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A probe into the Multifractal Behaviour of Total Ozone Time Series through Detrended Fluctuation Analysis

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Abstract

The present study reports a multifractal detrended fluctuation analysis of total ozone time series. Considering daily total ozone concentration (TOC) data ranging from 2015 to 2019, we have created a new profile by subtracting the trend. Subsequently we have divided the profile X_i into non intersecting segments of equal time scale varying from 25 to 30. Fitting a second order polynomial, we have eliminated the local trend from each segment and thereafter we have computed the detrended variance. Finally the multifractal behaviour has been identified and the singularity spectra has helped us in obtaining the generalised Hurst exponent which in this case has come out to be greater than 0.5.

Key words: Multifractal detrended fluctuation analysis; total ozone concentration; detrended variance

1. INTRODUCTION

The physics laws governing the atmospheric phenomena, are usually non-linear and hence the application of the conventional approaches on the time series of the atmospheric quantities reveals usual non-stationary behaviour of the time series (Varotsos, 2005). The dynamics of global total ozone (TO) has been thoroughly studied in Kondratyev and Varotsos (1996), where the various aspects of tropospheric and stratospheric ozone on the global climate have been demonstrated. The study of climate and its variations over time and distance has been a significant area of research for a very long time. Analysis of climatic behaviour over any region helps in understanding and predicting or forecasting future climatic conditions. The results of such studies are useful for governments and policy makers in nations for many causes such as predicting a possible natural calamity, planning resource managements and many purposes of such kind. It is well documented in several literatures that Natural Systems display fluctuations that can be characterised by long-range power law correlations (Király et al.,2006; Ivanova and Ausloos, 1999; Meyer and Kantz, 2019; Crato et

al.,2010). Identifying the presence and analysis of power law correlations would help in quantifying the dynamics of the underlying process. The study of any climatological parameter using the traditional time series process produces unreliable results due to highly non stationary nature of the respective parameters. The non-stationarity of various climatological parameters makes the study of long-range power law correlation using traditional techniques like Auto-correlation function or Power Spectrum difficult (Koutsogiannis et al.,2020; Kantelhardt et al., 2001; Varotsos, 2005). To study the dynamics of such complex processes several specific methodologies have been developed by scientists over the past few decades and one of such kind of process is the Detrended Fluctuation Analysis (DFA), which is going to be implemented in the current study with the endeavour of understanding the fractal behaviour of some time series associated with climatological processes. The detailed methodology and implementation procedure would be elaborated in the subsequent sections. At this juncture we discuss some existing literatures that have contributed significantly towards understanding complex meteorological processes through DFA.

Detrended Fluctuation Analysis (DFA) has been an area of major interest in recent years in the field of hydrology and climatology to understand intrinsic complexity of the associated processes (Kantelhardt et. al 2002; Talkner and Weber, 2000). Matsoukas et al. (2000) in their detailed study of rainfall and streamflow by applying to it the technique of DFA reported that rainfall exhibits power law correlations. Their study shows that the DFA is a much reliable procedure for the estimation of power law exponent in contrary to traditional Time Series analysis such as Power Spectrum method. The DFA method produces much stable plot and hence allows more detailed and accurate study of the exponent behaviour. Upon comparison of scaling exponents between rainfall and streamflow it was observed that the dampening effect of the land surface transforms the rainfall into streamflow. Telesca et al. (2011) in their study found that rainfall patterns in different parts of Argentina are largely affected by their respective geographical locations. The rainfall pattern shows both temporal and spatial variations. The results of their study indicate how the scaling behaviour of the rainfall time series is a good indicator of the climatology of any particular area. Mallick et al. (2020) have reported in their work an analytic study of annual rainfall variability and trend in 30 meteorological stations of the Asir region for the period of 1970–2017, using the Mann-Kendall (MK) test, Modified Mann-Kendall (MMK) test, Trend Free Pre-Whitening Mann-Kendall (TFPW MK) test, and the Innovative Trend Analysis (ITA). Upon performing a comparative study among the trend detection techniques using correlation coefficient, it was

found that the ITA was more consistent and reliable technique than the other tests for detecting rainfall trend in the region taken into consideration as the test was able to detect significant negative trends which were prevalent in the rainfall patterns in some stations which the other tests failed to do. The DFA study of the same based on the past data predicted a decrease in the amount of future rainfall over the Asir region.

Among various climatological parameters, the study of Ozone dynamics had gained immense importance over the past few decades especially after the discovery of the Ozone hole over Antarctica in the year 1985. Many a great researcher started working and studying the causes and consequences of the variations of the Ozone concentration in the atmosphere. The atmospheric phenomena have been observed to be governed by non linear physical laws quite significantly. The findings of the study of time series of atmospheric quantities based on Fourier spectral analysis suggest that these are usually non-stationary. These non-stationarities hinders the detection of intrinsic dynamic properties such as correlations and thereby calls for the need of techniques that can systematically eliminate trends and cycles in the data. Several such methods have been used they are Rescaled Range Analysis (R/S), Wavelet Techniques (WT), Artificial Neural Networking and Detrended Fluctuation Analysis (DFA) and many others. A detailed study by Chattopadhyay and Chattopadhyay (2020) on association between Total Column Ozone (TCO) and Surface Temperature (ST) over Kolkata during the time period of September–November through spectral analysis approach reveal that TOC exhibits lower degree of variability than ST but there is an apparent similarity in the basic pattern of the spectra of TOC and ST. Varotsos (2005) in his study of power law correlations in column ozone over Antarctica applied DFA on springtime daily column ozone edge and into the Antarctic column ozone hole. The literature reports extreme column ozone fluctuations obey a power-law with exponents, implying that large fluctuations are more likely to occur into the ozone hole than at its edge. It was also observed that for time-scales longer than one year, persistent long-range power-law correlations in the column ozone fluctuations were more pronounced during 1979–1992. However, by eliminating the long-term trend, antipersistence (persistence) for time lags more (less) than ten days was detected for the entire data record. Other literatures in this direction include Varotsos (2005) and Varotsos et al. (2012, 2013, 2016, 2020a, 2020b).

Varotsos et. al (2011) discussed the observations of the investigation of potential effects of increased urbanisation in the city of Athens on the intrinsic properties of the temporal fluctuations of the Surface Ozone Concentration (SOC). It has been reported in the literature as a finding of the study that despite the then present-day SOC doubling with

respect to historic SOC, its fluctuations exhibit long-range power-law persistence, with similar features in time periods of 1901-1940 and 1987-2007 that were taken into consideration in the study. In the present work, we have carried out a multifractal detrended fluctuation analysis to understand the intrinsic complexity of the total ozone time series. The detailed methodology and the outcomes are presented in the subsequent sections.

2. METHODOLOGY

The DFA was introduced in early 1990s as a method for analyzing fractal properties of underlying data. The method was later majorly popularized in the long-range correlations and multi-fractal analyses. The procedure of DFA is explained in this section in details.

In our work we consider the daily Total Ozone Concentration (TOC) data ranging from 1st January 2015 to 31st May 2019 where each data point is considered as x_k where k = 1,...,N. In this application, the index k corresponds to the time of the measurements. Our interest lies in analysing the correlation of the values x_k and x_{k+s} for varying time lags. First we calculate the mean

$$< x >= \frac{1}{N}\sum_{k=1}^{N} x_k \tag{1}$$

and subtract it from the entire data set to eliminate the constant offset in the data and thereby create a separate profile, (Tzanis et al. 2020).

$$X_i = \sum_{i=1}^{N} x_k - < x > \tag{2}$$

Thereafter we divide the profile X_i into $N_s = int\left({}^{N}/_{s}\right)$ non intersecting segments of equal time scale s varying s from 30 to 25. It may be noted that not all s is a factor of N, and as a result a small portion of the data at the end remain outside the computational procedure. To make sure that this portion of the data is not discarded we repeat the same procedure starting from the end which results in the creation of $2N_s$ segments.

We then fit a second order polynomial $\widetilde{X_v}$ to each segment v = 1,2,.., $2N_s$ with the help of which we calculate the local trend for each point of time pertaining to each segment. The local trend is then subtracted from the profile, and by doing so we successfully eliminate the second order trend from the profile. The detrended variance is then calculated as follows (Tzanis et al. 2020)

$$F^2(s,v) = \begin{cases} \frac{1}{s}\sum_{i=1}^{s}\{X[(v-1)s+i]-\tilde{X}_v(i)\}^2, & for\ v = 1,..,N_s \\ \frac{1}{s}\sum_{i=1}^{s}\{X[N-(v-N_s)s+i]-\tilde{X}_v(i)\}^2, & for\ v = N_s+1,..,2N_s \end{cases} \quad (3)$$

After that we calculate the qth order fluctuation by using the formula mentioned below,

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_{v=1}^{2N_s} [F^2(s,v)]^{\frac{q}{2}} \right\}^{\frac{1}{q}} \quad (4)$$

$F_q(s)$ is computed for all values of s.

Table 1: Tabular presentation of $F_q(s)$ values for varying values of q and s.

q	$F_q(30)$	$F_q(29)$	$F_q(28)$	$F_q(27)$	$F_q(26)$	$F_q(25)$
3	17.74	40.54	12.14	13.13	206.32	26.34
4	26.17	59.88	17.99	18.44	307.90	39.41
5	33.04	75.68	22.77	22.61	391.49	50.19
6	38.60	88.46	26.65	25.91	459.48	58.97
7	43.13	98.89	29.82	28.55	515.16	66.17
8	46.87	107.52	32.44	30.7	561.31	72.14
9	50.01	114.74	34.64	32.49	600.04	77.15
10	52.67	120.87	36.51	34	77.15	81.41

We examine the scaling behaviour of $F_q(s)$ through the plot of log($F_q(s)$) against log(s) for each moment q.

For time series that are long-range correlated, $F_q(s)$ follows a power law.

$$F_q(s) \sim s^{h(q)} \quad (5)$$

To calculate the $h(q)$ value we calculate the ratio of $\log\left(F_q(s)\right)$ to log(s) by keeping q fixed and varying s each time. This procedure is repeated for $q = 3, \ldots, 10$. By doing that we get a

$h(q)$ value with respect to each q value. After that we tried to check the correlation between q and $h(q)$ and plotted a graph representing values of q over x-axis against values of $h(q)$ over y-axis as shown in the figure below. From the figure it can be observed that the $h(q)$ is strongly correlated with q. To confirm this observation we computed the correlation coefficient between q and $h(q)$ and the value was 0.94 (approx).

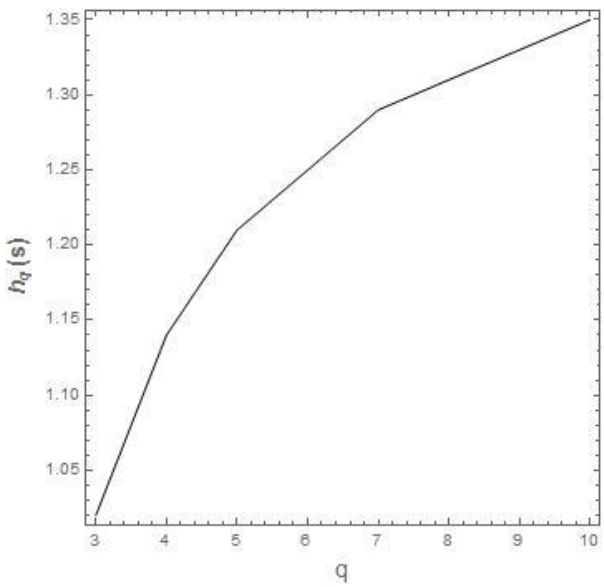

Fig 1: Line diagram representing q vs $h(q)$ values

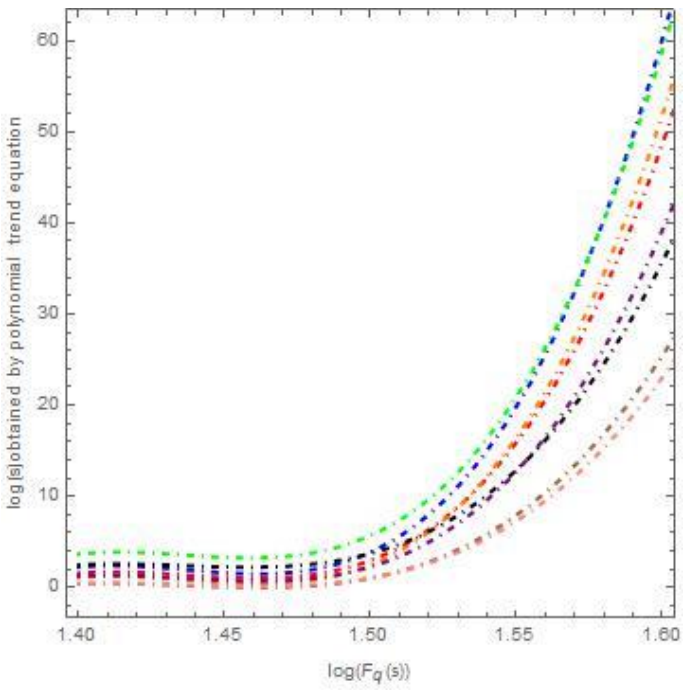

Fig 2: A schematic showing the behaviour of $\log\left(F_q(s)\right)$ against log(s) for the Total Ozone Time Series under consideration.

3. RESULTS AND DISCUSSIONS

In this work we have applied MF-DFA on the time series of Total Ozone (TO) in the univariate framework. In Eq. (1) we have calculated the mean of the time series that has been applied to eliminate the constraint of the data and create a separate profile in Eq. (2). Afterwards we have divided the new profile into non-intersecting segments of equal time scale varying from 30-25. To each segment we have fitted a second order polynomial to calculate the local trend for each point of time corresponding to each segment. In the subsequent phrase the local trend has been subtracted from the profile in order to eliminate the second order trend. The detrended profile has been calculated for its detrended variance using Eq.3. Finally we have computed the qth order fluctuation using Eq. (5). The qth order fluctuation $F_q(s)$ computed for all values of s has been presented in Table-1. Afterwards the scaling behaviour of $F_q(s)$ has been demonstrated in Figure-1, shows the strong dependence of h(q) on q. This figure has shown that h(q) is having an almost linear association with q. This is further quantified by the Pearson Correlation Coefficient of 0.94. The study has revealed the Multi-Fractal character of TO because h(q) and q are so strongly correlated. Furthermore h(q) is greater than 0.5 for all moments q. Hence long range correlations are identified in TO time series. In Figure-2 we have displayed a schematic presentation of log ($F_q(s)$) against log(s) obtained through polynomial form of trend equation. In this plot

different colours represent different values of s and in all the cases we have obtained similar pattern of the $\log(F_q(s))$-$\log(s)$ curve. Hence Multi Fractal behaviour of the TO time series is established through Detrended Fluctuation Analysis.

4. CONCLUDING REMARKS

The rigorous study presented above has revealed a Multi fractal Behaviour of Total Ozone time series for different moments. The cause of multifractality may lie in the intrinsic complexity of the Total Ozone and this behaviour indicates that to study the Total Ozone time series the fractal dimension is not enough to describe its dynamics instead a continuous spectrum of exponents are required. The singularity spectra computed in Eq. (4) has helped us in obtaining the generalised Hurst exponent which in this case comes out to be greater than 0.5. To further look into this source of multifractality we need to look into the multifractal behaviour of climatic parameters that influence the TO time series.

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Declarations:

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Conflicts of interest/Competing interests: The authors hereby declare that there is no conflict of interest associated with this work.
Availability of data and material : The Total Ozone Concentration data have been obtained from OMDOAO3e: OMI/Aura Ozone (O3) DOAS Total Column L3
Code availability: Computation has been carried out using Mathematica.
Authors' contributions: Both the authors have equal contribution to the paper.
Ethics approval: This is an original work and is submitted to this journal only.
Consent to participate: As this submission does not belong to Life Sciences, there is nothing relevant to declare in this section.
Consent for publication: Once accepted, the authors have the full consent to the publisher to publish the work.

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