Peer influence and sexual debut : A mathematical approach

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Research note

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Peer influence and sexual debut : A mathematical approach

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Abstract

Objectives: Early sexual debut is one of the major causes of sexually transmitted infections (STIs) and unwanted pregnancy. A deterministic mathematical model to attempt to understand the role of prostitution and peer influence on early sexual debut is developed and analyzed.

Results: The thresholds known as the reproduction number and equilibria for the model are determined and stabilities analyzed. Analysis of the reproduction number suggests that prostitution and economic hardships enhance early sexual debut. Sensitivity analysis of the reproduction number is carried out and it suggests that a reduction in prostitution has the greatest impact in reducing the reproduction number. Using the multi-domain pseudo-spectral relaxation method (MD-SRM), numerical simulations are carried out. Numerical simulations suggest that prostitution and peer influence enhances early sexual debut.

Key Words: sexual debut, prostitution, peer influence, reproduction number, stability, MD-SRM

Introduction

In sub-Saharan Africa, about 40 – 80% of young people would have sexually debuted by the age of 18 [1]. Early sexual debut is associated with unprotected and non-consensual sex which increases the risk of unwanted pregnancies, HIV/AIDS, and other sexually transmitted infections (STI's) [3]. Individuals involved in early sexual debut tend to continue indulging in risky sexual behaviors, including having multiple sexual partners [4]. Early sexual debut, is commonly defined as having had first sexual intercourse at or before age 14 [3]. Besides being an important determinant of HIV infection, early age at sexual debut has negative effects on academic outcomes which can extend beyond secondary school, although concurrent changes in other psycho-social risk factors have not been investigated [5]. Poverty is also a driving force of child prostitution in Africa and so is associated with early sexual debut [6, 7]. Children in poverty may be pushed into sugar daddy and sugar mommy relationships, which increases the risk of infection as these partners are not only older but tend to be in multiple sexual relationships. In this situation, young people are forced into early sexual debut and are sometimes powerless to negotiate safe sex, which leads to early STI's [8, 9]. Currently, a lot of Zimbabwean women are turning to prostitution in Botswana and South Africa due to poverty and unemployment in their home country [10, 11].

Traditionally, aunties and uncles socialized girls and boys into adulthood including sex and marriage [12]. However, due to modernization, coupled with the HIV epidemic, these have destroyed this traditional institution leaving young people without a potentially valuable resource, thus possibly shifting this social responsibility elsewhere. The HIV epidemic has robbed the younger generations of uncles and aunts through death and they end up seeking advice from other inexperienced peers and people of loose morals. This has

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led to less supervision and monitoring which are typical factors of moral decay, which is the failure to uphold sound morality in society. Moral decay has also contributed greatly to the increase in the number of divorce cases leading to many adolescents not living with both parents [13]. Moral decay is strongly associated with young people copying bad habits from some individuals, mainly through peer influence. In sub-Saharan Africa just like anywhere else in the world, losing a parent through a divorce and/or remarriage can present a major disruption to any child, more so for the adolescents who are at their critical stage of life. Young people particularly adolescents who are at their critical stage of development being raised by single parents fail to get: (i) enough discipline and basic family needs mostly provided by fathers in case of adolescents staying with the mother as the only parent, (ii) adequate nurturing and emotional support mostly provided by mothers in the case of adolescents staying with the father as the only parent [13]. In this manuscript, ideas borrowed from epidemiological models are applied to study the dynamics of social and behavioral processes as done in some previous studies [14, 15, 16, 17, 18, 38]. This manuscript is an attempt to understand the role of prostitution and peer influence on sexual debut from a mathematical framework. The manuscript mainly focuses on African settings where prostitution/commercial sex work is illegal.

The paper is outlined as follows; in *Main text* Section, we formulate and establish the basic properties of the model. The model stability analysis is then carried out in the *Model analysis* Section. In the *Results* Section, we present the parameter estimation, carry out some numerical simulations and the analysis of the reproduction number. The paper is concluded in the *Discussion* Section.

Main text

Model formulation

The model sub-divides the population based on age at which one experienced his/her first sexual encounter. The population is divided into the following sub-groups:

$S(t)$: Susceptible individuals, meaning those who have not partnered with any one in sex.

$I_y(t)$: Individuals who experienced first sexual intercourse at a young age.

$I_m(t)$: Individuals who experienced first sexual contact when they are mature and are not of loose morals.

$I_p(t)$: Adults of loose morals, for lack of a better word, whom we shall refer to as prostitutes.

For the purpose of our study, early sexual debut would be taken as any penetrative sexual exposure at or before the age of 14 years, as also presented in other researches [3, 19, 20]. In this model we assume that peer influence is the driving force behind early sexual encounters. The total population is given by

$$N(t) = S(t) + I_y(t) + I_m(t) + I_p(t). \quad (1)$$

Individuals in different human subgroups suffer from natural death at a per capita rate μ . We assume that interaction is homogeneous. In this paper, we target peer influence effects on early sexual intercourse. We model such effects as pure imitation process (peer influence). Prostitutes further experience deaths related to their profession (homicide, murder, drug-misuse) at a rate ν [40]. The peer influence force necessary for a susceptible individual to be coerced into sexual intercourse is given by $\lambda(t)$ with

$$\lambda(t) = \frac{\beta_y I_y(t)}{N(t)} + \frac{\beta_p I_p(t)}{N(t)}. \quad (2)$$

In equation (2), β_y is the product of the contact rate and the probability that one susceptible individual becomes sexually active as a result of interaction with sexually active peers; β_p is the product of the contact rate and the probability that one susceptible individual becomes sexually active as a result of interaction with people of loose morals (prostitutes). It is worth mentioning here that $\beta_p \geq \beta_y$ since those who spend

time with prostitutes are more likely to engage in early sexual relations leading to more sexual intercourses than others.

Individuals are recruited into the susceptibles through birth at a rate Λ . Susceptibles become sexually active at a rate λ defined in equation (2) with a proportion f doing it at a young age and the complementary $(1 - f)$ doing it when already mature with minimum outside influence to enter $I_y(t)$ and $I_m(t)$, respectively. Individuals in $I_y(t)$ leave the $I_y(t)$ -stage at a rate ρ with the proportion σ becoming prostitutes and the complementary $(1 - \sigma)$ becoming sober minded individuals of high integrity (not prostitutes) $I_m(t)$. Some leave prostitution at a rate θ due to being gainfully employed (as most people in Zimbabwe become prostitutes due to economic hardships) and enter the $I_m(t)$ -class. Due to economic hardships, most individuals in $I_m(t)$ become prostitutes at a rate ϕ . It is worth noting that some become prostitutes due to moral decay. In this manuscript, we will only assume that individuals become prostitutes through economic hardships. Based on these assumptions the following system of differential equations describe the model

$$\begin{aligned} S'(t) &= \Lambda - (\lambda + \mu)S(t), \\ I_y'(t) &= f\lambda S(t) - (\mu + \rho)I_y(t), \\ I_m'(t) &= (1 - f)\lambda S(t) + (1 - \sigma)\rho I_y(t) - (\mu + \phi)I_m(t) + \theta I_p(t), \\ I_p'(t) &= \sigma\rho I_y(t) + \phi I_m(t) - (\mu + v + \theta)I_p(t). \end{aligned} \tag{3}$$

Positivity and boundedness

In this subsection, we study the basic properties of the solutions of the model system (3).

Theorem 1. *The solutions $S(t)$, $I_y(t)$, $I_m(t)$ and $I_p(t)$ of model system (3) are non-negative for $t \geq 0$.*

Proof. Letting the initial values of the variables for model system (3) be strictly positive. We prove that the solution component of $S(t)$ is positive. Assuming that there exists a first time $t_1 : S(t_1) = 0$, $S'(t_1) < 0$ and

$$I_y(t) > 0, I_m(t) > 0, I_p(t) > 0 \text{ for } 0 < t < t_1.$$

From the first equation of model system (3), we have

$$\frac{dS(t)}{dt} = \Lambda > 0, \tag{4}$$

which happens to be a contradiction and consequently, $S(t)$ remains positive. The remaining variables are also proved in the same way. Therefore, the solutions of model system (3) are non-negative whenever $t \geq 0$. $\square$

Theorem 2. *The region $\Omega = \left\{ (S(t), I_y(t), I_m(t), I_p(t)) \in \mathbb{R}_+^4 : N(t) \leq \max\left\{N_0, \frac{\Lambda}{\mu}\right\} \right\}$ is positively invariant and attracting with respect to the model.*

Proof. Let $(S(t), I_y(t), I_m(t), I_p(t)) \in \mathbb{R}_+^4$ represent any solution of model system (3) with non-negative initial conditions given by $(S(0), I_y(0), I_m(0), I_p(0))$. $N(t)$ represents the total active population. Adding all the equations in model system (3), we have

$$\frac{dN}{dt} = \Lambda - \mu N(t) - v I_p. \tag{5}$$

For $I_p \geq 0$ with $t \geq 0$, we have

$$\frac{dN}{dt} + \mu N(t) \leq \Lambda. \tag{6}$$

and

$$N(t) \leq \frac{\Lambda}{\mu} + \left(N_0 - \frac{\Lambda}{\mu} \right) e^{-\mu t}, \tag{7}$$

where N_0 represents the value of model system (3) evaluated at the initial values of the respective variables. Two scenarios arise:

Scenario 1 : If $N_0 > \frac{\Lambda}{\mu}$ then (7) implies that $N(t) \leq N_0$ for all values of t .

Scenario 2 : If $N_0 < \frac{\Lambda}{\mu}$ then (7) implies that $N(t) \leq \frac{\Lambda}{\mu}$ for all values of t .

Therefore, $N(t) \leq \max \left\{ N_0, \frac{\Lambda}{\mu} \right\}$. Every feasible solution of the model that starts in the region

$$\Omega = \left\{ (S(t), I_y(t), I_m(t), I_p(t)) \in \mathbb{R}_+^4 : N(t) \leq \max \left\{ N_0, \frac{\Lambda}{\mu} \right\} \right\}, \quad (8)$$

remains in the region for all the values of t . Hence, the region is biologically feasible and positively invariant. $\square$

Therefore, the model is well posed mathematically and epidemiological.

Model analysis

Equilibrium states

By equating the right-hand side of each of the equations in the system (3), we can find the equilibrium points. We obtain the sex-free equilibrium point denoted by SFE and the equilibrium point where individuals are losing their virginity, known as the sex endemic equilibrium denoted by SEE. The SFE refers to the situation in which no individual is losing virginity in the community. The SEE equilibrium refers to a situation in which sex exists in the community and the susceptibles are losing their virginity.

Sex-free equilibrium and stability analysis

The sex-free equilibrium for model system (3) is given by

$$\mathcal{E}^0 = (S^0, I_y^0, I_m^0, I_p^0) = \left(\frac{\Lambda}{\mu}, 0, 0, 0 \right). \quad (9)$$

In modeling infectious diseases, the basic reproduction number $\mathcal{R}_0$, is among the most vital threshold quantities, which depict mathematical problems concerning infectious diseases. It is valuable in deciding if an infectious disease will spread within a population or not. $\mathcal{R}_0$, the basic reproduction number has been used in modeling of social epidemics [24, 25, 26, 27, 28].

In order to compute the basic reproduction number $\mathcal{R}_0$ for the system (3) we make use of the next generation technique as proposed in [21, 22, 23] and also summarized in Appendix A. The infectious classes in our system are I_y and I_p . Progression from I_y to I_p and I_y to I_m are not considered to be new sexual intercourse infections, but rather a progression of a sexually active individual through various compartments [21, 23]. Applying the notation as outlined in [21] for the next-generation matrix, we have the population dynamic differential equations.

$$\mathcal{F} = \begin{bmatrix} f\lambda S \\ (1-f)\lambda S \\ 0 \end{bmatrix} \quad \text{and} \quad \mathcal{V} = \begin{bmatrix} (\mu + \rho)I_y, \\ -(1-\sigma)\rho I_y + (\mu + \phi)I_m - \theta I_y, \\ -\sigma\rho I_y - \phi I_m + (\mu + v + \theta)I_p. \end{bmatrix}. \quad (10)$$

Additionally, the non-negative matrix F that represents the generation of new sexual intercourse and the non-singular matrix V that denotes the sexual activity transfer terms among compartments are respectively

given by

$$F = \begin{bmatrix} f\beta_y & 0 & f\beta_p \\ (1-f)\beta_y & 0 & (1-f)\beta_p \\ 0 & 0 & 0 \end{bmatrix} \text{ and } V = \begin{bmatrix} (\mu + \rho) & 0 & 0 \\ -(1-\sigma)\rho & (\mu + \phi) & -\theta \\ -\sigma\rho & -\phi & (\mu + v + \phi) \end{bmatrix}. \quad (11)$$

Then, the reproduction number $\mathcal{R}_S$ of model system (3) is the spectral radius of the next generation matrix FV^{-1} , given by

$$\begin{aligned} \mathcal{R}_S &= \frac{f\beta_y}{\mu + \rho} + \frac{\beta_p [\mu\phi(1-f) + \rho(\phi + f\mu\sigma)]}{(\mu + \rho)(\mu\theta + (\mu + v)(\mu + \phi))} \\ &= \mathcal{R}_Y + \mathcal{R}_P \end{aligned} \quad (12)$$

where

$$\mathcal{R}_Y = \frac{f\beta_y}{\mu + \rho} \text{ and } \mathcal{R}_P = \frac{\beta_p [\mu\phi(1-f) + \rho(\phi + f\mu\sigma)]}{(\mu + \rho)(\mu\theta + (\mu + v)(\mu + \phi))}.$$

From the work in [21], we know that the sex-free equilibrium is locally asymptotically stable when $\mathcal{R}_S < 1$, and unstable when $\mathcal{R}_S > 1$. Actually, we can establish below a stronger result regarding the global dynamics of the sex-free equilibrium. We will utilize the approach of Lyapunov functions [29, 30, 31, 32] in the analysis of the global asymptotic stability.

Theorem 3. *If $\mathcal{R}_S \leq 1$, the sex-free equilibrium is globally asymptotically stable in Ω . If $\mathcal{R}_S > 1$, the system is uniformly persistent.*

Proof. Let $\mathcal{H}(t) = (I_y(t), I_m(t), I_p(t))$. Since

$$\begin{aligned} I'_y(t) &= f\lambda S - (\mu + \rho)I_y, \\ I'_m(t) &= (1-f)\lambda S + (1-\sigma)\rho I_y - (\mu + \phi)I_m + \theta I_y, \\ I'_p(t) &= \sigma\rho I_y + \phi I_m - (\mu + v + \theta)I_p, \end{aligned} \quad (13)$$

it follows that

$$\dot{\mathcal{H}}(t) \leq (F - V)\mathcal{H},$$

where F and V are defined in equation (11). Inspired by [31], we present a Lyapunov function of the form

$$\mathcal{L} = \omega^T V^{-1} \mathcal{H}.$$

Taking the derivative of $\mathcal{L}$ along the solutions of (3), we have

$$\begin{aligned} \dot{\mathcal{L}} &= \omega^T V^{-1} \dot{\mathcal{H}} \\ &\leq \omega^T V^{-1} (F - V) \mathcal{H} \\ &= (\mathcal{R}_S - 1) \omega^T \mathcal{H} \leq 0, \text{ if } \mathcal{R}_S \leq 1. \end{aligned} \quad (14)$$

It can be easily established that the largest invariant subset of Ω where $\dot{\mathcal{L}} = 0$ is the singleton $\{\mathcal{E}^0\}$. Therefore, by LaSalle's invariance principle [34], $\mathcal{E}^0$ is globally asymptotically stable in Ω when $\mathcal{R}_S \leq 1$.

If $\mathcal{R}_S > 1$, then by continuity, $\dot{\mathcal{L}} > 0$ in the neighbourhood of $\mathcal{E}^0$ in the interior of Ω . Solutions in the interior of Ω sufficiently close to $\mathcal{E}^0$ move away from the sex-free equilibrium implying that the sex-free equilibrium is unstable. $\square$

We now examine the existence and stability of the endemic equilibrium point.

Stability and existence of the endemic equilibrium point

The endemic equilibrium, is the equilibrium point where sexual intercourse exists in the society termed as the *endemic equilibrium point*. Represented in terms of the force of peer influence, λ^* , the endemic equilibrium point is given by $\mathcal{E}^* = (S^*, I_y^*, I_m^*, I_p^*)$, with

$$\begin{aligned} S^* &= \frac{\Lambda}{\lambda^* + \mu}, \quad I_y^* = \frac{f\Lambda\lambda^*}{h_1(\lambda^* + \mu)}, \quad I_m^* = \frac{\Lambda\lambda^* [f\theta\rho\sigma + (f\rho(1-\sigma) + (1-f)h_1)h_3]}{h_1(\lambda^* + \mu)(h_2h_3 - \theta\phi)}, \\ I_p^* &= \frac{\Lambda\lambda^* [(1-f)\phi h_1 + f\rho(1-\sigma)\phi + \sigma h_2]}{h_1(\lambda^* + \mu)(h_2h_3 - \theta\phi)}, \quad \text{where} \quad h_1 = \mu + \rho, \quad h_2 = \mu + \phi, \quad h_3 = \mu + v + \theta. \end{aligned} \quad (15)$$

Lemma 1. *The endemic equilibrium point $\mathcal{E}^*$ exists whenever $\mathcal{R}_S > 1$.*

Proof. Substituting equation (15) into the force of peer influence, that is equation (2), we have

$$\begin{aligned} \lambda^* &= \frac{f\beta_y\lambda^*(h_2h_3 - \theta\phi) + \beta_p\lambda^*[\phi\mu(1-f) + (\phi + f\mu\sigma)\rho]}{h_1(h_2h_3 - \theta\phi) + A\lambda^*}, \quad \text{where} \\ A &= (h_3 + \phi)[h_1(1-f) + f\rho(1-\sigma)] + f[\theta(\mu + \rho\sigma) + h_2(\mu + v)]. \end{aligned} \quad (16)$$

It follows from (16) that

$$A\lambda^{*2} + \lambda^*[h_1(h_2h_3 - \theta\phi) - (f\beta_y(h_2h_3 - \theta\phi) + \beta_p[\phi\mu(1-f) + (\phi + f\mu\sigma)\rho])] = 0. \quad (17)$$

It follows from equation (17) that $\lambda^* = 0$ corresponds to the sex-free equilibrium state and

$$\lambda^* = \frac{h_1(h_2h_3 - \theta\phi)(\mathcal{R}_S - 1)}{A},$$

corresponding to the sex endemic equilibrium point which exists when $\lambda^* > 0$, thus $\mathcal{R}_S > 1$. Thus, the sex endemic equilibrium only exists when $\mathcal{R}_S > 1$. $\square$

Now, we prove the local stability of $\mathcal{E}^*$ provided the reproduction number $\mathcal{R}_S$ is sufficiently close to 1. We make use of the center manifold theory [49]. The center manifold theory, has been applied in many papers and we will deliberately omit it and not state it, since we are only interested in its application.

Theorem 4. *The unique endemic equilibrium point $\mathcal{E}^*$ is locally asymptotically stable if $\mathcal{R}_S > 1$ but close to 1.*

The proof for Theorem 5 is outlined in Appendix B.

Results

Estimation of parameters

Estimation of parameters Parameter values used for numerical simulations are given in Table 1.

Next, we explore the effects of some of the constituents of the reproduction number on it in order to make reasonable judgment.

Analysis of the reproduction number

Effects of becoming a prostitute

- **For those deflowered early**

If all young people who lost their virginity early become prostitutes ($\sigma = 1$) then

$$\begin{aligned}\lim_{\sigma \rightarrow 1} \mathcal{R}_S &= \mathcal{R}_{S_1} = \mathcal{R}_Y + \frac{\beta_\rho [\mu\phi(1-f) + \rho(\phi + f\mu\sigma)]}{(\mu + \rho)[\mu\theta + (\mu + v)(\mu + \phi)]}, \\ \Delta_{S_1} &= \mathcal{R}_{S_1} - \mathcal{R}_S = \frac{\beta_\rho f\mu(1-\sigma)}{(\mu + \rho)[\mu\theta + (\mu + v)(\mu + \phi)]} > 0.\end{aligned}\tag{18}$$

The fact that $\Delta_{S_1} > 0$, suggests that prostitution enhances early sexual debut.

- **For the mature due to economic hardships**

Taking the derivative of $\mathcal{R}_S$ with respect to ϕ we obtain

$$\frac{\partial \mathcal{R}_S}{\partial \phi} = \frac{\beta_\rho [\theta\mu(\mu(1-f) + \rho) + (\mu(1-f) + (1-f\sigma)\rho)(\mu + v)\mu]}{(\mu + \rho)((\mu + v)(\mu + \phi) + \theta\mu)^2} > 0.\tag{19}$$

$\frac{\partial \mathcal{R}_S}{\partial \phi} > 0$ suggests that economic hardships which force people into prostitution leads to an increase in the number of individuals becoming sexually active at a young age.

- **Once a prostitute always a prostitute**

In this case, the prostitutes will not leave prostitution ($\theta = 0$). Thus,

$$\begin{aligned}\lim_{\theta \rightarrow 0} \mathcal{R}_S &= \mathcal{R}_{S_2} = \mathcal{R}_Y + \frac{\beta_\rho [\mu\phi(1-f) + \rho(\phi + f\mu\sigma)]}{(\mu + \rho)(\mu + v)(\mu + \phi)}, \\ \Delta_{S_2} &= \mathcal{R}_{S_2} - \mathcal{R}_S = \frac{\mu\theta\mathcal{R}_P}{(\mu + v)(\mu + \phi)} > 0.\end{aligned}\tag{20}$$

$\Delta_{S_2} > 0$, also suggests prostitution enhances early sexual intercourse.

Effects of quitting prostitution

Taking the derivative of $\mathcal{R}_S$ with respect to θ we obtain

$$\frac{\partial \mathcal{R}_S}{\partial \theta} = -\frac{\mu\mathcal{R}_P}{(\mu + v)(\mu + \phi) + \theta\mu} < 0.\tag{21}$$

$\frac{\partial \mathcal{R}_S}{\partial \theta} < 0$, shows that quitting prostitution results in a decrease of $\mathcal{R}_S$. A decrease in $\mathcal{R}_S$ suggests a decrease in the number of young people becoming involved in sexual intercourse at an early age.

We now carry out the sensitivity analysis of the reproduction number.

Sensitivity analysis

To further examine the results of our foregoing analysis, we simulated model system (3) making use of the parameters in Table 1. Regrettably, the shortage of data on peer influence and prostitution limits our ability to calibrate, however, some of the parameter values are assumed within the realistic range for illustrative purposes. These miserly assumptions reflect the shortage of information currently available on peer influence, prostitution, and early sexual debut. Reliable data on peer influence, prostitution, and early sexual debut would enhance our understanding and be of great assistance in the possible intervention strategies to be executed. Before we present our numerical simulations, we shall first explore the sensitivity indices of $\mathcal{R}_S$

based on perturbation of fixed point estimates.

In countless models of epidemiology, the size of the reproductive number is related to the level of infection. The same applies with model system (3) since we are also examining sexual debut as an infection. Sensitivity analysis examines the quantity and type of change inherent in the model as captured by the terms that define the reproductive number ($\mathcal{R}_S$). If ($\mathcal{R}_S$) is exceedingly sensitive to a certain parameter, then a perturbation of the states that connects the dynamics to such may prove useful in identifying policies or intervention strategies that reduce epidemic prevalence.

We now present the normalized forward sensitivity index of $\mathcal{R}_S$, as outlined in Arriola and Hyman [50], for our model parameters in Table 1. The comprehensive sensitivity indices of $\mathcal{R}_S$ resulting from the assessment to other model parameters are also shown in Table 1, in the last column. For example, the sensitivity index concerning θ is,

$$\Upsilon_{\theta}^{\mathcal{R}_S} = \frac{\partial \mathcal{R}_S}{\partial \theta} \times \frac{\theta}{\mathcal{R}_S} = -0.601884.$$

In Table 1, on the sensitivity index column, it is shown that the increase of prostitutes interaction has the highest positive contribution to the growth of $\mathcal{R}_S$ suggesting prostitution has the potential to lure young people to experience early sexual debut. Measures that may encourage individuals to quit prostitution and also which influence prostitutes not to indulge in sexual acts with young persons, would be vital. The higher the proportion of early sexual debuts, the bigger $\mathcal{R}_S$ is. However, maturity tends to have a negative effect on $\mathcal{R}_S$. This result tends to suggest that as people get older they become wiser and know the dangers associated with early sexual debut, and may play a leading role in discouraging early sexual debut. It is worth stating that increasing the rate at which individuals quit prostitution by 10%, has an effect of reducing the reproduction number by 6.02%

Table 1: Model parameters and their interpretations

Definition	Symbol	Baseline values(Range)	Source	Sensitivity Index
Recruitment rate	Λ	0.03	[41]	-
Natural mortality rate	μ	0.02yr^{-1}	[39]	-0.798405
Prostitution related mortality rate	ν	0.035yr^{-1}	[40]	-0.011836
Probability of reaching adulthood	ρ	$0.97(0.1 - 1)$	[39]	-0.252791
Prostitution quitting rate	θ	$0.5(0.0 - 1)\text{yr}^{-1}$	[15]	-0.601884
Product of effective contact rate and probability of becoming sexually active due to influence of prostitutes	β_p	$0.125(0.011 - 0.95)$	[15]	0.899976
Product of effective contact rate and probability of becoming sexually active due to peer influence	β_y	$0.025(0.0 - 1)$	[42]	0.153205
Rate of becoming a prostitute due to economic hardships	ϕ	$0.1(0.0750 - 0.95)\text{yr}^{-1}$	Assumed	0.006023
Proportion of early sexual debutees	f	$0.33(0.01 - 1)\text{yr}^{-1}$	Assumed	0.008713
Proportion of early sexual debutees maturing into prostitution	σ	$0.33(0.01 - 1)\text{yr}^{-1}$	Assumed	0.021013

Numerical simulations

For our numerical simulations, we shall make use of the multi-domain pseudo spectral relaxation method (MD-SRM) [47]. The MD-SRM makes use of Legendre-Gauss-Lobato grid points, Gauss-Seidel relaxation method, and the pseudo-spectral collocation technique in approximating functions defined by Lagrange interpolation. The method was developed for a general system of n linear population dynamic differential equations, and it is very precise.

To apply this method, it is helpful to rescale our model. Let

$$\Lambda = \mu N, \quad x_1 = \frac{S}{N}, \quad x_2 = \frac{I_y}{N}, \quad x_3 = \frac{I_m}{N}, \quad x_4 = \frac{I_p}{N}, \quad (22)$$

and we now denote our total population by n , given by

$$n = x_1 + x_2 + x_3 + x_4 = 1,$$

so that model system (3) reduces to

$$\begin{aligned} x_1' &= \mu - (\beta_y x_2 + \beta_p x_4)x_1 - \mu x_1, \\ x_2' &= f(\beta_y x_2 + \beta_p x_4)x_1 - (\mu + \rho)x_2, \\ x_3' &= (1 - f)(\beta_y x_2 + \beta_p x_4)x_1 + (1 - \sigma)\rho x_2 + \theta x_4 - (\mu + \phi)x_3, \\ x_4' &= \sigma\rho x_2 + \phi x_3 - (\mu + v + \phi)x_4. \end{aligned} \quad (23)$$

Without loss of generality, we express system (23) in the form

$$\begin{aligned} \frac{dx_1}{dt} &= x_1 f_1(x_2, x_3, x_4) + g_1(x_1, x_2, x_3, x_4), \\ \frac{dx_2}{dt} &= x_2 f_2(x_1, x_3, x_4) + g_2(x_1, x_2, x_3, x_4), \\ \frac{dx_3}{dt} &= x_3 f_3(x_1, x_2, x_4) + g_3(x_1, x_2, x_3, x_4), \\ \frac{dx_4}{dt} &= x_4 f_4(x_1, x_2, x_3) + g_4(x_1, x_2, x_3, x_4), \end{aligned} \quad (24)$$

where

$$\begin{aligned} f_1(x_2, x_3, x_4) &= -(\lambda + \mu), \quad g_1(x_1, x_2, x_3, x_4) = \Lambda, \quad f_2(x_1, x_3, x_4) = -(\mu + \rho) + f\beta_y x_1, \\ g_2(x_1, x_2, x_3, x_4) &= f\beta_p x_1 x_4, \quad f_3(x_1, x_2, x_4) = -(\mu + \phi), \\ g_3(x_1, x_2, x_3, x_4) &= (1 - f)\lambda x_1 + (1 - \sigma)\rho x_2 + \theta x_4, \\ f_4(x_1, x_2, x_3) &= -(\mu + v + \theta), \quad g_4(x_1, x_2, x_3, x_4) = \sigma\rho x_2 + \phi x_3. \end{aligned} \quad (25)$$

Applying the MD-SRM method we get the following matrices,

$$\mathbf{A}_1 \mathbf{X}_{1,r+1}^{(l)} = \mathbf{R}_1^{(l)}, \quad \mathbf{A}_2 \mathbf{X}_{2,r+1}^{(l)} = \mathbf{R}_2^{(l)}, \quad \mathbf{A}_3 \mathbf{X}_{3,r+1}^{(l)} = \mathbf{R}_3^{(l)}, \quad \mathbf{A}_4 \mathbf{X}_{4,r+1}^{(l)} = \mathbf{R}_4^{(l)}, \quad (26)$$

where

$$\begin{aligned} \mathbf{A}_1 &= \mathbf{D} - \mathbf{f}_1[\mathbf{X}_{2,r}^{(l)}(t_j), \mathbf{X}_{3,r}^{(l)}(t_j), \mathbf{X}_{4,r}^{(l)}(t_j)]\mathbf{I}, \\ \mathbf{A}_2 &= \mathbf{D} - \mathbf{f}_2[\mathbf{X}_{1,r+1}^{(l)}(t_j), \mathbf{X}_{3,r}^{(l)}(t_j), \mathbf{X}_{4,r}^{(l)}(t_j)]\mathbf{I}, \\ \mathbf{A}_3 &= \mathbf{D} - \mathbf{f}_3[\mathbf{X}_{1,r+1}^{(l)}(t_j), \mathbf{X}_{2,r+1}^{(l)}(t_j), \mathbf{X}_{4,r}^{(l)}(t_j)]\mathbf{I}, \\ \mathbf{A}_4 &= \mathbf{D} - \mathbf{f}_4[\mathbf{X}_{1,r+1}^{(l)}(t_j), \mathbf{X}_{2,r+1}^{(l)}(t_j), \mathbf{X}_{3,r+1}^{(l)}(t_j)]\mathbf{I}, \end{aligned} \quad (27)$$

and the right hand side of (26) given by

$$\begin{aligned}
\mathbf{R}_1^{(l)} &= \mathbf{g}_1[\mathbf{X}_{1,r}^{(l)}(t_j), \mathbf{X}_{2,r}^{(l)}(t_j), \mathbf{X}_{3,r}^{(l)}(t_j)] - \mathbf{D}_{jN} \mathbf{X}_{1,r}^{(l)}(t_N), \\
\mathbf{R}_2^{(l)} &= \mathbf{g}_2[\mathbf{X}_{1,r+1}^{(l)}(t_j), \mathbf{X}_{2,r}^{(l)}(t_j), \mathbf{X}_{3,r}^{(l)}(t_j)] - \mathbf{D}_{jN} \mathbf{X}_{2,r}^{(l)}(t_N), \\
\mathbf{R}_3^{(l)} &= \mathbf{g}_3[\mathbf{X}_{1,r+1}^{(l)}(t_j), \mathbf{X}_{2,r+1}^{(l)}(t_j), \mathbf{X}_{3,r}^{(l)}(t_j)] - \mathbf{D}_{jN} \mathbf{X}_{3,r}^{(l)}(t_N), \\
\mathbf{R}_4^{(l)} &= \mathbf{g}_4[\mathbf{X}_{1,r+1}^{(l)}(t_j), \mathbf{X}_{2,r+1}^{(l)}(t_j), \mathbf{X}_{3,r+1}^{(l)}(t_j)] - \mathbf{D}_{jN} \mathbf{X}_{4,r}^{(l)}(t_N).
\end{aligned} \tag{28}$$

The following numerical simulations were plotted in Matlab using the parameters in Table 1 and the following initial conditions for the population sizes $x_1(0) = 0.7$, $x_2(0) = 0.2$, $x_3(0) = 0.099$, $x_4(0) = 0.001$.

Figure 1: Time series plots showing the impact of increasing the proportion of early sexual debutees maturing into prostitution on the population of prostitutes, $I_p(t)$.

Figure 1 shows the impact of increasing the proportion of early sexual debutees maturing into prostitution. As expected, we can see from Figure 1 that an increase of early sexual debutees maturing into prostitution has an impact of increasing the numbers of prostitutes. It is worth noting that for the first 2 years, there is a sharp increase. Hence, this is vital for policy formulators in knowing that the proportion of early sexual debutees maturing into prostitution, results in the highest number of prostitutes within the first 2 years. After the 2 years, the population begins to increase steadily.

Figure 2: Time series plots showing the impact of increasing the proportion of early sexual debutees maturing into prostitution on the population of early sexual debutees, $I_y(t)$.

Figure 2 depicts the impact of increasing the proportion of early sexual debutees maturing into prostitution on the population of early sexual debutees, $I_y(t)$. It is worth noting that an increase of the proportion of early sexual debutees maturing into prostitution results in an increase of the prostitutes population and a decrease in the sober minded individuals of high integrity, $I_m(t)$, and the converse is true. We note that increasing the proportion of early sexual debutees maturing into prostitution results in an increase in the number of prostitutes for the first 7 years only. After 7 years, an increase of the proportion of early sexual debutees maturing into prostitution results in a decrease in the population of early sexual debutantes. Furthermore, the difference after 7 years is not that significant. It is worth noting that the numerical simulations are in agreement with the sensitivity analysis, in that maturity has an impact of reducing the early sexual debutees.

Discussion

Early sexual debut is one of the major causes of STI's and HIV/AIDS. Thus, methods and channels to understand and curtail early sexual debut are needed. A mathematical model for the role of prostitution and peer influence on sexual debut is presented as a system of differential equations. Comprehensive and robust mathematical techniques have been used to analyze the model. The basic reproduction number of the model is computed and analyzed. Analysis of the reproduction number suggests that prostitution and economic hardships enhances early sexual debut. It has been established that the model has a sex-free equilibrium which is globally asymptotically stable when the associated reproduction number is less than unity. It was also established that sex persists when the reproduction number is greater than unity, and it was further proved using the persistence theory. The endemic equilibrium point is computed and shown to exist for $\mathcal{R}_S > 1$. With the aid of the centre manifold theory, the endemic equilibrium point is shown to be locally asymptotically stable when the reproduction number is greater than unity. Sensitivity analysis of the reproduction number has been carried out. Results from the sensitivity analysis of the reproduction number suggest that an increase of the effective contact rate in peer influence has the largest impact in increasing the reproduction number. Furthermore, sensitivity analysis suggests that maturity has an impact in the reduction of the reproduction number. It is worth noting that a reduction in prostitutes also has an impact in the reduction of the reproduction number $\mathcal{R}_S$. Hence, it would be vital to conduct some counselling services to reduce the number of prostitutes. A reduction of prostitutes, implies a reduction of $\mathcal{R}_S$ and which in turn results in a reduction of early sexual debutees. Numerical simulations are then carried out

using the MD-SRM method. The numerical simulations show that prostitutes have an impact in increasing the number of early sexual debutees. With the aid of sensitivity analysis, it was further noted that maturity of individuals has an impact of reducing early sexual debut. Hence, measures to reduce both peer influence and prostitution would be vital in reducing early sexual debut.

Limitations

Our study has a few limitations. Limited data exists with relation to peer influence, prostitution and sexual debut, particularly no mathematical models have been done. Therefore, some of our numerical estimates remain uncertain. In our model, we also have two equilibrium points, the sex free equilibrium and the existence of the sex equilibrium (termed the sex endemic equilibrium point). Thus, we assume the existence of the sex free equilibrium point within the community, although it seems a bit more unreasonable. However just like any other model, we cannot say the model is complete, it can be extended to include resource limited or resource given communities.

Abbreviations

MD-SRM: multi-domain pseudo-spectral relaxation method, STI: sexually transmitted infections, SFE: sexually transmitted infections, SEE: sex endemic equilibrium.

Declarations

Competing interests: The authors would like to declare that they have no conflicts of interests.

Financial disclosure: No funding was received in carrying out this research.

Availability of data: The data used to support the results of this study are included in the manuscript.

Authors contribution: All the work done by the authors.

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Figures

Figure 1

Time series plots showing the impact of increasing the proportion of early sexual debutees maturing into prostitution on the population of prostitutes, $I_p(t)$.

Figure 2

Time series plots showing the impact of increasing the proportion of early sexual debutees maturing into prostitution on the population of early sexual debutees, $I_y(t)$.

Supplementary Files

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