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Assessing Klebanoff's Data

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Abstract

In 1955, Klebanoff published the first full set of turbulence stress measurements in a zero pressure gradient boundary layer [1]. These results have achieved landmark status, and they are still widely used for comparisons with measurements and computations. The purpose of this paper is to show that these data are inaccurate in a number of ways, and that more recent data drawn from experiments and DNS should be used instead for future comparisons.

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1 Introduction

In 1955, Klebanoff published the first full set of turbulence stress measurements in a zero pressure gradient boundary layer [1] (hereafter K55). Figures 1 and 2 show the data in their original presentation, where u', v', and w' are the rms velocity fluctuations in the streamwise, wall-normal, and spanwise directions (x, y, and z), respectively, and $\overline{uv}$ is the (negative) shear stress. We will re-analyze these data and compare them with more recent experiments and computations. We find that the measurements are inaccurate in a number of ways, and that more recent data drawn from experiments and DNS should be used instead for future comparisons.

The K55 experiment was performed on a flat plate mounted on the centerline of the tunnel at a station located 10.5 ft (3.20 m) from the plate leading edge. The first 2 ft (0.61 m) was covered with #16 floor-sanding paper to trip the flow and artificially thicken the boundary layer. The measurements were obtained using a constant current anemometer with a compensation network, with a reported flat frequency response over the bandwidth of the amplifier (2 to 70,000 Hz). Some (unspecified) filtering was

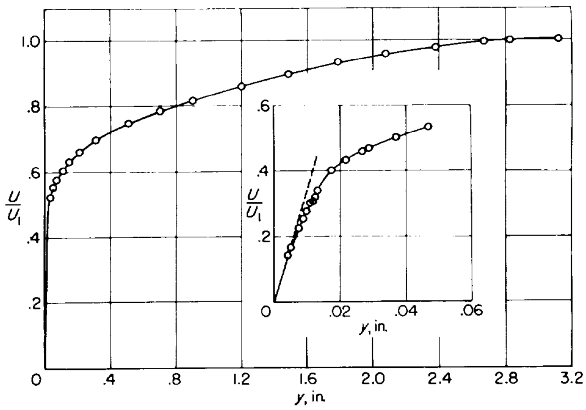

Fig. 1 Original Klebanoff mean velocity profile [1], reproduced with permission.

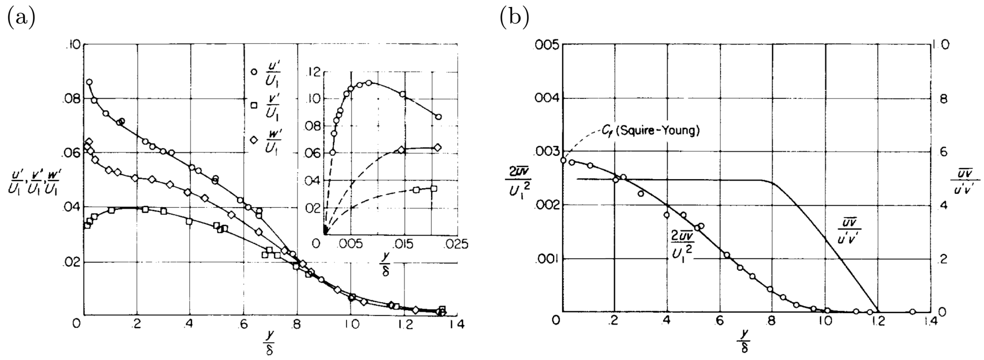

Fig. 2 Original Klebanoff turbulence profiles [1], reproduced with permission.

done to reduce noise. The wire sensors for all probes had a diameter $d = 2.5\mu$m with a length $\ell = 0.5$ mm ($\ell/d = 197$, with an estimated $\ell^+ \approx 18$). In some instances (unspecified), the diameter was reduced to 1.3μm. Given this information, it seems unlikely that the measurements were subject to any significant spatial or temporal filtering, except in the near-wall region where some spatial filtering is to be expected.

Table 1 lists the flow parameters pertinent to this experiment. Only the freestream velocity $U_1 = 50$ ft/s (15.24 m/s), the boundary layer thickness $\delta = 3$ in. (76.2 mm), and the Reynolds number based on the distance to the virtual origin x_V, that is, $Re_x = x_V U_1/\nu = 4.2 \times 10^6$, were given in the text, with $x_V = 14.5$ ft (4.42 m).

2 Data Analysis

To examine the data in more detail, we need additional information such as the skin friction coefficient, C_f, the Reynolds number based on the momentum thickness, Re_θ, and the friction Reynolds number, $Re_\tau = \delta u_\tau/\nu$. Here, as in the rest of this paper, δ is understood to be the 99% boundary layer thickness, that is, the distance from the wall where $U = 0.99U_1$. In addition, θ is the momentum thickness, $C_f = 2\tau_w/(\rho U_1^2)$

	U_∞ m/s	Re_x	Re_θ	δ mm.	δ^* mm	θ mm	C_f	Re_τ
Klebanoff (1955) [1]								
From text	15.24	4.2×10^6	6394[1]	76.2 (66.2[1])			0.00283[1]	2406[1]
From 1/7th power law			7360	75.8	9.47	7.37	0.00280	2755
Klebanoff & Diehl (1951) [2]								
From text	16.76	4.66×10^6	7820	78.7 (63.9[1])	10.1	7.16		2551[1]
From 1/7th power law							0.00275	
Adjusted to 15.24 m/s	15.24	4.2×10^6	7200	81.5 (66.2[1])	10.45	7.41	0.00280	2404[1]
DeGraaff & Eaton (2000) [3]								1692
DeGraaff & Eaton (2000) [3]								4336
Osaka *et al.* (1998) [4]								1750[1]
Sillero *et al.* (2013) [5] DNS								1848

Table 1 Boundary layer parameters ([1]denotes value estimated from original data).

is the skin friction coefficient, $u_\tau = \sqrt{\tau_w/\rho}$ is the friction velocity, τ_w is the shear stress at the wall, and ρ is the fluid density. Since the original records are lost, we used Datathief[1] to reconstitute the data.

The skin friction coefficient was found from the data point in figure 2b at $y = 0$ marked "Squire-Young," which gives $C_f = 0.00283$. Presumably, it was not measured directly but inferred from that correlation. There is some historical and circumstantial evidence that 1/7th power laws were used in this investigation (see, for example, the calculation of the boundary layer thickness — K55 page 16). Using the 1/7th power law, $C_f = 0.0592/Re_x^{0.2} = 0.00280$, in good agreement with the Squire-Young value.

As to the boundary layer thickness, the 1/7th power law relationship gives $\delta = 0.37x_V/Re_x^{0.2} = 2.99$ in (75.8 mm), in good agreement with the value of 3 in (76.2 mm) reported by K55. It would follow then that $\delta^* = \delta/8 = 9.47$ mm, and $\theta = 7\delta/72 = 7.37$ mm. In the absence of other information, we then get $Re_\theta = 7360$ and $Re_\tau = 2755$. As to the value of δ given by K55, however, it seems incompatible with the velocity distribution shown in figure 1, where we estimate that the 99% thickness is closer to 2.61 in. (66.2 mm), which then yields $Re_\theta = 6394$ and $Re_\tau = 2406$.

Additional support for our K55 estimates is provided by the earlier results obtained by Klebanoff & Diehl [2] using the same experimental configuration as in K55, but at a 10% higher freestream velocity (see table 1). The 1/7th power laws were used to scale these data to the lower velocity, and we found good agreement with the K55 values inferred here, as shown in table 1. As for K55 the 99% thickness for Klebanoff & Diehl was found directly from the velocity profile.

Therefore, our best estimates for K55 are $\delta = 66.2$ mm, $C_f = 0.00283$, $Re_\theta = 6394$, and $Re_\tau = 2406$. Surprisingly, these essential parameters have not been reported previously for this iconic experiment.

3 Data Comparisons

Figures 3, 5, 6, and 7 show how the K55 data compare with the experiments of DeGraaff & Eaton [3] and Osaka *et al.* [4], and the DNS of Sillero *et al.* [5] (see table 1). In our notation, $\overline{u^2}^+ = \overline{u^2}/u_\tau^2$, and the overbar denotes time-averaging. Similarly,

[1] B. Tummers, DataThief III (2006) <https://datathief.org/>

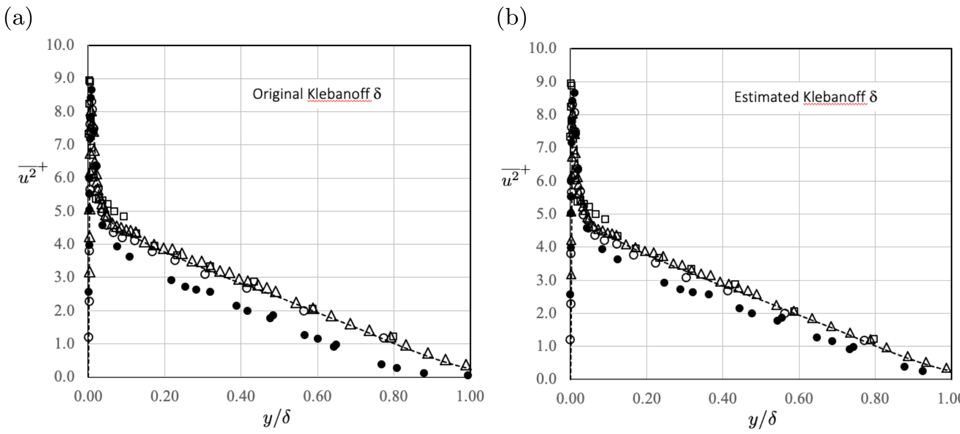

Fig. 3 Comparison in outer scaling for $\overline{u^2}^+$. ●, Klebanoff [1] $Re_\tau = 2406$; ○, DeGraaff & Eaton [3] $Re_\tau = 1692$; □, DeGraaff & Eaton [3] $Re_\tau = 4336$; △, Osaka *et al.* [4] $Re_\tau = 1750$; - - - - -, Sillero *et al.* [5] $Re_\tau = 1848$.

$\overline{v^2}^+ = \overline{v^2}/u_\tau^2$, $\overline{w^2}^+ = \overline{w^2}/u_\tau^2$, and $-\overline{uv}^+ = -\overline{uv}/u_\tau^2$. These particular data sets were chosen because they were taken at broadly similar Reynolds numbers to K55, and because they are among the very few high-quality sets that report all components of the Reynolds stress tensor. It should be noted that Osaka *et al.* [4] used the 99.5% thickness, which is about 4% larger than the 99% thickness. The value of $Re_\tau = 1750$ given in table 1 for this data set uses the 99% thickness estimated here.

3.1 Streamwise turbulence distribution

Figures 3 shows the comparisons in outer scaling for $\overline{u^2}^+$. Figure 3a uses the original boundary layer thickness (76.2 mm), and we see that in the middle of the layer the K55 values are about 25% lower than the other results. In figure 3b we show the same data using the 99% thickness found here (66.2 mm). It is clear that changing the boundary layer thickness cannot explain all of the observed discrepancies.

Instead, we note that Klebanoff's experiment used an artificially thickened boundary layer. From [2], we estimate that in K55 the boundary layer thickness at the end of the sandpaper was about $\delta_i = 38$ mm, so that the measuring station was approximately $65\delta_i$ downstream of the rough to smooth transition. In terms of the mean flow, we would therefore expect the flow to be fully recovered from the step change [6], but this may not hold for the turbulence. For example, [7] found that in a pipe flow downstream of a similar step in roughness the turbulent stresses were exceedingly slow to adjust to the new wall condition (> 120 radii), and they first fell below their equilibrium values before seemingly asymptoting to the fully recovered state. The measurements by Klebanoff & Diehl [2] support a similar conclusion here. In that experiment, at $U_1 = 108$ ft/s (32.9 m/s), the authors found that the u'/U_1 profiles at 3, 5.5 and 8.5 ft downstream of the roughness (0.91, 1.68 and 2.59 m, respectively) collapsed onto a single curve. We would expect, however, that the profiles ought to collapse in friction velocity scaling, not in freestream scaling. This is illustrated by the collapse of the DeGraaff & Eaton data at $Re_\tau = 1692$ and 4336, as shown in figure 3.

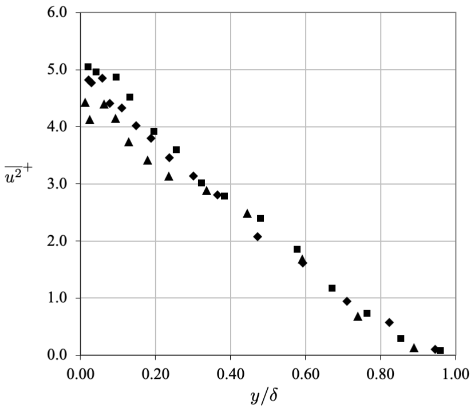

Fig. 4 Comparison in outer scaling for $\overline{u^2}^+$ at $U_1 = 108$ ft/s (32.9 m/s). Distance from the leading edge: ▲, 5 ft (1.52 m); ♦, $x = 7.5$ ft (2.29 m); ■, $x = 10.5$ ft (3.20 m). These locations correspond to distances of 0.91, 1.68 and 2.59 m downstream of the step change in roughness. Data from Klebanoff & Diehl [2].

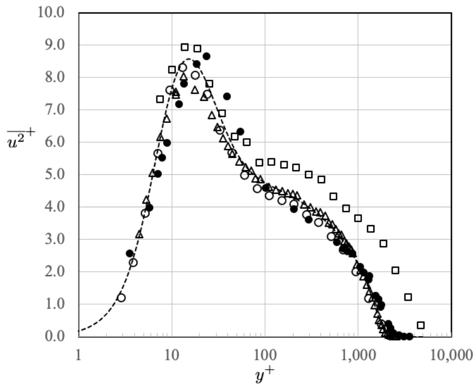

Fig. 5 Comparison in inner scaling for $\overline{u^2}^+$. ●, Klebanoff [1] $Re_\tau = 2406$; ○, DeGraaff & Eaton [3] $Re_\tau = 1692$; □, DeGraaff & Eaton [3] $Re_\tau = 4336$; △, Osaka *et al.* [4] $Re_\tau = 1750$; - - - - -, Sillero *et al.* [5] $Re_\tau = 1848$.

Yet the Klebanoff & Diehl profiles in friction velocity scaling are clearly still evolving with downstream distance, particularly for $y/\delta < 0.4$, as shown in Figure 4. It seems likely, therefore, that the turbulence in K55 is still recovering from the step change in roughness.

The discrepancies seen in outer scaling are less obvious in inner scaling (figure 5). We see that the inner peak maximum for K55 agrees well with the other data, although its position is closer to $y^+ = 25$ than the commonly accepted vale of 15.

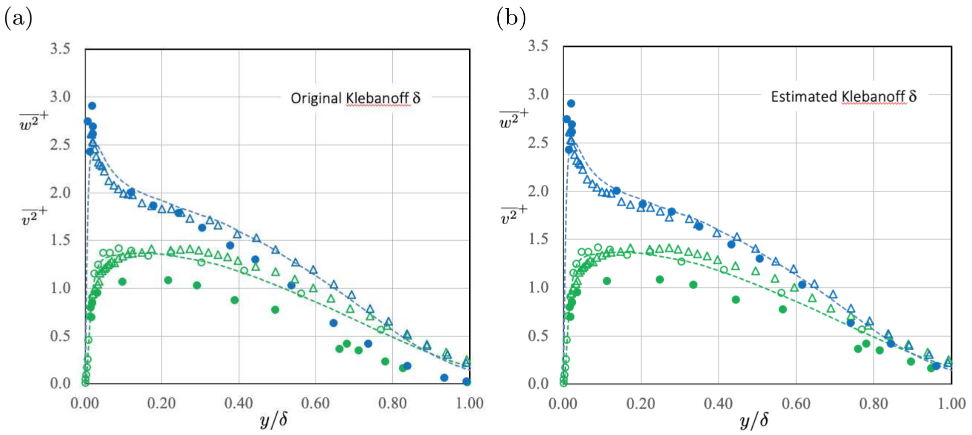

Fig. 6 Comparison in outer scaling for $\overline{v^2}^+$ (green) and $\overline{w^2}^+$ (blue). ●, Klebanoff [1] $Re_\tau = 2406$; ○, DeGraaff & Eaton [3] $Re_\tau = 1692$; △, Osaka *et al.* [4] $Re_\tau = 1750$; - - - - -, Sillero *et al.* [5] $Re_\tau = 1848$.

3.2 Wall-normal turbulence distribution

Figure 6a indicates that at about $y/\delta = 0.4$, the K55 value of $\overline{v^2}^+$ is approximately 30% too low, using the original boundary layer thickness given by K55. This discrepancy reduces to about 20% when using the 99% thickness estimated here (figure 6b).

3.3 Spanwise turbulence distribution

As to the spanwise turbulence levels, figure 6 demonstrates that the K55 levels agree well with the other data near the wall, and the agreement in the outer layer improves considerably when using the 99% thickness (comparing figures 6a and 6b).

3.4 Shear stress distribution

The shear stress follows the same trend as the spanwise stress, in that the K55 levels agree well with the other data near the wall. They then diverge from the consensus levels for $y/\delta > 0.2$, although the differences in the outer layer decrease when using the 99% thickness (comparing figures 7a and 7b). Notably, the DeGraaff & Eaton [3] data fall below the consensus levels by about 10 to 15% in the outer layer.

4 Conclusions

The Klebanoff K55 data [1] displays some serious shortcomings, and should not be used as a reference standard to compare with other experiments and computations. The distributions of $\overline{u^2}^+$, $\overline{v^2}^+$, and $-\overline{uv}^+$, all fall well below the current consensus levels in the outer layer, even when the "correct" boundary layer thickness is used. In addition, the inner peak in $\overline{u^2}^+$ is further from the wall than is now commonly accepted. Only $\overline{w^2}^+$ is in line with expectations. Apart from possible measurement

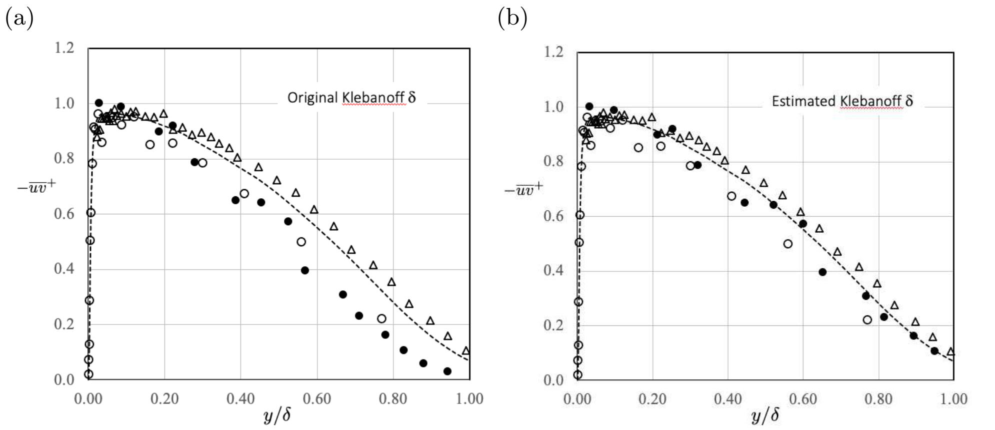

Fig. 7 Comparison in outer scaling for $-\overline{uv}^+$. ●, Klebanoff [1] $Re_\tau = 2406$; ○, DeGraaff & Eaton [3] $Re_\tau = 1692$; △, Osaka *et al.* [4] $Re_\tau = 1750$; - - - - -, Sillero *et al.* [5] $Re_\tau = 1848$.

errors, the discrepancies appear to be due to the slow decay of the effects of the upstream roughness used to artificially thicken the boundary layer.

As to a new standard, it seems clear that the DNS data by Sillero et al. [5] sets the benchmark for future comparisons, at least for Reynolds numbers comparable to K55. Although the highest friction Reynolds number for this data set is only 1848, it is within range of the K55 value of 2406, and this limit will undoubtedly increase in the near future. For comparisons at much higher Reynolds numbers, we suggest using the data sets obtained by [8], [9], and [10].

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Declarations

Ethical approval

Not applicable.

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Availability of data and materials

All data and materials are available in the open literature.

Competing interests

No financial or non-financial interests are directly or indirectly related to the work submitted for publication.

Authors' contributions

The author AJS is solely responsible for the content of this text.

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