}^* \mathbf{e}_\Omega^T (\mathbf{E}(\mathbf{R}, \mathbf{R}_d) \mathbf{e}_\Omega) \\
&\quad + \mathbf{S}^T \mathbf{d} - k_1 \|\mathbf{S}\| + \tilde{\boldsymbol{\theta}}^T \mathbf{P} (\mathbf{A}_{\vartheta 1} \tilde{\boldsymbol{\theta}} + \mathbf{B} \tilde{\boldsymbol{\theta}})
\end{aligned} \tag{37}$$

From (20), we have $\|\mathbf{E}(\mathbf{R}, \mathbf{R}_d)\| \leq \frac{\sqrt{2}}{2} \text{tr}(\mathbf{G})$. Furthermore, $\mathbf{A}_{\vartheta 1} + \mathbf{B} = \mathbf{A}_\vartheta$. Then (37) can be changed to

$$\begin{aligned}
\dot{V} &\leq -K_d \|\mathbf{e}_\Omega\|^2 - K_p c \|\mathbf{e}_R\|^2 - cK_d \mathbf{e}_R^T \mathbf{e}_\Omega + \frac{\sqrt{2}c}{2} \lambda_{\max}(\mathbf{J}^*) \text{tr}(\mathbf{G}) \mathbf{e}_\Omega^T \mathbf{e}_\Omega \\
&\quad + \|\mathbf{S}\| \bar{d} - k_1 \|\mathbf{S}\| - \tilde{\boldsymbol{\theta}}^T \mathbf{W} \tilde{\boldsymbol{\theta}}
\end{aligned} \tag{38}$$

where $\mathbf{W} > 0$ is the solution of $\mathbf{A}_\vartheta^T \mathbf{P} + \mathbf{P} \mathbf{A}_\vartheta = -\mathbf{W}$. Since $k_1 > \bar{d}$ is guaranteed in the controller, therefore, we have the following inequality

$$\dot{V} \leq -\left(K_d - \frac{\sqrt{2}c}{2} \lambda_{\max}(\mathbf{J}^*) \text{tr}(\mathbf{G})\right) \|\mathbf{e}_\Omega\|^2 - cK_p \|\mathbf{e}_R\|^2 + cK_d \|\mathbf{e}_\Omega\| \|\mathbf{e}_R\| - \lambda_{\min}(\mathbf{W}) \|\tilde{\boldsymbol{\theta}}\|^2 \tag{39}$$

This inequality can be rewritten as

$$\dot{V} \leq -\boldsymbol{\xi}^T \mathbf{Q}_2 \boldsymbol{\xi} \tag{40}$$

where

$$\mathbf{Q}_2 = \begin{bmatrix} cK_p & -\frac{cK_d}{2} & 0 \\ -\frac{cK_d}{2} & K_d - \frac{\sqrt{2}c}{2} \lambda_{\max}(\mathbf{J}^*) \text{tr}(\mathbf{G}) & 0 \\ 0 & 0 & \lambda_{\min}(\mathbf{W}) \end{bmatrix} \tag{41}$$

From Lemma 1, we have that if c satisfies

$$c < \min\left\{\frac{4K_p K_d}{2\sqrt{2}K_p \lambda_{\max}(\mathbf{J}^*) \text{tr}(\mathbf{G}) + K_d^2}, \frac{\sqrt{2}K_d}{\lambda_{\max}(\mathbf{J}^*) \text{tr}(\mathbf{G})}\right\} \tag{42}$$

then $\mathbf{Q}_2$ is positive. As a result, we have $\dot{V} \leq 0$. $V > 0$, and $\dot{V} \leq 0$ mean that V has a lower bound and it is nonincreasing. Therefore, $V, \dot{V} \in L_\infty$.

Integrating both sides of (40), one can obtain

$$V(t) \leq V(0) - \lambda_{\min}(\mathbf{Q}_2) \int_0^t \|\boldsymbol{\xi}\|^2 dt \tag{43}$$

which can be further expressed as

$$\|\boldsymbol{\xi}\|_{L_2} = (\lambda_{\min}(\mathbf{Q}_2) \int_0^t \|\boldsymbol{\xi}\|^2 dt)^{1/2} \leq (V(0) - V(t))^{1/2} \leq \sqrt{V(0)} \tag{44}$$

Hence, $\boldsymbol{\xi} \in L_2$, and $\boldsymbol{\xi} \in L_\infty$. According to Lemma 2, we have $\mathbf{e}_R, \mathbf{e}_\Omega, \tilde{\boldsymbol{\theta}} \rightarrow 0$ as $t \rightarrow \infty$.

If the attitude maneuver has been completed, which means $e_\Omega = \Omega = \tilde{\vartheta} = 0$, then Eq. (24) becomes

$$\dot{\vartheta} = A_{\vartheta 1} \vartheta \quad (45)$$

Then, another candidate Lyapunov function is considered

$$V_1 = \vartheta^T P \vartheta \quad (46)$$

Taking the time derivative of V_1 , and substituting Eq. (46) into the result, yield

$$\dot{V}_1 = \vartheta^T (A_{\vartheta 1}^T P + P A_{\vartheta 1}) \vartheta \quad (47)$$

For P , it satisfies $A_{\vartheta 1}^T P + P A_{\vartheta 1} < 0$, thus we have $\dot{V}_1 \leq -\lambda_{\min}(M) \|\vartheta\|^2$, where $M = -(A_{\vartheta 1}^T P + P A_{\vartheta 1}) > 0$. Then using similar stability analysis process as Eqs. (43) and (44) and Lemma 2, we have $\lim_{t \rightarrow \infty} \vartheta = 0$. In summary, the above analysis can be concluded by the following theorem

Theorem 1: Considering the flexible-spacecraft attitude error dynamics described by Eqs. (16)-(18), with the adaptive controller u_c in (27) and the active vibration-suppression control u_p in (23), if Assumptions 1 and 2 hold, and c satisfies

$$c < \min \left\{ \sqrt{\frac{2b_1 K_p \lambda_{\min}(\mathbf{J}^*)}{\lambda_{\max}^2(\mathbf{J}^*)}}, \frac{4K_p K_d}{2\sqrt{2}K_p \lambda_{\max}(\mathbf{J}^*) \text{tr}(\mathbf{G}) + K_d^2}, \frac{\sqrt{2}K_d}{\lambda_{\max}(\mathbf{J}^*) \text{tr}(\mathbf{G})} \right\} \quad (48)$$

then

- 1) The observer Eq. (25) can estimate the modal information η and $\dot{\eta}$, and $\lim_{t \rightarrow \infty} \tilde{\vartheta} = 0$.
- 2) The controller can guarantee the almost global asymptotic stability of the closed-loop system, i.e., $\mathbf{R} \rightarrow \mathbf{R}_d$ and $\Omega = 0$ as $t \rightarrow \infty$.
- 3) Active vibration-suppression is achieved, i.e., $\lim_{t \rightarrow \infty} \vartheta = 0$.

Remark 2: The proposed controller (27) includes an adaptive term $-\frac{k_1 \mathbf{S}}{\|\mathbf{S}\| + \epsilon k_2^2}$ with updated law (29). This part is applied to deal with the system disturbance. In addition, to eliminate the influence of the flexible deformation on attitude motion, two terms $\Omega^\times \mathbf{Z} \hat{\vartheta}$ and $-L_1 \hat{\vartheta}$ are added into the controller, where η and $\dot{\eta}$ are replaced by the observer estimated information. In [15], sign function is used to realize similar control objective with an assumption that the modal information has an upper bound, which will cause chattering phenomenon. Though tanh function is employed to replace sign function, it implies deterioration in control accuracy. Therefore, we can conclude that the control law of this paper is less conservative. Moreover, for the adaptive term, there is a problem that if k_2 goes to zero before the sliding surface $\mathbf{S}$, then it will turn into $-k_1 \text{sgn}(\mathbf{S})$. As a result, the chattering phenomenon cannot be avoided. Considering (29), we have $\gamma \frac{\epsilon k_1 k_2 \|\mathbf{S}\|}{\|\mathbf{S}\| + \epsilon k_2^2} \leq \gamma \epsilon k_1 k_2$, and then the inequality $\dot{k}_2 \geq -\gamma \epsilon k_1 k_2$ holds. Integrating both sides of this inequality, we can obtain $k_2(t) \geq k_0 \exp(-\gamma \epsilon k_1 t)$, where $k_2(t=0) = k_0 > 0$. We can conclude that $k_2(t) > 0$ holds all the time, and only tends to zero as $t \rightarrow \infty$. Consequently, it can be guaranteed that k_2 converges to some positive nonzero values in simulation by selecting γ and ϵ suitably.

Remark 3: As stated in the works of Liu [4] and Chen [26], spillover is an important problem in the vibration-suppression of flexible-spacecraft. In this paper, the proposed controller u_c and the active vibration-suppression

control $\mathbf{u}_p$ can achieve the control objective with the first N modes. By selecting N large enough, the effect of residual modes on the overall system performance can be neglected, which means the spillover effects can be ignored with the controller proposed in this paper. In addition, from the above stability analysis, $\mathbf{u}_p$ can realize active vibration-suppression if $\mathbf{A}_{\vartheta_1}$ is Hurwitz, and the values of $\mathbf{K}_1$ and $\mathbf{K}_2$ can be obtained by choosing $\mathbf{P}$ and $\mathbf{W}$ properly.

Remark 4: In Theorem 1, we declare that the closed-loop system yielded by $\mathbf{u}_c$ and $\mathbf{u}_p$ is almost globally asymptotically stable. $\Psi = 0$ means the attitude of the spacecraft has been rotated to the ideal attitude by the designed controller $\mathbf{u}_c$. However, it should be noted that $\Psi = 0$ has three critical points, which means that when $\mathbf{e}_R = 0$, $\mathbf{R}$ can converge to $\mathbf{R}_d$ or the other three critical points. If we show that these three undesired equilibria are unstable and the attraction regions to them have zero measure, then we can conclude that for almost all initial attitude errors, the spacecraft attitude can converge to the desired attitude $\mathbf{R}_d$. Therefore, the statement in Theorem 1 holds. At the first critical point $\mathbf{R}_1 = \mathbf{R}_d \exp(\pi \mathbf{e}_1^\times)$, we have $\Psi = g_2 + g_3$. Define $V_2 = K_p(g_2 + g_3) - V$, then $V_2 = 0$ at $\mathbf{R}_1$. Since Ψ is continuous, in the neighbourhood of $\mathbf{R}_1$ on $\text{SO}(3)$, there exists $\mathbf{R}_{1\Delta} \in U$ such that $K_p(g_2 + g_3) - \Psi(\mathbf{R}_{1\Delta}, \mathbf{R}_d) > 0$. If $\|\mathbf{e}_\Omega\|$, $\|\tilde{\boldsymbol{\vartheta}}\|$, and k_2/γ are sufficiently small, then $V_2 > 0$ can be guaranteed, since

$$V_2 \geq K_p(g_2 + g_3 - \Psi) - \frac{\lambda_{\max}(\mathbf{J}^*)}{2} \|\mathbf{e}_\Omega\|^2 - c\lambda_{\max}(\mathbf{J}^*) \|\mathbf{e}_\Omega\| \|\mathbf{e}_R\| - \frac{\lambda_{\max}(\mathbf{P})}{2} \|\tilde{\boldsymbol{\vartheta}}\|^2 - \frac{1}{2\gamma} k_2^2 \quad (49)$$

Moreover, in the domain U , $\dot{V}_2 = -\dot{V} > 0$. Therefore, the critical point $\mathbf{R}_1$ is unstable and the instabilities of the other two critical points can be proved by the similar way. Furthermore, to guarantee the global asymptotic stability of the closed-loop system, if the spacecraft attitude moves to the neighborhoods of these undesired points, the hybrid controller such as in [33] could be adopted.

B. Steering Law Design

In this paper, to reduce the impact of singularity, the SDA steering law in [30] is considered. A singularity measure $\kappa = \det(\mathbf{A}\mathbf{A}^T)$ can be defined to describe the singularity degree [5], and $\kappa = 0$ means the system encounters singular state. Modifications have been made to Eq. (9) to generate the so-called singularity-robust (SR) steering laws such as [2], [5] to avoid the singularity problem. However, it has been proved in [30] that the SDA steering law has less torque error and the rotational motion is much smoother than the SR law. Therefore, a modified SDA law will be adopted in this paper to generate the required control torques.

First, the Jacobian matrix $\mathbf{A}$ can be decomposed into the product of three special matrices by the singular value decomposition (SVD) method

$$\mathbf{A} = \mathbf{U}_a \mathbf{S}_a \mathbf{V}_a^T \quad (50)$$

where $\mathbf{U}_a$ is an 3×3 unitary matrix, $\mathbf{V}_a$ is an 4×4 unitary matrix, and $\mathbf{S}_a$ is an 3×4 diagonal matrix and the last column of $\mathbf{S}_a$ is zero. By discarding the zero column of $\mathbf{S}_a$ and $\mathbf{V}_a$, we have

$$\mathbf{A}^\dagger = \mathbf{V}_t \mathbf{S}_t^{-1} \mathbf{U}_a^T \quad (51)$$

where $\mathbf{V}_t$ and $\mathbf{S}_t$ are the truncated matrices.

For $\mathbf{S}_t$, its singular values are positive and ordered such that

$$\mathbf{S}_t[1, 1] \geq \mathbf{S}_t[2, 2] \geq \mathbf{S}_t[3, 3] \geq 0 \quad (52)$$

When a singularity occurs, the rank of $\mathbf{A}$ is 2, and $\mathbf{S}_t[3, 3] = 0$. Preventing $\mathbf{S}_t[3, 3]$ going to zero will make sure that the singularity is avoided. Therefore, the steering law is designed as follows

$$\dot{\delta} = \mathbf{V}_t \mathbf{S}_{\text{SDA}}^\dagger \mathbf{U}_a^T \mathbf{u}_c \quad (53)$$

where

$$\mathbf{S}_{\text{SDA}}^\dagger = \text{diag}\left(\frac{1}{\mathbf{S}_t[1, 1]}, \frac{1}{\mathbf{S}_t[2, 2]}, \frac{\mathbf{S}_t[3, 3]}{\mathbf{S}_t^2[3, 3] + \alpha}\right) \quad (54)$$

with $\alpha = \alpha_0 k_3 \sqrt{\kappa} / \sinh(k_3 \sqrt{\kappa})$, where $\alpha_0 > 0$, and $k_3 > 0$.

IV. SIMULATION RESULTS

In this section, simulations are conducted to illustrate the effectiveness of the proposed controller with an application to a flexible-spacecraft agile attitude stabilization scenario. Meanwhile, active vibration-suppression should be achieved simultaneously. The spacecraft operates in a low Earth orbit, and its attitude needs to stabilize to

$$\mathbf{R}_d = \begin{bmatrix} -0.766 & -0.643 & 0 \\ 0.455 & -0.542 & -0.707 \\ 0.455 & -0.542 & 0.707 \end{bmatrix}$$

Initially, the attitude, angular velocity, modal displacement and its derivative of the spacecraft are

$$\mathbf{R}(0) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{\Omega}(0) = [-0.001; -0.001; 0.001] \text{ rad/s}$$

$\boldsymbol{\eta}(0) = [0.001; 0.001; 0.001; 0.001]$, and $\dot{\boldsymbol{\eta}}(0) = [0.0005; 0.0005; 0.0005; 0.0005]$, respectively.

The model parameters of the rigid part are shown in Table I. It should be noted that the diagonal values of $\mathbf{J}$, namely $\mathbf{J}_0 = \text{diag}([350, 270, 190])\text{kg} \cdot \text{m}^2$, will be used to compute the control signal in order to show the robustness of the proposed controller against model uncertainty. In addition, four natural modes of the flexible appendages are considered in the simulation, and the main parameters of the flexible part are selected from [4], [6], [21] as follows. The coupling matrices $\boldsymbol{\Xi}$ and $\boldsymbol{\Xi}_p$ are

$$\boldsymbol{\Xi} = \begin{bmatrix} 6.45637 & 1.27814 & 2.15629 \\ -1.25619 & 0.91756 & -1.67264 \\ 1.11687 & 2.48901 & -0.83674 \\ 1.23637 & -2.6581 & -1.12503 \end{bmatrix} \sqrt{\text{kg} \cdot \text{m}}$$

$$\Xi_p = 0.01 \times \begin{bmatrix} 2.342552 \\ -0.422537 \\ 3.912984 \\ 7.026176 \end{bmatrix} \sqrt{\text{kg}} \cdot \text{m/s}^2/\text{V}$$

The natural frequencies are $\omega_1 = 0.7681$ rad/s, $\omega_2 = 1.1038$ rad/s, $\omega_3 = 1.8733$ rad/s, $\omega_4 = 2.5496$ rad/s, and the damping ratios are $\xi_1 = 0.005607$, $\xi_2 = 0.00862$, $\xi_3 = 0.01283$, $\xi_4 = 0.02516$. The external disturbance that the spacecraft suffers is chosen as

$$\mathbf{d} = \begin{bmatrix} 0.012 \sin(0.18t) + 0.005 \\ 0.02 \cos(0.15t) + 0.005 \\ 0.015 \sin(0.15t) + 0.005 \end{bmatrix} \text{N} \cdot \text{m}$$

TABLE I: Spacecraft parameters

Parameters	Values
Spacecraft inertia matrix $\mathbf{J}$	$\begin{bmatrix} 350 & 3 & 4 \\ 3 & 270 & 10 \\ 4 & 10 & 190 \end{bmatrix} (\text{kg} \cdot \text{m}^2)$
Angular momentum magnitude of CMG h_0	50 ($\text{kg} \cdot \text{m}^2/\text{s}$)
Initial gimbal angles $\boldsymbol{\delta}_{t=0}$	$[0 \ 0 \ 0]^\text{T}$ (deg)
Maximum of the gimbal rate $\dot{\delta}_{\max}$	10 (deg/s)
Skew angle β	54.73 (deg)

In this section, we consider two cases. In Case 1, the spacecraft attitude is governed by the proposed controller (27) and the modal information is estimated by the observer (25), and the parameters are: $K_p = 20$, $K_d = 100$, $k_1 = 8$, $k_2(0) = 1$, $c = 0.2$, $\epsilon = 0.005$, $\gamma = 0.02$, $\mathbf{A}_\vartheta^\text{T} \mathbf{P} + \mathbf{P} \mathbf{A}_\vartheta = -5000 \mathbf{I}_{8 \times 8}$, $\mathbf{K}_1 = [1000, -600, 100, -2.5]$, $\mathbf{K}_2 = [600, 800, -200, 5]$, $\mathbf{K} = \text{diag}([0.8, 1.25, 1])$, $\alpha_0 = 0.0001$, $k_3 = 10$, $\hat{\boldsymbol{\eta}}(0) = \mathbf{0}_{4 \times 1}$, and $\dot{\hat{\boldsymbol{\eta}}}(0) = \mathbf{0}_{4 \times 1}$. In the second case, a traditional PD-like controller similar as [28] with the following form

$$\mathbf{u}_{cpd} = \mathbf{u}_{eq} - K_p \mathbf{e}_R - K_d \mathbf{e}_\Omega + \boldsymbol{\Omega}^\times \mathbf{Z} \boldsymbol{\vartheta} - \mathbf{L}_1 \boldsymbol{\vartheta} \quad (55)$$

is employed for the purpose of comparison to show the advantages of the controller proposed in this paper. Though the controller in [28] is applied to a rigid body, assuming that the modal information is available and the spacecraft operates in an environment without disturbance, the controller can also guarantee the stability of the closed-loop flexible-spacecraft attitude stabilization system with the following candidate Lyapunov function

$$V_3 = \frac{1}{2} \mathbf{e}_\Omega^\text{T} \mathbf{J}^* \mathbf{e}_\Omega + K_p \Psi(\mathbf{R}, \mathbf{R}_d) + c \mathbf{J}^* \mathbf{e}_\Omega^\text{T} \mathbf{e}_R + \frac{1}{2} \boldsymbol{\vartheta}^\text{T} \mathbf{P} \boldsymbol{\vartheta} \quad (56)$$

The controller parameters are consistent with each other in these two cases.

The simulation results of Case 1 are shown in Figs. 2-11. Figs. 2 and 3 display the attitude and angular velocity errors of the flexible spacecraft in terms of $\mathbf{e}_R$ and $\mathbf{e}_\Omega$, from which it can be seen that both $\mathbf{e}_R$ and $\mathbf{e}_\Omega$ can converge

to within a small bound under controller (27) no more than 50s, and the control accuracy is $|e_{Ri}| \leq 4 \times 10^{-3}$ deg and $|e_{\Omega i}| \leq 5 \times 10^{-4}$ deg/s. Figs. 4 and 5 plot the estimation of modal coordinates and their estimation errors. It can be observed that the modal coordinates can be estimated by the proposed observer (25) with high accuracy. The time responses of the modal coordinates ϑ are shown in Fig. 6, which illustrate that the vibration is suppressed effectively by the proposed active vibration-suppression controller u_p . The plots of the control torques are given in Fig. 7, which are generated by the SGCMGs. The corresponding gimbal angles and gimbal rates are presented in Figs. 8 and 9, from which we can see that the limit on the gimbal rate in Table I is satisfied. Moreover, the time responses of the singularity measure are given in Fig. 10, which imply that the system avoids being caught in a singular state during the attitude maneuver. In addition, time responses of the adaptive gain k_2 are given in Fig. 11. It can be seen that k_2 decreases monotonically to nonzero values as illustrated in Remark 2. From the above simulation results, we can conclude that the tested results of Case 1 mean the control objective of this paper has been achieved with fine performance.

Fig. 2: Time responses of e_R Fig. 3: Time responses of e_{Ω} Fig. 4: Estimation of modal coordinates $\hat{\vartheta}_i$ Fig. 5: Estimation errors of modal coordinates $\tilde{\vartheta}_i$

Fig. 6: Responses of modal coordinates ϑ_i Fig. 7: Time responses of control torques u_c

Fig. 8: Time responses of gimbal angles

Fig. 9: Time responses of gimbal rates

Fig. 10: Time responses of singularity measure

Fig. 11: Time responses of the adaptive parameter k_2

The simulation results of Case 2 are presented in Figs. 12-18. Figs. 12 and 13 show the time responses of the attitude and angular velocity errors under controller (55). The steady-state errors are $|e_{Ri}| \leq 0.05$ deg and $|e_{\Omega i}| \leq 0.01$ deg/s. Moreover, the limit on the gimbal rate in Table I is satisfied and the system also avoids being caught in a singular state during the attitude maneuver. However, comparing the control accuracy of these two cases with each other, we can find that not only the control accuracy of the attitude stabilization maneuver has been

improved, but also the vibration-suppression performance is better than the traditional PD-like controller in (55). Meanwhile, the requirement for the modal information in the designed control law is released successfully.

Fig. 12: Time responses of e_R (Case 2)

Fig. 13: Time responses of e_{Ω} (Case 2)

Fig. 14: Time responses of control torques u_c (Case 2)

Fig. 15: Responses of modal coordinates ϑ_i (Case 2)

Fig. 16: Time responses of gimbal angles (Case 2)

Fig. 17: Time responses of gimbal rates (Case 2)

Fig. 18: Time responses of singularity measure (Case 2)

V. CONCLUSIONS

In this paper, a new control law for performing agile attitude stabilization maneuvers for flexible-spacecraft using CMGs is proposed. The control law is designed on the Special Orthogonal group $SO(3)$, which allows the problem to be formulated in a unique and singular-free way. The control law is able to provide high precision attitude stabilization and active vibration-suppression during an agile maneuver. This is accomplished without modal information and in the presence of external disturbances. An observer is designed to estimate the modal information, and coupled with an adaptive PD+ controller and an active vibration-suppression controller are able to guarantee stability of the closed-loop system. The robustness and chattering avoidance of the controller can be guaranteed with the aid of the adaptive law. Moreover, adding the active vibration-suppression controller to the overall control algorithm can reduce the adverse effects on the spacecraft caused by the vibrations. Theoretical proofs and numerical simulations are carried out to support the analysis of the designed control law, and the simulation results reveal that the attitude stabilization maneuver and vibration-suppression can be achieved with fine performance. In addition, comparisons with the PD-like controller also help to demonstrate the superiority of the proposed controller.

VI. DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Figures

Figure 1

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