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Prediction of Natural Convection Heat Transfer Phenomena of Nanofluids in Corrugated Annuluse

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Abstract— The natural convection heat transfer and fluid flow characteristic of water based Al_2O_3 nano-fluids in a symmetrical and unsymmetrical corrugated annulus enclosure has been studied numerically using CFD. The inner cylinder is heated isothermally while the outer cylinder is kept constant cold temperature. The study includes eight models of corrugated annulus enclosure with constant aspect ratio of 1.5. The governing equations of fluid motion and heat transfer are solved using stream-vorticity formulation in curvilinear coordinates. The range of solid volume fractions of nanoparticles extends from $\text{PHI}=0$ to 0.25, and Rayleigh number varies from 10^4 to 10^7 . Streamlines, isotherms, local and average Nusselt number of inner and outer cylinder has been investigated in this study. Sixty-four correlations have been deduced for the average Nusselt number for the inner and outer cylinders as a function of Rayleigh number have been deduced for eight models and five values of volume fraction of nano particles with an accuracy range 6-12 %. The results show that, the average heat transfer rate increases significantly as particle volume fraction and Rayleigh number increase. Also, increase the number of undulations in unsymmetrical annuli reduces the heat transfer rates which remain higher than that in symmetrical annuli. There is no remarkable change in isotherms contour with increase of volume fraction of nanofluid.

Key words: Natural convection, heat transfer, nanofluid, corrugated annulus.

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I. INTRODUCTION

Fluid field and heat transfer characteristics by free convection inside enclosure has been intensive attention due to wide practical applications in industry. Applications include effective cooling system for electronic components, solar collectors, thermal storage system, nuclear reactors, etc. In recent years, there are many attempts from researchers using different methods to enhance the heat transfer rate using of porous media [1-15] and nanotechnology [16-43]. The present study has used the second method. The aim is to obtain new fluid has higher thermal conductivity than normal fluids called 'nanofluid'.

Khalil et al. 2003, [16] analyzed heat transfer performance of nanofluids inside an enclosure and concluded that the nanofluid heat transfer rate enhances as the nanoparticles volume fraction increases. Hakan and Abu-Nada 2008, [17] showed that the natural convection inside a partially heated enclosure using nanofluids is higher at low aspect ratio than at high aspect ratio and the heater location

has a significant effect on the fluid and temperature characteristics. Abu-Nada et al. 2008, [18] concluded that using of water-based nanofluid with different volume fractions of Cu, Ag, Al_2O_3 and TiO_2 nanoparticles in horizontal enclosed annuli has adverse effects on heat transfer characteristics. Nanoparticles with high thermal conductivity cause great enhancement of heat transfer rates at high Rayleigh number. Elif 2009, [19] used water-based five types of nanoparticles: Cu, Ag, CuO, Al_2O_3 , and TiO_2 to study the laminar natural convection heat transfer in an inclined square enclosure. The left vertical wall was heated with a constant heat flux, while the right wall was cooled, and the other sides were kept adiabatic. The results show that the heat transfer rate enhances as the heater length increases. Abu-Nada and Hakan 2009, [20] showed that the effect of solid particle dispersion on heat transfer rate inside two-dimensional inclined enclosure filled with Cu-nanofluid is more pronounced at low volume fraction than at high volume fraction. Ghasemi and Aminossadati 2010, [21] examined the periodic natural convection inside an enclosure filled with

Cu-nanofluids. The left wall was heated by oscillating heat flux while the cold right wall is kept at constant temperature and the other walls were thermally insulated. The results show that utilization Cu nanoparticles, enhances Nusselt number especially at low Rayleigh numbers. **Mostafa Mahmoudi 2011, [22]** concluded that the horizontal positioned heater in a square cavity filled with various water based nanofluids gives higher heat transfer rate than the vertical positioned heater at low Rayleigh numbers while there is no effect for this position on the heat transfer rate at high Rayleigh numbers. **Bararnia et al. 2011, [23]** concluded that heat transfer characteristics and the vortex formation in the square enclosure strongly depend on the Rayleigh number and the position of inner heated elliptic cylinder. **Sheikholeslami et al. 2012, [24]** showed that the enhancement heat transfers by natural convection of Cu–water nanofluid around a hot inner sinusoidal circular cylinder enclosed by cold outer circular enclosure increases as Rayleigh number increases only with the presence of horizontal magnetic field. **Nasrin and Alim 2013, [25]** used water as a working fluid based nanofluid having two nanoparticles alumina and copper to study the free convective flow and fluid fields inside a complicated enclosure. The results show that increasing the Rayleigh and Prandtl numbers enhances the heat transfer at the heated diamond shaped cylinder. **Abouei et al. 2013, [26]** concluded that Nusselt number and the maximum stream functions inside cold circular cylinder contains a heated triangular cylinder increase by augmentation of solid volume fraction. The effect of nanoparticles was more remarkable at low Ra. **Habibi and Pop 2013, [27]** studied the laminar free convection heat transfer inside an eccentric horizontal annulus filled with Copper Cu–water nanofluid. The results show that the eccentricity ε is a good control parameter for both pure and nanofluid filled annulus. **Sourtiji et al. 2015, [28]** concluded that the percentage of heat transfer enhancement caused by Cu–water nanofluid around a heated inner circular cylinder inside a horizontal enclosure is more remarkable at lower Rayleigh numbers.

Ravnik and Škerget 2015, [29] showed that the enhanced thermal properties of Al_2O_3 ; Cu and TiO_2 nanofluids around a heated circular and elliptical cylinder enclosed by an inclined cooled cubic enclosure give highest heat transfer enhancement in the conduction dominated flow regime. **Sourtiji et al. 2015, [30]** noticed that the heat transfer rate inside a horizontal triangular cylindrical annulus filled with of Cu–water nanofluids increases as radius ratio increases at higher values of Rayleigh number and vice versa at lower Rayleigh numbers. **Hatami et al. 2016, [31]** concluded that the shape of the wavy wall of enclosure filled with nanofluids has a significant effect on the heat transfer process. **Hatami and Safari 2016, [32]** found the best location of heated circular cylinder located in a wavy-wall enclosure filled with nanofluid. It was found that concentric annulus enhances the heat transfer rate in both wavy side walls. **Kalidasan and Rajesh 2017, [33]** showed that the strength of the primary vortex inside open square cavity containing vertical diagonal heaters and filled with hybrid nanofluid of nano-diamond-Cobalt Oxide/Water increases as percentage of nano composites increase for all the Rayleigh numbers. **Mehryan et al. 2017, [34]** used Al_2O_3 -Cu water hybrid nanofluid in a

cavity filled with a porous medium. They observed decreasing in heat transfer rate much higher for hybrid nanofluid compared to the single nanofluid and for using nanoparticles in porous media. **Dogonchi et al. 2018, [35]** concluded that reduction the radius of the hot semicircle bottom wall of triangular cavity filled with Cu-water-nanofluid enhances the heat transfer rate at the absence of the magnetic field. **Mabrouk et al. 2018, [36]** used Cu-water and TiO_2 -water nanofluids in a partially heated horizontal cylindrical enclosure to prove that the length of heat source affects strongly the natural convection heat transfer. **Sadia et al. 2018, [37]** showed that the thicknesses of momentum and thermal boundary layers flow of a nanofluid past a sinusoidal semi-infinite vertical surface increase significantly as the amplitude of the wavy surface increases.

Thangavelu et al. 2018, [38] proved that the natural convection heat transfer rate increases with increasing heater length at both vertical and horizontal positions located inside an enclosure filled with nanofluid. Also, they showed that the heat transfer rate for Ag-water nanofluid is higher than CuO-water and Al_2O_3 -water nanofluids. **Aravindhane et al. 2019, [39]** concluded that the heat transfer rate around rectangular cylinder enclosed by an elliptical cylinder filled with CuO-water nanofluid at curved boundary conditions enhances as the solid volume fraction of nanofluid increases. **Ammar et al. 2019, [40]** used annulus with inner hot corrugated cylinder and outer cooled wavy- cylinder filled with two layers. The right and left layers were filled with Ag nanofluid and combined porous media-nanofluid; respectively. They concluded that the strength of the shear layer thickness increases with increasing Rayleigh and Darcy numbers. **Nepal 2019, [41]** found that the heat transfer rate of a nanofluid in a square cylinder enclosed by a wavy wall cylinder enhances with increasing volume fraction of nanoparticles and Rayleigh number.

Abdeslem et al. 2020, [42] noticed that placing the heaters on the left and the right sides of the inner cylinder wall for elliptic annulus filled with a Cu–water nanofluid gives higher heat transfer rates than other locations. **Yu-Peng et al. 2020, [43]** concluded that the radius ratio has a significant effect on the fluid and thermal fields in an annulus, while the volume of nanoparticle has a little effect.

Eight effective models of a symmetrical and unsymmetrical corrugated annulus filled with Al_2O_3 nanofluid have been used to study the free convection heat transfer process resulting from heating inner cylinder. To reach the aim, a mathematical model has been used for two-dimensional dimensionless equations written in the form of stream function-vorticity formulation. Numerical calculations using Galerkin finite element method (FEM) have been carried out to demonstrate the streamlines, isotherms, and the local and average Nusselt number at the inner and outer cylinders for various values of Rayleigh number ($\text{Ra}=10^3$ to 10^7) and volume fraction of nanoparticles ($\phi=0, 0.01, 0.02, 0.03, 0.04, 0.05, 0.1, 0.15, 0.2, 0.25$). As can be seen from the available above literatures, there are no researchers talk about study this phenomenon inside these eight geometries. So, this reason is motivated the present work.

II. THE MATERIAL AND METHOD

A. Mathematical Modeling

The proposed physical eight models of a two-dimension corrugated annulus enclosure filled with Al_2O_3 -water nanofluid incompressible and laminar is shown in Figure 1.A. The study applied for different values of nano-particles volume fraction ($\phi=0, 0.01, 0.02, 0.03, 0.04, 0.05, 0.1, 0.15, 0.2, 0.25$) and Rayleigh number ($Ra=10^3$ to 10^6). The inner cylinder of annulus was assumed to be heated at T_h and the outer cylinder is considered to be cooled at T_c . Figure 1 illustrates the schematic diagram of a corrugated annulus a representative physical domain.

Fig. 1 Geometry configuration of eight models.

The outer and inner corrugation circular cylinder are assumed to have identical centers at the origin of the Cartesian coordinate. the ratio between the diameter of the inner cylinder to the subdivision of the mean radius of the outer cylinder and the inner radius cylinder R_o/R_i is equal to 2.66; therefore, the characteristic length is equal $(R_o - R_i)$. The mathematical function is applied to have a corrugation inner and outer walls on the surface of a circular cylinder as shown below:

For symmetry simulation θ changed from $\frac{\pi}{2}$ to $\frac{3\pi}{2}$

$$x = R + \delta \sin \sin \left(\frac{N \cdot \theta \cdot \pi}{N} \right) \cdot (\cos \left(\theta \cdot \frac{\pi}{180} \right)) \quad (1)$$

$$y = R + \delta \sin \sin \left(\frac{N \cdot \theta \cdot \pi}{N} \right) \cdot (\sin \left(\theta \cdot \frac{\pi}{180} \right)) \quad (2)$$

Where

inner cylinder ($R = R_i$)

outer cylinder ($R = R_o$)

N = number of corrugations

δ = amplitude =1.

θ =angle of circular cylinder (deg)

GOVERNING EQUATION AND FORMULATION

Fig. 1 shows a schematic diagram of corrugated annulus enclosure. The fluid in the enclosure is a water-based nano-fluid containing with different volume fraction of nanoparticles. It is assumed that the base fluid (water) and nano-particles are in thermal equilibrium and no slip accure between them. The thermo-physical properties of the nano-fluid are given as shown in Table I [44]:

TABLE I
THERMAL PHYSICAL PROPERTIES OF BASE FLUID (WATER), NANOPARTICLES AND NANOFLUID [44]

Material	$C_p \left(\frac{J}{kg \cdot K} \right)$	$\rho \left(\frac{kg}{m^3} \right)$	$k \left(\frac{W}{m \cdot K} \right)$	$\beta \left(\frac{1}{K} \right)$
Water	4179	997.1	0.613	$21 \cdot 10^{-5}$
Al_2O_3	765	3970	40	$0.85 \cdot 10^{-5}$

For the nanofluid, the thermo-physical properties must be estimated. In this regard, the density ρ_{nf} , thermal expansion coefficient $(\rho\beta)_{nf}$, and specific heat energy $(\rho C_p)_{nf}$ are estimated as are estimated as follows [44]:

$$\rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s \quad (3)$$

$$\beta_{nf} = (1 - \phi)\beta_f + \phi\beta_s \quad (4)$$

$$(\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \quad (5)$$

Maxwell-Garnetts model was considered to calculate the thermal conductivity of nanofluid [44].

$$k_{nf} = k_f \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \quad (6)$$

the dynamic viscosity of nanofluid represented as Brinkman model [44] as follows:

$$\mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \quad (7)$$

Maxwell-Garnetts model and Brinkman model are used widely in the previous studies with good accurate results to modeling the thermal conductivity and viscosity in nanofluid respectively [45-48]. The following assumption are used in this study: steady state incompressible flow, single

phase, and Boussinesq approximations. the incompressible Navier-Stokes and energy equations in the Cartesian coordinate which are described as [35 and 49]:
Continuity equation

$$u_x + v_y = 0 \quad (8)$$

Momentum equations

$$\rho_{nf}(u.u_x + v.u_y) = -p_x + \mu_{nf}(u_{xx} + u_{yy}) \quad (9)$$

$$\rho_{nf}(u.v_x + v.v_y) = -p_y + \mu_{nf}(v_{xx} + v_{yy}) + \rho_{nf}\beta_{nf}g(T - T_c) \quad (10)$$

Energy equation

$$\rho_{nf}(u.T_x + v.T_y) = k_{nf}(T_{xx} + T_{yy}) \quad (11)$$

the stream function ψ and vorticity ω are define as:

$$u = \psi_y, \quad v = -\psi_x, \quad \omega = v_x - u_y \quad (12)$$

The dimensionless variables are defined as below:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad \Omega = \frac{\omega L^2}{\alpha_f}, \quad \Psi = \frac{\psi}{\alpha_f},$$

$$U = \frac{uL}{\alpha_f}, \quad V = \frac{vL}{\alpha_f}, \quad \Theta = \frac{T - T_c}{T_h - T_c}$$

By incorporating above equations, the dimensionless governing equations will become as:

$$\Psi_{XX} + \Psi_{YY} = -\Omega \quad (13)$$

$$\Psi_Y \Omega_X - \Psi_X \Omega_Y = \frac{\mu_{nf}}{\rho_{nf}} \cdot Pr \cdot Pr (\Omega_{XX} + \Omega_{YY}) + \frac{\beta_{nf}}{\beta_f} \cdot Ra \cdot Pr \cdot \Theta_X \quad (14)$$

$$\Psi_Y \Theta_X - \Psi_X \Theta_Y = \frac{\frac{k_{nf}}{k_f}}{\frac{(\rho C_p)_{nf}}{(\rho C_p)_f}} \cdot (\Theta_{XX} + \Theta_{YY}) \quad (15)$$

The dimensionless boundary conditions and parameters described as:

$\Theta = 0$ On the outer circular cylinder (cold cylinder)

$\Theta = 1$ On the inner circular cylinder (hot cylinder)

$\Psi = 0$ On the walls

$$Pr = \frac{\nu_f}{\alpha_f} \quad (16)$$

$$Ra = \frac{g\beta_f(T_h - T_c)L^3}{(\alpha_f \nu_f)} \quad (17)$$

the local and average Nusselt numbers evaluated at the hot and cold cylinder have been present below:

$$Nu_{loc} = \frac{k_{nf}}{k_f} \Theta_n \quad (18)$$

$$Nu_{ave} = \frac{1}{S} \int_0^S Nu_{loc} ds \quad (19)$$

where n is normal direction to the wall and S is the wall length.

B. Grid Generation and Coordinate Transformation

Grid Generation is decisive for time, cost, and results quality in any case study. Therefore, to transfer the physical domain into computational domain has a great influence on the solution. Elliptical Partial differential equations method is the most general, applicable and programmable method. This method produce grids with smoothly varying cell sizes and slopes of the grid lines. Furthermore, to control the orthogonality and the spacing near boundaries. Poisson

equation type of generating system was used in this study. The transformation function $\xi = \xi(X, Y), \eta = \eta(X, Y)$ [50]:

$$\xi_{XX} + \xi_{YY} = P(X, Y) \quad (20)$$

$$\eta_{XX} + \eta_{YY} = Q(X, Y) \quad (21)$$

Where $P(X, Y)$ and $Q(X, Y)$ are two arbitrary function specified to adjust the local density of the grids. FORTRAN code was built to solve above equations by iterative method finite difference discretization. Firstly, simple algebraic grid generation method is used to generate an initial grid in the interior domain. Secondly, starting solution for the iteration process by the initial grid. Where the final grid will be independent of the initial grid. finally, solve the Poisson grid generation equations iteratively by iteration method. The boundary grid points defines as a Dirichlet boundary conditions. In general, a sufficiently converged grid is obtained more than 100 iterations. Figure 2 shows the final grid generation shape from FORTRAN code.

Fig 2 Mesh geometry of eight models

Then, the set of equations (13-15) can be written in transformation form as follows:

$$A \cdot \Psi_{\xi\xi} + 2B \cdot \Psi_{\xi\eta} + C \cdot \Psi_{\eta\eta} + D \cdot \Psi_{\xi} + E \cdot \Psi_{\eta} = -J \cdot \Omega \quad (22)$$

$$\begin{aligned} & \Psi_{\eta} \cdot \Omega_{\xi} - \Psi_{\xi} \cdot \Omega_{\eta} = \\ & \frac{\mu_{nf}}{\rho_{nf}} \cdot Ra \cdot Pr \left(A \cdot \Omega_{\xi\xi} + 2B \cdot \Omega_{\xi\eta} + C \cdot \Omega_{\eta\eta} + D \cdot \Omega_{\eta} \right. \\ & \left. + E \cdot \Omega_{\xi} \right) + \frac{\beta_{nf}}{\beta_f} \cdot Ra \cdot Pr \cdot [Y_{\eta} \cdot \Theta_{\xi} - Y_{\xi} \cdot \Theta_{\eta}] \end{aligned} \quad (23)$$

$$\begin{aligned} \Psi_{\eta} \cdot \Theta_{\xi} - \Psi_{\xi} \cdot \Theta_{\eta} = & \frac{\frac{k_{nf}}{k_f}}{\frac{(\rho C_p)_{nf}}{(\rho C_p)_f}} \cdot (A \cdot \Theta_{\xi\xi} + 2B \cdot \Theta_{\xi\eta} + C \cdot \Theta_{\eta\eta} \\ & + D \cdot \Theta_{\eta} + E \cdot \Theta_{\xi}) \end{aligned} \quad (24)$$

Where

$$\begin{aligned} \Gamma &= X_{\eta}^2 + Y_{\eta}^2, \quad \gamma = X_{\xi}^2 + Y_{\xi}^2, \quad J = X_{\xi} \cdot Y_{\eta} - Y_{\xi} \cdot X_{\eta}, \\ \sigma &= X_{\xi} \cdot X_{\eta} + Y_{\xi} \cdot Y_{\eta}, \quad A = \frac{\Gamma}{J}, \quad B = \frac{\sigma}{J}, \quad C = \frac{\gamma}{J}, \\ D &= B_{\xi} + C_{\eta}, \quad E = A_{\xi} + B_{\eta} \end{aligned}$$

The boundary conditions will become:

$\Theta|_{\eta=1} = 0$ On the outer circular cylinder (cold cylinder)

$\Theta|_{\eta=0} = 1$ On the inner circular cylinder (hot cylinder)

$\Theta_{\eta}|_{\xi=0,1} = 0$ On the symmetry line

$\Psi|_{\eta=0,1} = 0$ On the walls

$$Nu_{loc} = \frac{-k_{nf}}{k_f} \frac{1}{\sqrt{r}} (\delta \cdot \Theta_{\xi} - \sigma \cdot \Theta_{\eta}) \quad (25)$$

$$Nu_{ave} = \frac{1}{S} \int_0^S Nu_{loc} d\xi \quad (26)$$

The above numerical integral was calculated using Simpson's rule 1/3 method.

C. Grids influence and Validation Tests

Table II demonstrates the influence of grids number for a test case of fluid confined within the present configuration at $Ra=10^6$, $AR=2.66$, and $\phi=0.1$. The results included the Nu_{ave} , and Ψ_{max} , Ψ_{min} on the inner cylinder and the enclosure with grid number respectively. it is clear that, the grid system of 41×41 is fine enough to obtain accurate results. Therefore, adopted a grid system of 41×41 with symmetric geometry.

The numerical solution of governing equations was carried out using a CFD code written in the FORTRAN programming language. To verify the accuracy of the present physical model, validation case is presented. The validation case has been compared with the works of **kim et al. [51]** and **Moukalled [52]** as shown in Table III. It is noticed that the values of average Nusselt number at constant value of Rayleigh number are close together with a very little difference range from 0.2 % to 1.7 %.

Also, the streamlines and isotherms are presented as shown in Fig. 3 for case study of [51], where the present results are in good agreement with results of [51] as shown in Fig. 3, therefore the numerical procedure is reliable and efficient.

TABLE II VARIATION OF AVERAGE NUSSLETT NUMBER, MINIMUM AND MAXIMUM STREAM FUNCTIONS FOR DIFFERENT GRIDS NUMBER AT $RA=10^6$, $\phi=0.1$

	Grids	21 × 21	31 × 31	41 × 41	51 × 51
Model-1	Nu_{ave}	5.202213	5.504034	5.604844	5.604850
	Ψ_{max}, Ψ_{min}	0.883, -20.54	0.933, -22.66	0.991, -23.71	0.990, -23.72
Model-2	Nu_{ave}	5.207575	5.400876	5.527194	5.527199
	Ψ_{max}, Ψ_{min}	0.01, -23.44	0.002, -25.88	0, -26.21	0, -26.20
Model-3	Nu_{ave}	5.098781	5.175389	5.200673	5.200680
	Ψ_{max}, Ψ_{min}	0.02, -26.01	0.001, -27.23	0, -28.19	0, -28.21
Model-4	Nu_{ave}	4.977645	5.149740	5.271204	5.271208
	Ψ_{max}, Ψ_{min}	0.01, -21.11	0.004, -26.21	0, -25.77	0, -25.76
Model-5	Nu_{ave}	5.276382	6.034176	6.154805	6.154808
	Ψ_{max}, Ψ_{min}	0.02, -25.32	0.01, -29.01	0, -30.15	0, -30.14
Model-6	Nu_{ave}	5.25436	6.006832	6.177319	6.177322
	Ψ_{max}, Ψ_{min}	0.01, -23.08	0.01, -24.55	0, -26.61	0, -26.62
Model-7	Nu_{ave}	7.165473	6.472319	6.105654	6.105656
	Ψ_{max}, Ψ_{min}	0.01, -31.13	0.01, -29.14	0, -28.10	0, -28.11
Model-8	Nu_{ave}	6.987235	6.353871	6.163538	6.163539
	Ψ_{max}, Ψ_{min}	0.01, -30.31	0.01, -26.77	0, -25.12	0, -25.11

TABLE III: COMPARISON OF THE PRESENT WORK WITH WORKS OF KIM ET AL [29] AND MOUKALLED [30].

Ra	Mean Nusselt number at hot wall				
	Present study	Kim et al. [29]	Moukalled [30]	Error with [29] %	Error with [30]. %
10^4	3.387476	3.414	3.331	-0.783	1.667206
10^5	5.164313	5.1385	5.08	0.499831	1.632606
10^6	9.355465	9.39	9.374	-0.36914	-0.19812
10^7	15.70126	15.665	15.79	0.230914	-0.5652

Fig. 3 Comparison numerical results with [29] by Isotherms and streamlines in the enclosure, $Pr=0.7$.

In order to more verify the accuracy of the present numerical code, a second validation between present results and the provided data by [53] is carried out as shown in Figure 4.

which shows the effect of Al_2O_3 -water nanofluid on heat transfer by free convection inside enclosures at $Ra=10^5$ and $\phi=1\%$, It can be seen that there is close similarity between the patterns of streamlines and isotherms.

$$\Psi_{min_{pur\ Fluid}} = -8.58, \Psi_{min_{nanofluid}} = -11.35$$

$$\Psi_{min_{pur\ Fluid}} = -11.86, \Psi_{min_{nanofluid}} = -15.8$$

[ref]

$$\Psi_{min_{pur\ Fluid}} = -8.89, \Psi_{min_{nanofluid}} = -11.85$$

$$\Psi_{min_{pur\ Fluid}} = -11.93, \Psi_{min_{nanofluid}} = -15.62$$

Present results

Fig 4 Comparison results of second validation test by streamlines and isotherms for $Ra=105$, $\phi=0.1$ and different geometry configuration.

D. Numerical Solution

The governing equation in the curvilinear coordinates (equations 22, 23, and 24) as well as boundary conditions were discretized by finite difference method. Central difference and upwind difference approximation are used for partial derivatives and convective terms respectively. Three point forward or backward difference formula to evaluate the derivative at the boundary. The explicit method is applied for flow and energy fields, while the stream function has been calculated by the successive over-relaxation (SOR) method with tolerance 10^{-6} . It is necessary to observe the variation of the local Nusselt number on the corrugate walls to show nanoparticle effect on heat transfer rate.

III. RESULTS AND DISCUSSION

A. Streamlines and isotherms

The effect of the volume fraction of nanoparticles ϕ and Rayleigh number Ra on the streamlines and isotherms of eight models is shown in Figs. 5 and 6, respectively. Generally, the intensity of streamlines increases with

increasing of the nanoparticles volume fraction. The streamlines are symmetric about the vertical centerline of the enclosure, so we can take one half only. In model one $N_i = N_o = 3$ (symmetric annulus), in each half of the annular gap, there are two vortices. The major vortex locates on the left and right upper undulation of inner cylinder while the other is a minor vortex which locates near the bottom undulation of inner cylinder. There are no vortices at the bottom region of annulus enclosure. The major vortex consists of two eddies emerge with each other as Rayleigh number increases. Increasing the volume fraction of nanoparticles leads to separate this eddy again. The minor vortex has only one eddy emerge also with major vortex as Rayleigh number increases. The left major vortex has positive stream function, but the right has negative value. The physical behavior of the flow motion can be explained as follows: the hot fluid moves upward near the wall of inner cylinder and then moves downward near the inner vertical wall of outer cylinder. As a result, it generates two flow directions of the fluid flow, counterclockwise in the left side and clockwise in the right side. In addition, adding the nanoparticles to the base fluid

causes increasing the thermal conductivity and the density of the nanofluid. So, the streamlines increase and the isotherms seem to be more smoothing as the volume fraction of nanoparticles increase. The figures show also that the streamlines and isotherms change with changing the number of undulations (N_i & N_o) of inner and outer cylinder. It is noticed that the intensity of streamlines decreases as the number of undulations the number of undulations increases and the temperature distributed greatly towards the outer cylinder. In model two $N_i = N_o = 4$ (symmetric annulus), in each half of the annular gap, there are three major vortices, two of them located near the sides of upper and lower undulations and one locates at the region bonded between the horizontal undulations of annular gap. These vortices emerge with each other as Rayleigh number increases. In model three $N_i = N_o = 6$ (symmetric annulus), in each half of the annular gap, there are one major vortex located at the middle of annulus and two minor vortices located at the upper and lower parts of annulus. In model four $N_i = N_o = 8$ (symmetric annulus), in each half of the annular gap, there is one only major vortex located at the middle of annulus. It creeps towards upward of annular gap as Rayleigh number increases. In model five $N_i = 3$ & $N_o = 6$ (asymmetric annulus), in each half of the annular gap, there are main major strong vortex located at the first quarter of annulus and mini vortex down the horizontal axis of annulus emerges with main vortex as Rayleigh number increases.

In model six $N_i = 6$ & $N_o = 3$ (asymmetric annulus), in each half of the annular gap, there are two main major vortices which one located at the first quarter of annulus and the other penetrates the horizontal axis of annulus. They emerge with each other as Rayleigh number increases. They emerge with each other as Rayleigh number increases.

In model seven $N_i = 4$ & $N_o = 8$ (asymmetric annulus), in each half of the annular gap, there is one mini vortices penetrates the horizontal axis of annulus and two major vortices located at above and below the horizontal axis of annulus. In model eight $N_i = 8$ & $N_o = 4$ (unsymmetrical annulus), in each half of the annular gap, there is one major vortex penetrates the horizontal axis of annulus and two minor vortices located above and below the horizontal axis of annulus. The major vortex and the minor down vortex emerge with each other as Rayleigh increases. The streamlines and isotherms are strongly affected by Rayleigh number. The magnitude of streamlines increases significantly as Rayleigh number increases. Where the conduction heat transfer regime is dominated, the isotherms arranges in a shape similar to the shape of enclosure walls for low Rayleigh number. This behavior of isotherms contour diffuses as Rayleigh number increase to 10^6 and it seems to be undulant because of the higher thermal convection of the warmer fluids. There is no remarkable change in isotherms contour with increase of volume fraction of nanofluid.

Fig 5 Streamlines contour for eight models at different Rayleigh numbers and volume fraction of nanofluid $\phi = 0.1$ and 0.25 .

Fig 6 Isotherms contour for eight models at $Ra=10^5$ and 10^6 and volume fraction of nanofluid $\phi=0.1$ and 0.25

B. Local Nusselt Number

The variation of local Nusselt number around the inner and outer cylinders from $\theta=0^\circ$ to $\theta = 180^\circ$ for model 1 and 6 at $Ra=10^5$ and 10^6 with different volume fraction of nanoparticles ($\phi = 0, 0.1$ and 0.25) are shown in Fig. 7 and Fig. 8; respectively. It is noticed from Fig. 7 that the number of peaks of maximum Nusselt number is the same as the number of undulations ($N_i = 3$ for model 1 and 6 for model 6). While on the outer cylinder, the maximum Nusselt number occurs downstream as shown in Fig. 8. It is shown also from two figures that the local Nusselt number increases with increase Rayleigh number and volume fraction of nanoparticles due to increase the heat exchange between the

solid particles and based fluid in addition to increasing of vortex intensity and buoyancy force. It is known that the increasing of thermal conductivity of the nanofluid ($\phi = 0.1$ and 0.25) which is higher than the base fluid ($\phi = 0.0$), leads to transport more heat to the cold outer cylinder compared to the hot inner cylinder. Besides, the induced buoyancy force displaces hot fluid towards upward near the inner cylinder while the colder fluid particles go downward near the outer cylinder walls. It is concluded also the behavior of heat transfer on the inner surface of outer cylinder at constant number of undulations ($N_o = 3$) is not affected much with increase number of inner cylinder undulations (from N_i 3 to 6). While this increasing causes rise of heat transfer rate on the outer surface of inner cylinder.

Fig 7 Local Nusselt number of inner cylinder for two models 1 and 6 at $Ra=10^5$ and 10^6 with different volume fractions of nanofluid

Fig 8 Local Nusselt number of outer cylinder for two models 1 and 6 at $Ra=10^5$ and 10^6 with different volume fractions of nanofluid

C. Average Nusselt number

The variations of the average Nusselt number at the inner cylinder versus the volume fraction of nanoparticles (ϕ) at different Rayleigh numbers for eight models are shown in Fig. 9. It is noticed that the Nusselt number increases linearly with increase the volume fraction of nanoparticles because of increase the heat capacity and the thermal conductivity of the nanofluid with increase the volume fractions of nanoparticles. It can be seen that the heat transfer rates increase as the Rayleigh number increases. For higher Rayleigh numbers, the stronger buoyancy driven flow makes the fluid particles transport heat more rapidly to the boundary surfaces. As a result, the Nusselt number accelerates at the inner and outer cylinder surfaces as Rayleigh number increases.

Hence, we want to know which models give higher heat transfer rates. Fig. 10 shows the variation of average Nusselt number in the inner and outer cylinders against Rayleigh number for eight models at $\phi=0.01, 0.1$, and 0.25 . It is shown that, there is a slight effect for increasing Rayleigh number from 10^4 to 10^5 on the heat transfer process. Beyond $Ra=10^5$ the average Nusselt number increases with increase Rayleigh number. From the available data obtained from the present work and as shown in Figure 8, the models that give higher heat transfer rates can be sequenced as follows: for inner cylinder (5, 6, 7, 8, 2, 1, 3, 4); for outer cylinder (6, 8, 7, 5, 2, 1, 3, 4). It is concluded that the unsymmetrical models (5, 6, 7, 8) give higher heat transfer rates than symmetric models (2, 1, 3, 4) and the model 4 gives lower rate of heat transfer than other models. Model 5 ($N_i = 3, N_o = 6$) has one a great main vortex with an intensity increases as a volume fraction of

nanoparticles increases. The wide enclosure space of model 5 relatively compared with other models makes the eddies grew easily inside it and get bigger then merge with each other to form this great main vortex at the upper part of enclosure. This leads to increasing the free convection currents in this region. Increasing the number of undulations in unsymmetrical annuli weakens the heat transfer process which remain relatively higher than that rates in the symmetrical annuli. Model 4 is the only model that gives a wavy vortex because of high number of undulations of inner and outer wavy cylinders ($N_i = N_o = 8$). This distortion in vortex weakens it and makes the convection currents moving in undulation paths cause lower heat transfer rates.

D. Average Nusselt number Correlations

The variations of logarithmic average Nusselt number for the inner and outer wavy cylinders with variations of logarithmic Rayleigh number are plotted at $\phi=0.01, 0.1, 0.15$, and 0.25 . Fig. 11 shows some of these results only at $\phi=0.01, 0.1$ and 0.25 ; respectively. This figure gives more markedly insight to indicate the preference for the heat transfer coefficient of the eight models. The range of Rayleigh number extends from 10^4 to 10^7 . From these subfigures related to Fig. 11, sixty four correlations for the average Nusselt number as a function Rayleigh number have been deduced to give the general following equation:

$$Nu = c Ra^n \quad (27)$$

Where c and n are constants given in Table IV.

Fig 9 Average Nusselt number of inner cylinder versus volume

Fig 10 Average Nusselt number versus Rayleigh number for eight models at different volume fractions.

Fig 11 Logarithmic average Nusselt number for inner and outer cylinder versus logarithmic Rayleigh number for eight models at different volume

TABLE IV CONSTANT VALUES OF EQ. (23) FOR INNER AND OUTER CYLINDER OF EIGHT ANNULUS MODELS.

model	Inner cylinder		Outer cylinder		Inner cylinder		Outer cylinder	
	$\phi = 0.01$				$\phi = 0.1$			
	c	n	c	n	c	n	c	n
1	1.2045	0.0992	0.9816	0.0759	1.0058	0.0969	0.8858	0.072
2	1.16707	0.1048	0.9788	0.0831	0.9603	0.1019	0.8663	0.0787
3	1.0944	0.1133	0.9411	0.0887	0.8782	0.1104	0.8163	0.0848
4	1.0945	0.1129	0.9552	0.0844	0.8831	0.1088	0.8339	0.0822
5	1.1632	0.1213	0.9891	0.0748	0.9141	0.1175	0.8879	0.0728
6	1.1407	0.1254	1.0319	0.0677	0.8815	0.1225	0.9491	0.065
7	1.1402	0.1226	0.9722	0.086	0.8887	0.1197	0.8504	0.0823
8	1.1127	0.1262	1.0098	0.0762	0.8589	0.1231	0.9103	0.0724
	$\phi = 0.15$				$\phi = 0.25$			
1	1.0761	0.0945	0.9473	0.0697	1.2374	0.0880	1.0851	0.0644
2	1.0293	0.099	0.9276	0.0762	1.1892	0.0915	1.0648	0.0702
3	0.9404	0.1076	0.8737	0.082	1.0841	0.1004	1.0015	0.0768
4	0.9466	0.1059	0.8911	0.0802	1.0915	0.0988	1.0202	0.0749
5	0.9783	0.1149	0.9480	0.0708	1.0937	0.1104	1.0426	0.0743
6	0.9446	0.1196	1.0136	0.0629	1.0840	0.1130	1.1639	0.0572
7	0.9508	0.1172	0.9097	0.080	1.1047	0.1194	1.0427	0.0743
8	0.9207	0.1201	0.9739	0.070	1.0613	0.1129	1.1166	0.0644

VII. CONCLUSION

The present work displays numerical study of natural convection heat transfer of water based Al_2O_3 nano-fluids inside a symmetrical and unsymmetrical corrugated annulus enclosure at different values of volume fraction of nanoparticles and Rayleigh number. In view of the obtained results, the following conclusions may be summarized:

1. Adding the nanfluid particles to the base fluid enhances the flow strength and heat transfer rate.
2. The heat transfer process enhances as Rayleigh number and volume fraction of nanoparticles increase.
3. Increasing the number of undulations in unsymmetrical annuli generates wavy vortex weakens the heat transfer process which remain

relatively higher than that rates in the symmetrical annuli.

4. Model 5 which has number of undulations in the inner and outer cylinder $m_i=3$ and $m_o=6$ gives higher heat transfer rates than other models.
5. Model 4 which has equal number of undulations in the inner and outer cylinder $m_i=m_o=8$ gives lower heat transfer rates than other models.
6. Adding the nanfluid particles to the base fluid enhances the flow stream and heat transfer rate.
7. At low Rayleigh number, the conduction heat transfer regime is dominated and the isotherms arrange in a shape similar to the shape of enclosure walls. It distorted as Rayleigh number increases because of dominated natural convection.
8. There is no remarkable change in isotherms contour with increase of volume fraction of nanofluid.

NOMENCLATURE

A, B, C, D, E	Transformation parameters
C_p	Specific heat [kJ/kg-K]
g	Gravitational acceleration [m/s ²]
k	Thermal conductivity [W/m.K]
p	Pressure [Pa]
PHI	Nanofluid volume fraction
Pr	Prandtl number [-]
R	Radius cylinder [m]
Ra	Rayleigh number [-]
T	Temperature [K]
ω	Vorticity [s ⁻¹]
L	characteristic length ($R_o - R_i$) [m]
N	Number of corrugations [-]
Nu_{loc}	Local Nusselt number
Nu_{ave}	Average Nusselt number
J	Jacobian []
u, v	The components of dimension and
U, V	dimensionless velocities in space coordinates [m/s], [-]
x, y	Dimension and dimensionless in space
X, Y	coordinates [m], [-]

Greek Symbols

α	Diffusivity of heat (m ² /s)
ρ	Density [kg/m ³]
Γ, γ, σ	Transformation functions
δ	Amplitude [m]
θ	Angle of circular cylinder [deg.]
β	coefficient of thermal expansion (K ⁻¹)
μ	Dynamic viscosity [Pa.s]
ν	kinematic viscosity [m ² /s]
ϕ	Nanofluid volume fraction
ψ	Dimensional stream function
Ψ	Dimensionless stream function
Ω	Dimensionless vorticity [s ⁻¹]
θ	Dimensionless temperature.
ξ, η	Curvilinear coordinates (Transformation coordinates)

Subscript

t

c, h	Cold and hot cylinder
i, o	Inner and outer cylinder
f	Pure Fluid
s	Solid
nf	Nanofluid
x, y	First derivative respect to
X, Y, ξ, η	x, y, X, Y, ξ, η
xx, yy	Second derivative respect to xx, yy
XX, YY	$XX, YY, \xi\xi, \eta\eta, \xi\eta$
$\xi\xi, \eta\eta, \xi\eta$	

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Authors' contributions

The idea of research was by Israa Alesbe. The theoretical work was done by Sattar Aljobair. Writing of research and discussion of results were done by Akeel Abdullah Mohamed and Israa Alesbe.

Competing interests

The authors declare that they have no competing interests.

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Figures

Figure 1

Geometry configuration of eight models.

Figure 2

Mesh geometry of eight models.

Figure 3

Comparison numerical results with [29] by Isothermals and streamlines in the enclosure, $Pr=0.7$.

$$\Psi_{min_{pur\ Fluid}} = -8.58, \Psi_{min_{nanofluid}} = -11.35$$

$$\Psi_{min_{pur\ Fluid}} = -11.86, \Psi_{min_{nanofluid}} = -15.8$$

[ref]

$$\Psi_{min_{pur\ Fluid}} = -8.89, \Psi_{min_{nanofluid}} = -11.85$$

$$\Psi_{min_{pur\ Fluid}} = -11.93, \Psi_{min_{nanofluid}} = -15.62$$

Present results

Figure 4

Comparison results of second validation test by streamlines and isotherms for $Ra=105$, $\phi=0.1$ and different geometry configuration.

Figure 5

Streamlines contour for eight models at different Rayleigh numbers and volume fraction of nanofluid $\phi=0.1$ and 0.25 .

Figure 6

Isotherms contour for eight models at Ra=105 and 106 and volume fraction of nanofluid $\phi=0.1$ and 0.25.

Figure 7

Local Nusselt number of inner cylinder for two models 1 and 6 at $Ra=10^5$ and 10^6 with different volume fractions of nanofluid.

Figure 8

Local Nusselt number of outer cylinder for two models 1 and 6 at Ra=105 and 106 with different volume fractions of nanofluid.

Figure 9

Average Nusselt number of inner cylinder versus volume fraction of nanofluid for eight models at different Rayleigh numbers.

Figure 10

Logarithmic average Nusselt number for inner and outer cylinder versus logarithmic Rayleigh number for eight models at different volume fractions ($\phi=0.01,0.1,0.25$) fraction of nanofluid for eight models at different Rayleigh numbers.

Figure 11

Average Nusselt number versus Rayleigh number for eight models at different volume fractions.

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