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¹Large Bending Deformation of Piezoelectric Nanobeam with Surface Energy Model

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Abstract. In the present paper, we analyze the influence of surface effect on the nonlinear bending behavior of piezoelectric nanobeams by Euler-Bernoulli model and surface energy model. Based on the principle of minimum potential energy and the electromagnetic theory, the nonlinear governing equation as well as the boundary conditions is deduced incorporated surface effects and piezoelectric effects. The effect of surface effect on bending deformation and induced charge of piezoelectric beams under different boundary conditions is discussed under the condition of small deformation and large deformation. The results show that, under the condition of small deformation, the surface effect reduces the bending deformation and the inductive charge, under the condition of large deformation, the surface effect has little effect on the bending deformation of piezoelectric nanobeams. Furthermore, the external constraints and cross-section width b of the piezoelectric nanobeam have significant effect on the bending deformation stiffness. these conclusions have important guiding significance for the design and correction of nanobeams-based nanomechanical systems and piezoelectric nanofunctional devices.

Keywords: surface energy, boundary conditions, nanobeams, nonlinear deformation

I. INTRODUCTION

Rapid development in nanotechnology has led to the Micro/Nano electromechanical system (MEMS/NEMS) has become an important part of new electromechanical system[1]. Due to its many advantages, such as small size, low energy consumption, high precision, and low cost, the application range of nano devices is becoming more and more extensive, They are widely used in actuators, transistors, nano switches, and resonators and sensors and other devices[2-6].

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Nanobeams are important materials in the manufacture of micro/nano electromechanical systems, and occupy an important position in the development of micro/nano electromechanical systems. The application of nanobeams in these devices may be subjected to in-planar loads, and consequently, bending and vibration behavior becomes possible.

Through the new analytical methodology, Abdelrahman[7,8] analyzed the coupled effect of the microstructure and the surface energy on the bending and buckling behavior of perforated nanobeams. Yang[9] analyzed the large deformation of nanobeams with considering the influence of surface energy by surface-energy-based model and the core-shell model. The governing equation of the nanobeam considering the nonlinear curvature and surface effects is established, Zhao[10] studied the surface effects on quasi-periodical vibration is significant. Shahrokh[11] studied the damping vibration behavior of two-phase local/nonlocal viscoelastic nanobeams in conjunction with surface effects and the governing equation of Euler - Bernoulli nanobeam and corresponding constitutive boundary conditions are presented. Based on the nonlocal elasticity theory considering surface effects, Ebrahimi[12] analyzed vibration characteristics of a flexoelectric nanobeam in contact with Winkler-Pasternak foundation.

The piezoelectric effects and the surface effects are important physical properties of nanobeams, Therefore, this is crucial that piezoelectric effects and surface effects of nanobeams are studied. Based on the surface elasticity theory and surface energy model, Yang[13] analyzed the influence of surface effect on the linear bending of cantilever piezoelectric nanobeams. Mohtashami[14] used a finite element method to study the vibrations and buckling of a piezoelectric nanobeam. A new continuum model is established, Yue[15] studied the influence of surface stress on the behaviors of piezoelectric nanobeams. Considering surface and magnetic field effects is performed, Ebrahimi[16] analyzed the vibration of axially preloaded nonlocal flexoelectric nanobeams. Through parametric research, Salman[17] studied the bending, buckling and vibrational behavior of size-dependent piezoelectric nanobeams under thermo-magneto-mechano-electrical environment with surface effects. Based on the couple stress piezoelectric theory including flexoelectric effects, Liu[18] studied the size-dependent electromechanical responses of a bilayer piezoelectric microbeam.

In this work, Based on the Euler-Bernoulli model and the surface energy model, considering the piezoelectric effects and the surface effects, the large deformation of nanobeams is analyzed. the principle of minimum potential energy and the electromagnetic theory are used to deduce the governing equation and the overall boundary conditions of nonlinear bending deformation of the piezoelectric nanobeam. As an example, the nonlinear bending deformation of the piezoelectric nano-cantilever beam is studied using our theory.

II. LARGE DEFORMATION OF PIEZOELECTRIC NANOBEAMS WITH SURFACE ENERGY

In this paper, we consider a slender piezoelectric nanobeam model based on Euler-Bernoulli beam model with length l . Consider establishing a Cartesian coordinate system at the origin of the left end, In an undeformed structure, The position of any point on the nanobeam is indicated by $(x,0)$. The displacement along the x and y axes is represented by $u(x)$ and $v(x)$, $u(x)$ and $v(x)$ are both functions of the space coordinate x of piezoelectric nanobeams. The piezoelectric nanobeams with several common boundary conditions are shown in figure 1.

Figure 1 Boundary conditions of piezoelectric nanobeams

Figure 2 Cross-section of Piezoelectric nanobeams

Based on surface elasticity theory and core-shell model, the surface of piezoelectric nanobeam can be regarded as a thin film with negligible thickness. Such as, the effective bending stiffness $(EI)^*$ can be interpreted as[19]

$$(EI)^* = \begin{cases} \frac{1}{12} E b h^3 + \frac{1}{2} E^s b h^2 + \frac{1}{6} E^s h^3 \\ \frac{\pi}{64} E D^4 + \frac{\pi}{8} E^s D^3 \end{cases}, \quad (1)$$

and the effective tension-compression stiffness $(EA)^*$

$$(EA)^* = \begin{cases} Ebh + 2E^s(b+h) \\ \frac{\pi}{4}ED^2 + E^s\pi D \end{cases}, \quad (2)$$

where E is the Young's modulus of the bulk of beam and E^s is the surface elastic modulus of the nanobeam, which can be acquired by atomistic simulations or experiments.

Assuming that the piezoelectric nanobeam undergoes geometric nonlinear deformation under external load, According to Kirchhoff assumption, We conclude that the strain at any point on the beam

$$\varepsilon_x = \frac{du(x)}{dx} + \frac{1}{2} \left(\frac{dv(x)}{dx} \right)^2 - \frac{d^2v(x)}{dx^2} y \quad (3)$$

Considering the one-dimensional piezoelectric nanobeam structure, assuming polarization along the y axis of the piezoelectric nanobeam, the stress-strain relationship is as follows[20]:

$$\sigma_x = C_{22}\varepsilon_x - e_{32}E_y \quad (4)$$

where C is stiffness coefficient of piezoelectric materials, e is piezoelectric constants of piezoelectric materials, and E is electric field intensity matrix.

the formula of electric displacement of the piezoelectric nanobeam is as follows according to the electromagnetic theory

$$D_y = e_{32}\varepsilon_x + \lambda_{33}E_y, \quad D_x = \lambda_{22}E_x$$

$$E_x = -\frac{\partial\phi}{\partial x}, \quad E_y = -\frac{\partial\phi}{\partial y} \quad (5)$$

where D is the potential shift of piezoelectric materials, λ is the dielectric constant of piezoelectric materials and ϕ is the electric potential.

According to the Electrical Equilibrium Equation

$$\frac{\partial D_y}{\partial y} + \frac{\partial D_x}{\partial x} = 0 \quad (6)$$

The electric field intensity in the x -axis direction is much less than that in the y -axis direction in one-dimensional piezoelectric nanobeam structure, that is, $E_x \ll E_y$. Then the electric displacement D_y can be neglected.

Combined formula (3), formula (4), formula (5) and formula (6), the following formal equations can be obtained

$$\phi = \frac{e_{32}}{\lambda_{33}} y \frac{du(x)}{dx} + \frac{e_{32}}{\lambda_{33}} y \left(\frac{dv(x)}{dx} \right)^2 - \frac{e_{32}}{\lambda_{33}} \frac{y^2}{2} \frac{d^2v(x)}{dx^2} + a_1 y - a_2$$

$$E_z = \frac{e_{32}}{\lambda_{33}} \frac{du(x)}{dx} + \frac{e_{32}}{\lambda_{33}} \left(\frac{dv(x)}{dx} \right)^2 - \frac{e_{32}}{\lambda_{33}} y \frac{d^2v(x)}{dx^2} + a_1 \quad (7)$$

where a_1, a_2 are undetermined coefficient. When the displacement $v(x)=0$ and $u(x)=0$, the electric potential $\varphi=0$. therefore, the undetermined coefficient a_1 and a_2 are zero.

During the deformation process, the surface energy is the energy required for the rearrangement and reorganization of the surface atoms of the piezoelectric nanobeam. The most intuitive effect of this part of the energy is the increase in the surface area of the piezoelectric nanobeam. In order to better measure the surface energy, we introduce the concept of surface energy density γ . To simplify, we consider that the surface energy density γ of the entire nanobeam surface is a constant, so the surface energy expression is:

$$U_s = \gamma S_0 + H \int_0^l \left[1 + \frac{du(x)}{dx} + \frac{1}{2} \left(\frac{dv(x)}{dx} \right)^2 \right] dx \quad (8a)$$

After performing variation operations, the Eq. (8a) becomes as follows

$$\delta U_s = H \left\{ \delta u(x) \Big|_0^l + \frac{dv(x)}{dx} \delta v(x) \Big|_0^l - \int_0^l \frac{d^2v(x)}{dx^2} \delta v(x) dx \right\} \quad (8b)$$

where $S_0 = \pi D^2/2 + \pi D l$ is the initial surface area and $H = \pi D \gamma$ is the constant for a nanobeam of circular cross section with diameter D . Similarly, for a nanobeam of rectangular cross section with width b and thickness h , the initial surface area $S_0 = 2(b+h)l$ and the constant $H = 2(b+h)$.

The strain energy of nonlinear deformation of the piezoelectric nanobeam

$$U_b = \int_V \int_0^\varepsilon \sigma d\varepsilon dV \quad (9a)$$

After performing variation operations, the Eq. (9a) becomes as follows

$$\delta U_b = \int_0^l N_x \delta \left(\frac{du(x)}{dx} \right) dx + \int_0^l N_x \frac{dv(x)}{dx} \delta \left(\frac{dv(x)}{dx} \right) dx - \int_0^l M_x \delta \left(\frac{d^2v(x)}{dx^2} \right) dx \quad (9b)$$

Where N_x is the axial force, M_x is the bending moment on the cross section. Its can be expression as

$$(N_x, M_x) = \int_A \sigma(1, x) dA \quad (10)$$

Assume that the piezoelectric nanobeam is subjected to the distributed loading $q(x)$, a shear force F and a transverse force P are applied the free end of piezoelectric nanobeam.

$$W = \int_0^l q(x)v(x)dx + Fv(x) \Big|_{x=l} + Pu(x) \Big|_{x=l} \quad (11a)$$

After performing variation operations, the Eq. (11a) becomes as follows

$$\delta W = \int_0^l q(x) \delta(v(x)) dx + F \delta(v(x)) \Big|_{x=l} + P \delta(u(x)) \Big|_{x=l} \quad (11b)$$

So the total potential energy can be expressed as

$$\Pi = U_b + U_s - W \quad (12a)$$

After performing variation operations, the Eq. (12a) becomes as follows

$$\delta \Pi = \delta U_b + \delta U_s + \delta W \quad (12b)$$

According to the principle of minimum potential energy, $\delta \Pi(v)=0$ and $\delta \Pi(u)=0$, the governing equation of nonlinear bending deformation of the piezoelectric nanobeam is obtained as

$$\begin{cases} \frac{d^2 M_x}{dx^2} + \frac{dN_x}{dx} \frac{dv(x)}{dx} + (N_x + H) \frac{d^2 v(x)}{dx^2} - q(x) = 0 \\ \frac{dN_x}{dx} = 0 \end{cases} \quad (13)$$

The overall boundary conditions

$$\begin{aligned} & \left\{ (N_x + H) \frac{dv(x)}{dx} + \frac{dM_x}{dx} - F \right\} \delta(v(x)) \Big|_0^l = 0 \\ & \{N_x + H - P\} \delta(u(x)) \Big|_0^l = 0 \\ & M_x \delta \left(\frac{dv(x)}{dx} \right) \Big|_0^l = 0 \end{aligned} \quad (14)$$

Combined formulas (13) and (14) can calculate the deflection of nonlinear bending of the piezoelectric nanobeam considering surface effect. when the surface energy density $\gamma=0$ and the surface elastic modulus $E^s=0$, the differential governing equation (13) and the overall boundary condition (14) degenerate into a nonlinear bending problem for the piezoelectric nanobeam when without considering the surface effect.

According to equation (14), these four boundary conditions of piezoelectric nanobeam as shown in figure 1 are

$$\begin{aligned} & v(x) \Big|_{x=0} = 0, \frac{\partial v(x)}{\partial x} \Big|_{x=0} = 0, \frac{\partial^2 v(x)}{\partial x^2} \Big|_{x=l} = 0, \\ & (N_x + H) \frac{dv(x)}{dx} + \frac{dM_x}{dx} - F \Big|_{x=l} = 0, N_x + H - P \Big|_{x=l} = 0, \\ & v(x) \Big|_{x=0} = 0, \frac{\partial^2 v(x)}{\partial x^2} \Big|_{x=0} = 0, \frac{\partial v(x)}{\partial x} \Big|_{x=\frac{l}{2}} = 0, \end{aligned} \quad (15a)$$

$$(N_x + H) \frac{dv(x)}{dx} + \frac{dM_x}{dx} - F \Big|_{x=\frac{l}{2}} = 0, \quad N_x + H - P \Big|_{x=\frac{l}{2}} = 0 \quad (15b)$$

$$v(x) \Big|_{x=0} = 0, \quad \frac{\partial v(x)}{\partial x} \Big|_{x=0} = 0, \quad \frac{\partial v(x)}{\partial x} \Big|_{x=\frac{l}{2}} = 0,$$

$$(N_x + H) \frac{dv(x)}{dx} + \frac{dM_x}{dx} - F \Big|_{x=\frac{l}{2}} = 0, \quad N_x + H - P \Big|_{x=\frac{l}{2}} = 0 \quad (15c)$$

$$v(x) \Big|_{x=0} = 0, \quad \frac{\partial v(x)}{\partial x} \Big|_{x=0} = 0, \quad v(x) \Big|_{x=l} = 0,$$

$$(N_x + H) \frac{dv(x)}{dx} + \frac{dM_x}{dx} - F \Big|_{x=\frac{l}{2}} = 0, \quad N_x + H - P \Big|_{x=\frac{l}{2}} = 0 \quad (15d)$$

releatively. combined formulas (13) and (15) can calculate the nonlinear bending deflection of the piezoelectric nanobeam can be obtained when considering surface effect.

The upper and lower surfaces of piezoelectric nanobeams can be regarded as two parallel plates, which can generate negative charges on the compressed surface and positive charges on the tensile surface, thus forming potential difference. According to the theory of electromagnetism

$$Q = C\varphi = CE_y h \quad (16)$$

where C is the capacitor, the formula expression is:

$$C = \frac{\varepsilon_0 S}{d} = \frac{\lambda_{33} b l}{h} \quad (17)$$

where S is the area of capacitor plate. d is the distance between capacitor plates. ε_0 is the dielectric constant.

Combined formulas (16) and (17), The induced charge generated by nonlinear bending of the piezoelectric nanobeam is:

$$Q = e_{32} b l \left[\frac{du(x)}{dx} + \left(\frac{dv(x)}{dx} \right)^2 - h \frac{d^2 v(x)}{dx^2} \right] \quad (18)$$

III. RESULTS AND DISCUSSIONS

Assume that the piezoelectric nanobeam with a cross-section height $h=20$ nm, width $b=20$ nm, and length $l=400$ nm. a shear force $F/B=35$ is applied the free end of piezoelectric nanobeam. In order to facilitate calculation, the dimensionless parameters are substituted into calculation

$$\bar{x} = \frac{x}{h}, \bar{v} = \frac{v}{h}, \bar{H} = \frac{12\lambda_{33} F l^3}{b h^4 (C_{22} \lambda_{33} - e_{32}^2)}, B = \frac{12\lambda_{33} l^3}{b h^4 (C_{22} \lambda_{33} - e_{32}^2)} \quad (19)$$

where B is dimensionless parameters. The physical parameters of the piezoelectric nanobeam during calculation are shown in Table 1.

Table 1 Matreial parameters of piezoelectric nanowire[21]

E / GPa	γ / J·m ⁻²	λ_{33} / F·m ⁻¹	e_{32} / C·m ⁻²
207	1.6	-7.88×10^{-11}	-0.51

Figure 3 displays the deflection of the piezoelectric nanobeam of the slenderness ratio $l/h=20$. When the amount of deformation is small (the upper half of Fig .2), the surface effect has a significant effect on the bending deformation of piezoelectric nanobeams. However, when the deformation is large (the lower part of Fig .2), the surface effect has little effect on the bending deformation of piezoelectric nanobeams, and the geometric nonlinear deformation of beam occupies a dominant position. At the same position on the piezoelectric nanobeam, as the cross-sectional width b of the piezoelectric nanobeam increases, the nonlinear deformation of the piezoelectric nanobeam decreases, and the nonlinear deformation trend also decreases. Especially, for piezoelectric nanobeams with one end fixed and one end hinged, the cross-sectional width b has little effect on the amount of deformation.

(a) Cantilever piezoelectric nanobeam

(b) Simply supported piezoelectric nanobeams

Figure 3. Deflection of the piezoelectric nanobeam

Figure 4 displays the relationship between dimensionless load F/B and the maximum deformation. It can be seen from the figure that as the dimensionless F/B increases, the maximum deflection of the piezoelectric nanobeam also increases. When the shear conditions are the same, the surface effect has little effect on the maximum bending deflection. As the cross-sectional width b increases, the nonlinear bending deformation of the piezoelectric nanobeam decreases, and its deformation trend also decreases. Additionally, When the dimensionless load F/B is the same, the deformation of the piezoelectric nanobeam under the piezoelectric nanobeam with one end fixed and one end hinged is the smallest, and the deformation under the cantilever piezoelectric nanobeam structure is the largest, indicating that external constraints have a significant effect on the deformation of the piezoelectric nanobeam.

(c) Fixed supported piezoelectric nanobeams

(d) Piezoelectric nanobeam with one end fixed and one end hinged

Figure 4. Deflection of the piezoelectric nanobeam ($F/B=35$)

Figure 5 displays the variation of induced charge with the load. The surface effect has little influence on the induced charges of the piezoelectric nanobeam. With the increase of the dimensionless load F/B , the induced charge of the piezoelectric nanobeam also increases with the increase in deformation. There is a quadratic relationship between the dimensionless load F/B and the induced charge of the piezoelectric nanobeam under the cantilever structure. there is a linear relationship between the dimensionless load F/B and the induced charge of the piezoelectric nanobeam under the other boundary conditions. With the cross-section width b of the piezoelectric nanobeam increases, the bending deformation of the piezoelectric nanobeam decreases and the generation of inductive charge decreases. Especially, the cross-section width b has little effect on the inductive charge of the piezoelectric nanobeam under one end fixed and one end hinged structure. This results are consistent with figure 3.

(a) Cantilever piezoelectric nanobeams

(b) Simply supported piezoelectric nanobeams

(c) Fixed supported piezoelectric nanobeams

(d) Piezoelectric nano-beam with one end fixed and one end hinged

Figure 5. The variation of induced charge with load

IV. CONCLUSIONS

Based on the surface energy model, using the principle of minimum potential energy and electromagnetic theory, the governing equation and the overall boundary conditions of nonlinear bending deformation of piezoelectric nanobeams are obtained. the effect of surface effect on nonlinear bending behavior is analyzed. results show that under small deformation conditions, the surface effect has a significant effect on the bending behavior of piezoelectric nanobeams, which increases the stiffness of piezoelectric nanobeams, decreases the inductive charge. However, under the condition of large deformation, because the geometric nonlinear deformation of piezoelectric nanobeams occupies the dominant position, the surface effect has little

effect on the bending of piezoelectric nanobeams. On the other hand, the external constraints and cross-section width b of the piezoelectric nanobeam have significant effect on the bending deformation stiffness. the results of this study have important guiding significance for the design and correction of nanobeam-based nanomechanical systems and nano-functional devices.

References

- [1] Attia MA, Emam SA. Electrostatic nonlinear bending, buckling and free vibrations of viscoelastic microbeams based on the modified couple stress theory[J]. *Acta Mech*, 2018,229:3235–3255.
- [2] Kino H, Kiyota A, Miyamoto N, et al. Basic Study of Heating Response Measurement for Nanosheet Particle/Polymer Composite Gel Actuator with Anisotropic Contraction[J]. *Key Engineering Materials*, 2019, 804:17-21.
- [3] Tan Q, Wang Q, Liu Y, et al. Single-Walled Carbon Nanotube Dominated Micron-Wide Stripe Patterned-Based Ferroelectric Field-Effect Transistors with HfO₂ Defect Control Layer[J]. *Nanoscale Research Letters*, 2018,13(1):127.
- [4] Yang F, Wang GF, Long JM, et al. Influence of Surface Energy on the Pull-in Instability of Electrostatic Nano-Switches[J]. *Journal of Computational & Theoretical Nanoence*, 2013,10(5):1273-1277.
- [5] Kazmi S N R , Hajjaj A Z , Hafiz M A A , et al. Highly Tunable Electrostatic Nanomechanical Resonators[J]. *IEEE Transactions on Nanotechnology*, 2018,17(1): 113-121.
- [6] Mousavi M, Zand MM, Siahpour S. Effect of added mass distribution on the dynamic PI and frequency shifting in MEMS and NEMS biosensors[J]. *Microsystem Technologies*, 2020:1-10.
- [7] Abdelrahman AA , Mohamed NA , Eltaher M A . Static bending of perforated nanobeams including surface energy and microstructure effects[J]. *Engineering With Computers*, 2020,5:1-21.
- [8] Abdelrahman AA, Eltaher MA. On bending and buckling responses of perforated nanobeams including surface energy for different beams theories[J]. *Engineering with Computers*, 2020.
- [9] Yang F, Li CB. A theoretical analysis of large deformation of nanobeams with surface energy[J]. *Microsyst Technol*, 2020,26:2887–2891.
- [10] Zhao DM, Hao P, Wang JW, et al. Surface effects on the quasi-periodical free vibration of the nanobeam: semi-analytical solution based on the residue harmonic balance method[J]. *Meccanica*, 2020,55: 989–1005.
- [11] Shahrokh HH, Shahin H, Mahmood F. Vibration analysis of two-phase local/nonlocal viscoelastic nanobeams with surface effects[J]. *European Physical*

Journal Plus, 2020, 135(2):190.

[12] Ebrahimi F, Barati MR. Surface effects on the vibration behavior of flexoelectric nanobeams based on nonlocal elasticity theory[J]. European Physical Journal Plus, 2017,132(1):19.

[13] Yang F. Effect of surface energy on the bending of piezoelectric nanowires [J]. Electronic components & Materials , 2016, 35(08):46-49.

[14] Mohtashami M, Beni YT. Size-Dependent Buckling and Vibrations of Piezoelectric Nanobeam with Finite Element Method[J]. Iran J Sci Technol Trans Civ Eng, 2019,43: 563–576.

[15] Yue, Y, Xu K, Zhang X, et al. Effect of surface stress and surface-induced stress on behavior of piezoelectric nanobeam[J]. Appl. Math. Mech.-Engl. Ed, 2018,39:953–966.

[16] Ebrahimi F, Barati MR. Nonlocal and Surface Effects on Vibration Behavior of Axially Loaded Flexoelectric Nanobeams Subjected to In-Plane Magnetic Field[J]. Arab J Sci Eng, 2018,43:1423–1433.

[17] Salman EN, Mahya B. Comprehensive nonlocal analysis of piezoelectric nanobeams with surface effects in bending, buckling and vibrations under magneto-electro-thermo-mechanical loading[J]. Materials Research Express, 2018,5(3):035028.

[18] Liu Y, Zhou S, Wu K, et al. Size-dependent electromechanical responses of a bilayer piezoelectric microbeam[J]. Int J Mech Mater Des, 2020,16: 443–460.

[19] Wang GF. Effects of surface energy on the mechanical performance of nanosized beams[J]. Journal of Computational and Theoretical Nanoscience, 2011,8(7):1173-1177.

[20] Wang GF, Feng XQ. Effect of surface stresses on the vibration and buckling of piezoelectric nanowires[J]. Europhys Letters, 2010,91(5): 56007-1-4.

[21] Kim SM, Sohn JI, Kim HJ, et al. Radially dependent effective piezoelectric coefficient and enhanced piezoelectric potential due to geometrical stress confinement in ZnO nanowires/nanotubes[J]. Applied Physics Letters, 2012,101(1):013104-1-3.

Figures

Figure 1

Boundary conditions of piezoelectric nanobeams

Figure 2

Cross-section of Piezoelectric nanobeams

(a) Cantilever piezoelectric nanobeam

(b) Simply supported piezoelectric nanobeams

(c) Fixed supported piezoelectric nanobeams

(d) Piezoelectric nanobeam with one end fixed and one end hinged

Figure 3

Deflection of the piezoelectric nanobeam

(a) Cantilever piezoelectric nanobeams

(b) Simply supported piezoelectric nanobeams

(c) Fixed supported piezoelectric nanobeams

(d) Piezoelectric nanobeam with one end fixed and one end hinged

Figure 4

Deflection of the piezoelectric nanobeam ($F/B=35$)

(a) Cantilever piezoelectric nanobeams

(b) Simply supported piezoelectric nanobeams

(c) Fixed supported piezoelectric nanobeams

(d) Piezoelectric nano-beam with one end fixed and one end hinged

Figure 5

The variation of induced charge with load