Mathematical study on a dynamical predator-prey model with constant prey harvesting and proportional harvesting in predator

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Research Article

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Posted Date: March 31st, 2021

DOI: https://doi.org/10.21203/rs.3.rs-377501/v1

Noname manuscript No.
(will be inserted by the editor)

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Received: date / Accepted: date

Abstract A dynamical predator-prey model with constant prey harvesting, proportional harvesting in predator has been studied. The square root functional response also has been incorporated in the system to describe the prey herd behaviour, assuming the average handling time is zero. The existence and the local stability of equilibria of the system have been discussed. It is examined that, two types of bifurcation occur in the system. The two types of bifurcations have been analyzed, and it has been found by analyzing the saddle-node bifurcation that, there is a maximum sustainable yield. It is observed that if harvesting rate is greater than the maximum sustainable yield, the prey population abolish from the system and then extinction of the predator population happen. But if harvesting rate is lesser than the maximum sustainable yield, the extinction of the prey population can not be possible. By analyzing the Hopf bifurcation, it is obtained that, there exists an unstable limit cycle around the interior equilibrium point. Several numerical simulations are performed to check the results.

Keywords Predator-prey system · Harvesting · Square root functional response · Stability · Bifurcation

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1 Introduction

To describe the interactions between different species in the nature, predator-prey system is one of the most important and useful models. Lotka (1) and Voterra (2) separately offered basic models to describe the interaction between two species and proposed a two dimensional system of ordinary differential equations. They introduced a functional response and used in their model which is proportional to the density of predator. Later, many researchers studied on the classical Lotka-Volterra (1) (2) model and also produced different types of functional responses, e.g., C.S Holling (3) introduced three types of functional responses, Crowley-Martin (4), Beddington-DeAngelis (5) (6) offered different functional responses, etc. In the mentioned studies the authors used these functional responses by assuming that the predator can take any prey for their food i.e., they considered solitary prey behaviour.

But, in nature some preys live in herd behaviour to fight with the predators for their benefits. For that type of herd behaviour, the above mentioned functional responses are not appropriate to describe the interaction. To investigate the model with that type of prey herd behaviour, a model with a new functional response was proposed by Ajraldi et al.(7) and the author states that, it is more appropriate to model the functional response for prey herd behaviour as a square root function of prey density. Later, Braza in his paper(8) introduced a new functional response considering the square root of prey density in the Holling Type II functional response (3) and assuming the average handling time is zero he proposed a model with this functional response. Further many researchers studied that model to investigate the interaction between predator and prey with the functional response (9) (10) (11) (12) (13) (14). In this paper, this type of functional response has been included in the model.

Nowadays, people are very much interested to harvest the species for economic interest which very much affects on the dynamics of any ecological or biological system. For example, we can see in any marine ecosystem people are harvesting fishes for their economic interest which affects on the dynamics of a marine ecosystem. To tackle that type of situation, many researchers worked on a predator-prey system with harvesting effects (15) (16) (17) (18). Basically researchers used three types of harvesting and the classification is constant harvesting (19), proportional harvesting (20) and non-linear harvesting (21). In this paper we consider harvested predator and prey population. Prey populations are harvested by constant rate and predator populations are harvested by proportional rate. The main aim of the paper is to investigate the dynamical behaviour of a predator prey system with constant harvesting in prey, proportional harvesting in predator and also with square root functional response assuming average handling time is zero. The organization of the rest of the paper in this way:

In Sec.2 we formulate a mathematical model with constant harvesting in prey, proportional harvesting in predator and also with square root functional response assuming average handling time is zero. The condition for existence of equilibria and their local stability of system (3) have been discussed in Sec.3.

In Sec.4 two types of bifurcations of the system (3) have been analyzed. Sec.5 includes numerical examples. Finally, Sec.6 contains the conclusion.

2 Mathematical Model Formulation

The basic predator-prey model with the square root functional response (8), constant harvesting in prey and proportional harvesting in predator is as follows:

$$\begin{cases} \dfrac{dx(t)}{dt} = rx(1-\dfrac{x}{N}) - \alpha\sqrt{x}y - h, \\ \dfrac{dy(t)}{dt} = -\beta y + m\alpha\sqrt{x}y - E_q y. \end{cases} \tag{1}$$

Where the prey population denoted by x and the predator population denoted by y, r is the growth rate of prey, m is the biomass conversion rate, α denotes the predator's search efficiency for prey. N is the environment carrying capacity, h is the constant prey harvesting and β is the predator's natural death rate in the prey free situation and E_q is harvesting effort. For the dynamic analysis of the model, the variables and the parameters are scaled as, $\bar{x} = \dfrac{x}{N}, \bar{y} = \dfrac{\alpha y}{r\sqrt{N}}, \bar{t} = rt, c = \dfrac{m\alpha\sqrt{N}}{r}, d = \dfrac{\beta}{r}, E = E_q\dfrac{\sqrt{N}}{\alpha}$
Then the system (1) becomes:

$$\begin{cases} \dfrac{d\bar{x}(t)}{dt} = \bar{x}(1-\bar{x}) - \sqrt{\bar{x}}\bar{y} - h, \\ \dfrac{d\bar{y}(t)}{dt} = -d\bar{y} + c\sqrt{\bar{x}}\bar{y} - E\bar{y}. \end{cases} \tag{2}$$

Excluding the bars of the system (2) we formulate the following system:

$$\begin{cases} \dfrac{dx(t)}{dt} = x(1-x) - \sqrt{x}y - h, \\ \dfrac{dy(t)}{dt} = -dy + c\sqrt{x}y - Ey. \end{cases} \tag{3}$$

Where c, d, h, E all are positive.

3 Existence and stability of equilibria

3.1 Existence of equilibria

The system (3) has been defined on the following set considering biological background:
$\Gamma = \{(x, y) \in \mathbb{R}^2; x \geq 0, y \geq 0\}$.
To identify the equilibrium points of the system (3), the following system has

been solved:

$$\begin{cases} x(1-x) - \sqrt{x}y - h = 0, \\ -dy + c\sqrt{x}y - Ey = 0. \end{cases} \tag{4}$$

The above system has three equilibrium points and we get the following theorems regarding their existence.
Note that:

$$\begin{cases} x_1 = \dfrac{1+\sqrt{1-4h}}{2}, x_2 = \dfrac{1-\sqrt{1-4h}}{2}, \\ x_* = \left(\dfrac{d+E}{c}\right)^2, \quad y_* = \sqrt{x_*}(1-x_*) - \dfrac{h}{\sqrt{x_*}} \end{cases} \tag{5}$$

Theorem 1 *[(i)]*

When $0 < 4h < 1$, the system (3) has two axial equilibrium points $E_i = (x_i, 0), (i = 1, 2)$, which are different.

2. When $4h = 1$, two equilibria collide and produce a unique axial equilibrium point $E_0 = (x_0, 0) = \left(\frac{1}{2}, 0\right)$.

3. When $\sqrt{x_*}(1-x_*) - \frac{h}{\sqrt{x_*}} > 0$, then the equilibrium point of the system (3) exists uniquely which is denoted by $E_* = (x_*, y_*)$.

Proof. If $4h > 1$, then $\frac{dx(t)}{dt} = x(1-x) - \sqrt{x}y - h < 0$ in Γ. This implies that prey population will go extinct and then extinction of predator population will happens. It is obvious that in Γ the system (3) has no equilibria. Now for predator free (i.e., y=0) equilibrium points we have solved the following quadratic equation:

$$x^2 - x + h = 0. \tag{6}$$

Two different roots x_1, x_2 of the equation (6) are mentioned in (6) are positive when $4h < 1$ for positiveness of the equilibrium points. And if $4h = 1$, two equilibria collide and the unique value is $\frac{1}{2}$. For interior equilibrium point we solve the system (4). From second equation of the system (4) we get, $\sqrt{x_*} = \left(\frac{d+E}{c}\right)$, and after putting the value of x_* from first equation of the system (4) we get, $y_* = \sqrt{x_*}(1-x_*) - \frac{h}{\sqrt{x_*}} > 0$. ■

3.2 Stability of equilibria

This subsection includes the dynamic analysis of the system (3) around the equilibrium points. The following matrix J is the Jacobian matrix of the system

(3):

$$J = \begin{pmatrix} -2x - \dfrac{y}{2\sqrt{x}} + 1 & -\sqrt{x} \\ \dfrac{cy}{2\sqrt{x}} & c\sqrt{x} - (d+E) \end{pmatrix}. \tag{7}$$

Now, the local stability of the system (3) are analyzed in the neighbourhood of E_1 and E_2. And regarding this we have the following theorems.

Theorem 2 *When $4h < 1$ the system (3) has two different equilibria E_1 and E_2.*

[(i)]If $c\sqrt{x_1} > (d+E)$, then E_1 is hyperbolic saddle. If $c\sqrt{x_1} < (d+E)$, then E_1 is stable.

Proof. It can be seen that from Theorem (1) that, the system (3) has two different equilibria E_1 and E_2. Now the following matrix $J(E_1)$ is the Jacobian matrix at E_1 of the system (3)

$$J(E_1) = \begin{pmatrix} 1 - 2x_1 & -\sqrt{x_1} \\ 0 & c\sqrt{x_1} - (d+E) \end{pmatrix}. \tag{8}$$

The eigenvalues λ_1, λ_2 of the Jacobian matrix $J(E_1)$ are as follows:
$\lambda_1 = 1 - 2x_1 = -\sqrt{1-4h} < 0$, since $4h < 1$,
$\lambda_2 = c\sqrt{x_1} - (d+E) > 0$, if $c\sqrt{x_1} > (d+E)$. Hence, E_1 is hyperbolic saddle.
Now, if $c\sqrt{x_1} < (d+E)$, then $\lambda_2 = c\sqrt{x_1} - (d+E) < 0$. Hence, E_1 is stable.
■

2. **Theorem 3** *When $4h < 1$ the system (3) has two different equilibria E_1 and E_2.*

[(i)]If $c\sqrt{x_2} > (d+E)$, then E_2 is unstable. If $c\sqrt{x_1} < (d+E)$, then E_2 is hyperbolic saddle.

Proof. From Theorem (1) it can be observed that, the system (3) has two different equilibria E_1 and E_2. The following matrix $J(E_2)$ is the Jacobian matrix at E_2 of the system (3)

$$J(E_2) = \begin{pmatrix} 1 - 2x_2 & -\sqrt{x_2} \\ 0 & c\sqrt{x_2} - (d+E) \end{pmatrix}. \tag{9}$$

The eigenvalues μ_1, μ_2 of the Jacobian matrix $J(E_2)$ are as follows:
$\mu_1 = 1 - 2x_2 = \sqrt{1-4h} > 0$, since $4h < 1$,
$\mu_2 = c\sqrt{x_2} - (d+E) > 0$, if $c\sqrt{x_2} > (d+E)$. Hence, E_2 is unstable.
If $c\sqrt{x_2} < (d+E)$, then $\mu_2 = c\sqrt{x_2} - (d+E) < 0$. Hence, E_1 is hyperbolic saddle.
Now, the local stability of the unique interior equilibrium point E_* are analyzed and the following theorem is obtained.

2. **Theorem 4** *When $\sqrt{x_*}(1-x_*) - \dfrac{h}{\sqrt{x_*}} > 0$, then there exists a unique equilibrium point $E_* = (x_*, y_*)$ of the system (3).*

[(i)]If $h = h_1$, then E_* is center. E_* is source at $h > h_1 > 0$. E_* is sink at $h < h_1$.

Note: $h_1 = 3x_*^2 - x_*$

Proof. Theorem (1) shows that, when $\sqrt{x_*}(1 - x_*) - \dfrac{h}{\sqrt{x_*}} > 0$, then there exists a unique interior equilibrium point $E_* = (x_*, y_*)$ of the system (3). The following matrix $J(E_*)$ is the Jacobian matrix at E_* of the system (3)

$$J(E_*) = \begin{pmatrix} 1 - 2x_* - \dfrac{y_*}{2\sqrt{x_*}} & -\sqrt{x_*} \\ \dfrac{cy_*}{2\sqrt{x_*}} & 0 \end{pmatrix}. \tag{10}$$

$\text{Trace}(J(E_*)) = -\dfrac{y_*}{2\sqrt{x_*}} - 2x_* + 1$ and $\text{Det}(J(E_*)) = \dfrac{cy_*}{2} > 0$. Now, $\text{Trace}(J(E_*))$ $= -\dfrac{y_*}{2\sqrt{x_*}} - 2x_* + 1 = -\dfrac{\sqrt{x_*}(1 - x_*) - \dfrac{h}{\sqrt{x_*}}}{2\sqrt{x_*}} + 1 - 2x_* = \dfrac{-(3x_*^2 - x_*) + h}{2x_*}$.

Taking $h_1 = 3x_*^2 - x_*$, we get, $\text{Trace}(J(E_*)) = \dfrac{-h_1 + h}{2x_*}$.

Now, E_* is center when $h = h_1$, since the Trace of the Jacobian matrix (10) is zero at $h = h_1$.

E_* is source when $h > h_1 > 0$, since the Trace of the Jacobian matrix (10) is greater than zero at $h > h_1 > 0$.

E_* is sink when $h < h_1$, since the Trace of the Jacobian matrix (10) is less than zero at $h < h_1$ ■

4 Bifurcation Analysis

Form Theorem (1) and Theorem (4), it can be seen that the system (3) may exhibits saddle-node and Hopf bifurcation. So, in this section, the conditions for two different types of bifurcations have been analyzed.

4.1 Saddle-node bifurcation

From Theorem (1), it is observed that two equilibria of the system (3) collide and produce a unique equilibria point $E_0 = \left(\dfrac{1}{2}, 0\right)$. Now, the following matrix $J(E_0)$ is the Jacobian matrix at E_0 of the system (3

$$J(E_0) = \begin{pmatrix} 0 & -\sqrt{\dfrac{1}{2}} \\ 0 & c\sqrt{\dfrac{1}{2}} - (d + E) \end{pmatrix}. \tag{11}$$

Easily it can be seen that the above Jacobian matrix (11) has a zero eigenvalue. This means that, at E_0 the stability analysis of the system (3) is not possible

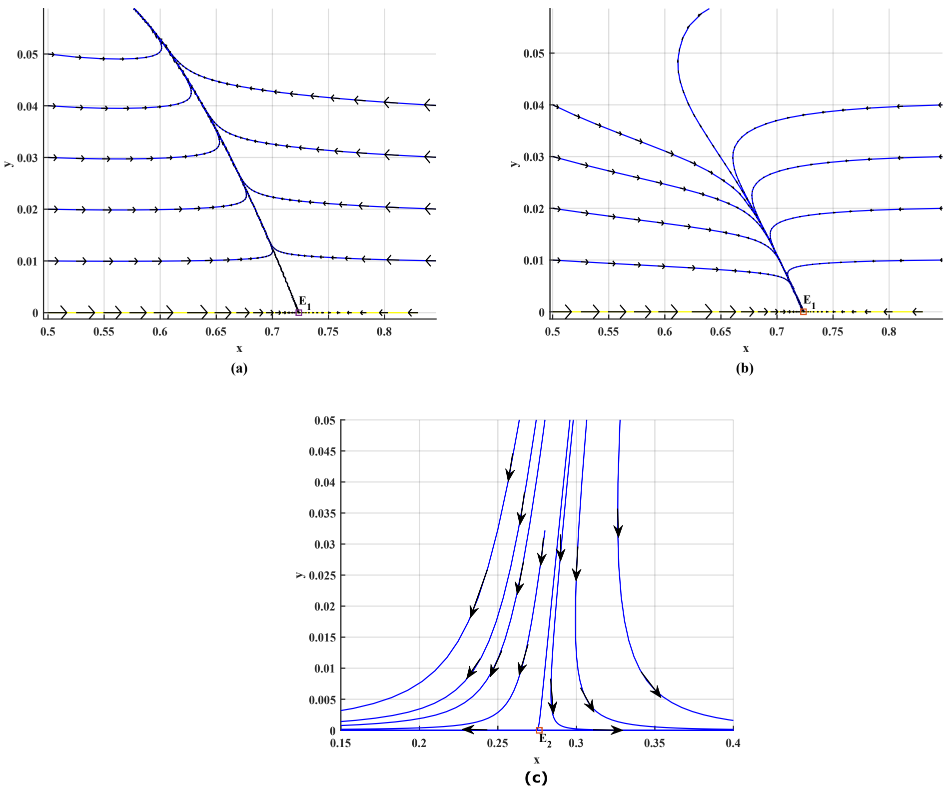

Fig. 1: (a) The Phase plane diagram of the system (3) when $c = 0.2, h = 0.2, d = 0.1, E = 0.05$, where E_1 is saddle. (b) The Phase plane diagram of the system (3) when $c = 0.2, h = 0.2, d = 0.1, E = 0.1$, where E_1 is stable. (c) The Phase plane diagram of the system (3) when $c = 0.5, h = 0.2, d = 0.6, E = 0.6$, where E_2 is saddle.

by linearization technique. Therefore, the system (3) exhibits a saddle-node bifurcation in the neighbourhood of E_0, as two equilibria collide at the bifurcation parameter $h = \frac{1}{4}$. The following theorem describe the saddle-node bifurcation.

3. **Theorem 5** *A saddle-node bifurcation occurs in the system (3) around E_0 at the bifurcation point $h = \frac{1}{2}$, when $4h = 1$.*

Proof. As $J(E_0)$ contains a zero eigenvalue, according to Sotomayor's theorem(22) the necessary condition for saddle-node bifurcation around E_0 at $h = \frac{1}{4}$ satisfies. We consider U, V as the eigenvectors of the matrix $J(E_0)$ and $J(E_0)^T$ corresponding to the zero eigenvalues, respectively.

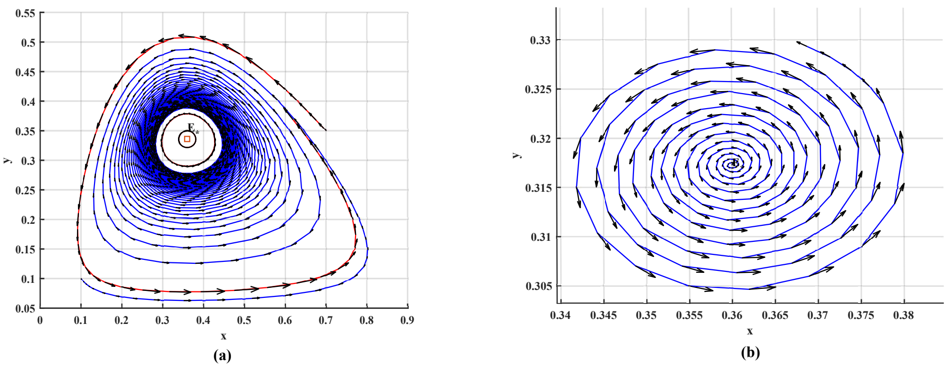

Fig. 2: (a) Phase plot of the system (3) for $c = 1, h = 0.0288 = h_1, d = E = 0.3$, where E_* is center. (b) Phase plot of the system (3) for $c = 1, h = 0.0400 > h_1, d = E = 0.3$, where E_* is source.

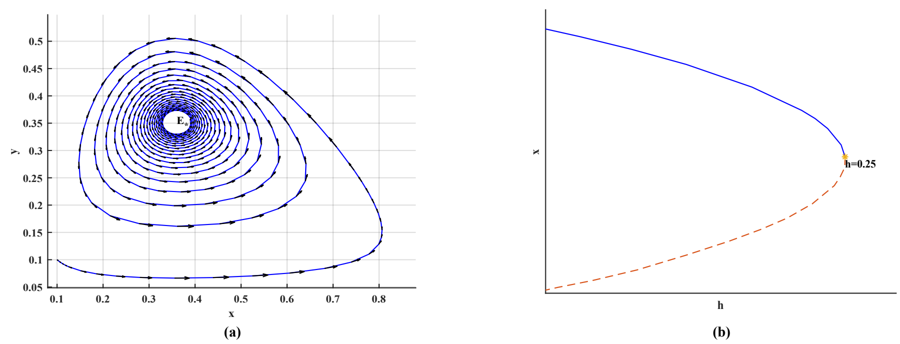

Fig. 3: (a) Phase plot of the system (3) for $c = 1, h = 0.0288 = h_1, d = E = 0.3$, where E_* is sink. (b) The diagram for saddle-node bifurcation of system (3) at $h = 0 : 4225$: Stable equilibrium points are represented by the solid blue curve and unstable equilibrium points are represented by green dashed curve.

$$U = \begin{pmatrix} U_1 \\ U_2 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \; W = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} c\sqrt{\dfrac{1}{2}} - (d+E) \\ \sqrt{\dfrac{1}{2}} \end{pmatrix}.$$

Now, we can get $g_h(E_0, h) = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ at $h = \dfrac{1}{4}$,

$D^2 g_h(E_0, h)(U, U) = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$. It is obvious that, U and V satisfy the transversality conditions:

$V^T g_h(E_0, h) = (d + E) - c\sqrt{\frac{1}{2}} \neq 0,$

$V^T D^2 g_h(E_0, h) = 2[(d+E) - c\sqrt{\frac{1}{2}}] \neq 0$, for saddle-node bifurcation existence in the system (3) at the bifurcation parameter h (22). Biologically we can conclude that, there is a maximum sustainable yield $h_{MSY} = \frac{1}{4}$ and if $h > h_{MSY}$, the prey population will go extinct and it will causes the extinction of predator population. But if $0 < h < \frac{1}{4}$, the prey population do not go extinct. ■

4.2 Hopf bifurcation

Theorem (1) shows that, Hopf bifurcation may occur in the system (3). Now we get $\frac{d}{dh}\text{Trace}(J(E_*))=\frac{1}{2x_*} \neq 0$ which implies that the transversality condition satisfies for occurring Hopf bifurcation. Hence, Hopf bifurcation occur in the system (3) around E_* at $h = h_1$ (22). Now, the equilibrium point E_* is translated to origin to find the Hopf bifurcation direction. Using the transformation $(X, Y) = (x - x_*, y - y_*)$ and expanding the system (3) in a power series around origin, we get

$\frac{dX}{dt} = a_{10}X + a_{01}Y + a_{11}XY + a_{20}X^2 + a_{02}Y^2 + a_{21}X^2Y + a_{12}XY^2 + a_{30}X^3 + a_{03}Y^3 + P(X, Y)$

$\frac{dY}{dt} = b_{10}X + b_{01}Y + b_{11}XY + b_{20}X^2 + b_{02}Y^2 + b_{21}X^2Y + b_{12}XY^2 + b_{30}X^3 + b_{03}Y^3 + Q(X, Y),$

where $P(X, Y)$ and $Q(X, Y)$ are minimum four order smooth functions of X, Y and

$a_{10} = 0, a_{01} = -\sqrt{x_*}, a_{11} = -\frac{1}{\sqrt{x_*}}, a_{20} = -2 + \frac{y_*}{4x_*^{\frac{3}{2}}}, a_{02} = a_{12} = a_{03} = 0, a_{21} = \frac{1}{4x_*^{\frac{3}{2}}}, a_{30} = -\frac{3y_*}{8x_*^{\frac{5}{2}}}.$

$b_{01} = b_{03} = b_{12} = 0, b_{10} = \frac{cy_*}{2\sqrt{x_*}}, b_{11} = \frac{c}{\sqrt{x_*}}, b_{20} = -\frac{cy_*}{4x_*^{\frac{3}{2}}}, b_{21} = -\frac{c}{4x_*^{\frac{3}{2}}}, b_{30} = \frac{3cy_*}{8x_*^{\frac{5}{2}}}.$

Now, we calculate first Lypunov number(22):

$l = -\frac{3\pi}{2a_{01}\Delta^{\frac{3}{2}}}N = \frac{3\pi}{2\sqrt{x_*}\Delta^{\frac{3}{2}}}N$

where $\Delta = \frac{(d+E)y_*}{2\sqrt{x_*}} > 0$, and $N = \frac{cy_*}{4\sqrt{x_*}}\left(\frac{y_*}{2x_*^{\frac{3}{2}}} + \frac{c}{\sqrt{x_*}}\right) + \frac{cy_*}{2}\left(c^2 + \frac{y_*}{4x_*^{\frac{5}{2}}} + \frac{9y_*}{8x_*^2} + \frac{c}{4x_*}\right) > 0$. Therefore, from the above expression of Δ and N it can be conclude that $l > 0$, that means the Hopf bifurcation is sub-critical at $h = h_1$. And a unstable limit cycle present in the system (3) around E_* (22). ■

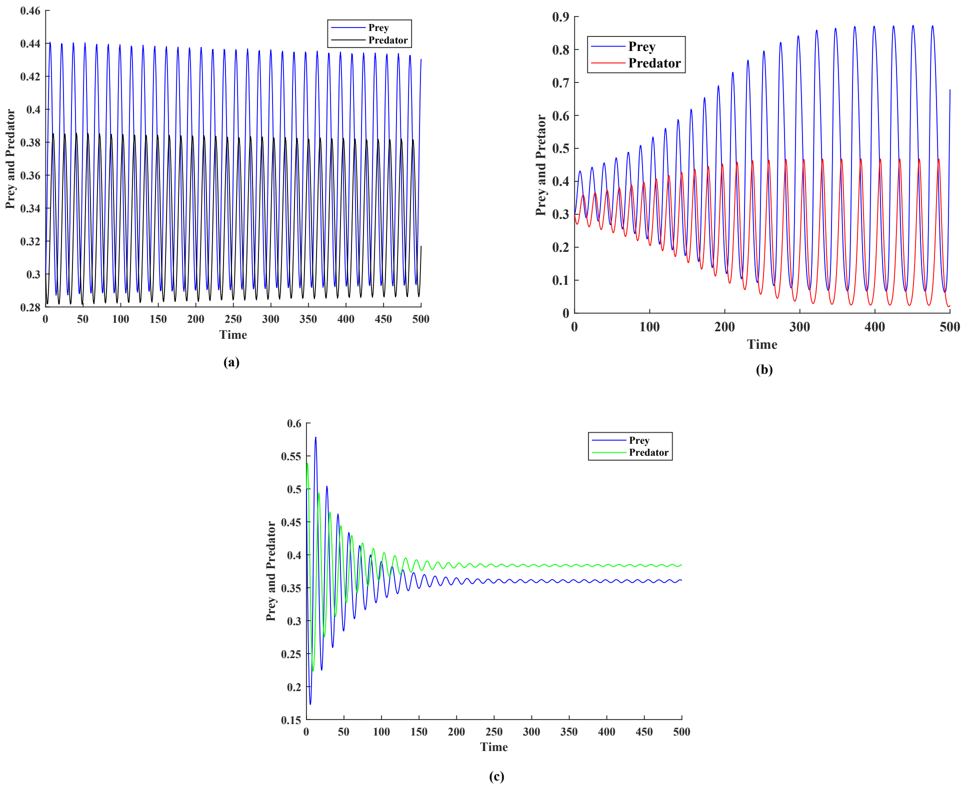

Fig. 4: (a) Time series at $h = 0.0288 = h_1$. (b) Time series at $h = 0.0400 > h_1$. (c) Time series at $h = 0.0195 < h_1$.

5 Numerical Examples

Some numerical examples to verify the analytical results have been discussed in this section.

- For $c = 0.2, h = 0.2, d = 0.1, E = 0.05$, the eigenvalues of $J(E_1)$ are $\lambda_1 = -0.4472 < 0, \lambda_2 = 0.0201 > 0, E_1 = (0.7236, 0)$ is saddle (Figure 1a).
- For $c = 0.2, h = 0.2, d = 0.1, E = 0.1$, the eigenvalues of $J(E_1)$ are $\lambda_1 = -0.4472 < 0, \lambda_2 = -0.0299 < 0, E_1 = (0.7236, 0)$ is stable (Figure 1b).
- For $c = 0.5, h = 0.2, d = 0.05, E = 0.05$, the eigenvalues of $J(E_2)$ are $\mu_1 = 0.4472 > 0, \mu_2 = 0.0051 > 0, E_2 = (0.2764, 0)$ is unstable. For $c = 0.5, h = 0.2, d = 0.6, E = 0.6$, the eigenvalues of $J(E_2)$ are $\mu_1 = 0.4472 > 0, \mu_2 = -0.1.0949 < 0, E_1 = (0.2764, 0)$ is saddle (Figure 1c).
- For $c = 1, h = 0.0288 = h_1, d = E = 0.3$, the Trace of $J(E_2)$ is zero. $E_* = (0.3600, 0.3360)$ is center (Figure 2a).

- For $c = 1, h = 0.0400 > h_1, d = E = 0.3$, the Trace of $J(E_2)$ is $0.0156 > 0$. $E_* = (0.3600, 0.3173)$ is source (Figure 2b).
- For $c = 1, h = 0.0195 < h_1, d = E = 0.3$, the Trace of $J(E_2)$ is $-0.0129 < 0$. $E_* = (0.3600, 0.3515)$ is sink (Figure 3a).
- The system (3) exhibit saddle-node bifurcation for $c = 0.2, h = \frac{1}{4} = 0.25, d = 0.1, E = 0.05$ (Figure 3b).

6 Conclusions

A dynamical predator-prey model with the square root functional response to describe the prey herd behaviour assuming the average handling time is zero has been studied. Also we have incorporated constant prey harvesting, proportional harvesting in predator in the model. Conditions for the existence of the equilibria of the system (3) has been discussed. The local stability of two different predator free equilibria and a unique equilibrium point the system (3) has been analyzed. It has been observed that, two types of bifurcations occur. The saddle-node and Hopf bifurcation has been analyzed and it has been found that, there is a maximum sustainable yield $h_{MSY} = \frac{1}{4}$. At $h > h_{MSY}$, the prey population will go extinct and it will be the cause for extinction of the predator population. But if $0 < h < \frac{1}{4}$, the prey population do not go extinct. An unstable limit cycle exists around the interior equilibrium point. It has been examined that if harvesting rate is chosen at $h \leq h_1$, both population will coexist and will maintained ecological balance. The calculation of first Lypunov number cleared that the Hopf bifurcation is super-critical. To enhance a good knowledge about the interaction between two species prey and predator, the results of this paper may help.

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Ethics approval (include appropriate approvals or waivers)
Consent to participate (include appropriate statements)
Consent for publication (include appropriate statements)
Availability of data and material (data transparency)
Code availability (software application or custom code)

7 Declarations

7.1 Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

7.2 Conflicts of interest

The authors have no conflict of interest to declare.

7.3 Ethics approval

Not applicable.

7.4 Consent to participate

Not applicable.

7.5 Consent for publication

Not applicable.

7.6 Availability of data and material

Not applicable.

7.7 Code availability

Not applicable.

7.8 Authors' contributions

Conceptualization: [Md Golam Mortuja, Mithilesh Kumar Chaube]; Methodology: [Md Golam Mortuja]; Formal analysis and investigation: [Md Golam Mortuja, Mithilesh Kumar Chaube, Santosh Kumar]; Writing - original draft preparation: [Md Golam Mortuja]; Supervision: [Mithilesh Kumar Chaube, Santosh Kumar].

Figures

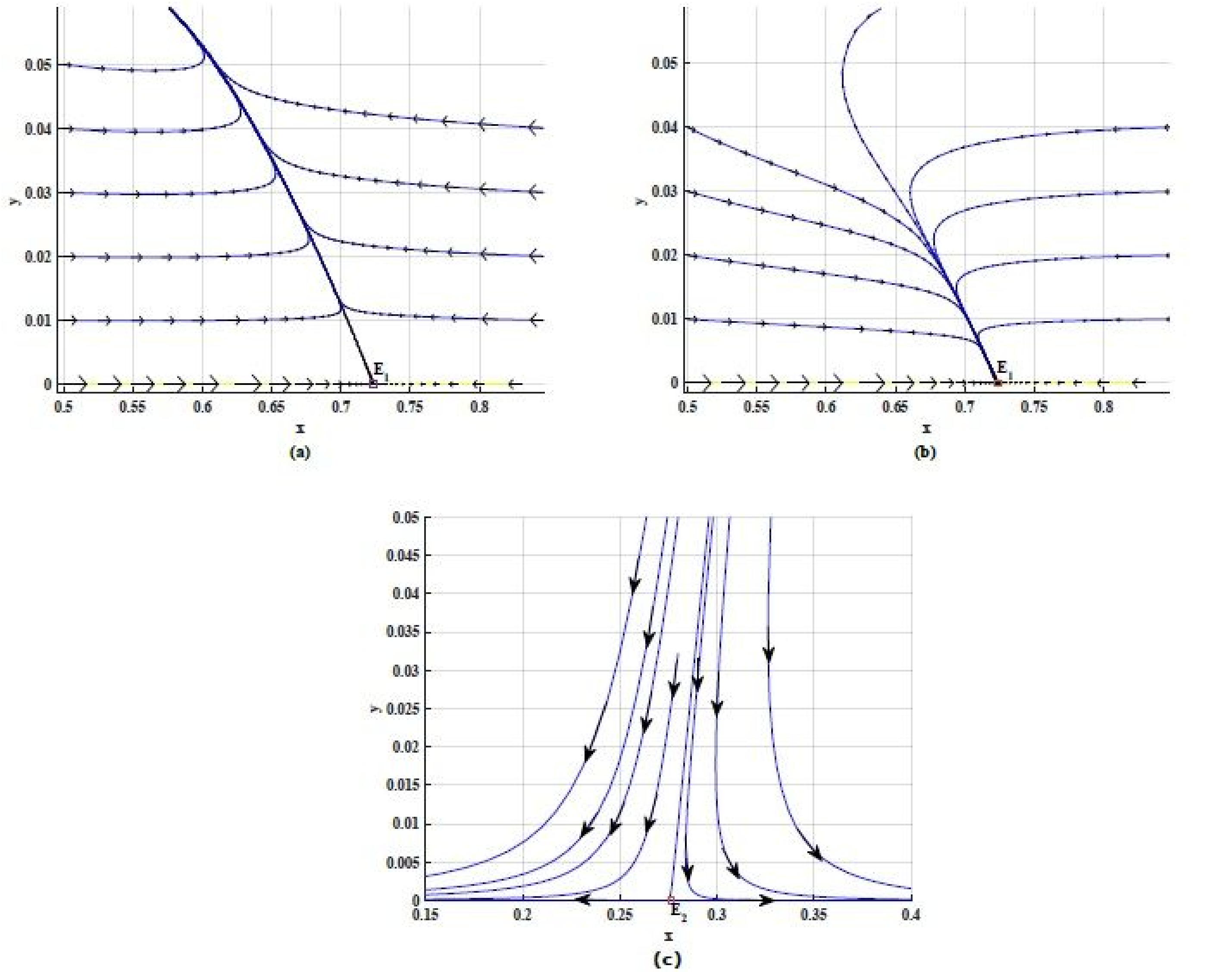

Figure 1

(a) The Phase plane diagram of the system (3) when c = 0:2; h = 0:2; d = 0:1;E = 0:05, where E1 is saddle.
(b) The Phase plane diagram of the system (3) when c = 0:2; h = 0:2; d = 0:1;E = 0:1, where E1 is stable.
(c) The Phase plane diagram of the system (3) when c = 0:5; h = 0:2; d = 0:6;E = 0:6, where E2 is saddle.

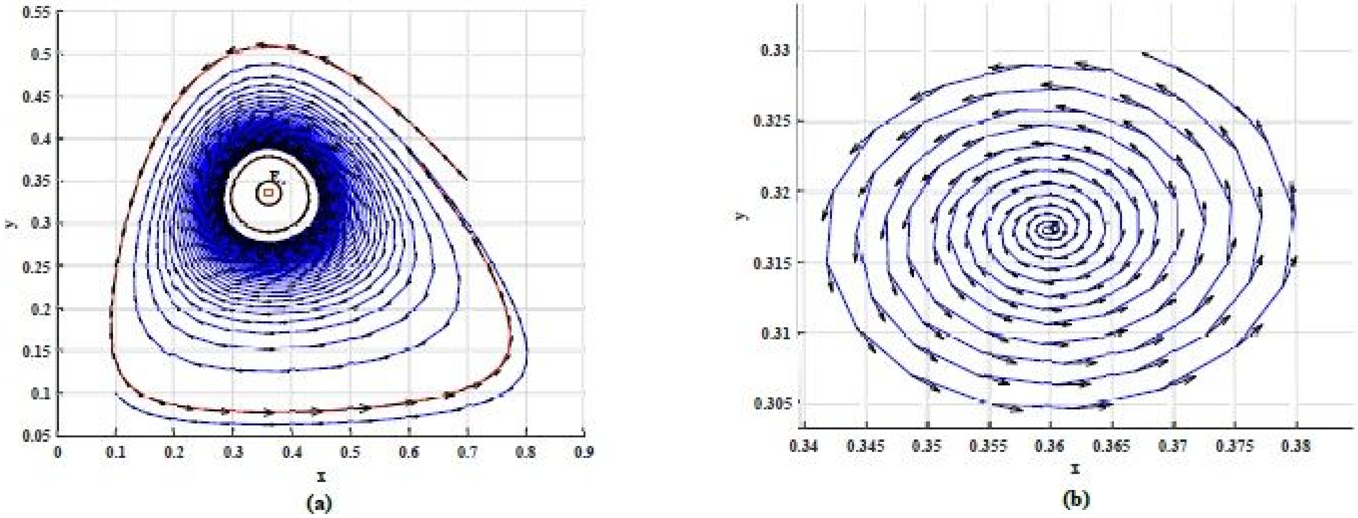

Figure 2

(a) Phase plot of the system (3) for c = 1; h = 0:0288 = h1; d = E = 0:3, where E* is center. (b) Phase plot of the system (3) for c = 1; h = 0:0400 > h1; d = E = 0:3, where E* is source.

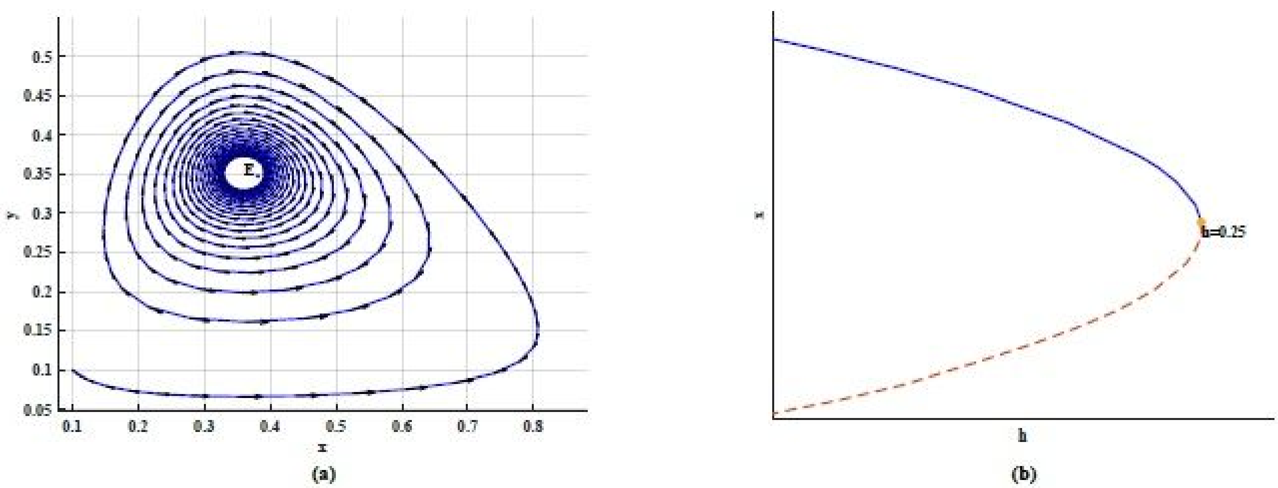

Figure 3

(a) Phase plot of the system (3) for c = 1; h = 0:0288 = h1; d = E = 0:3, where E* is sink. (b) The diagram for saddle-node bifurcation of system (3) at h = 0 : 4225: Stable equilibrium points are represented by the solid blue curve and unstable equilibrium points are represented by green dashed curve.

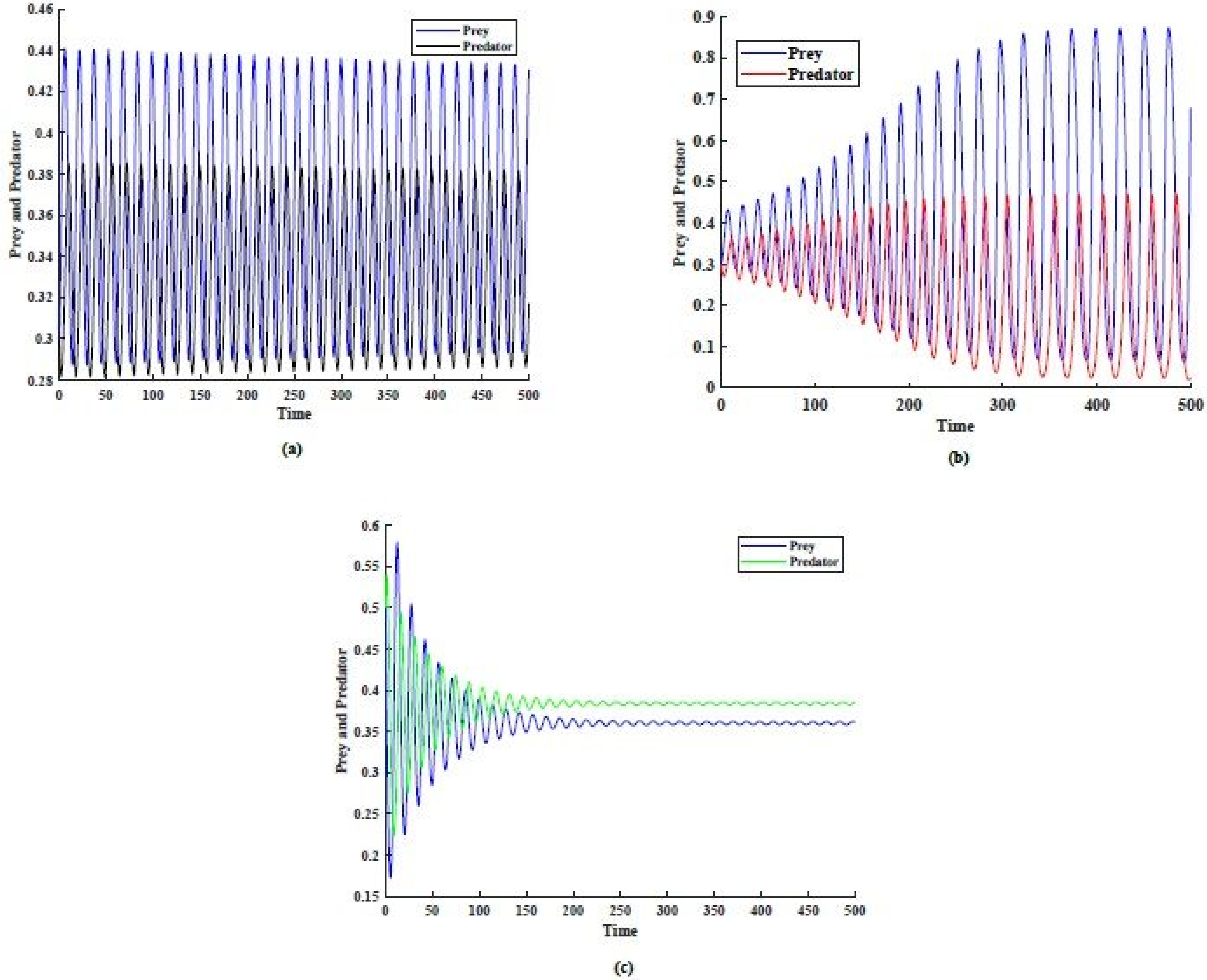

Figure 4

(a) Time series at h = 0:0288 = h1. (b) Time series at h = 0:0400 > h1. (c) Time series at h = 0:0195 < h1.