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A Novel Tooth Tip Relief Method for Reducing Micro-pitting of Spur Gears

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Abstract: Micro-pitting is a common fatigue failure mode of spur gears. To reduce it, this paper proposes a novel tooth tip relief method. Gear meshing simulation is firstly performed considering the actual contact path, showing that there are four Hertz contact stress peaks in the transition areas of the tooth flank. Equations are then developed for the novel tooth tip relief method: one focuses on the smooth transition between the involute profile and the tip relief region, the other on the smooth transition between the tip relief region and the tooth tip. The results of curvature radius and Hertz contact stress show that the proposed novel method can effectively reduce the Hertz contact stress peaks, which indicates this tip relief method could benefit for reducing the micro-pitting. Finally, both wear simulations and bench tests are conducted, verifying the effectiveness of the proposed method for reducing the micro-pitting of spur gears.

Key words: Tooth tip relief; Micro-pitting; Gear meshing simulation; Hertz contact stress; Smooth transition.

1. Introduction

Spur gears are widely used in heavy-load machinery, such as vehicles, railway equipment, ships, and wind turbines. In the meshing process of spur gears, various fatigue modes occur on the tooth surface owing to the complex and harsh working environment. Among them, micro-pitting is a predominant early stage fatigue mode in most engineering applications. Micro-pitting, manifest as matt patches in tooth-flank areas, causes unexpected vibration and noise [1,2], and may propagate into macro-pitting, spall, and even tooth fracture [3], ultimately leading to the failure of the gears.

Recently, numerous studies on micro-pitting have been carried out, including contact stress, material properties, microstructure, tribological conditions and load, etc. Using a finite element elastic plastic contact model [4], Wang et al. [5] found that rolling contact fatigue decreases as tensile stress increases. Roy et al. [6] found that greater retention of austenite leads to reduce micro-pitting with the same cycles. In addition, studies on the microstructure of the gear surface [7-9] have revealed that the anisotropy and randomness of the crystal plasticity result in variations in the crack initiation

Nomenclature

a	Center distance
b	Tooth width
C_a	Amount of tooth tip relief
C_l	Starting positions of the arctangent proportional tip relief
c	Constant coefficient of arctangent proportional tip relief
d_a	Tip diameter
F	Force on the tooth
i_G	Transmission ratio
L	Backlash of the gears
l_{eff}	Effective tooth width
l_r	Starting positions of the progressive fillet
m_n	Normal modulus
p	Hertz contact stress
R_a	Average roughness
R_s	Crack criterion
r	Flank radius
r_k	Radius of tooth tip fillet
W	Comprehensive deformation of the meshing tooth at the contact point
$w_{tb}, w_{ts}, w_{tr}, w_r$	Different kind of tooth deformation
X_a	Height coordinate of arctangent proportional tip relief
X_b	Height coordinate of progressive fillet
x, y	Coordinate of the point
z	Tooth number
α	Slope angle
α_n	Normal pressure angle
$\Delta_{s,1}$	Profile deviation by arctangent-proportional tip relief
$\Delta_{s,2}$	Profile deviation by progressive fillet
δ	Position angle
θ_{oil}	Lubricate temperature
ν_0	Kinematic viscosity
ρ	Curvature radius
ρ_{15}	Density of lubricant
φ	Rotation angle
A, E	Position of engaging-in and out
B, D	Transition position between single and double-tooth contact
C	Pitch point
<i>Suffixes</i>	
P	Refers to contact point
1,2	Refers to the pinion and the wheel
<i>ini</i>	Refers to the initial position
H	Refers to the Hertz contact

position. Tribological [10] and mixed lubrication conditions [11] also play important roles in the initiation and propagation of micro-pitting in the tooth flank. Zhang et al. [12] found that the root of the mean square of the surface roughness significantly influenced the competition between micro-pitting and pitting. In a study by Morales-Espejel et al. [13], the contact sliding speed, pressure and specific film thickness influenced the final damage level of micro-pitting based on a stable lubrication condition. However, a remarkable study by Li et al. [14] revealed the dynamic relationship between contact stress and lubrication conditions, suggesting a dynamic model for micro-pitting. The effects of micro-scale surface irregularities on micro-pitting were also discussed by Moorthy et al. [15].

Many methods to reduce micro-pitting have been proposed, and improvement of surface [16] and tribological [17] qualities are the most widely applied. The hindering of austenite retention in the micro-pitting of carburized steel was investigated by Roy et al. [18] under boundary lubrication conditions. An ultrasonic nanocrystal surface modification technique by Qin et al. [19] can be used to reduce the surface roughness to reduce micro-pitting. Furthermore, surface coatings [20-22] have also been proved effective in mitigating micro-pitting. Different lubricant additives [23] added in a base of low viscosity polyalphaolefin also influenced the micro-pitting with different degrees.

Previous studies [24-26] have indicated that contact stress contributes to micro-pitting, and that this stress is determined by the instantaneous meshing state of the gear pair. Many papers have also reported that the load distribution [27] was improved by influencing the meshing state through proper tooth tip relief. Although it is not the same as reducing the micro-pitting of the tooth surface directly, using tooth tip relief to influence the meshing state will also affect the contact stress, thereby affecting the micro-pitting. In fact, a proper tooth tip relief is a relatively economical method to prevent micro-pitting in engineering practice. Optimal tooth tip relief [28] results in better lubrication behaviors for reducing micro-pitting of gears, as has been proved by rigorous experiments. Simon [29] presented a linear function for the simultaneous calculation of optimal tooth tip relief and tooth crowning for spur and helical gears, which improved the load and stress distribution considerably. Ni et. al [30,31] improved the contact performance by a parabolic tip relief by a numerical design approach based on the gear-manufacturing and meshing theory. The dynamic responses of low contact ratio spur gear by linear and parabolic tip relief [32] have been compared for the minimum dynamic load of gears under different working conditions. The width modification of a spur gear [33] can reduce the micro-pitting by eliminating the Hertz contact stress. Mao [34] considered not only tooth tip relief but also lead crowning in the face-width direction to be highly accurate modification methods for reducing gear surface pitting, but this research lacked bench test verification. Bruyere et al. [35] analyzed the combination of profile relief and lead crowns in narrow-faced helical gears.

Linear tip relief, widely used in engineering applications, is an effective and economical method for reducing micro-pitting, but it leads to an increase in the local Hertz contact stress at the transition area between the involute profile and the tip relief region. Thus, the micro-pitting risk of this area increases considerably. Parabolic tip relief achieves a smooth transition between the tip relief region and the involute profile, but the Hertz contact stress is quite large when the gear comes into engagement because less material is removed at the tip relief region. Actually, parabolic tip relief is rarely used in theoretical and practical applications because of its complex manufacturing method and limited reduction of micro-pitting.

In order to reduce the stress peaks in the tip relief transition areas, a novel tooth tip relief method for spur gears is proposed in this paper. The main contributions of this paper are: (1) Considering the actual contact path, the gear meshing process is simulated for curvature radius, contact path and Hertz contact stress; (2) A new method including an arctangent-proportional tip relief and a progressive fillet is modelled mathematically, and key parameters are recommended; (3) The tooth profile deviation is calculated based on a wear rate model, and bench tests are carried out to verify the effectiveness of the proposed tooth tip relief method compared with that of the linear tip relief.

The rest of this paper is organized as follows: Section 2 describes the gear meshing simulation considering the actual path of contact, which can be used to investigate the curvature radius and Hertz contact stress along the contact path after tooth deformation and tooth tip relief. Section 3 discusses the stress peaks caused by linear tip relief. Then, the equations for tooth tip relief are introduced, and values of the key parameters of the equations are recommended. A simulation model using tooth profile deviation to evaluate micro-pitting is established. Experiments to verify the effectiveness of the tooth tip relief method are described in Section 4. Section 5 presents the conclusions.

2. Gear meshing simulation

2.1 Contact point

Theoretically, the contact point of a gear pair moves along the line of action. However, advanced engaging-in and lagged engaging-out would occur due to tooth deformation, which contributes to micro-pitting. Fig. 1 shows the initial position in the meshing process and a later random one. The origin of the coordinate system is the rotational center of the pinion. Subscripts 1 and 2 denote pinion and wheel, respectively. The initial position is defined as when the contact point $P(x_p, y_p)$ is located on the center line of the gear pair. For a random position, the position angles between the center line and the point are $\delta_{1/2}$. P will be added to the subscript when the symbol specifies the contact point. For the initial position, the position angles of the contact point are:

$$\begin{aligned} \delta_{P,1,ini} &= 0 \\ \delta_{P,2,ini} &= 0 \end{aligned} \quad (1)$$

Then the $\delta_{1/2, ini}$ of any points of the gears can be obtained by geometrical calculation.

Fig. 1. Gear pair in (a) initial and (b) random position in the meshing process. P: point of contact. φ : rotation angle of the gears

When the pinion is at a random position with a rotation angle φ_1 (Fig. 1 (b)), the conditions are as follows:

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} r_1(\varphi_1) \sin(\delta_1) \\ r_1(\varphi_1) \cos(\delta_1) \end{bmatrix}, \quad (2)$$

$$r_{P,2} = \sqrt{x_p^2 + (a - y_p)^2}, \quad (3)$$

$$\delta_{P,2} = \arctan\left(\frac{x_p}{a - y_p}\right), \quad (4)$$

$$\delta_{1/2} - \delta_{1/2,ini} = \varphi_{1/2}, \quad (5)$$

where $r_{1/2}$ are the flank radius of the point and a is the center distance. The position angles $\delta_{1/2}$ of any point corresponding to the radius $r_{1/2}$ are obtained based on the initial position by geometrical analysis during the meshing process.

According to the gear meshing principle, the normal vectors of the contact points of the gear pair are collinear. As illustrated in Fig. 1, this means

$$\alpha_{P,1} - \alpha_{P,2} = \pm 180^\circ, \quad (6)$$

where $\alpha_{P,1/2}$ are the slope angles of the contact point.

Also, for any $\varphi_{1/2}$:

$$\alpha_{1/2}(\varphi_{1/2}) - \alpha_{1/2,ini} = \varphi_{1/2}, \quad (7)$$

$$\alpha_{1/2,ini} = \arctan\left(-\frac{x'_{1/2}}{y'_{1/2}}\right), \quad (8)$$

where the prime represents the derivative with respect to time. Thus, the functional relation between $\alpha_{1/2}$ and $r_{1/2}$ can be established.

The contact point P is found by the iteration of $r_{P,1}$ from the tooth root r_f to the tooth tip r_a . When the pinion is at any rotation angle φ_1 , the slope angle $\alpha_{P,1}$ of the hypothetical contact point P₁ with the flank radius $r_{P,1}$ can be obtained by Eq. (7); then, the flank radius $r_{P,2}$ of the corresponding point P₂ in the wheel is obtained by Eq. (3); third, the slope angle $\alpha_{P,2}$ is obtained; finally, the contact point P=P₁=P₂ will be confirmed if the slope angles $\alpha_{P,1/2}$ satisfy Eq. (6). Otherwise, the next calculation will be conducted. It is found that the contact point is determined by the tooth profile geometry, thus this method can be applied to tooth with tip relief and deformation.

The curvature radius of the contact point (in spite of the effect of tooth deformation, tooth tip relief and wear) can be calculated by

$$\rho = \left| \frac{\left[(x')^2 + (y')^2 \right]^{3/2}}{x' y'' - y' x''} \right|, \quad (9)$$

where the double prime represents the second derivative with respect to time.

The backlash of the gears is given by

$$L(\varphi_1) = \varphi_2 - \varphi_1 \cdot i_G, \quad (10)$$

where i_G is the transmission ratio.

Moreover, the meshing tooth of the gear pair and the path of contact can be confirmed based on the flank radius $r_{P,1}$ of contact point P on the pinion.

2.2 Tooth deformation at the contact point

As shown in Fig. 2, tooth deformations include bending w_{tb} , shear w_{ts} , compression w_r , and body deformation w_{tk} , where black line is the undeformed tooth and blue line is the deformed tooth. The displacement deformations at point P by these deformations along the normal direction are, respectively

$$w_{tb} = \int_{r_f}^{r_p} \int_{r_f}^r \frac{\cos \alpha_n}{E'} \frac{F_u [y(r_p) - y(r)] - F_r x(r_p)}{ly(r)} dr dr, \quad (11)$$

$$w_{ts} = \cos \alpha_n \int_{r_f}^{r_p} \tan \left(\frac{\chi F_u}{G' A(r)} \right) dr, \quad (12)$$

$$w_r = \cos \alpha_n (\tan \alpha_n - \nu) \int_{r_f}^{r_p} \frac{F_r}{E' A(r)} dr, \quad (13)$$

where r is the flank radius, A is the section area, x and y represent the coordinates of the load point, F_u and F_r are the component forces of the normal force along the coordinate axis, and E' , G' , and χ are the material parameters.

The body deformation [36] is $w_{tk} = w_{KP} + w_{KT}$, where w_{KP} and w_{KT} respectively represent the parallel displacement and tilting rotation of the tooth, are given according to engineering experience:

$$w_{KP} = \frac{4.8 F_u}{E \cdot l_{eff}} \left[\frac{(1-\nu^2)}{\pi} (1 + 0.294 \tan^2 \alpha_n) \right], \quad (14)$$

$$w_{KT} = \frac{2 F_u}{E \cdot l_{eff}} \left[\frac{9(1-\nu^2)}{\pi} \frac{y(r_p) - x(r_p) \tan \alpha_n - y(r_f)}{2x(r_f)} + (1+\nu)(1-2\nu) \left(\frac{y(r_p) - x_p \tan \alpha_n - y(r_f)}{2x_f} \right) \right], \quad (15)$$

where E and ν are the material parameters, and l_{eff} is the effective tooth width.

Fig. 2. The tooth deformations by load F_n applied at point P.

Therefore, the deformation angle of P is

$$w_p = \frac{[w_{tb} + w_{ts} + w_r + w_{tk}] \cos \alpha_n}{r_p}. \quad (16)$$

Moreover, the local Hertz contact deformation will appear at the contact point by the load, as shown in Fig. 3. The local displacement deformations along the contact line of the gears are

$$w_{H,1/2} = 2 \frac{F_n}{l_{eff}} \frac{1-\nu^2}{E' \pi} \left[\ln \left(\frac{2h_{1/2}}{b_H} \right) - \frac{\nu}{2(1-\nu)} \right], \quad (17)$$

where F_n is the meshing force, $h_{1/2}$ are the depths of force, and b_H is the half contact width

$$b_H = \sqrt{\frac{8F_n(1-\nu^2)\rho_{P,1}\rho_{P,2}}{\pi l_{eff}E'(\rho_{P,1} + \rho_{P,2})}}. \quad (18)$$

Fig. 3. Local Hertz contact deformation due to meshing force F_n . b_H : half contact width. $h_{1/2}$: depth of force.

Based on Eqs. (16) and (17), the tooth deformation and local Hertz contact deformation contribute to the actual deformation of the profile at the contact point during the gear meshing process. The comprehensive deformation of the meshing tooth at the contact point by the load F_n is

$$W = \frac{w_{P,1}}{\cos \alpha_n} \cdot (-i_G) + \frac{w_{P,2}}{\cos \alpha_n} + \frac{w_{H,1} \cos \alpha_n}{r_{P,1}} \cdot (-i_G) + \frac{w_{H,2} \cos \alpha_n}{r_{P,2}}. \quad (19)$$

Therefore, F_n and W are related by Eq. (19) when the pinion is at a rotation angle of φ_1 . it is noted that the comprehensive tooth deformation W may lead to the double contact as shown in Section 3, which may increase the risk of micro-pitting.

Based on a traversal iteration method for the flank radius r_1 , the theoretical contact points and contact path are confirmed during meshing. However, the contact path of the gear pair will change because of tooth deformation. Furthermore, only when the comprehensive deformation W is larger than the backlash L will the teeth contact each other and the load be transmitted. The Hertz contact stress is

$$p_H = \frac{2}{\pi} \frac{F_n}{b_H l_{eff}}. \quad (20)$$

In this section, meshing simulation of gears with tip relief is conducted considering the actual contact path. The risk of micro-pitting can be discussed according to Hertz contact stress for gears with or without tip relief.

3. Equations for tooth tip relief

3.1 Effects of linear tip relief

On the tooth flank, **A** and **E** are the positions of engaging-in and out, **B** and **D** are the transition positions of single and

double-tooth contact, and **C** is the pitch point. In this study, the tooth tips of both pinion and wheel are modified with a tip relief amount of 170 μm and start position of **D** on the tooth flank. But note that the discussions in this paper are conducted based on the pinion.

Based on the gear meshing process in Section 2, Fig. 4 discusses the effects of linear tip relief on the curvature radius and Hertz contact stress along the contact path, where profile deviation is the distance between the actual and involute tooth profile along the meshing line. As shown in Fig. 4 (a), serious Hertz contact stress peaks occur at **A** and **E** on the tooth flank without tooth tip relief. This is because tooth deformation causes meshing interference between the tooth tip of the wheel and the tooth root of the pinion when the gear pair starts meshing. Moreover, it is also found that double contact by tooth deformation occurs at the tooth root according to the path of contact, increasing the risk of micro-pitting. As shown in Fig. 4 (b), after linear tip relief, the Hertz contact stress peaks are reduced by 56.3% and 60.9% at **A** and **E**, respectively, because the removed material of the tooth tip mitigates the mesh interference. Moreover, mitigating double contact of the tooth root also decreases the risk of micro-pitting. However, the Hertz contact stress peaks increase significantly at **B** and **D** by 21.6% and 13.4%, respectively, which would result in the initiation of micro-pitting at the transition area between the tip relief region and the involute profile. It is evident that the sharp decrease in the curvature radius at point **D** causes stress peaks. In summary, micro-pitting is reduced at **A** and **E** but increased at **B** and **D** by linear tip relief.

Fig. 4. Profile deviation, curvature radius, and Hertz contact stress (a) without tip relief and (b) with linear tip relief

3.2 Equation for arctangent-proportional tip relief

In order to reduce the stress peaks at the transition areas between the tip relief region and the involute profile while retaining the advantages of linear tip relief, the arctangent-proportional tip relief is proposed in this paper for a smoother transition and illustrated in Fig. 5. The governing equation for the method is

$$\Delta_{s,l}(X_a) = C_a \cdot \left(\frac{X_a}{C_l} \right) \cdot \frac{\arctan \left[c \cdot (X_a/C_l)^2 \right]}{\arctan(c)}, \quad (21)$$

where $\Delta_{s,l}$ represents the profile deviation by arctangent-proportional tip relief, X_a is the height coordinate of the arctangent-proportional tip relief with the range from 0 mm to C_l , C_a is the size of the tooth tip relief, and c is a constant coefficient.

Fig. 5. Flank radius versus profile deviation in arctangent-proportional tip relief

Fig. 6 illustrates the profile deviation, curvature radius and Hertz contact stress along the contact path with arctangent-proportional tip relief, where 50, 5 and 0.5 are selected as representative large, moderate, and small values of c , respectively. As shown in Fig. 6 (a), the arctangent-proportional tip relief achieves a smoother transition at **D**. Thus, the Hertz contact stress peaks at transition areas **B** and **D** are reduced by 6.7% and 9.1% for $c = 50$, respectively, compared with those obtained by linear tip relief. Fig. 6 (b) shows that for c is 5, the tooth flank at **D** is smoother with a larger curvature radius. The Hertz contact stress peaks at **B** and **D** are reduced by 15.1% and 10.7%, respectively, compared with those obtained by linear tip relief. However, as shown in Fig. 6 (c), a smaller c leads to larger Hertz contact stresses in the tip relief region with little reduction in the Hertz contact stress at transition areas. Actually, the arctangent-proportional tip relief with smaller c is similar to the parabolic tip relief, which would produce less removed material at the tip relief region causing serious micro-pitting. As a result, a moderate c reaches an agreement about the Hertz contact stresses at the transition areas **B** and **D** and the tooth tip region. However, the Hertz contact stress peaks at **A** and **E** and the double contact due to tooth deformation at the tooth root are still predominant, promoting the initiation and propagation of micro-pitting.

(a) Arctangent-proportional tip relief with $c=50$.

(b) Arctangent-proportional tip relief with $c=5$.

(c) Arctangent-proportional tip relief with $c=0.5$.

Fig. 6. The effects on the tooth meshing of arctangent-proportional tip relief with different c

3.3 Equation for the progressive fillet

To reduce double contact of the tooth root and the Hertz contact stress peaks at the transition area between the tip relief region and the tooth tip by the arctangent-proportional tip relief, a progressive fillet is further proposed based on the arctangent-proportional tip relief for a smoother transition, as shown in Fig. 7:

$$\Delta_{s,2}(X_b) = \frac{X_b}{r_k} \cdot \left(r_k - \sqrt{r_k^2 - X_b^2} \right), \quad (22)$$

where $\Delta_{s,2}$ represents the profile deviation by progressive fillet, X_b is the height coordinate of the progressive fillet with the range from 0 mm to l_r , r_k is the radius of the progressive fillet.

Fig. 7. Flank radius versus profile deviation in the progressive fillet

Fig. 8 shows the effects of the geometrical parameter r_k of the progressive fillet on the curvature radius, Hertz contact stress and path of contact. As shown in Fig. 8 (a), the reductions of the Hertz contact stress at **A** and **E** and of the double contact are limited for $r_k = m_n/3$, where m_n is normal modulus. This is because the curvature radius at the transition area between the tooth tip and tip relief region is too small. Furthermore, as shown by the profile deviation, the sudden change of the profile curve at the start position of the progressive fillet would increase the risk of micro-pitting there. Fig. 8 (b) shows that when r_k is increased to $m_n/2$, the Hertz contact stress peaks at **A** and **E** and the contact path are ameliorated compared with those shown in Fig. 8 (a). The tooth flank is smoother at the start position of the progressive fillet with a larger curvature radius. As shown in Fig. 8 (c), the Hertz contact stress peaks at **A** and **E** and the double contact of the tooth root are eliminated with $r_k = m_n$. Fig. 8 indicates that a larger r_k produces a greater reduction in the Hertz contact stress and double contact, suggesting a lower risk of micro-pitting. However, excessive r_k is not recommended despite this trend because it will result in too little material of the tooth tip, leading to a significant decrease in the strength. Therefore, $r_k = m_n$ is recommended in this study for minimizing the risk of micro-pitting.

Fig. 8. The effects on the tooth meshing by the progressive fillet with different geometrical parameters r_k

4. Simulation verifications and bench tests

4.1 Wear simulation results

The feathers of micro-pitting are the micro-pits on the tooth surface. Therefore, micro-pitting has been evaluated by surface wear in previous studies [24,25] and it is demonstrated effective. Cracks causes surface wear but short cracks are

discussed in this paper because only they lead to micro-pitting. Based on the Dang Van multiaxial fatigue criterion [37] and the fretting fatigue crack initiation theory [38], Xu et al. proposed a crack criterion model [10] to predict the crack initiation. The limiting value of the criterion was $R_s = 1$, indicating the initiation of the first cracks on the tooth flank surface. The risk of micro-pitting increases when $R_s < 1$; thus, the crack criterion value R_s can also be a safety factor for the tooth flank surface.

The crack criterion is used to predict the initiation of the first crack, regardless of the number of loading cycles. In practice, with an increase in the number of loading cycles, new cracks initiate, and cracks on the tooth flank surface will increase and expand in depth. Besides, surface roughness Ra , sliding velocity v_s , rolling velocity v_r , surface hardness H_s and minimum film thickness h_{min} all affect the wear rate. Based on an empirical formula for calculating tooth surface fatigue wear [38], Xu et al. proposed a wear rate model [10] including the crack density

$$\frac{dV}{dN} = k_s \cdot k_w \cdot \left(\frac{p_H}{p_{H,R}} \right)^\alpha \cdot \left(\frac{|v_s|}{|v_r|} \right)^\beta \cdot \left[1 - \left(\frac{Ra_\Sigma}{Ra_{\Sigma,R}} \right)^\kappa \cdot \frac{v_s}{v_r} \right]^\chi \cdot \left(\frac{Ra_\Sigma}{h_{min}} \right)^\gamma \cdot \left(\frac{H_s}{H_{s,R}} \right)^\delta \cdot \frac{1}{N_R} + \frac{cd_R}{N_R} \cdot [\arctan(\zeta \cdot cd)]^\varepsilon, \quad (23)$$

where k_s and k_w are the parameters determined by lubricant and material properties; the subscript R denotes the reference value as listed in ref. [10]; Ra_Σ represents the arithmetic mean center roughness by $Ra_\Sigma = (Ra_1 + Ra_2)/2$; N_R is the reference number of load cycle as 10^6 ; dV/dN is the wear rate in $\mu\text{m}/10^6$ load cycle; and $\alpha, \beta, \gamma, \delta, \varepsilon, \zeta, \kappa$, and χ are coefficients determined by experiment. It is noted that the parameters about meshing conditions can be obtained by meshing simulation in Section 2.

Thus, the amount of wear after ΔN_k cycles is

$$V_k = \left(\frac{dV}{dN} \right)_k \cdot \Delta N_k, \quad (24)$$

where the wear amounts are calculated for the profile deviation to evaluate the micro-pitting. Moreover, the wear simulation is carried out based on the newest tooth profile, contact path and load distribution after certain cycles. This is because too much calculation time is needed if the wear simulation is conducted after every cycle, and the simulation results are unauthentic when all calculations are based on the initial tooth profile. Therefore, the wear can be simulated under different conditions including running-in for micro-pitting.

A spur gear pair was chosen for verifying the tooth tip relief method proposed in this paper. The basic gear and the tooth tip relief parameters are listed in Table 1, and the lubricant parameters are listed in Table 2.

Table 1 Tested spur gear parameters and tooth tip relief parameters

Parameter		Value	
		Pinion	Wheel
Tooth number	z	16	24
Tip diameter	d_a	402.69 mm	507.62 mm
Profile shift coefficient	x	0.1817	0.1715
Center distance	a	447.33 mm	
Normal modulus	m_n	22 mm	
Tooth width	b	100 mm	
Normal pressure angle	α_n	20°	
Average roughness	R_a	1 μm	
amount of tooth tip relief	C_a	170 μm	
starting positions of the progressive fillet	l_r	$m_n/5$	
radius of progressive fillet	r_k	m_n	
Surface hardness	HV_s	760HV	
Core hardness	HV_c	353HV	

Table 2 lubricant parameters

Parameter		Value
Oil type	-	Mineral oil
Density of lubricant	ρ_{15}	907kg/m ³
Lubricate temperature	θ_{oil}	90°C
Kinematic viscosity	ν_{θ}	213.7mm ² /s at 40°C; 18.29 mm ² /s at 100°C

Simulation results of a load-stage test according to FVA 54 [39] for both linear tip relief and arctangent-proportional tip relief with a progressive fillet are presented in Fig. 9. The results for the crack criterion R_S show that micro-pitting is concentrated in the area from **A** to **C**. However, it would also occur at point **D** with more cycles by the linear tip relief, owing to the smaller R_S there. The maximum and submaximal profile deviations due to the linear tip relief are 12 μm and 7 μm , respectively, whereas the maximum profile deviations due to the novel tooth tip relief are only 3 μm . Compared with linear tip relief, this novel tooth tip relief method eliminates the most serious wear at **B**. Fig. 9 illustrates that the

arctangent-proportional tip relief and the progressive fillet reduce both the initiation and propagation of cracks effectively, indicating a significant reduction in micro-pitting. This is because the proposed tooth tip relief method can reduce the Hertz contact stress peaks and double contact in the gear meshing process. Moreover, based on the results in Section 3, it is found that the arctangent-proportional tip relief and the progressive fillet reduce the maximum and submaximal profile wear, respectively. In addition, with the process of gear meshing cycles, this novel method achieves less wear with higher quality of the tooth surface; thus, the factor of safety is further improved and the micro-pitting on the tooth flank is further reduced.

(a) Linear tip relief

(b) Arctangent-proportional tip relief with a progressive fillet

Fig. 9. Effects on micro-pitting of (a) linear tip relief and (b) arctangent-proportional tip relief with a progressive fillet

4.2 Test results

The bench tests were carried out by a back-to-back gearbox test rig designed by Ruhr-Universität Bochum. A touch-probe system was applied to measure the roughness of the gear-flank surface. Parameters of test gear pair are the same with Table 1, and the material is of chemical composition 18CrNiMo7-6. The modified gears were manufactured using a grinding-wheel dresser with the quality of five. The lubricant is mineral oil with the property in Table 2.

The procedure of tests was according to the load stage test as specified in FVA 54 [39]. The procedure contains six steps. In each load stage, the Hertz contact stress at the pitch point should be guaranteed as the specified value by FVA 54 [39]. The running time of the 1-5 load stage is about 16 h with the pinion revolutions of 2.1 million, but the time of running-in stage is 1 h with the pinion revolutions of 0.13 million. Moreover, the circumferential velocity is set as 8.3m/s. The detailed operation conditions of each load test stage are followed as Table 3.

Table 3 Operation conditions of each load test stage

Load stage	Hertz contact stress at the pitch point	Toque on pinion	Pinion speed
Running-in	795.1 MPa	11,675 Nm	442.7 rpm
1	945.1 MPa	16,487 Nm	442.7 rpm
2	1093.9 MPa	22,098 Nm	442.7 rpm
3	1244.9 MPa	28,616 Nm	442.7 rpm
4	1395.4 MPa	35,931 Nm	442.7 rpm
5	1547.3 MPa	44,197 Nm	442.7 rpm

Fig. 10 shows the measured profile deviations with the proposed tooth tip relief method and the linear tooth tip relief after the loading tests. The measured profile consistent with the simulation results is displayed in Fig. 10. In particular, as shown in Fig. 10 (b), the tooth surface is kept intact and surface damage is avoided as much as possible. Moreover, there is a good agreement between the simulation in Fig. 9 and the experiments in Fig. 10 for the profile deviation. In addition, the pictures in Fig. 10 are the tooth surfaces after bench tests and the areas surrounded by red frame are the most worn areas. It is obvious that the surface wear area is reduced significantly with this novel method compared with that by linear tip relief. By comparing the profile deviations and pictures in Fig. 10, it can be concluded that the level of micro-pitting in the tooth flank is minimized and that the proposed tooth tip relief is valid.

Fig. 10. Bench test results of profile deviation after tooth tip relief

5. Conclusion

Considering both tooth deformation and tooth tip relief, this study first develops a gear meshing simulation model. The gear contact path, curvature radius, and local Hertz contact stress can be calculated considering the actual contact path despite the tooth tip relief and tooth deformation. In order to eliminate the stress peaks in the transition areas of the tooth flank and thus reduce the micro-pitting of spur gears, arctangent-proportional tip relief and progressive fillet are proposed. The effects of the key parameters of the tip relief on the gear contact path, curvature radius and Hertz contact stress are discussed, then the recommended parameters are given. Theoretical simulations and bench tests are carried out to verify the effectiveness of the proposed novel tooth tip relief method in reducing the micro-pitting of the tooth flank. The following conclusions are drawn from this study:

- (1) Linear tip relief leads to the reduction of stress peaks at **A** and **E**, but a significant increase in those at **B** and **D**, causing serious micro-pitting. Moreover, micro-pitting would occur at **D** after more cycles.
- (2) Arctangent-proportional tip relief achieves a smooth transition between the tip relief region and the involute profile. Furthermore, the Hertz contact stresses at **B** and **D** are reduced more with the recommended parameter value $c = 5$.
- (3) Progressive fillet reduces the Hertz contact stress peaks at **A** and **E** by the smooth transition between the tip relief region and the tooth tip, and the key parameter $r_k = m_n$ is recommended. And it can eliminate the double contact of tooth root for further reduction of micro-pitting.
- (4) According to the theoretical wear model and bench tests, this novel tooth tip relief method reduces micro-pitting significantly compared with linear tip relief. Furthermore, the arctangent-proportional tip relief and the progressive fillet

reduce the maximum and submaximal micro-pitting, respectively.

With this tooth tip relief method, the stress distribution is improved and micro-pitting phenomena in the tooth flank of spur gears is reduced considerably. Moreover, this proposed tip relief method can be achieved by grinding wheel dressers. Therefore, the tooth tip relief method proposed in this paper can be used in engineering practice to improve gear stability by reducing micro-pitting.

Authors' Contributions

XX and PD were in charge of the whole trial; YL and SZ finished the modeling, data analysis and wrote the manuscript; YL assisted with the manuscript checking; SW and ZW reviewed and edited the manuscript. All authors read and approved the final manuscript.

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Competing Interests

The authors declare that they have no competing interests.

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Figures

Figure 1

Gear pair in (a) initial and (b) random position in the meshing process. P: point of contact. φ : rotation angle of the gears

Figure 2

The tooth deformations by load F_n applied at point P.

Figure 3

Local Hertz contact deformation due to meshing force F_n . b_H : half contact width. $h_1/2$: depth of force.

(a) Without tip relief

(b) Linear tip relief

Figure 4

Profile deviation, curvature radius, and Hertz contact stress (a) without tip relief and (b) with linear tip relief

Figure 5

Flank radius versus profile deviation in arctangent-proportional tip relief

(a) Arctangent-proportional tip relief with $c=50$.

(b) Arctangent-proportional tip relief with $c=5$.

(c) Arctangent-proportional tip relief with $c=0.5$.

Figure 6

The effects on the tooth meshing of arctangent-proportional tip relief with different c

Figure 7

Flank radius versus profile deviation in the progressive fillet

(a) $r_k = m_n/3$

(b) $r_k = m_n/2$

(c) $r_k = m_n$

Figure 8

The effects on the tooth meshing by the progressive fillet with different geometrical parameters r_k

(a) Linear tip relief

(b) Arctangent-proportional tip relief with a progressive fillet

Figure 9

Effects on micro-pitting of (a) linear tip relief and (b) arctangent-proportional tip relief with a progressive fillet

(a) The linear tip relief

(b) The arc tangent-proportional tip relief with a progressive fillet

Figure 10

Bench test results of profile deviation after tooth tip relief