Spatial-Temporal Rainfall Models Based On Poisson Cluster Processes

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Spatial-temporal rainfall models based on Poisson cluster processes

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Abstract We fit stochastic spatial-temporal models to high-resolution rainfall radar data using Approximate Bayesian Computation (ABC). As a baseline we fit a model of Cox, Isham and Northrop, which we then generalise in a variety of ways. Of central importance is the use of ABC, as it is not possible to fit models of this complexity using previous approaches. We also introduce the use of Simulated Method of Moments (SMM) to initialise the ABC fit.

Keywords Rainfall; spatial-temporal; spatiotemporal; Approximate Bayesian Computation.

1 Introduction

Our interest in spatial-temporal rainfall models comes from the use of rainfall simulators to understand the responses of hydrological systems to rainfall events. It has been argued by many authors (for example Wheeler et al. [18], Segond et al. [14], Chander et al. [5]) that using simulated rainfall with realistic spatial and temporal variation is an effective way of understanding the range and frequency of responses from any given hydrological system. In particular, extreme responses such as flash floods and debris flows are regularly triggered by storm cells with an area of 4 km^2 and a duration of 30 minutes [7], so we want our models to have this level of detail.

Spatial-temporal rainfall models can typically be broken down into two main parts: a model for the frequency, duration and extent of rainfall events, and a separate model for the spatial and temporal structure of a single event. In this paper we are concerned with the latter, for which we consider stochastic cluster-type models. These models are structured to mimic the cell-like structure of rainfall events, and are straight-forward to simulate. We start with the model of Cox & Isham [6] as extended by Northrop [10], which we call the CIN model,

and then introduce a number of extensions. The CIN model and our extensions of it are all stationary models for the interior of a rainfall event; definitions are given in Section 2. To fit them we use high-resolution rainfall radar data, which gives rainfall intensity on a 1 km^2 grid every 6 minutes. In what follows we use a single rainfall event covering an area of $180 \times 180 \text{ km}^2$ for a duration of 4 hours, described in Section 1.1. The thesis of Aryal [1] considers other rainfall events at the same and different locations and obtains similar results.

Because it has an intractable likelihood function, in the past the CIN model has been fitted using the Generalized Method of Moments (GMM) [18]. GMM fitting matches theoretical and observed moments of the process, and thus is restricted to moments for which you have an analytic expression. However we do not have analytic expressions for the moments of our extended processes, so GMM fitting is no longer an option and we instead use Approximate Bayesian Computation (ABC). ABC fitting compares the observed process to simulations, and places no restrictions on the statistics used to compare them. It also has the advantage of providing credible intervals for the estimated parameters. We give a brief description of ABC in Section 3, before dealing with the specifics of the case in hand. In [2] the authors give a comparison of GMM and ABC fitting for the Bartlett-Lewis point rainfall model, demonstrating the advantages of ABC.

There is an extensive literature on modelling and simulating rainfall, though not so much on high-resolution high-frequency spatial-temporal models. For cluster-type processes such as the CIN model, a good place to start is with the surveys of Onof et al. [11] and Wheeler et al. [17]. The main alternative to these types of models use random fields rather than cluster processes. Some recent examples of these can be found in the papers of Paschalis et al. [12] and Benoit et al. [4].

1.1 The data

The CIN model and our generalisations are stationary and are used to model the ‘interior’ of a rainfall event. We will suppose that we have observations of the rainfall in some finite space-time window $A \times [0, T]$, where T is chosen so that the leading and trailing edges of the rainfall event are not observed. For this study we used radar data collected at Laverton, Melbourne, on 24th September 2016 from 12:54 to 16:48 hours, calibrated by the Australian Bureau of Meteorology using rain-gauge data. The data gives rainfall depth in mm averaged over 1 km^2 pixels every 6 minutes for a period of 4 hours. The radar covers a circular region of 128 km radius, but we restrict ourselves to a square study area of size $180 \times 180 \text{ km}^2$. To reduce the noise in the data, rainfall depths of less than 0.01 mm for a six minute interval are rounded down to 0.

A contour plot of the spatial rainfall intensity at a single time-point is given in Figure 1(a). Looking at the spatial maximum and mean over the study period, and the percentage of the study area covered by rain (Figure 1(b–d)), an assumption of stationarity seems reasonable.

Fig. 1: Rainfall on 24th September 201 from 12:54 to 16:48 hours at Laverton, Melbourne.

2 The Cox-Isham-Northrop model and extensions

The Cox-Isham-Northrop (CIN) rainfall model is a spatial-temporal stochastic model for a rainfall event, constructed using a cluster point process. The cluster process is constructed by taking a primary point process, called the storm arrival process, and then attaching to each storm center a finite secondary point process, called a cell process. To each cell center we then attach a rain cell, with an associated area, duration and intensity. The storm and cell centers all share a common velocity. The total rainfall intensity at point (x, y) and time t is then the sum of the intensity at (x, y) of all cells active at time t [6, 10].

The storm arrival process is taken to be a Poisson process in $\mathbb{R}^2 \times [0, \infty)$ with homogeneous rate λ_s . Let $\mathbf{v} = (v_x, v_y)$ be the velocity of the rainfall event, so if a storm center arrives at $(\mathbf{u}, s)$ then at time $s + t$ it will be at $(\mathbf{u} + t\mathbf{v}, s + t)$. Storm durations are random with an $\exp(\gamma_s)$ distribution.

While a storm is active it produces cells at a rate λ_c in time, starting with a cell at the moment the storm begins. If the storm arrives at $(\mathbf{u}, s)$ and produces a cell at time $s + t$, the cell will be centered at $\mathbf{u} + t\mathbf{v} + \mathbf{w}$, where $\mathbf{w}$ comes from a Gaussian distribution with mean $\mathbf{0}$ and covariance Σ . The cell centre then also moves with velocity $\mathbf{v}$. We parameterise Σ using the storm

Fig. 2: Schematic of the temporal structure of the CIN model: a) Storm process (storm origins at T_i) and cell processes (the solid points show cell origins). The S_i^j note the times from the storm origins to cell origins; b) Storm durations D_i , cell durations L_i^j , and cell intensities X_i^j .

diameter d_s , eccentricity e and orientation ω . d_s^{-1} has a gamma distribution with mean μ_{1/d_s} and coefficient of variation CV_{1/d_s} .

Individual cells have random durations, distributed as $\exp(\gamma_c)$, and random diameters d_c (the major axis). Rain cells are elliptical, with the same eccentricity e and orientation ω as the storms. d_c^{-1} has a gamma distribution with mean μ_{1/d_c} and coefficient of variation CV_{1/d_c} . It is convenient to use μ_A , the expected area of a rain cell, instead of CV_{1/d_c} , where

$$\mu_A = \pi \sqrt{1 - e^2} \mu_{1/d_c} (1 + CV_{1/d_c}^{-2}).$$

For the CIN model the intensity of a rain cell is constant over the area and duration of the cell, with an exponential distribution mean μ_X . The displacement, duration, diameter, and intensity of a cell are all independent, and independent of other cells. We give a schematic of the temporal structure of the CIN model in Figure 2 and of the spatial structure in Figure 3.

All together the CIN model has 13 parameters: velocity $\mathbf{v} = (v_x, v_y)$; eccentricity e ; orientation ω ; storm rate λ_s ; mean storm duration $1/\gamma_s$; storm diameter given by μ_{1/d_s} and CV_{1/d_s} ; cell rate λ_c ; mean cell duration $1/\gamma_c$; cell diameter given by μ_{1/d_c} and μ_A ; and mean cell intensity μ_X .

2.1 Generalisations of the CIN model

We extend the model in two stages. In both cases the temporal structure of the process is unchanged and we refine the spatial structure of the rain cells.

The first stage we call the CIN-1 model. With this model we primarily seek to capture an observed variation in the eccentricity of the cells: some cells appear circular in shape while some are long and thin. Accordingly we

Fig. 3: Schematic diagram of the spatial structure of the CIN model. The centre point is a storm centre (which has a constant velocity). Stars are cell centres and the lines indicate the displacement of cell centres from the storm centre. The dashed curves indicate the cell areas, and the dotted curve gives a 95% prediction region for the storm area.

suppose that cell eccentricity has a normal distribution with mean μ_e and variance σ_e^2 , truncated to $[0, 1]$.

Also to provide a better match with observations, we suppose that the rainfall intensity decreases continuously from the centre of a cell to the edge, rather than acting as a step function. If a and b are the lengths of the semi-major and semi-minor axis of a rain cell, and X is the intensity at the cell centre (c_x, c_y) , then we model the intensity at (x, y) as

$$X \sqrt{1 - \frac{(x - c_x)^2}{a^2} - \frac{(y - c_y)^2}{b^2}}.$$

For the second stage CIN-2 model we introduce further changes aimed at making it easier to capture high localised intensities. Specifically, for cell intensity X and cell diameter d_c , we suppose that

$$\log \begin{pmatrix} X \\ d_c \end{pmatrix} \sim N \left(\begin{pmatrix} \mu_X \\ \mu_{d_c} \end{pmatrix}, \begin{pmatrix} \sigma_X^2 & \rho_{X,d_c} \sigma_X \sigma_{d_c} \\ \rho_{X,d_c} \sigma_X \sigma_{d_c} & \sigma_{d_c}^2 \end{pmatrix} \right)$$

Figure 4 represents the intensity of a single cell in (a) the CIN model and (b) the CIN-1 and CIN-2 models.

3 Model Fitting Using ABC

ABC is a likelihood-free Bayesian inference technique, which uses simulations from the likelihood of interest in the absence of an analytic form. The technique developed from numerically intensive techniques for estimating population genetics models [13], and has since seen steadily increasing use in a variety of applications. The recent collection edited by Sisson, Fan and Beaumont [16] gives a comprehensive introduction to the subject. In what follows we use

Fig. 4: Cell intensities for (a) CIN model (b) CIN-1 and CIN-2 models

ABC-MCMC, introduced by Marjoram et al. [8], which uses Markov Chain Monte Carlo to speed up the effective sampling rate of vanilla ABC.

We suppose that we have an observation D from some model $f(\cdot|\theta)$, depending on parameters θ , and that we are able to simulate from f . Let π be the prior distribution for θ and $S = S(D)$ a vector of summary statistics for D , then ABC generates samples from $f(\theta|\rho(S(D^*), S(D)) < \epsilon)$, where $D^* \sim f(\cdot|\theta)$, $\theta \sim \pi$, and ρ is some distance function. If S is a sufficient statistic, then as $\epsilon \rightarrow 0$ this will converge to the posterior $f(\theta|D)$. ABC-MCMC adds a proposal chain with density q and an additional rejection step, to generate a sample $\{\theta_i\}$. The algorithm is as follows:

FOR $i = 1$ to N

- 1 Given current state θ_i propose a new state θ^* using $q(\cdot|\theta_i)$
- 2 Put $\alpha = \min \{1, (\pi(\theta^*)q(\theta_i|\theta^*)) / (\pi(\theta_i)q(\theta^*|\theta_i))\}$
- 3 Go to 4 with probability α , otherwise set $\theta_{i+1} = \theta_i$ and return to 1
- 4 Simulate data $D^* \sim f(\cdot|\theta^*)$
- 5 If $\rho(S(D^*), S(D)) \leq \epsilon$ then set $\theta_{i+1} = \theta^*$, otherwise set $\theta_{i+1} = \theta_i$

END FOR

Unlike [8] we put the MCMC rejection step 3 before the ABC comparison step 5, to avoid unnecessarily running the simulation in step 4. Note that if we wish to use a non-uniform kernel in step 5 (see for example Sisson and Yan [15] §4.3) then we can no longer separate the MCMC rejection step and the ABC comparison step, which increases the simulation burden. Sisson and Yan argue that using a kernel with unbounded support can improve the mixing of the Markov chain, however we did not find this to be a problem. In particular, using a regression adjustment [3] allows some relaxation of the threshold ϵ , to increase the acceptance rate without deliteriously impacting the posterior.

Practically, if θ_0 has very low posterior probability then ABC-MCMC can fail to accept any new sample points. Previous authors have suggested using

a separate ABC step (without MCMC) to find a θ_0 with large posterior probability; we found that using Simulated Method of Moments (SMM) instead requires much less computation time. SMM is a variant of the Generalised Method of Moments (GMM) that uses Monte-Carlo estimates of moments, rather than analytic expressions (McFadden 1989 [9]). Thus, like ABC, using SMM we have much more freedom in the choice of moments used to fit the model to the data, and we found that it worked well using the same summary statistics S that we use for the ABC fitting.

Following Wheeler et al. [18], for the CIN model the velocity $\mathbf{v}$, eccentricity e and orientation ω were all estimated using temporal and spatial autocovariance estimates, and then fixed. For the CIN-1 and CIN-2 models we used our estimate of e for μ_e but also need to estimate σ_e^2 . To do this we divided the study region spatially into four equal parts, then estimated the eccentricity for each part at each time point, giving four time-series of estimates for μ_e . Treating each series as an AR(1) process with mean μ_e , we can correct for the autocorrelation to get the usual moment-based estimate for σ_e^2 .

The remaining parameters are all estimated using ABC. We used the same set of 23 summary statistics for the CIN, CIN-1, and CIN-2 models:

- The overall mean and standard deviation of rainfall, taken over all pixels and all times.
- The spatial-temporal auto-correlation, with lags of (x, y, t) , where x and y are measured in pixels and t is in units of 6-minutes. We take $t = 0$, $x \in \{-1, 0, 1\}$, $y \in \{-1, 0, 1\}$, and $t = 1$, $x \in \{-1, 0, 1\} + v_x$, $y \in \{-1, 0, 1\} + v_y$. Here v_x and v_y are the velocity components, in units of pixels per 6-minutes. Note that the lag $(0, 0, 0)$ auto-correlation is just the variance and so has already been included.
- The probability of an arbitrary pixel and time being dry.
- The ratio of dry/wet area and mean and standard deviation of wet area, averaged over time.

For the distance function ρ we used a weighted sum of squares $\rho(S(D^*), S(D)) = \sum_i w_i (S^*(i) - S(i))^2$, where $S^*(i)$ and $S(i)$ are the i -th components of $S(D^*)$ and $S(D)$ respectively. We found empirically that a good choice for w_i is to take it inversely proportional to the variance of $S^*(i)$ conditioned on using a θ with high posterior probability. Given that θ_0 was chosen using SMM to have maximal posterior probability, we used a separate sample of $S(D^*)$ given θ_0 to estimate the w_i .

The choice of summary statistics S and distance metric ρ plays a large part in the performance of ABC. Ideally S should be sufficient, but certainly it should reflect those aspects of the real process considered most important. However choosing S too large reduces the efficiency of ABC, though this can be mitigated to some extent by using a regression adjustment, for which we used the approach of Beaumont et al. [3].

The remaining parameters were transformed to reduce dependence and skewness, and mapped to $(-\infty, \infty)$. This makes it easier for the proposal chain to spend its time in regions of high posterior probability. Vague normal

priors are used for all the transformed parameters, and for the proposal chain we used a random walk with $N(0, 0.2^2 I)$ steps.

For the CIN and CIN-1 model our new ABC-parameters are

$$\begin{aligned}\theta(1) &= \log(\lambda_s \gamma_s^{-1}), & \theta(2) &= \log(\lambda_s \gamma_s), \\ \theta(3) &= \log(\lambda_c \gamma_c^{-1}), & \theta(4) &= \log(\lambda_c \gamma_c), \\ \theta(5) &= \log(\mu_X \mu_A), & \theta(6) &= \log(\mu_X \mu_A^{-1}), \\ \theta(7) &= \log(\mu_{1/d_c}), & \theta(8) &= \log(\mu_{1/d_s}), \\ \theta(9) &= \log(CV_{1/d_s}),\end{aligned}$$

For the CIN-2 model we replace parameters μ_{1/d_c} and μ_A by μ_{d_c} and $\sigma_{d_c}^2$, and gain parameters σ_X^2 and ρ_{X,d_c} .

$$\begin{aligned}\psi(1) &= \log(\lambda_s \gamma_s^{-1}), & \psi(2) &= \log(\lambda_s \gamma_s), \\ \psi(3) &= \log(\lambda_c \gamma_c^{-1}), & \psi(4) &= \log(\lambda_c \gamma_c), \\ \psi(5) &= \log(\mu_X), & \psi(6) &= \log(1/\sigma_X^2), \\ \psi(7) &= \log(\mu_{d_c}), & \psi(8) &= \log(1/\sigma_{d_c}^2), \\ \psi(9) &= \log\left(\frac{\rho_{X,d_c} + 1}{1 - \rho_{X,d_c}}\right), & \psi(10) &= \log(\mu_{1/d_s}), \\ \psi(11) &= \log(CV_{1/d_s})\end{aligned}$$

3.1 Comparison of Fitted Models

Using the spatial autocorrelation function we estimated $\mathbf{v} = (v_x, v_y) = (0.10, 29.9)$, $e = \mu_e = 0.86$, $\sigma_e^2 = 0.04$ and $\omega = 39^\circ$.

In the appendix Figures 10–12 plot the posterior sample traces for models CIN, CIN-1 and CIN-2. In each case the chains appear stationary and exhibit good mixing.

For the CIN model using a threshold of $\epsilon = 10$ the overall acceptance rate was approximately 4.1%. Increasing ϵ improves the acceptance rate and the degree of mixing, at the expense of reduced accuracy for the posterior approximation. For the CIN-1 and CIN-2 models the acceptance rates were approximately 5% and 11% respectively, using thresholds of $\epsilon = 14$ and 8. For ABC-MCMC rather than the acceptance rate, a better measure of sampling efficiency is the “effective sample rate” for each parameter. That is, the effective sample size over number of simulations required to produce them. For the CIN, the effective sample rate varied from 0.002 to 0.004 depending on the parameter. For the CIN1 and CIN2 models the effective sample rate varied from 0.001 to 0.003 and from 0.001 to 0.005 respectively.

Posterior plots for models CIN, CIN-1 and CIN-2 are given in Figures 5–7. In all three cases we get nice peaks on our posteriors.

Fig. 5: CIN model: posteriors for $\theta(i), i = 1, 2, \dots, 9$. Priors are given by the dashed lines.

Fig. 6: CIN-1 model: posteriors for $\theta(i), i = 1, 2, \dots, 9$. Priors are given by the dashed lines.

Fig. 7: CIN-2 model: posteriors for $\psi(i)$, $i = 1, 2, \dots, 11$. Priors are given by the dashed lines.

Posterior summaries for the original parameters are given in Tables 1–3 in the Appendix. The fitted CIN-2 model shows moderate evidence of negative correlation between cell intensity and diameter: the posterior mean for ρ_{X,d_c} is -0.69 with a 95% credible interval of $(-0.93, 0.10)$.

To compare the performance of our three models we use posterior predictive probabilities to judge how close each model is to the original data, as measured by our distance ρ . That is, we compare for each model the distribution of $\rho(S(D^*), S(D))$ where D is the original data and D^* is generated by the model when θ is distributed according to the posterior. We can sample from this distribution by sampling θ from the posterior, generating D^* from θ and the model, and then calculating $\rho(S(D^*), S(D))$. We can then estimate the c.d.f. of $\rho(S(D^*), S(D))$ using the e.d.f. of a suitably large sample.

Figure 8 plots the estimated c.d.f. of the posterior for $\rho(S(D^*), S(D))$ under models CIN and CIN-1. We see that under CIN-1 smaller distances are more

Fig. 8: Posterior distribution of $\rho(S(D^*), S(D))$ for the CIN model (blue) and CIN-1 model (red). The CIN-1 model is more likely to produce simulations D^* close to D , as measured by ρ .

likely, that is the fitted CIN-1 model is more likely to produce data closer to our observation.

We can also construct posterior predictive distributions for $(S^*(i) - S(i))^2$. That is, we can compare the fit of each model as measured by individual components of the summary statistic S . They are given in Figures 13 and 14 in the Appendix. Generally the CIN-1 model performs better, though not uniformly, but without any obvious pattern to the exceptions.

Quantitatively the CIN-1 model is an improvement over the CIN model. We can also see this qualitatively with some sample plots. In Figure 9 we give sample contour plot of the spatial rainfall intensity at a single time-point for (a) the data; (b) the fitted CIN model; and (c) the fitted CIN-1 model.

Acknowledgements Datasets for this research are available from the Australian Bureau of Meteorology, GPO Box 1289, Melbourne VIC 3001. <http://www.bom.gov.au/climate/data-services/data-requests.shtml>. The Bureau charges according to the Australian Government Cost Recovery Guidelines.

Fig. 9: (a) Calibrated rainfall radar data, courtesy of the Australian Bureau of Meteorology. (b) A simulation from the fitted CIN model. (c) A simulation from the fitted CIN-1 model.

Appendix

Fig. 10: ABC-MCMC fitting for CIN model: posterior chains for $\theta(i)$, $i = 1, 2, \dots, 9$.

Fig. 11: ABC-MCMC fitting for CIN-1 model: posterior chains for $\theta(i)$, $i = 1, 2, \dots, 9$.

Fig. 12: ABC-MCMC fitting for CIN-2 model: posterior chains for $\psi(i), i = 1, 2, \dots, 11$.

Parameter	Mean	Median	95 % Credible Interval
λ_s	0.0008	0.0006	(0.0003, 0.0026)
γ_s^{-1}	14.8159	13.2333	(2.9086, 38.5852)
λ_c	0.6968	0.6201	(0.2321, 1.7451)
γ_c^{-1}	5.3733	4.6462	(2.6768, 13.1802)
μ_1/d_s	0.5199	0.4006	(0.0738, 1.8749)
CV_1/d_s	1.9438	1.5953	(0.1467, 6.0694)
μ_X	0.1946	0.1917	(0.0792, 0.3332)
μ_A	37.2997	32.6773	(11.8394, 93.7327)
μ_1/d_c	0.0595	0.0548	(0.0255, 0.1211)

Table 1: Posterior estimates of the ABC-parameters for the CIN model.

Parameter	Mean	Median	95 % Credible Interval
λ_s	0.0008	0.0005	(0.0002 0.0036)
γ_s^{-1}	12.7340	12.3910	(1.0938 38.4967)
λ_c	0.6166	0.5722	(0.1171, 1.8787)
γ_c^{-1}	10.7880	7.6703	(4.0369, 35.9604)
μ_1/d_s	0.8022	0.7224	(0.3761, 1.6064)
CV_1/d_s	1.2535	1.2105	(0.5601, 2.1132)
μ_X	0.2850	0.2731	(0.1508, 0.4848)
μ_A	33.0017	31.5407	(4.6533, 92.1609)
μ_1/d_c	0.0840	0.0771	(0.0441, 0.1617)

Table 2: Posterior estimates of the ABC-parameters for the CIN-1 model.

Parameter	Mean	Median	95% Credible Interval
λ_s	0.0006	0.0006	(0.0002, 0.00150)
γ_s^{-1}	31.797	32.462	(4.4944, 64.524)
λ_c	0.1887	0.1801	(0.0457, 0.4429)
γ_c^{-1}	13.883	11.407	(5.6338, 42.133)
μ_1/d_s	1.3331	1.1647	(0.1671, 4.3280)
CV_1/d_s	1.2102	1.1901	(0.6291, 1.9917)
μ_X	0.1882	0.1258	(0.0426, 0.7603)
σ_X^2	0.5235	0.4519	(0.0671, 1.5503)
μ_{d_c}	2.7478	2.7523	(0.7829, 5.6306)
$\sigma_{d_c}^2$	0.8602	0.7859	(0.4335, 1.7103)
ρ_{X,d_c}	-0.6906	-0.7564	(-0.9327, 0.0996)

Table 3: Posterior estimates of the ABC-parameters for the CIN-2 model.

Fig. 13: Predictive probability for the CIN model (blue line) and CIN-1 model (red line). Plots (a) and (b) are for mean and standard deviation summaries. Plots (c) to (j) are spatial correlations $\rho(-1, -1, 0)$, $\rho(-1, 0, 0)$, $\rho(1, 1, 0)$, $\rho(0, -1, 0)$, $\rho(0, 1, 0)$, $\rho(1, -1, 0)$, $\rho(1, 0, 0)$, and $\rho(1, 1, 0)$. Plots (k) and (l) are of $\rho(-1 + v_x, -1 + v_y, 1)$, and $\rho(-1 + v_x, 0 + v_y, 1)$.

Fig. 14: Predictive probability for the CIN model (blue line) and CIN-1 model (red line). Plots (a) to (g) are of spatial autocorrelations $\rho(-1 + v_x, 1 + v_y, 1)$, $\rho(0 + v_x, -1 + v_y, 1)$, $\rho(0 + v_x, 0 + v_y, 1)$, $\rho(0 + v_x, 1 + v_y, 1)$, $\rho(1, +v_x, -1 + v_y, 1)$, $\rho(1, +v_x, 0 + v_y, 1)$, and $\rho(1, +v_x, 1 + v_y, 1)$. Plots (h) and (i) are of dry probability of an arbitrary pixel and dry and wet area ratio. Plots (j) and (k) are of mean wet area over time and standard deviation of wet area over time. Plot (l) is of total distance from all summaries.

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Figures

(a) Sample rainfall for a single time point.

(b) Spatial maximum rainfall (mm) over time.

(c) Spatial mean rainfall (mm) over time.

(d) Percentage of sample area covered by rainfall over time.

Figure 1

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Figure 2

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