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Epidemic Big Bang: development and quarantine measures for Covid-19

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New discrete approximation for the infection spread is constructed based on COVID-19 epidemic data. We consider the epidemic as dependent upon four key parameters: the size of population involved, the mean number of dangerous contacts of one infected person per day, the probability to transmit infection due to such contact and the mean duration of disease. In the simplest case of free epidemic in an infinite population, the number of infected rises exponentially day by day. Here we show the model for epidemic process in a closed population, constrained by isolation, treatment and so on. The four parameters introduced here have the clear sense and are in association with the well-known concept of reproduction number in the continuous susceptible-infected-susceptible model. We derive these parameters from the adequate statistical data. On this basis, we also found the corresponding basic reproduction number mentioned above. Our approach allows evaluating the influence of quarantine measures on free pandemic process. We found a good correspondence of the theory and reliable statistical data. The model is quite flexible and it can be expanded for situations that are more complex.

PACS number(s): 02.50.-r, 05.60.-k, 82.39.-k, 87.19.Xx

I. INTRODUCTION

The virus SARS-CoV-2 (causing COVID-19 disease) appeared first in China in 2019 and a few months later it spreads around the world. 11 March 2020 the World Health Organization (WHO) recognize the pandemic. There are various scenario and models for the description of the epidemic process. Models of epidemic transmission of the infection go back to the pioneering work of Kermack and McKendrick [1], where the so-called susceptible-infected-susceptible (SIS) has been formulated and analyzed. More recent investigations have been devoted to the developing of epidemic models. Such works aim to predict the spread of disease in the given population, which accounts the specific peculiarities of the epidemic. On this basis, formulation of the quarantine strategy is desirable. The SIS model and its variants are the basic themes in the mathematical epidemiology [2], [3]. The review on epidemic processes have been done in [4]-[6] (see also references therein). Description of the balance between the susceptible and infected individuals in population under the various conditions of infection transfer, stochastic approaches and fluctuations influence have been developed in [7]-[9]. Kinetic approach can be applied based on [10]. However, the situation with the COVID-19 has the specific features [11]-[14] and here we consider some of them based on the discrete model [15], [16].

There are essential efforts to slow down the epidemic rate, so it is a partly controlled process. Model we offer here takes into account some specific features the COVID-19 epidemic such as a long duration, asymptomatic cases, high contagiousness and mortality. We consider the epidemic in the real finite population in which acts many other circumstances and features like population density, national habits and much more. In general we know these factors only partially. Nevertheless, the model makes it possible, if necessary, to take into account the additional factors of influence beyond those that we consider in this article.

II. FREE EPIDEMIC IN A LIMITED POPULATION

We consider a limited population without the immune members (i.e., when anyone can get sick). By day $l = 1$ the first infected appears in that population. On this day, this person may have some contacts and may infect some people. The contacts that lead to infection, we define as dangerous ones. Let n_c denote the average number of the dangerous contacts of one infected person per day, and k denote the probability of infection during the such contact. The parameter k depends on the average susceptibility to the disease in given population. So, the product $p = n_c k$ is the average number of people infected by one virulent person per day. Therefore, the number of infected on the l

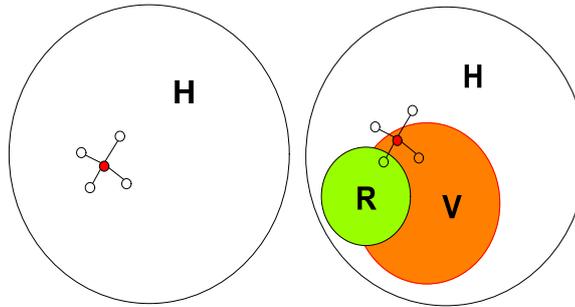


Figure 1: Effective contacts of an infectious person with the surrounding ones at the start of epidemic (left picture) and during its development (right picture). Here H , V and R are explained in the text. Only two "legs" of the four-legged infected "spider" in the area H are effective.

day becomes [6]

$$N_I(n_c; k; l) = (n_c k + 1)^l, \quad (1)$$

The value p may be greater than one (see [6], MedRxiv) or in general case arbitrary (as in [6], Research Gate). Obviously, the number of infections increases exponentially in time with the exponent equals to $l \cdot \ln(p + 1)$. Let d is the characteristic duration of the disease. So, when $l > d$ the removed people R (recovered and dead) appears in the population. The number of infectious will become less by amount R .

$$N_I(n_c; k; l) = (n_c k + 1)^l \left[1 - \frac{1}{(n_c k + 1)^d} \right]. \quad (2)$$

After d days of epidemic we can consider the population as a set N , containing several subsets: H - healthy people ("preys"); V - infectious people (virus carriers, "predators"); R - removed people that can neither infect nor be infected. Let the people once infected in the course of an epidemic is A (affected). It's obvious (see, figure 1) that

$$N = H + V + R = H + A, \quad (3)$$

$$A = V + R, \quad (4)$$

$$H = N - A. \quad (5)$$

We assume that the infection spreads through contacts of virus carriers V with people from the set H . According to (1) the number of infected on the first day is $N_I(n_c; k; l = 1) = n_c k + 1$, in the second $N_I(n_c; k; l = 2) = (n_c k + 1) + (n_c k + 1)n_c k$, etc. (which leads to (1), according to [6], [7]). Consider first the free spread of infection without any protective measures. As the epidemic develops and the number of virus carriers in the set V increases, the circle of people in contact with those infectious may include not only healthy persons H , but also already infected ones. The number of infected people grows only by contact with people from the set H . Such contacts are effective. That is, during the epidemic, the number of dangerous contacts of an actual virus carrier may remain the same, but some of these contacts become ineffective (fig. 1).

Therefore, we consider n_c as a time-dependent variable. This dependence is determined by the ratio of the number of infected people prior to the current day $N_I(l)$ to the size of population N . The number of infected is described by the relation

$$N_I(l) = N_I(l - 1) + N_I(l - 1)n_c k [1 - N_I(l - 1)/N] - N_I(l - d + 1), \quad (6)$$

where $N_I(l - d + 1) = R$ when $l > d$, else $R = 0$. During the course of the epidemic, the variable n_c gradually decreases both naturally and under the influence of artificial protective measures like quarantine, while the sustainability k is associated with the personal hygiene like the face masks, medications and vaccinating in future. The sample graph of function Eq. (6) is in figure 2.

Obviously, the number of dangerous contacts n_c is a key factor determining the duration of the epidemic in a population of a given size N in a given locality (country, district, city). The following figure (figure 3, left side) shows how the free-running epidemic process depends upon n_c ceteris paribus. The course of the epidemic also depends on the mean duration of disease d (figure 3, right side). The curves on graph are depict the number of infections during the free-running epidemic process in the population of $N = 10^7$ people. It is clear that the longer is duration d , the higher and later is the peak of curve on graph.

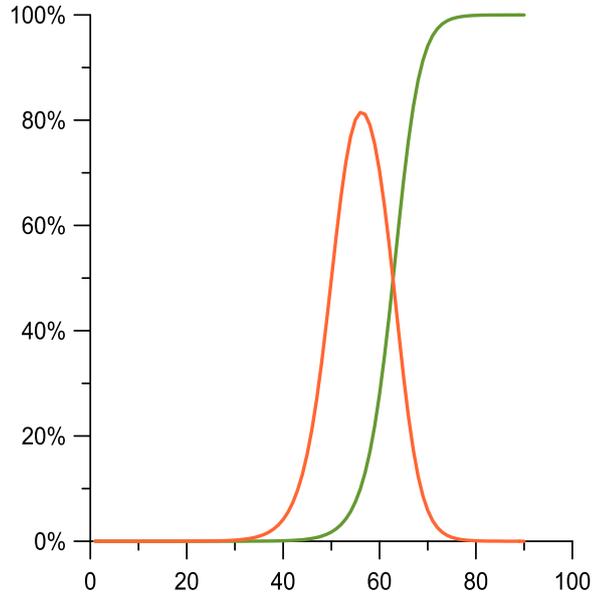


Figure 2: The number of infections and recoveries during the free-running epidemic. X-axis shows the days after the first infection, Y-axis shows the percentage of population $N = 10^7$ with $n_c = 2.2$, $k = 0.1$, $d = 14$.

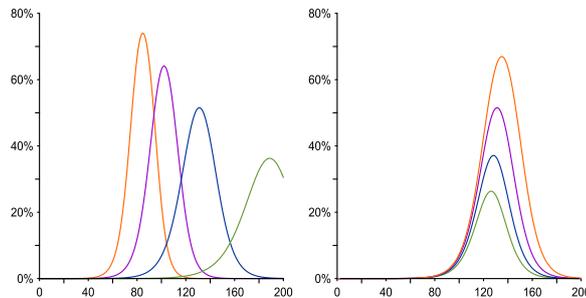


Figure 3: The number of infections during the free-running epidemic in dependence of n_c (left) and d (right). X-axis shows the days after the first infection, Y-axis shows the percentage of population N . On the left $d = 20$ days, green, blue, purple, orange curves for $n_c = 0.8, 1.2, 1.6, 2.0$ respectively. On the right $n_c = 1.6$ - green, blue, purple, orange curves for $d = 10, 14, 20, 28$ days respectively.

III. EPIDEMIC WITH COUNTERMEASURES

We have found that the development of the epidemic is largely determined by the behavior of n_c , which slightly decreases during the process due to natural causes. It is possible to show rigorously that equation (6) is valid for the case of replacement n_c on some function, depended on day $n_c(l)$. However, in the course of epidemic the time-dependent variable $n_c(l)$, can be forced to drastically decrease by the some protective measures like quarantine, isolation etc. This is an artificial impact and therefore it has to be found phenomenologically. The epidemic model in the closed population, presented here, allows to do this based on real official data from the governmental sources of some countries. We consider the cases of Germany as a big country (population $N \simeq 8 \cdot 10^7$) and Israeli as an smaller one (population $N \simeq 8 \cdot 10^6$). So, we use the generalized result (6) with $n_c(l)$ to calculate day by day the German and Israeli data on the total confirmed cases of infections in course of the pandemic 2020 (figure 4). We denote by l_0 the date of the introduction of administrative measures against the epidemic.

It is obvious that the point l_0 divides each graph in two parts. The short period $l < l_0$ at the earliest stage of epidemic looks like a very noisy almost stochastic process near somewhat steady mean value of n_c . The large scatter of values over these early days is logically explained by the fact that at first it was not clear what was happened, the borders were not locked so the country could not be considered as a closed population, accurate accounting was not set up, etc. As follows from the statistical data [12] and figure 4 during the much longer second and final stage of the

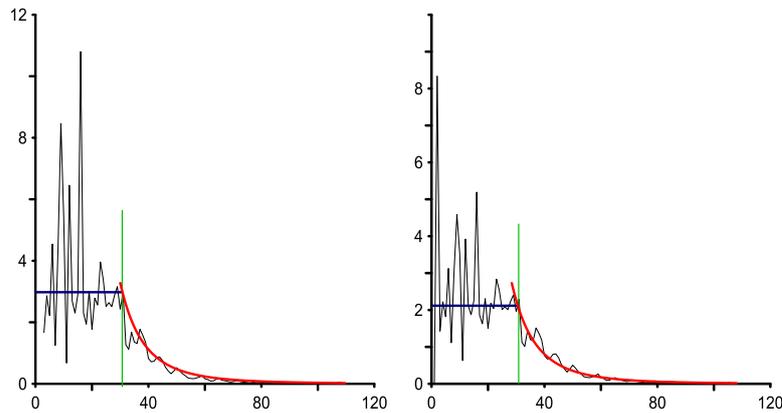


Figure 4: The n_c values by days after the first infection, computed from official German (left) and Israeli (right) data. The step point l_0 (green line) divides each graph into short free process at the earliest stage of epidemic and a much longer second one under control. The parameters used for Germany: $N = 8 \cdot 10^7$, $k = 0.1$, $d = 16$ days; step $l_0 = 28$ days, $n_c = 3.5$ before the step l_0 (blue line). The parameters used for Israel $N = 8 \cdot 10^6$, $k = 0.1$, $d = 20$ days; step $l_0 = 28$ days, $n_c = 2.9$ before the step l_0 (blue line). The approximating functions after l_0 (red curves) are like $y = \alpha l^{-4}$, where α of Germany differs from α for Israel.

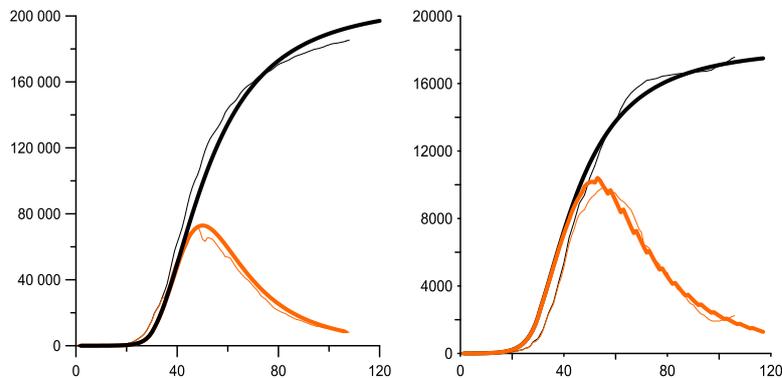


Figure 5: The total confirmed cases of infection (black curves) and the actual cases of infected people (orange curves) both computed with model described here (thick lines) in comparison with the published official German (left) and Israeli (right) data on epidemic (thin curves). The parameters used was: for Germany $N = 8 \cdot 10^7$, $k = 0.1$, $d = 16$ days; step $l_0 = 32$ days, $n_c = 3.5$ before the step l_0 and else $n_c = 2.93 \cdot 10^6 \cdot l^{-4}$ and for Israel $N = 8 \cdot 10^6$, $k = 0.1$, step $l_0 = 31$ days, $d = 20$ days, $n_c = 2.9$ before the step l_0 and else $n_c = 1.86 \cdot 10^6 \cdot l^{-4}$.

epidemic the values of n_c shows a monotonous decrease.

The calculations made by equation (6) with variable $n_c(l)$ instead a constant n_c (Fig. 5) demonstrates a good agreement with statistics. Not only the maximal value of the active cases and the position of the peak, but also all the curve of daily cases are reproduced with a high accuracy. Thus, based on the data from Germany and Israel, for the gradual model we found the phenomenological law for $n_c(l)$ in the form $n_c(l) = n_c$ for $l < l_0$ and $n_c(l) = \alpha/l^4$ for $l > l_0$ (16). The parameter α is different for these countries, however they are in the expected intervals for $n_c, p = n_c \cdot k$ and l_0 . The agreement testifies the good description of the epidemic COVID-19 by the presented general theory.

Such calculations as were presented here, can show how the quarantine measures can restrain the number of virus carriers. Obviously, effective drugs for SARS-CoV-2 virus must decrease in the average duration of disease d could significantly reduce the peak values of active cases (hospitalized and staying at home), thereby reducing quarantine measures aimed to reducing n_c . However, now, as far as we know, there are no clinically proven effective medicines to reduce the duration of virus SARS-CoV-2 contagiousness.

IV. CONCLUSIONS

The description of the real-world epidemic course requires discrete approach to the problem, which can be useful for determination of the strategy of the quarantine measures for various epidemic processes. Instead the SIS-type model, based on the differential equations, we introduced the equations in discrete time (days). This approach corresponds to the real daily statistical reports about epidemic course. In the suggested model, we considered the free epidemic process for various values of the average dangerous contacts n_c of one infected individual per day. The idealistic case of infinite population is considered. We demonstrate how for the fixed N the free course of the epidemic duration increases and the pick of active cases decreases with reduction of n_c . We have mention the relation between the parameters in the model under consideration and the reproduction parameter R_0 of the SIS model $R_0 = n_c k d$.

As known, the duration of the COVID-19 depends on its severity. In more than 60 percent of cases, the infectious period (active cases, or virus carriers) is $22 \div 25$ days. For the free course of the epidemic the shape of the virus carriers curve is investigated for fixed N and n_c and the increase the peak with the increasing duration of the disease is found. In the case of free course of the epidemic for the reasonable values of the parameters n_c and k , typical for the virus SARS-CoV-2 and its mutating variants transmission, the pick of active cases corresponds to the values $50 \div 70$ percent of the whole population. This level is unacceptable for the modern society and stipulates the necessity of the rigorous quarantine measures.

The advantage of the developed approach is the opportunity to take into account the various specific quarantine measures via the introduced current number of the dangerous contacts $n_c(l)$ ("influence function") instead the constant one n_c . The explicit expression for the total cases as function of day l after beginning of the epidemic is found and applied to describe the existing statistical data fixed by the governmental offices and in the site [12]. The generalization of the expressions for the case of free epidemic course for the quarantine measures case is presented.

We considered the daily statistical data for two countries Germany and Israeli and found the phenomenological expression for the $n_c(l)$ which is constant for initial period $l < l_0$ of the COVID-19 and possesses power-law decrease $n_c(l) = \alpha/l^4$ for $l \geq l_0$. We also determine approximately the date l_0 of the quarantine measures introduction. The parameters l_0 for both investigated countries are close, the power of the "influence function" is the same. The comparison of the calculated curves for the active cases and total cases of the COVID-19 are in a good agreement with the statistical data.

V. USED PROGRAMS

Numerical codes, simulation data and data descriptors are available at Image editing program Xara Designer Pro X and the spreadsheet plotting application Golden Software Grapher 11.

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Declarations:

Competing interests: The authors declare no competing interests.

Figures

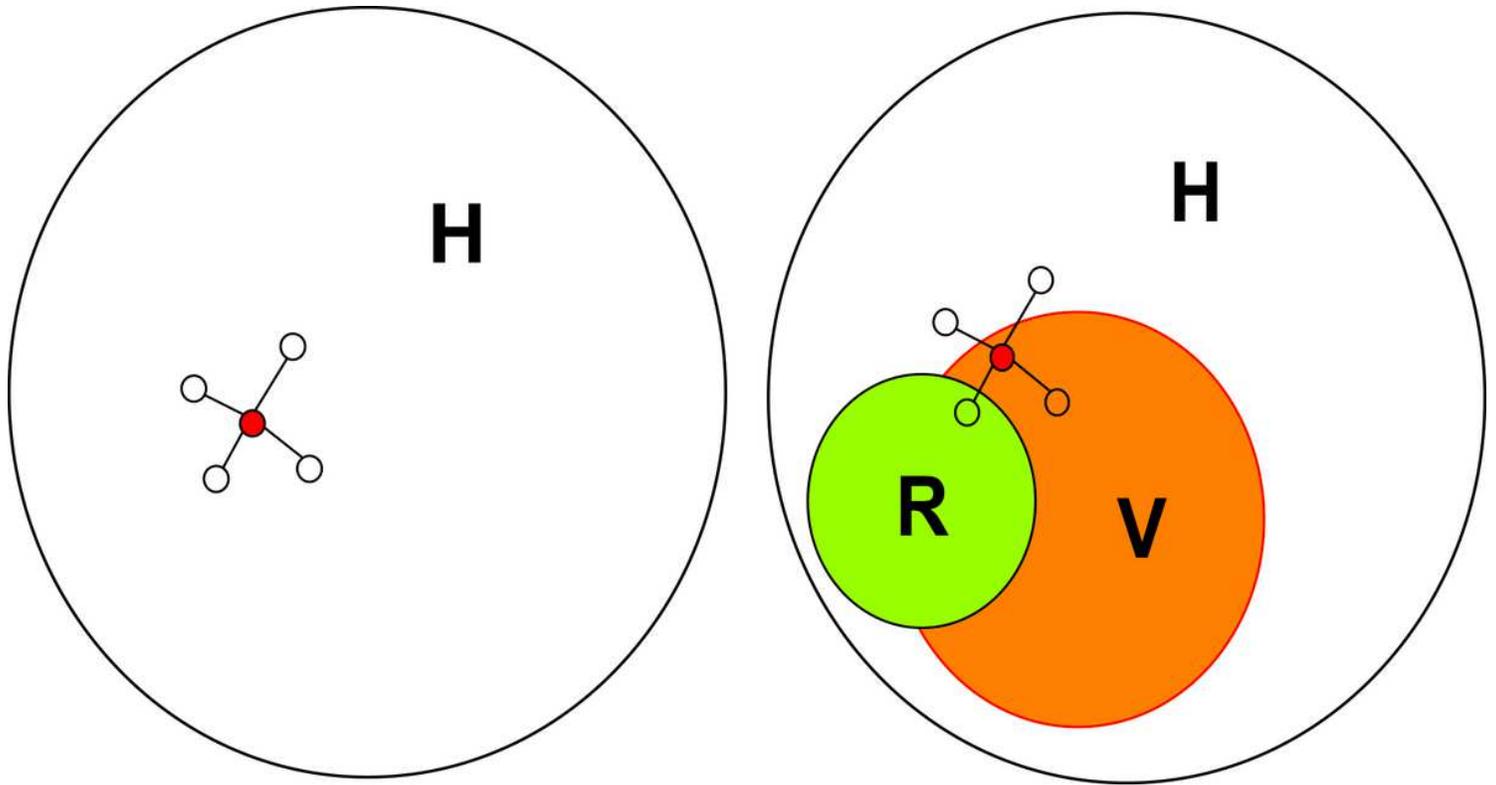


Figure 1

Effective contacts of an infectious person with the surrounding ones at the start of epidemic (left picture) and during its development (right picture). Here H (health part of population), V (virus carriers) and R (removed - recovered or dead). Only two "legs" of the four-legged infected "spider" in the area H are effective.

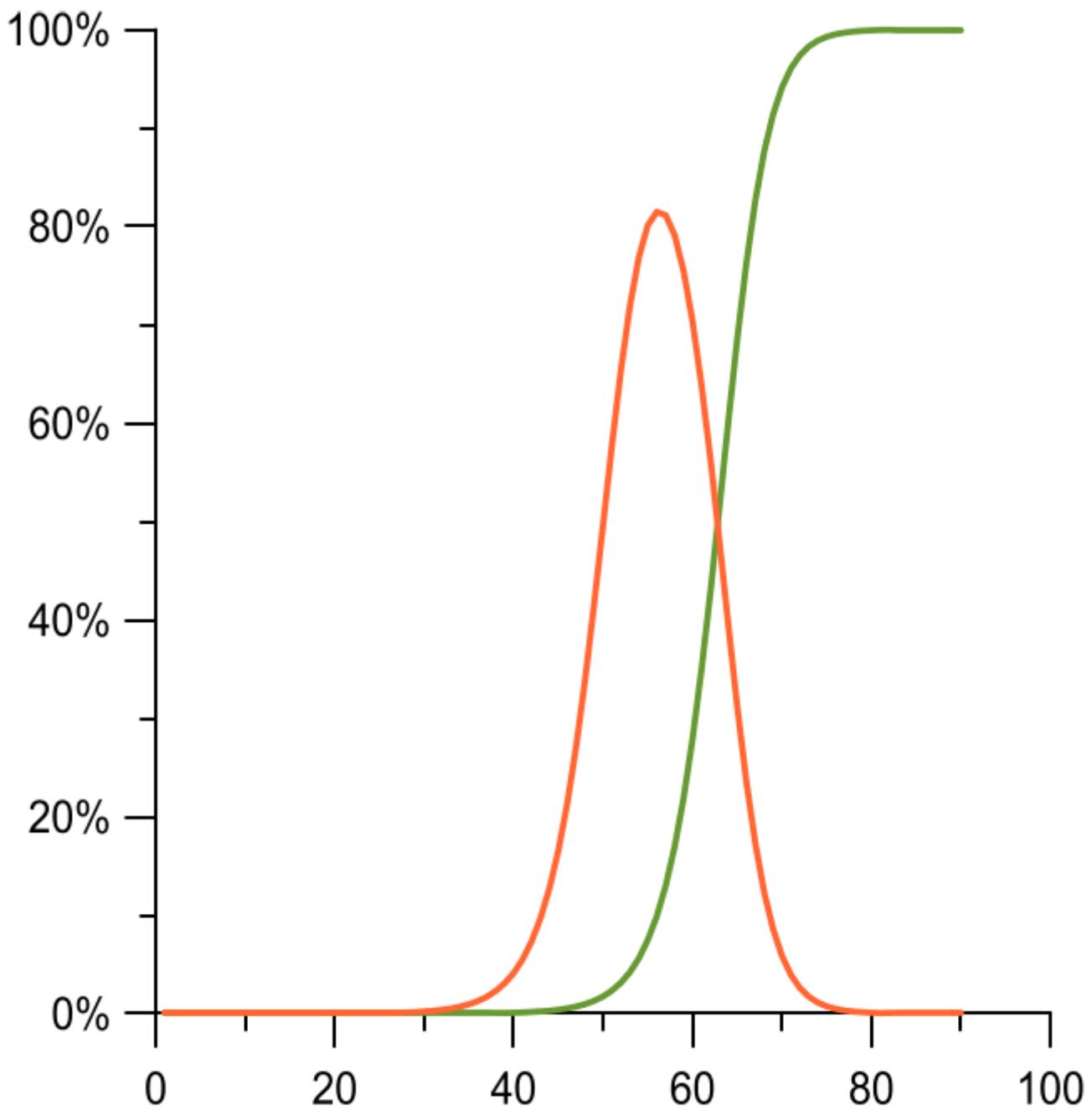


Figure 2

The number of infections and recoveries during the free-running epidemic. X-axis shows the days after the first infection, Y-axis shows the percentage of population $N = 10^7$ with $n_c = 2.2$, $k = 0.1$, $d = 14$ (dangerous contacts, average susceptibility, average duration of the disease, respectively)

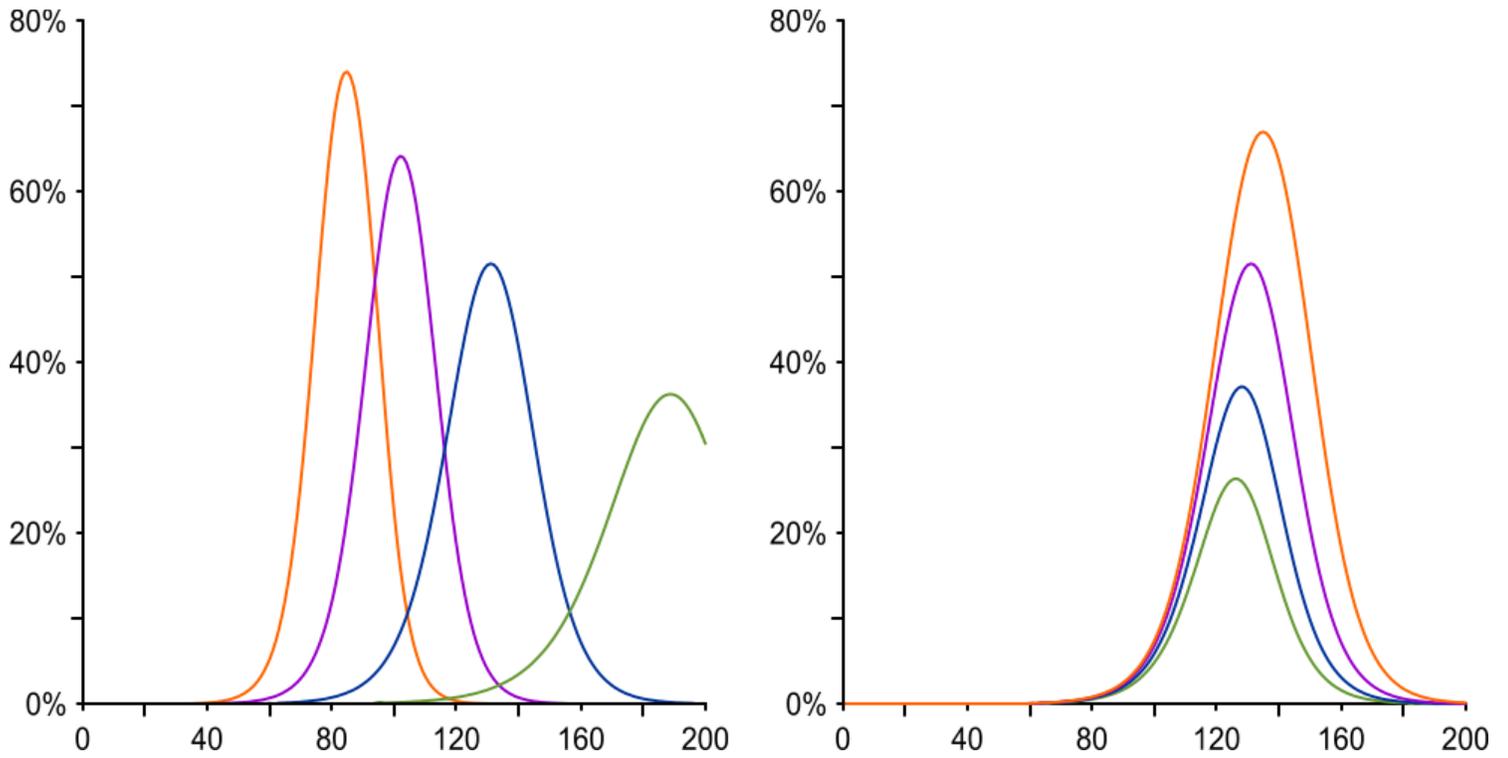


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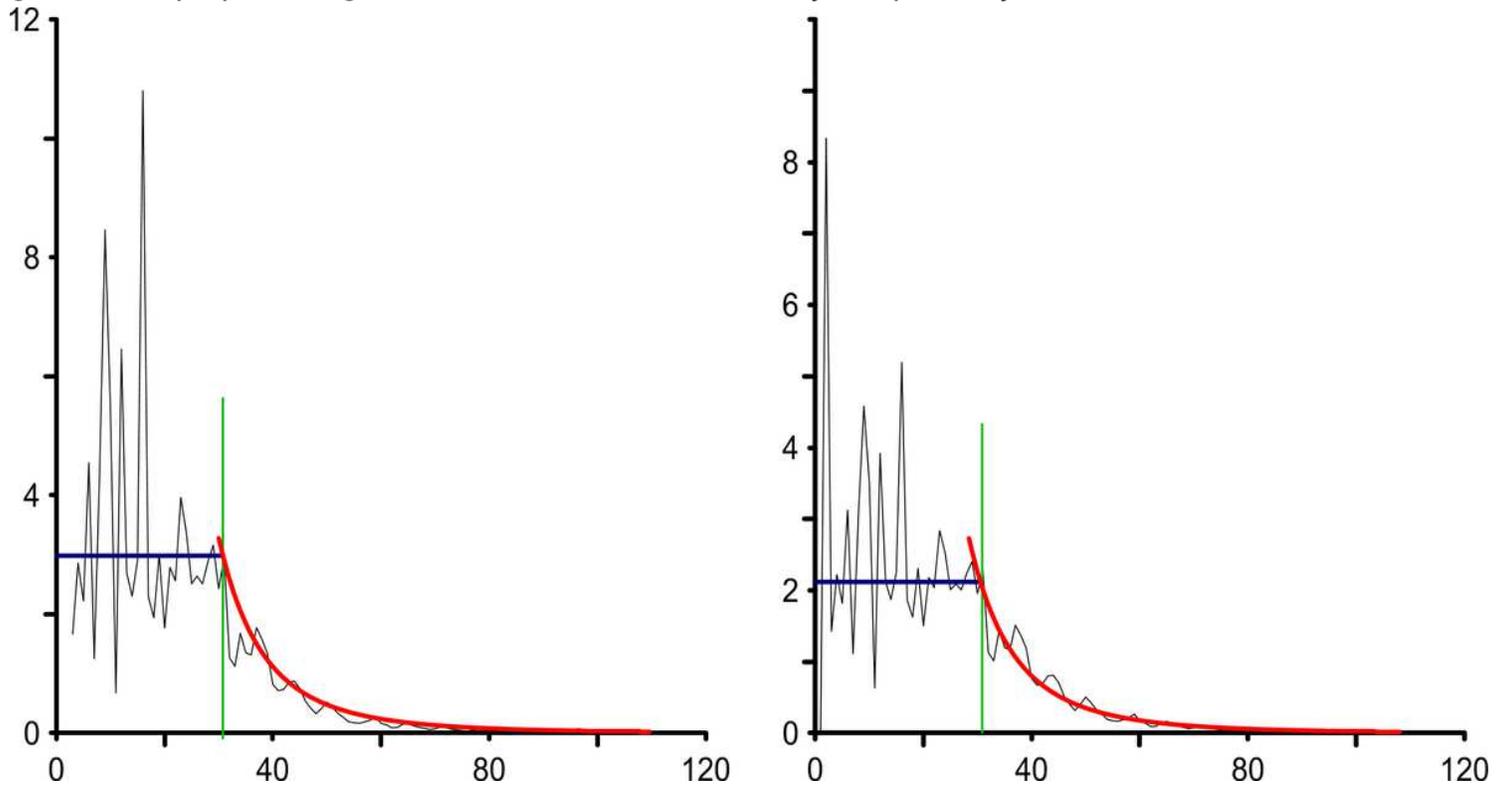


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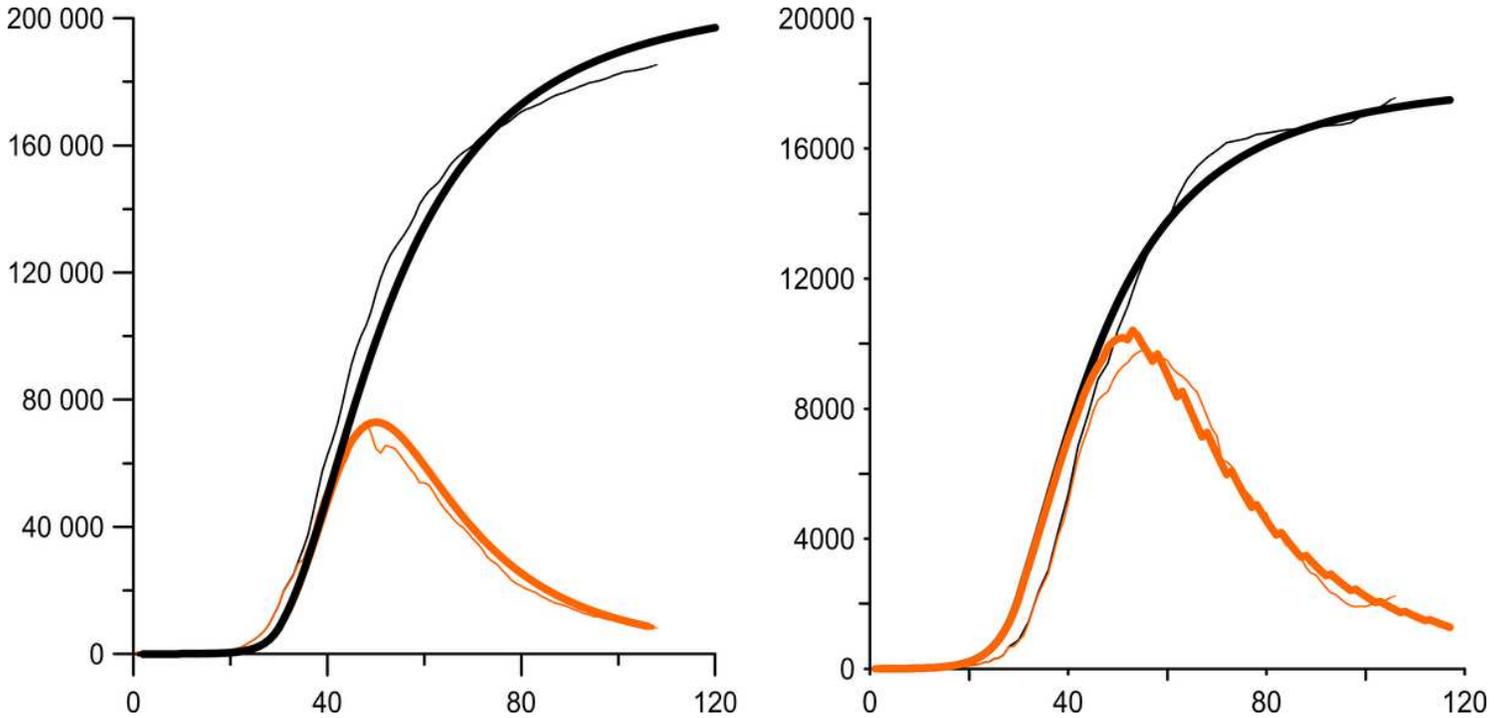


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Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [RGInitialstageofCOVID19no2.pdf](#)
- [RGCOVID19Roleoftheasymptomaticcases.pdf](#)