Energy-Optimal Braking Control Using a Double-Layer Scheme for Trajectory Planning and Tracking of Connected Electric Vehicles

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Title page

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ORIGINAL ARTICLE

Energy-Optimal Braking Control Using a Double-Layer Scheme for Trajectory Planning and Tracking of Connected Electric Vehicles

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Abstract: To improve the energy efficiency of electric vehicles (EVs), this paper presents an energy-optimal braking strategy (EOBS) considering shared braking intention. First, a double-layer control scheme is formulated. In the upper-layer, an energy-optimal braking problem with braking intention is formulated and solved by distance-based dynamic programming, which could derive the energy-optimal braking trajectory. In the lower-layer, a linear time-varying model predictive (LTV-MPC) controller is designed to track the desired trajectory while ensuring braking comfort and safety. Second, several simulations are conducted to demonstrate the regeneration energy efficiency improvement of proposed EOBS. Finally, the energy-optimal braking mechanism of EVs is investigated based on the analysis of braking deceleration, battery charging power, and motor efficiency.

Keywords: Connected electric vehicles • Energy optimization • Velocity planning • Regenerative braking • Dynamic programming • Model predictive control

1 Introduction

Increasingly stringent fuel economy and emission standards in recent years promote the development of electric vehicles (EVs) [1]. However, the driving range of EVs is still too short compared with the internal combustion engine vehicle, because the battery energy density is insufficient, it is the main obstacles for wide diffusion of EVs [2]. To extend the driving range, one of the popular ways is increasing the battery capacity, however, the battery size and vehicle price are increased simultaneously [3]. Recycling vehicle kinetic energy in the braking process is another prominent approach to extend the driving range, which is called the regenerative braking in some literature [4].

The regenerative braking is the prior choice in some traffic conditions for EVs, e.g., approaching an

intersection with red signal, following the slowed preceding vehicle, and reaching a destination, because the EVs can transfer vehicle kinetic energy to electric energy by regenerative braking while without additional battery energy consumption. Since approximately 30% ~ 50% of driving energy can be recycled in urban traffic [5], several studies have investigated the regenerative braking in recent decades. Some studies tried to optimize the powertrain configuration and vehicle components for increasing regeneration energy. Hellgren et al. [6] demonstrated that the distributed-driven configuration of EVs has superior energy recycle potential than a central driven. Joy et al. [7] presented an integrated power circuit and electronic commutator control strategy to improve the generating energy of EVs. Kiddee et al. [8] designed a hybrid energy storage system, which uses the super-capacitor and lithium-ion battery to enhance the harvesting of braking energy. Zhang et al. [9] proposed an electrical-hydraulic integrated brake configuration based on the electronic stability program to achieve an efficient and safe braking system.

Another approach to improve the regeneration energy is the regenerative braking strategy (RBS) design. The conventional RBS focus on each wheels (axles) braking force distribution, which cooperating the motor braking and friction braking force to improve regeneration energy, while ensuring braking safety, stability, and comfort [10]. Sun et al. [11] integrated the braking control strategy to met with the requirement of high braking stability and energy regeneration rate, which including three braking force operating modes. Xu et.al [12] presented two braking regenerative energy optimization controllers to promote regenerative energy, it considering motor efficiency to distribute the friction and motor braking torque of the front and rear wheels. Li et al. [13] proposed a composite RBS to optimize regenerative and plugging

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braking simultaneously with the driver's intention recognition. Lian et al. [14] designed an optimal braking force distribution strategy, while uses a safety distance model to avoid collision. In addition, the rule-based algorithm [9], PID algorithm [15], sliding mode control [16], fuzzy logic control [17], design of experiment method [11], model predictive control (MPC) [12], and dynamic programming (DP) [18] were adopted to refine the RBS.

Recently, the emerging of connected vehicle technologies, i.e., vehicle-to-everything (V2X) communication, provides possibilities to further improve energy efficiency and reduce emission [19]. The traffic, road, and vehicle information can be accessed by V2X, which used to plan the energy-optimal velocity, i.e., the surrounding vehicle states [20], road slope [21], road curvature [22], and traffic signal phase and timing [23]. However, most researches focus on the RBS design in vehicle components control and braking torque distribution, few combine the connected vehicle technologies into braking velocity planning. If the braking intention is accessed by the V2X (i.e., braking distance and terminal velocity), the EVs could plan the braking velocity for recovering more energy.

Therefore, this paper proposes an energy-optimal braking strategy (EOBS), where the key is using shared braking intention to planning energy-optimal braking velocity. The major contributions of this paper are threefold. First, a double layer control scheme is proposed: In the upper-layer, the energy-optimal braking trajectory is planned by the distance-based DP with only one state variable and one control variable for fast computation. Second, to follow the optimal trajectory, the lower-layer designs a tracking controller using the linear time-varying MPC (LTV-MPC), to minimize the trajectory tracking errors. Third, the mechanism of energy-optimal braking is derived. The energy-optimal braking trajectory is consists of multiple phases, which can be converted to a rule-based braking strategy.

The remainder of this paper is organized as follows. Section 2 introduces the EVs model and regenerative braking strategy. Section 3 formulates the energy-optimal braking problem and double-layer EOBS. The EOBS performance is verified and energy-optimal braking mechanism is analyzed in Section 4. Section 5 concludes this paper and proposes future work.

2 Vehicle model and regenerative braking strategy

2.1 Vehicle model

This paper optimizes the braking velocity of four-wheel-independent-driven electric vehicles, which is powered by four in-wheel motors (IWM).

2.1.1 Vehicle dynamics

For energy-optimal braking research, only the longitudinal dynamics is modeled, as shown in Eq. (1).

$$\dot{x} = f(x, u) = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{0.5C_D A_r \rho v^2 + mgf \cos \theta + mg \sin \theta}{\delta m v} \end{bmatrix} x + \begin{bmatrix} 0 \\ \frac{1}{\delta m} \end{bmatrix} u \quad (1)$$

where m is the vehicle mass, δ is the vehicle rotational inertia coefficient, g is the acceleration of gravity, θ is the road slope, f is the rolling resistance coefficient, C_D is the aerodynamic drag coefficient, A_r is the frontal area, and ρ is the air density. $x = [d \ v]^T$ is the states variable, d is the distance, v is the velocity. $u = F$ is the control variable, F is the vehicle force, positive for traction force and negative for braking force.

In the traction process, the vehicle force is supplied by IWM, i.e., $F = \sum F_m$. In the braking process, the vehicle force including motor force F_m and friction braking force F_f , i.e., $F = \sum F_m + \sum F_f$. The friction braking force is generated by a hydraulic brake system (HBS). We use the first-order lag relation to represent the HBS dynamic response, i.e.,

$$\dot{p}_w + p_w = p_d \quad (2)$$

with

$$p_d = F_f r_w \kappa$$

where p_w is the brake wheel cylinder pressure, κ is the conversion coefficient of wheel cylinder pressure, τ is the HBS time constant coefficient, r_w is the radius of wheel.

2.1.2 Energy consumption

The power of IWM is calculated by Eq. (3).

$$P_m = \frac{F_m r_w n}{9.55} \quad (3)$$

where P_m is the motor power, positive for propulsion power and negative for generating power. n is the motor rotational speed.

This paper ignores the energy losses in electric wires and considers the auxiliary power P_a . Then the battery power P_b is

$$P_b = \sum P_m \eta_m^{-\text{sign}(P_m)} \eta_b^{-\text{sign}(P_m)} + P_a \eta_b^{-1} \quad (4)$$

where η_b is the battery efficiency, η_m is the motor efficiency which is determined by the motor torque and speed as shown in Figure 1, $\text{sign}()$ is the signum function.

Figure 1 The efficiency map and peak torque of IWM

The battery is modeled as a simple equivalent circuit [24], which generating current I_b is expressed as Eq. (5).

$$I_b = \frac{E - \sqrt{E^2 - 4R_o P_b}}{2R_o} \quad (5)$$

where E is the open-circuit voltage, and R_o is the internal resistance. The battery state of charge (SOC) can be calculated by Eq. (6).

$$SOC = SOC_0 + \frac{\int I_b dt}{3600C_b} \quad (6)$$

where SOC_0 is the battery initial SOC value, C_b is the battery capacity.

2.2 Regenerative braking strategy

The RBS including twofold: braking force distribution on the front and rear axles, motor and friction braking force distribution on each wheels. The distributed braking force on the front and rear axles affect braking stability, then we use ‘ideal distribution strategy’ to prevent wheel locking [25]:

$$F_{bf} = \beta F \quad (7)$$

$$F_{br} = (1 - \beta)F \quad (8)$$

where F_{bf} and F_{br} are the braking force on the front and rear axle, respectively. β is the ideal distribution ratio, which is proportional to vehicle axle normal forces:

$$\beta = \frac{(L_b + zh_g)}{L_w} \quad (9)$$

where L_b is the distance from the rear axle to the center of vehicle mass, L_w is the vehicle wheelbase, h_g is the distance from the ground to the center of vehicle mass, z is the brake strength.

We assume the vehicle driving in the straight road, then the braking force on the left and right wheels are identical. The motor force is preferred to recycle more energy, and HBS provides the remained force while the motor force can not meet the demand. For an example of the front-rear wheel, the motor and friction braking force are calculated by Eqs. (10) and (11).

$$F_m = \min(F_{m\max}, 0.5F_{bf}) \quad (10)$$

$$F_f = 0.5F_{bf} - F_m \quad (11)$$

where $F_{m\max}$ is motor maximum braking force.

3 Energy-optimal braking strategy

This section formulates the energy-optimal braking problem and proposes a double-layer EOBS to improve regeneration energy while ensuring braking comfort and safety. In the upper-layer, the energy-optimal braking trajectory is planned by solving the optimal control problem. In the lower-layer, a tracking controller is used to track the optimized trajectory accurately while ensuring braking comfort and safety. Figure 2 shows the scheme of

the double-layer EOBS.

3.1 Energy-optimal braking trajectory planning

The energy-optimal braking trajectory is planned by the DP algorithm. Since the DP has been widely used in the energy optimization problem [19], which breaks the optimal control problem into simpler sub-problems and calculates recursively based on the Bellman Optimality Principle [26].

3.1.1 Optimization problem formulation

Figure 3 shows the scenario of braking, where D is the distance to the destination (can be a traffic light or a preceding vehicle), V_s and V_e are the initial and desired terminal velocity. Note that the D and V_e are accessed by V2X communication, the V_s is obtained using on-board sensors.

In this paper, the braking distance D is fixed, thus the whole problem is discretized in distance-domain by distance step Δd . The total number of distance N_D is determined by the D and Δd , i.e., $N_D = D/\Delta d - 1$. The velocity is selected as the state variable, i.e., $x_D = v$, and the braking deceleration a is selected as the control variable, i.e., $u_D = a$. Since the distance step is small, the braking deceleration of each sub-problem is assumed to be constant. Thus, the energy-optimal braking problem is expressed in (12) indexed by k .

$$\begin{aligned} & \text{Minimum}_{u_D \in U} J_D(x_D, u_D) \\ & = \sum_{k=1}^{k=N_D} P_b(k) \Delta t_D(k) + \alpha (v(N_D) - V_b)^2 \end{aligned} \quad (12)$$

s.t.

$$\begin{aligned} x_D(0) &= V_s, \quad x_D(N_D) = V_b \\ v(k) &\in [v_{\min}, v_{\max}] \\ u_D(k) &\in [a_{\min}, a_{\max}] \end{aligned}$$

where $v_{\min}$ and $v_{\max}$ are the minimum velocity for efficient passing and the maximum velocity for road limit, respectively. $a_{\min}$ and $a_{\max}$ are the deceleration depend on comfort and vehicle resistance, respectively. Δt_D is the time interval. Note that the terminal condition of the velocity has been relaxed by converting them to soft constraints and merged into the objective function as Eq. (12), with weighting factor α for the latter term.

Besides, the state transfer of vehicle velocity and time interval in k step are calculated by Eqs. (13) and (14), respectively.

$$v(k+1) = \begin{cases} \sqrt{v^2(k) + 2a(k)\Delta d} & v^2(k) + 2a(k)\Delta d > 0 \\ 0 & v^2(k) + 2a(k)\Delta d \leq 0 \end{cases} \quad (13)$$

$$\Delta t_D(k) = \frac{v(k+1) - v(k)}{a(k)} \quad (14)$$

Figure 2 The EOBS scheme

Figure 3 The process of braking

3.1.2 DP solving

Denote the transitional cost-to-go function from k step to $k+1$ step $W(u_D(k), x_D(k))$ and the penalty function for the terminal states $G(x_D(N_D))$, i.e.,

$$W(u_D(k), x_D(k)) = P_b(k) \Delta t_D(k) \quad (15)$$

$$G(x_D(N_D)) = \alpha(v(N_D) - V_s)^2 \quad (16)$$

The formulated control problem is solved recursively by following the Bellman Optimality Principle [26].

$N_D - 1$ step:

$$J_{N_D-1}^*(x_D(N_D-1)) = \min_{u_D \in U} \{W(u_D(N_D-1), x_D(N_D-1)) + G(x_D(N_D))\} \quad (17)$$

k step:

$$J_k^*(x_D(k)) = \min_{u_D \in U} \{W(u_D(k), x_D(k)) + J_{k+1}^*(x_D(k+1))\} \quad (18)$$

where $J_k^*(x_D(k))$ is the minimum cost-to-go value at the moment k . Then, the energy-optimal braking trajectory which starting from any given arbitrary initial and terminal states can be obtained by backward calculation.

3.2 Tracking controller

3.2.1 Tracking problem formulation

The objective of the tracking controller is to minimize the tracking error between the actual and desired trajectory for ensuring high regeneration efficiency, while adapts to traffic conditions for braking comfort and safety. In this paper, the state error and driving force increment are defined as soft constraints to ensure tracking accuracy

and driving comfort, thus, the tracking problem is shown in Eq. (19).

$$\begin{aligned} & \text{Minimum } J_M(u_M | k) \\ & u_M \in U \\ & = \sum_{i=1}^{N_p} \|\tilde{x}(k+i|k)\|_Q^2 + \sum_{j=1}^{N_c-1} \|\Delta u(k+j|k)\|_R^2 + \beta \varepsilon^2 \quad (19) \end{aligned}$$

s.t.

$$\begin{aligned} u_M(k) & \geq u_{\min}(k) \\ D-L & \leq d(N_M) \leq D \\ V_e - \vartheta & \leq v(N_M) \leq V_e \end{aligned}$$

where $\tilde{x} = [d-d_{ref}, v-v_{ref}]$ is the states tracking error, d_{ref} and v_{ref} are the desired distance and velocity, respectively, which are derived from the upper-layer optimization. Δu is the increment of the control variable, $(u_M|k)$ represents the derived control variable of the controller at k moment. Q , R , and β are the weighing factor matrix, ε is the relaxation factor, N_p and N_c are the prediction horizon and control horizon, respectively. N_M is the tracking problem horizon, which determined by the length of the desired trajectory. L is the vehicle length, ϑ is the velocity slack.

In tracking problem Eq. (19), the constraints including twofold. First, the vehicle force is subject to physical limits $u_{\min}$, which determined by the maximum comfort braking force and road adhesion. Secondly, the virtual stop region that including distance and velocity is defined as a buffer space that relies on safety distance and minimum steady velocity of EVs, then the actual terminal

braking distance and velocity are meet the second and third constraints.

3.2.2 Linear time-varying state prediction model

Since the vehicle dynamics in Eq. (1) is nonlinear, it greatly increases the computational burden of MPC. Thus, we employ the approximate linearization method [27] to transform the nonlinear system to a linear system. Thus, the linear time-varying (LTV) vehicle state error model is

$$\dot{\tilde{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{C_D A_r \rho v_{ref}}{\delta m} \end{bmatrix} \tilde{x} + \begin{bmatrix} 0 \\ 1 \\ \delta m \end{bmatrix} \tilde{u} \quad (20)$$

where $\tilde{x} = [d - d_{ref} \quad v - v_{ref}]^T$, $\tilde{u} = [F - F_{ref}]$, F_{ref} is the desired braking force. In addition, the MPC calculates the control rule discretely, then the LTV system in Eq. (20) is converted to the discrete-time system using Euler method [28]:

$$\tilde{x}(k+1) = \begin{bmatrix} 1 & \Delta t_M \\ 0 & 1 - \frac{C_D A_r \rho v_{ref}(k) \Delta t_M}{\delta m} \end{bmatrix} \tilde{x}(k) + \begin{bmatrix} 0 \\ \Delta t_M \\ \delta m \end{bmatrix} \tilde{u}(k) \quad (21)$$

where Δt_M is the time step.

Denote a state prediction matrix:

$$\varsigma(i+1|k) = \begin{bmatrix} \tilde{x}(i|k) \\ \tilde{u}(i-1|k) \end{bmatrix} \quad (22)$$

where $(i|k)$ represents the i -step-ahead predicted error at the k -th step. Substituting Eq. (21) into Eq. (22), a new state-space model is:

$$\begin{cases} \varsigma(i+1|k) = A_{k,i} \varsigma(i|k) + B_{k,i} \Delta \tilde{u}(i|k) \\ v(i|k) = C_{k,i} \varsigma(i|k) \end{cases} \quad (23)$$

with

$$A_{k,i} = \begin{bmatrix} 1 & \Delta t_M & 0 \\ 0 & 1 - \frac{C_D A_r \rho v_{ref}(i|k) \Delta t_M}{\delta m} & \frac{\Delta t_M}{m} \\ 0 & 0 & 1 \end{bmatrix}$$

$$B_{k,i} = \begin{bmatrix} 0 & \frac{\Delta t_M}{\delta m} & 1 \end{bmatrix}^T, \quad C_{k,i} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Finally, the state prediction model is listed in Eq. (24) by converting Eq. (23).

$$\tilde{Y}(k) = \Gamma_k \varsigma(k) + \Xi_k \Delta \tilde{U}(k) \quad (24)$$

with

$$\Gamma_k = \begin{bmatrix} C_{k,i} A_{k,i} & \cdots & C_{k,i} A_{k,i}^{N_c} & \cdots & C_{k,i} A_{k,i}^{N_p} \end{bmatrix}_{1 \times N_p}$$

$$\Xi_k = \begin{bmatrix} C_{k,i} B_{k,i} & 0 & \cdots & 0 \\ C_{k,i} A_{k,i} B_{k,i} & C_{k,i} B_{k,i} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ C_{k,i} A_{k,i}^{N_c} B_{k,i} & C_{k,i} A_{k,i}^{N_c-1} B_{k,i} & \cdots & C_{k,i} A_{k,i} B_{k,i} \\ \vdots & \vdots & \ddots & \vdots \\ C_{k,i} A_{k,i}^{N_p-1} B_{k,i} & C_{k,i} A_{k,i}^{N_p-2} B_{k,i} & \cdots & C_{k,i} A_{k,i}^{N_p-N_c-1} B_{k,i} \end{bmatrix}_{N_p \times N_c}$$

$$\tilde{Y}(k) = [v(k+1|k) \quad \cdots \quad v(k+N_c|k) \quad \cdots \quad v(k+N_p|k)]_{1 \times N_p}^T$$

$$\Delta \tilde{U}(k) = [\Delta \tilde{u}(k|k) \quad \Delta \tilde{u}(k+1|k) \quad \cdots \quad \Delta \tilde{u}(k+N_c|k)]_{1 \times N_c}^T$$

where $\tilde{Y}(k)$ and $\Delta \tilde{U}(k)$ are output and control increment of the prediction model, respectively. Noted

that according to the principle of MPC, the $N_c \leq N_p$, and the control input that beyond N_c is always the last control input of N_c .

3.2.3 Quadratic programming solving

The tracking problem is solved using the quadratic programming (QP) algorithm. Then, the objective function and constraints in Eq. (19) are converted to standard QP form [29], i.e.,.

$$J_M(u_M | k) = \frac{1}{2} \begin{bmatrix} \Delta \tilde{U}(k)^T \\ \varepsilon \end{bmatrix} \begin{bmatrix} 2(\Xi_k^T Q \Xi_k + R) & 0 \\ 0 & 2\beta \end{bmatrix} \begin{bmatrix} \Delta \tilde{U}(k)^T \\ \varepsilon \end{bmatrix} + \begin{bmatrix} 2E_k^T Q \Xi_k \\ 0 \end{bmatrix}^T \begin{bmatrix} \Delta \tilde{U}(k)^T \\ \varepsilon \end{bmatrix} \quad (25)$$

s.t.

$$D - L \leq d(N_M) \leq D$$

$$V_e - \vartheta \leq v(N_M) \leq V_e$$

$$\begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}_{N_c \times N_c} \Delta \tilde{U}(k) + 1_{N_c} \Delta \tilde{u}(k-1) \geq U_{\min}$$

where 1_{N_c} is the column vector with N_c rows, $U_{\min}$ is the minimum value set of control input:

$$U_{\min} = [u_{\min}(k) \quad u_{\min}(k+1) \quad \cdots \quad u_{\min}(k+N_c-1)]_{1 \times N_c}^T \quad (26)$$

A series of control input increments $\Delta \tilde{U}_k^*$ in the control horizon can be obtained by solving the QP problem. The predicted control input $\tilde{u}^*(k)$ is the first term of control sequence plus previous control input $\tilde{u}(k-1)$:

$$\tilde{u}^*(k) = \tilde{u}(k-1) + [1 \quad 0 \quad \cdots \quad 0]_{1 \times N_c} \Delta \tilde{U}_k^* \quad (27)$$

Finally, the predicted vehicle braking force $u^*(k)$ is

$$u^*(k) = \tilde{u}^*(k) + F_{ref}(k) \quad (28)$$

4 Simulation results and discussion

In this section, two braking scenarios are simulated to evaluate the regeneration energy efficiency of the proposed strategy, and the constant deceleration braking strategy (CDBS) is simulated as the benchmark. For CDBS [30], the vehicle will brake by fixed deceleration, and the lower-layer uses the same LTV-MPC controller as the proposed strategy to track the fixed deceleration. In addition, the index α_r is defined to evaluate the performance of regeneration efficiency as described in Eq. (29) [9]:

$$\alpha_r = \frac{\int P_b dt}{0.5 m \delta (v_s^2 - v_e^2)} \times 100\% \quad (29)$$

where v_e is the actual terminal velocity.

4.1 Simulation setup

A simulation is conducted by jointing MATLAB

(version 9.4, R2018a) and CarSim (version 20959, 2016.1) on a PC with an Intel Core i7-8700 @ 3.20GHz CPU, the joint simulation structure as shown in Figure 2. The EVs model is established using C-Class hatchback 2012 vehicle in CarSim, and the main parameters of EVs are $f = 0.016$, $m = 1421$ kg, $C_D = 0.3$, $A_r = 2.22$ m², $\rho = 1.206$, $r_w = 0.325$ m, $g = 9.8$ m·s⁻², $\delta = 1.022$, $L = 4.85$ m, $L_w = 2.91$ m, $L_b = 1.9$ m, $h_g = 0.54$ m, and $P_a = 300$ W. The specifications of IWM are the peak propulsion power and generating power are 20.75 kW and -20.53 kW, respectively. The peak propulsion torque and generating torque are 312.50 Nm and -311.50 Nm, respectively, and the peak rotational speed is 1600 rpm. The specifications of battery are $E = 360$ V, $C_b = 140$ Ah, $R_o = 0.45$ Ω , $SOC_0 = 0.8$, and $\eta_b = 0.9$. According to the HBS module in CarSim, the κ of wheels in front axle and rear axle are 250MPa/(N·m) and 150MPa/(N·m), the τ of all wheels is 0.06.

In the DP optimization, the distance interval $\Delta d = 0.01$ m, the grid step of state variable v and control variable a are 0.1 m·s⁻¹ and 0.05 m·s⁻¹, respectively. In LTV-MPC controller, the sampling time is $\Delta t = 0.01$, the prediction and control horizon are $N_p = 100$ and $N_c = 100$, respectively. The weight matrices are $Q = 1000I_{2N_p \times 2N_p}$ and $R = 200I_{N_c}$.

(a) Velocity trajectory

(b) Distance trajectory

Figure 4 The braking trajectory of EVs **Figure 5** The braking force of EVs **Figure 6** The slip rate of EVs by LTV-MPC

Two scenarios are simulated and referred as Scenario A and Scenario B, i.e., normal braking with small deceleration and emergency braking with big deceleration. The main parameters of these two scenarios are listed in Table 1.

Table 1 Braking scenarios

	Scenario A	Scenario B
Initial velocity V_s	34 m·s ⁻¹	20 m·s ⁻¹
Terminal velocity V_e	20 m·s ⁻¹	0 m·s ⁻¹
Distance to destination D	204 m	50m
Adhesion coefficient ϕ	0.85	
Velocity slack ω	0.2 m·s ⁻¹	
Safety distance slack ϑ	4.85 m	
Road adhesion θ	0	

4.2 Performance of proposed EOBS

4.2.1 Tracking performance

The tracking performance of the LTV-MPC controller will be verified in Scenario A, and compared with the regular PID controller. Figures 4-6 and Table 2 are the simulation results.

Table 2 Simulation result of LTV-MPC and PID controller

	Regeneration energy efficiency α_r	Braking duration	Distance d	Terminal velocity v_e
Desired trajectory	68.21%	7.30s	198.85m	20.09m·s ⁻¹
LTV-MPC controller	68.18%	7.32s	198.88m	20.10m·s ⁻¹
PID controller	64.35%	7.68s	208.81m	20.13m·s ⁻¹

Figure 4 shows the vehicle trajectory of distance and velocity, Figure 5 and Figure 6 shows the vehicle braking force and slip rate, respectively. The regeneration energy, vehicle terminal states are listed in Table 2. In Figure 4 and Table 2, the actual terminal velocity and braking distance error of LTV-MPC controller are $0.01\text{m}\cdot\text{s}^{-1}$ and 0.03m respectively, and $0.72\text{m}\cdot\text{s}^{-1}$ and 9.96m of PID controller. The error data shows better tracking performance of the LTV-MPC controller than the PID controller. The distance exceeds the maximum permissible distance ($201.58\text{m} \sim 196.73\text{m}$) in PID controller, which causes collision possibly. Figure 5 shows the braking force both do not exceed constraints of LTV-MPC and PID controller, however, the PID controller leads to a large tracking error, which is the cause of state tracking error as shown in Figure 4. Figure 6

shows the small slip rate of each wheel, which indicates the braking stability is still ensured by employing the LTV-MPC.

As shown in Table 2, the vehicle with the LTV-MPC controller achieves regeneration energy efficiency increase by 3.83% compared with the PID controller, and decreases by 0.03% compared with the desired trajectory. Thus, the regeneration energy efficiency of the LTV-MPC controller is superior to the PID controller, because of the excellent tracking performance.

4.2.2 Regeneration energy efficiency

The Regeneration energy efficiency of EOBS is evaluated by compared with CDBS in Scenarios A and B. The simulation results are shown in Figure 7 and Table 3.

Figure 7 The simulation results of braking deceleration

Table 3 Simulation results of EOBS and CDBS

		Regeneration energy efficiency α_r	Improvement	Distance d	Terminal velocity v_e
Scenario A	EOBS	68.18%	11.42%	198.88m	$20.10\text{m}\cdot\text{s}^{-1}$
	CDBS	56.76%	N/A	198.94m	$20.10\text{m}\cdot\text{s}^{-1}$
Scenario B	EOBS	44.61%	3.19%	46.97m	$0.10\text{m}\cdot\text{s}^{-1}$
	CDBS	41.42%	N/A	47.13m	$0.11\text{m}\cdot\text{s}^{-1}$

Figure 7 shows the braking deceleration. The braking deceleration of CDBS is constant (e.g. Scenario A is $-1.89\text{m}\cdot\text{s}^{-2}$ and Scenario B is $-4.26\text{m}\cdot\text{s}^{-2}$, which are the mean deceleration of EOBS), while EOBS derives a varied deceleration for energy-optimal consideration. Table 3 shows the regeneration energy and vehicle terminal states of EOBS and CDBS. We can observe the vehicle could stay within the defined region in Scenarios A and B, and the proposed EOBS has improved the regeneration energy in Scenario A and Scenario B by 11.42% and 3.19% than CDBS.

These above-mentioned results show that the proposed EOBS guarantees the braking safety, while improving regeneration energy, especially in normal braking scenario. Further, the energy-optimal braking mechanism of EVs will be analyzed in detail in Section 4.3.

4.3 Energy-optimal braking mechanism

Figures 8-10 are the motor braking force, braking cylinder pressure of HBS (the sum of every wheel), and battery charging power, respectively. The motor efficiency during the regenerative braking and its average value are listed in Table 4.

Scenario A is the normal braking scenario, the mean deceleration is $-1.89\text{m}\cdot\text{s}^{-2}$. As shown in Figure 7(a), the deceleration of EOBS including three phases: increasing firstly, then decreasing, and increasing again. The wheel cylinder pressure increases firstly and then drops to approximate zero in the second and third phases (see Figure 9(a)). In the first phase, a large braking deceleration contributes to three advantages: (1) The velocity drops rapidly to decrease the aerodynamic drag (the relationship of aerodynamic drag with velocity is an exponential function); (2) Peak generating power of the motor (see Figure 10(a)) can be fully used for regenerative braking to recover more energy. The second and last

phases of EOBS should be discussed together because these two phases are complementary compared to CDBS. Figures 8(a) and 10(a) illustrate complementary relationship: the motor braking force, battery charging power of EOBS less than CDBS in the second phase, but

larger than CDBS in the last phase. In the second phase, the braking deceleration decreases quickly to reduce friction braking force (see Figure 9(a)). In the last phase, to make full use of motor (see Figures 8(a), 10(a)), the braking deceleration of EOBS increases.

(a) Scenario A

(b) Scenario B

Figure 8 The braking torque of motor

(a) Scenario A

(b) Scenario B

Figure 9 The wheel cylinder pressure of HBS

(a) Scenario A

(b) Scenario B

Figure 10 The battery charging power

Table 4 The efficiency of motor

		Front axle	Rear axle	Mean efficiency
		left/right wheel	left/right wheel	
Scenario A	EOBS	90.59%	87.73%	89.16%
	CDBS	90.55%	87.75%	89.15%
Scenario B	EOBS	77.99%	77.93%	77.96%
	CDBS	79.49%	79.49%	79.49%

Scenario B is the emergency braking scenario, the mean deceleration is $-4.26\text{m}\cdot\text{s}^{-2}$. As shown in Figure 7(b), the braking acceleration of EOBS can be divided into three phases same as Scenario A and for the same reasons. The first and second phases are discussed together, in the first phase, the deceleration of EOBS decreases with the velocity drops and increases in the second phase. In the third phase, the braking deceleration decreases because in low-speed operating conditions, the motor generating force is gradually dwindling and replaced by the friction braking force (see Figures 8(b) and 9(b)). As shown in Figure 10(b), the battery charging power of EOBS and CDBS is similar, which causes the regeneration energy improvement of EOBS is not obvious in the emergency braking case (see Table 3).

In addition, Table 4 shows the motor efficiency of EOBS and CDBS. The mean motor efficiency in EOBS is greater than CDBS in Scenario A, but it less than CDBS in Scenario B. This phenomenon indicates only focus on the maximum motor efficiency does not mean maximum regeneration energy. The regeneration energy is related to motor generating power and motor efficiency, and the co-optimization of motor generating power and motor efficiency is the key to achieve the energy-optimal braking control. Therefore, the energy-optimal braking control requires reducing vehicle resistance and increasing motor braking force for improving the motor generating power, and operating the motor working at high-efficiency point at the same time.

5 Conclusions

This paper presented a double-layer energy-optimal braking strategy to improve the regeneration energy using shared braking intention. First, the energy-optimal braking trajectory is derived by distance-based dynamic programming in the upper-layer. Second, the linear time-varying model predictive controller is formulated to follow the desired trajectory which derived from the upper-layer, to ensures regeneration energy efficiency, braking safety, and comfort. The simulation results demonstrate the proposed energy-optimal braking strategy achieves prominent regeneration energy improvement than regular constant deceleration braking strategy, and the trajectory tracking performance of the linear time-varying model predictive controller is superior to regular PID controller. The analysis of the energy-optimal braking mechanism indicates the optimal regeneration energy operation is required to reduce the vehicle resistance and friction braking force while ensures motor working at high-efficiency point as much as possible.

7 Declaration

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Availability of data and materials

The datasets supporting the conclusions of this article are included within the article.

Authors' contributions

YG provided fundamental ideas and all support conditions of this research. HD was in charge of the trial and wrote the manuscript. WZ assisted with building up the framework of the research. LX and YW assisted the trial and simulations. FW and YL conducted proof reading and made some critical revisions. All authors read and approved the final manuscript.

Competing interests

The authors declare no competing financial interests.

Consent for publication

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Ethics approval and consent to participate

Not applicable

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Figures

Figure 1

The efficiency map and peak torque of IWM

Figure 2

The EOBS scheme

The braking force of EVs

Figure 6

The slip rate of EVs by LTV-MPC

Figure 7

The simulation results of braking deceleration

Figure 8

The braking torque of motor

(a) Scenario A

(b) Scenario B

Figure 9

The wheel cylinder pressure of HBS

(a) Scenario A

(b) Scenario B

Figure 10

The battery charging power