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Research Article

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Posted Date: April 9th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-396288/v1>

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Identification of aquifer pollution's point sources with the Reciprocity Principle

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Received: date / Accepted: date

Abstract The principle of reciprocity, called Maxwell-Betti theorem, initially used in mechanics in an elastic structure, establishes a relation of equality between two distinct strains under different loads. In this paper, we extend and apply this principle to flow and solute transport equations in porous media, in order to perform the pollution sources identification in aquifers.

We developed general 2D expressions of the reciprocity principle for transient transport problems. This model leads to a linear equations set, with point sources coordinates, concentrations and associated water fluxes as unknowns

The proposed model is then applied to the Rocky Mountain Arsenal aquifer [16], where polluted water is injected into a well in the domain. The proposed inverse technique successfully recovered the position and the pollutant concentration in addition to the associated water flux.

In addition, we developed and implemented the inverse method for different knowledge levels of the degrees of the aquifer contamination, i.e. more or less data available in the field. Multiple pollution point sources and noisy data situations are also developed and tested with high efficiency.

The proposed method would be easy and useful to be implemented in the modeling software now widely used by researchers and groundwater managers. It can thus be applied in real case studies, to help authorities and regulators to efficiently identify the polluters and the contamination process,

i.e. its location, onset, duration and the associated mass and water fluxes.

Keywords Hydrogeology · Advection-diffusion equation · Inverse problem · Reciprocity principle · Pollution point sources recovery · Groundwater quality, · Rocky Mountain Arsenal aquifer.

1 INTRODUCTION

Nowadays, the industrial development and the intensification of the agricultural activities, introduce continuously in the environment new molecules, more or less carefully produced and used. This requires the utmost vigilance on behalf of the sanitary and water management authorities [18,25, 28]. Indeed, a major part of the world population relies, to some extent, on groundwater for drinking, and for crops and food production. The protection of these resources should thus be of a great concern.

For many years, it was believed that the layers of soil and sediment above an aquifer act as a natural filter that retains pollutants and thus protects the groundwater. However, it has been widely recognized that the capacity of these soil layers to retain pollutants can be exceeded very quickly [27]. In addition, since an aquifer is polluted, it may become unusable for decades. The remediation of contaminated groundwater is inherently complex and expensive and can require long periods of time and sometimes centuries [20].

Actually, decontamination of polluted groundwater is a huge challenge in Hydrogeology, in relation with the environmental and health requirements. A main challenge in any rehabilitation action is the evaluation of the degree of contamination. This includes the identification of unknown sources of contamination and the corresponding water and solute fluxes that led to the present state of pollution.

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This process is important for both understanding the implementation of adequate remedial measures and for the identification of causes and responsibilities. In fact, the identification of the location and the level of pollution sources is crucial for the application of the polluter-pay principle adopted by the United Nation Conference of Rio de Janeiro Environment and Development declaration in its article 16 [30]. In addition, the contaminant source identification could also be a means of dissuading potential infringements of laws for pollutant discharge and waste repository managements.

In this context, many works have considered this inverse problem in hydrogeology. Most of the existing studies concerns the recovering of the points-sources locations and/or the contaminant release histories [3, 19] and/or the number of these points-sources [5].

Atmadja and Bagtzoglou [4] presented a review on mathematical methods that have been developed for the study of identifying sources of contamination. Authors classified these methods as deterministic or stochastic and associated to an optimization model [2, 19, 22–24, 12, 17, 26]. Other heuristic approaches based on genetic algorithm are proposed in [2, 5].

In this paper, we introduce a new method based on the reciprocity principle, that allows the simultaneous identification of pollution point-sources locations and their pollutants concentrations, from the concentration's measurements in the aquifer domain. This principle, also known as the Maxwell-Betti theorem [8], was first introduced in mechanics for linear problems [1]. It stipulates that for a linear elastic structure subject to two forces F and G , the work resulting from the application of the force F on the displacement field, yielded by the force G , is equal to the work resulting from the application of the force G on the displacement field yielded by the force F .

From a phenomenological perspective, this principle establishes strong relationships between different sets of forces and the consequent displacements applied to a given structure.

In mechanics, the reciprocity principle is usually applied to obtain displacements due to complex forces by using proxy problems with simpler forces that are more easily solved. Within the framework of groundwater flow scenario, reciprocity between two interference pumping tests was analyzed by Bruggeman [9] for Darcian flows in an unbounded, heterogeneous porous medium. Hariga and al [13–15, 7] have also applied the reciprocity principle in groundwater flows by using sources and boundary conditions as forcing terms and the resulting head field as a consequence.

In this research, the reciprocity principle is applied to the transport equation to recover the features of the pollutant point sources in aquifers. We show that this method allows to evaluate the position, solutes concentration and the associated injected water flux, for point solutes sources, from the

knowledge of the cumulative mass flux through the boundary at any time in the considered interval and the concentrations on all the domain at the given time. The proposed method can then be considered as a "direct" one according to the Neuman classification [21].

Obviously, in the real-world scenarios of contamination problems, since the concentrations are measured in a finite number of points, these should first be interpolated throughout the considered domain.

We show that the accuracy of the present identification method, depends on the number of available data. However, reasonable results are also obtained with few data which is a valuable insight to guide managers in the process of pollution sources identification.

The remainder of this paper is structured as follows. After a general review of the reciprocity principle (section 2.1), we derive it for the advection-diffusion equation with a constant injection during a given time (section 2.2). We then illustrate it in point-source pollution identification (section 3) for four scenarios: with complete data, with few observations, with noisy data and with multiply point sources.

2 THE RECIPROCITY PRINCIPLE APPLIED TO THE ADVECTION-DIFFUSION EQUATION

2.1 General formulation and interpretation of the reciprocity principle

Built on the Maxwell-Betti principle [8], for the sake of simplicity we express the reciprocity principle in a general mathematical framework for linear elliptic problems. Let \mathcal{V} be a Hilbert space associated to a domain Ω , α a bilinear form on \mathcal{V} , assumed to be symmetric, continuous and coercive and l_i a linear form defined on \mathcal{V} for $i = 1, 2$ assumed to be continuous. Then we define the following variational problem :

$$\text{Find } u_i \text{ on } \mathcal{V} \text{ such that } \alpha(u_i, \varphi) = l_i(\varphi) \text{ for all } \varphi \in \mathcal{V}. \quad (1)$$

With (u_i, φ) successively equal to (u_1, u_2) and (u_2, u_1) and using the symmetry of operator α , the reciprocity principle can be expressed by the identity:

$$l_1(u_2) = l_2(u_1) \quad (2)$$

The reciprocity principle is fundamentally derived from the symmetry and the bi-linearity of the form α . From a physical perspective, it relates the responses to different external and internal forcing terms (source/sink terms, boundary conditions) of a given phenomenon on a fixed structure represented by form α . Finally, the reciprocity principle is similar

to Green's second identity giving way to the boundary element methods.

The crucial point of the method is the relevant choice of the test functions. The test functions should ideally be closely related to the initial problem but should also lead to much simpler and, if possible, analytical solutions.

2.2 The transport and flow equations in porous media

The advection-diffusion equation governing solute transport in a domain over a time interval $[t_0, t_f]$ is:

$$\begin{cases} \omega \frac{\partial C}{\partial t} - \text{div}(\overline{\overline{D}} \text{grad} C - VC) = Q & \text{in } \Omega \times [t_0, t_f], \\ \overline{\overline{D}} \text{grad} C - VC \cdot n & = \Phi_N \text{ on } \Gamma_N \times [t_0, t_f], \\ C & = C_D \text{ on } \Gamma_D \times [t_0, t_f], \\ C(x, y, t_0) & = C_0 \text{ in } \Omega. \end{cases} \quad (3)$$

with $C(x, y, t)$, the solute concentration $[ML^{-3}]$; ω the aquifer porosity; $\overline{\overline{D}}$ the hydrodynamic diffusion-dispersion tensor $[L^2T^{-1}]$; V the Darcy velocity $[LT^{-1}]$; C_D the prescribed concentration at the Dirichlet boundary Γ_D ; Φ_N the prescribed flux at the remaining Neuman boundary Γ_N ; C_0 the initial concentration distribution and Q is the source term.

In this work, we consider the case of point-sources pollutant injected during a finite time, so that Q is expressed by the following equation:

$$Q = \sum_{i=1}^{i=N_p} Q_{si} C_{si} \delta(x - S_i) \Pi\left(\frac{t_i - \frac{l_i}{2} - t}{l_i}\right) \quad (4)$$

Where N_p is the number of pollutant point-sources; Q_{si} the fluid volume flux rate at the i^{th} point-source $[T^{-1}]$; C_{si} the pollutant concentration of the injected water at the i^{th} point-source $[ML^{-3}]$ which starts at instant t_i and stops at $t_i + l_i$; so l_i represents the time during which the pollutant is injected in the aquifer. $\delta(x - S_i)$ is the Dirac function which is non zero only in the point-source located at $S_i = (x_i, y_i)$ and Π is a rectangular function defined as:

$$\Pi(t) = H(t + 1) - H(t - 1) \quad (5)$$

with H the Heaviside step function.

The transport problem (3) is coupled with the groundwater flow model via the Darcy velocity:

$$V = -T \text{grad}(h) \quad (6)$$

In a stationary 2D case, the hydraulic head h is solution of the following problem:

$$\begin{cases} -\text{div}(T(x, y) \text{grad}(h)) = \sum_{j=1}^{j=N_f} Q_{sj} \delta(x - S_j) & \text{in } \Omega, \\ T \text{grad}(h) \cdot n & = Q_N \quad \text{on } \Gamma'_N, \\ h & = h_D \quad \text{on } \Gamma'_D. \end{cases} \quad (7)$$

with N_f the number of flow point-source ($N_f > N_p$); $T(x, y)$ the transmissivity field, Q_{sj} the fluid volume flux rate at the j^{th} point-source $[T^{-1}]$, h_D the prescribed heads at the flow Dirichlet boundary Γ'_D , Q_N the prescribed flux at the remaining flow Neuman boundary Γ'_N .

Note that systems (3) and (7) are coupled via the Darcy velocity (6) and that the pollutant point-sources form a sub-set of the pumping point-sources.

Let us recall that the considered **inverse problem's unknowns** are: C_{si} the concentration of the pollutant released during the time interval $[t_i, t_i + l_i]$ and the corresponding point source's position $S_i = (x_i, y_i)$. In the other hand we have hydraulic head's measurements and pollution concentration's measurements in some points of the domain Ω .

We hereafter establish the reciprocity expression for the transient transport equation in a generic 2D domain .

2.3 Reciprocity principle with the advection-diffusion equation

The Reciprocity principle can be applied to the advection-diffusion equation using test functions ϕ that verify:

$$\text{div}(\overline{\overline{D}} \text{grad}(\phi)) = 0 \quad \text{in } \Omega . \quad (8)$$

Multiplying the first equations of systems (3) by ϕ and integrating it over Ω then applying two times Green's first identity, lead to the following equations:

$$\begin{aligned} \int_{\Omega} \omega \frac{\partial C}{\partial t} \phi - \int_{\Omega} \text{div}(\overline{\overline{D}} \text{grad} C - VC) \phi &= \sum_{i=1}^{i=N_p} \langle Q_{si} C_{si} \delta(x - S_i) \Pi\left(\frac{t_i - \frac{l_i}{2} - t}{l_i}\right), \phi \rangle \\ \frac{\partial}{\partial t} \int_{\Omega} \omega C \phi + \int_{\Omega} \overline{\overline{D}} \text{grad} C \text{grad} \phi - \int_{\partial \Omega} \overline{\overline{D}} \text{grad} C \cdot n \phi - \int_{\Omega} VC \text{grad} \phi + \\ + \int_{\partial \Omega} CV \cdot n \phi &= \sum_{i=1}^{i=N_p} Q_{si} C_{si} \phi(S_i) \Pi\left(\frac{t_i - \frac{l_i}{2} - t}{l_i}\right) \\ \frac{\partial}{\partial t} \int_{\Omega} \omega C \phi - \int_{\partial \Omega} (\overline{\overline{D}} \text{grad} C - CV) \cdot n \phi + \int_{\partial \Omega} \overline{\overline{D}} CV \text{grad} \phi \cdot n - \\ - \int_{\Omega} VC \text{grad} \phi &= \sum_{i=1}^{i=N_p} Q_{si} C_{si} \phi(S_i) \Pi\left(\frac{t_i - \frac{l_i}{2} - t}{l_i}\right) \end{aligned}$$

Then, using the fact that the test function verify equation (8), leads to:

$$\begin{aligned} \frac{\partial}{\partial t} \int_{\Omega} \omega C \phi - \int_{\partial \Omega} (\overline{\overline{D}} \text{grad} C - CV) \cdot n \phi + \int_{\partial \Omega} \overline{\overline{D}} CV \text{grad} \phi \cdot n - \\ - \int_{\Omega} VC \text{grad} \phi = \sum_{i=1}^{i=N_p} Q_{si} C_{si} \phi(S_i) \Pi\left(\frac{t_i - \frac{l_i}{2} - t}{l_i}\right) \end{aligned} \quad (9)$$

Integrating equation (9) over the time range $[t_0, t_f]$ leads to the expression of the reciprocity principle for the advection-diffusion equation :

$$\begin{aligned} \int_{\Omega} \omega (C_f - C_0) \phi - \int_{t_0}^{t_f} \int_{\Omega} VC \text{grad} \phi - \int_{t_0}^{t_f} \int_{\partial \Omega} (\overline{\overline{D}} \text{grad} C - CV) \cdot n \phi + \\ + \int_{t_0}^{t_f} \int_{\partial \Omega} \overline{\overline{D}} CV \text{grad} \phi \cdot n = \sum_{i=1}^{i=N_p} Q_{si} C_{si} \phi(S_i) \int_{t_i}^{t_i + l_i} \Pi\left(\frac{t_i - \frac{l_i}{2} - t}{l_i}\right) dt \end{aligned} \quad (10)$$

where $C_f = C(x, y, t_f)$.

Note that for a given test function φ , the left hand side of equation (10) is known. Then we can determine the point-source concentrations (C_{si}) and locations (x_i, y_i) by solving a linear system constitutes with equation (10) with different functions φ , as we will show it in the next example.

So in conclusion, the reciprocity principle relates the concentration and flux values at the boundary, the concentration values in the domain at any time with the pollution point-sources parameters.

However, since the wells water flow are also the pollutant point-source injection and as the flow equations are steady ones, it will be easier to apply the reciprocity principle to the system (7) as a first step of the identification procedure to recover the wells position(see [13–15]). Then the equation (10) is exploited to recover the pollutant concentration. This identification process will be illustrated in the following examples.

3 ILLUSTRATION OF THE METHOD FOR POLLUTION SOURCES IDENTIFICATION

First, we check the methodology with a single point-source pollution in the case of a hydrogeologic configuration inspired from the work by Konikow [16] who predicted long term pollutants dispersion in groundwater flow due to a leaky chemical pond under the Rocky Mountain arsenal in Colorado, US. The model setup is directly inspired from a test case in SUTRA code developed by Voss [31]. The porous media is supposed with isotropic heterogeneous properties. The geometry, with boundary conditions data, is sketched on figure (1). The domain is a 6100 m per 4880 m heterogeneous rectangle with a porosity $\omega = 0.2$, a transmissivity $T = 2.510^{-4}m^2/s$ and two less permeable zones of $T = 2.510^{-8}m^2/s$. The aquifer is bounded upstream by a lake at the north with a constant head $h_n = 75m$, a river downstream at the south with linear head, varying from 5m to 23.5m and two impervious lateral borders. The aquifer is exploited by three (1, 2 and 3) pumping wells with a volumetric fluxes $Q_{out}^{1,3} = -0.00810^{-2}m^3/s$ and $Q_{out}^2 = -0.01610^{-2}m^3/s$. The longitudinal dispersivity is $\alpha_L = 30m$ and the transversal dispersivity is $\alpha_T = 3m$ in the entire domain.

Solute transport boundary conditions are zero prescribed concentration on upstream of the lake border and zero convective-dispersive flux on the lateral impervious border. A constant concentration ($C_s = 1kg/m^3$) is assigned to the injected water at the contaminated pond, situated at (2745m,4270m), with a water flux of $Q_{in}^4 = 0.002m^3/s$. We suppose that pollution injection starts at t_1 and continues for l_1 hours as shown on figure (2). The initial concentration is null over the entire domain.

The coupled flow and transport equations are solved using

the FEM software Comsol Multiphysics [10]. The domain is meshed into 652 linear triangular elements and 364 nodes. Integrals are numerically evaluated using the Trapeze numerical integration.

The objective is to find the injected concentration of pollution C_s and the position (x_s, y_s) of the point source, i.e. the pond, from the piezometric heads and the concentrations in the domain and the concentration flux over the boundaries at different times. As the problem has three unknowns, the reciprocity method requires only three virtual fields φ_1, φ_2 and φ_3 . We choose three simple polynomial functions:

$\varphi_1 = 1, \varphi_2 = Real(x + iy) = x$ and $\varphi_3 = Im(x + iy) = y$. Their derivatives in the direction of the normal to the boundary of the domain are given by:

$$\frac{\partial \varphi_1}{\partial n} = 0, \frac{\partial \varphi_2}{\partial n} = n_x \text{ and } \frac{\partial \varphi_3}{\partial n} = n_y.$$

Where $n = (n_x, n_y)$ the outward normal to the boundary. Applied to these test functions, equation (10) and using the fact that:

$$\int_{t_1}^{t_1+l_1} \Pi\left(\frac{t_i - \frac{l_1}{2} - t}{l_1}\right) d\tau = l_1$$

leads to the following equations:

$$\int_{\Omega} \omega C_f - \int_{t_0}^{t_f} \int_{\partial\Omega} (\overline{D} \text{grad} C - CV) \cdot n = 2l_1 Q_{in}^4 C_s \quad (11)$$

$$\int_{\Omega} \omega C_f \cdot x - \int_{t_0}^{t_f} \int_{\Omega} VC \cdot \text{grad} \phi - \int_{t_0}^{t_f} \int_{\partial\Omega} (\overline{D} \text{grad} C - CV) \cdot n \cdot x + \int_{t_0}^{t_f} \int_{\partial\Omega} \overline{DC} \cdot n_x = l_1 Q_{in}^4 \cdot x_p C_s \quad (12)$$

$$\int_{\Omega} \omega C_f \cdot y - \int_{t_0}^{t_f} \int_{\Omega} VC \cdot \text{grad} \phi - \int_{t_0}^{t_f} \int_{\partial\Omega} (\overline{D} \text{grad} C - CV) \cdot n \cdot y + \int_{t_0}^{t_f} \int_{\partial\Omega} \overline{DC} \cdot n_y = l_1 Q_{in}^4 \cdot y_p C_s \quad (13)$$

From equation (11), we note that we directly obtain the injected concentration C_s from the knowledge of the cumulative mass flux through the boundary at any time in the interval $[t_0, t_f]$ and the concentration on all the domain Ω at the final time t_f . Then we replace this value in equation (12) to obtain x_p and in equation (13) in order to obtain y_p .

In equations (12) and (13), we need to have the concentrations field at each time to identify the point-source position. However, as in this example, the pollution point-source is also a source point for the flow equation, we can apply the reciprocity principle to the **stationnary** problem (7) as done by Hariga et al in [13–15] to identify the point-source position. It's more easy and it consists in multiplying the first

equation of system (7) by simple test functions ($\phi_1 = x$ and $\phi_2 = y$) and then using the Green formula twice times. So that at the end, we find the following expression for x_p and y_p :

$$\begin{aligned} x_p &= \frac{1}{Q_{in}^+} \int_{\partial\Omega} T(h.n_x - x.grad(h).n) \\ y_p &= \frac{1}{Q_{in}^+} \int_{\partial\Omega} T(h.n_y - y.grad(h).n) \end{aligned} \quad (14)$$

For the numerical study we define the relative errors as:

$$\varepsilon_S = \frac{\|S_{exact} - S_{compute}\|_{L^2}}{\|S_{exact}\|_{L^2}}, \quad (15)$$

for position's identification, where S the vector position and $\| \cdot \|_{L^2}$ the euclidean norm. and :

$$\varepsilon_f = \frac{\|f_{exact} - f_{compute}\|_{L^1}}{\|f_{exact}\|_{L^1}}, \quad (16)$$

for fluxes' identification, where f is the injected pollution's concentration and $\| \cdot \|_{L^1}$ the absolute value.

3.1 Identification with complete data over the domain

For the injected concentration recovering we use equation (11), whereas for the point-source position identification we compare the two methodologies: the transport one by the use of equations (12) and (13) and the stationary flow one by applying equation (14). We consider $t_1 = 25$ days and $l_1 = 5$ days.

Figure (3), figure (4) and figure (5) show, respectively, the hydraulic head, the concentration distribution and the total flux at the pond over the domain.

On table (1) and on figure (6), we show the injected concentration recovered values for different period ($t_f = 45, t_f = 60, t_f = 90$ and $t_f = 180$ days) and note that when the ratio $\frac{t_f}{t_1}$ increases the parameter recovering is hard.

On table (2) we give the different point-source location identification. We note that the recovering with stationary flow reciprocity (equations (14)) is better than the one with reciprocity principle applied to the transport problem (equations (12) and (13)). This is due to the presence of time integration in the two last equations which lead to numerical errors.

3.2 Pollution source identification with few observation points

In this section, we perform the identification methodology with the sole knowledge of some 'measurements' for the hydraulic heads at the initial time (as we consider a stationary

flow) and for the concentration at different times. Then, hydraulic heads and concentrations on the domain and on its boundaries are obtained by kriging [29]. With interpolated heads, the resulting point-source position remains close to their reference at around 10% (identified location is (2489.7 m, 4470.7 m)). For the injected concentration identification with kriged concentrations, the error values are shown in table (3) and on figure (7), for different number of retained observations. We note that as expected the error increases when the number of observations decreases until it reaches 27% for only 20 observations.

3.3 Noise sensibility Pollution source identification

Sensitivity to noise is performed on the last problem. We have tested the effect of different levels of noise on the identification process, by adding an uniform white noise, with zero mean, to the pollution concentration measurement. Table (4) shows the relative errors for different noise levels in the case with complete data. We note that the error remains acceptable (max.23%) until 8% of noise.

3.4 Multiply point-source pollution identification

In this case, we consider that in addition to the pond, pumping wells are also point-source pollution. We change the volumetric flux's sign for the wells and affect them the following pollution concentration: $C_s^1 = 0.5 \text{ kg/m}^3$, $C_s^2 = 0.1 \text{ kg/m}^3$ and $C_s^3 = 0.5 \text{ kg/m}^3$ (exact positions and concentrations are summarized on table (5)). So the studied inverse problem is to identify the four concentrations from the knowledge of the cumulative mass flux through the boundary at any time in the considered interval and the concentration on all the domain at the final time considered. We identify the positions concentrations by using the reciprocity principle respectively with the darcean equations and the advection-diffusion. As shown on table (6), positions as well as concentrations are identified with a satisfying errors which don't exceed 8%.

4 CONCLUSION

In this article, we have considered the principle of reciprocity, derived from the field of mechanics and generally applicable to linear problems, and we have developed and applied it to the identification of pollution point sources in the advection-diffusion equation for the transport of solutes in aquifers.

As shown in this paper, in the case of pollution transport in a Darcian flow, this principle yields a strong relationship between the forcing terms and the resulting fields for two different forcing sets.

In the framework of this research and for relevance and simplification purposes, we have limited the scope of our work to the case involving the conservative transport of pollutants. But cases including adsorption and/or degradation can also be developed. Besides, we believe that the method developed here can easily be implemented in hydrogeological modeling codes, SUTRA, MODFLOW, FEFLOW, ..., which are now widely used by managers of water resources and environment, all around the world. Even with limited knowledge and available data on the state of the aquifer contamination, the results of the identification procedure are satisfactory.

The main interest of the reciprocity principle lies in the simplicity of the resulting identification process. The computational cost of this direct method is very low, since the computations are restricted to solve small linear systems, the coefficients of which are given by some numerical evaluation of integrals. Whereas, the required large amount of data, including the full knowledge of concentration and fluxes at the boundaries and interfaces (generally not available), constitutes its main drawback. However, using incomplete data sets remains possible as shown in [6]. These simplified direct identification methods can be used in conjunction with other advanced parameter identification methods. They can constitute a first assessment before launching other costly field and laboratory investigations. These can also constitute prime tools of persuasion for decision-makers and for the competent authorities

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$\frac{t_f}{t_1}$	1.8	2.4	3.6	7.2
t_f [days]	45	60	90	180
Computed C_s [kg/m ³]	0.964	0.953	1.072	1.29
Relative Error (%)	3.6	4.7	7.2	29

Table 1 Recovered injected concentration for different final time with $t_1 = 25$ days and $l_1 = 5$ days ($C_s, exact = 1\text{kg/m}^3$)

	x_p [m]	y_p [m]
Exact Value	2745	4270
Computed value with equ(14)	2741	4198
Relative Error (%)	0.12	1.67
Computed value with equ(12,13)	2484.3	3952.9
Relative Error (%)	9.49	7.42

Table 2 Point-source location identification with stationary and transient equations.

Observations' number	364	182	91	20
Computed C_s [kg/m ³]	0.964	0.89	1.19	1.27
Relative Error (%)	3.6	11	19	27

Table 3 Recovering concentration's error for different number of observations ($t_1 = 45$ days, $t_f = 25$ days and $l_1 = 5$ days)

Noise level (%)	0	2	4	6	8
Computed C_s [kg/m ³]	1.01	0.98	1.043	0.874	0.77
Relative Error (%)	1	2	4.3	12.6	23

Table 4 Recovered injected concentration for different noise's level.

	S_1	S_2	S_3	S_4
S (m)	(915, 1220)	(2135, 1220)	(3355, 1220)	2747, 4270)
C_s [kg/m ³]	0.5	0.1	0.5	1.0

Table 5 Exact Point-sources' position and concentration in the case of 4 sources

	S_1	S_2	S_3	S_4
Computed S (m)	(885, 1195)	(2096, 1197)	(3398, 1190)	2637, 4241)
Computed C_s [kg/m ³]	0.494	0.09	0.492	0.82
ϵ_S (%)	2.5	1.8	1.6	2.2
ϵ_f (%)	1.2	1.8	1.6	0.01

Table 6 Computed wells position and flux from over-specified boundary data and 20 interior observations in the case of 4 sources

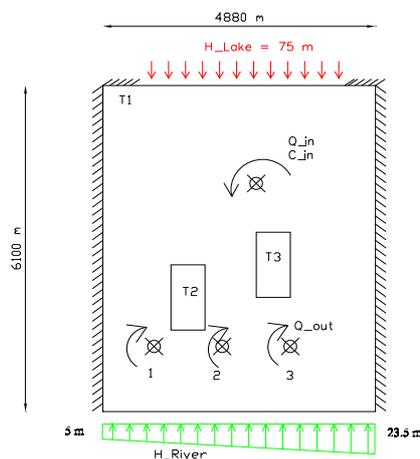


Fig. 1 Rocky Mountain aquifer properties.

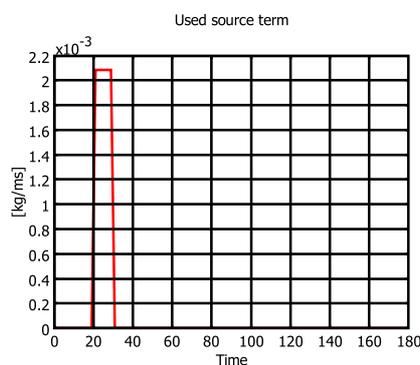


Fig. 2 The used source term in equation (4).

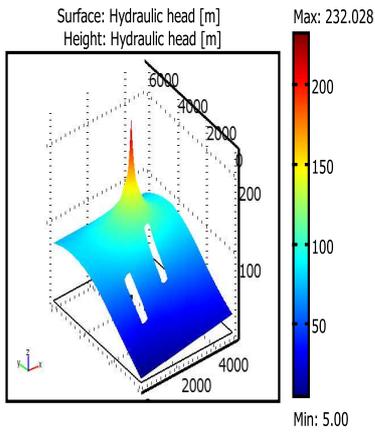


Fig. 3 Rocky Mountain aquifer hydraulic head distribution.

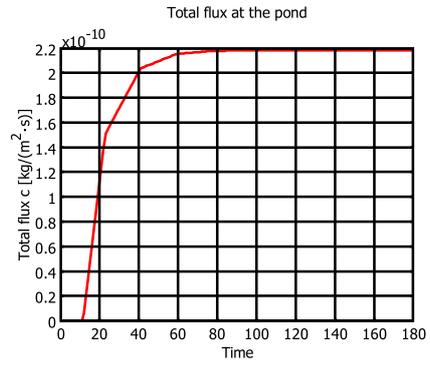


Fig. 5 Total flux's pollutant distribution at the pond.

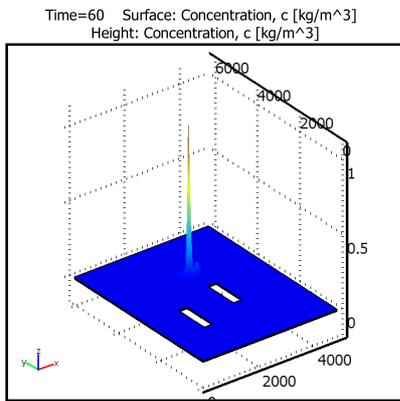


Fig. 4 Rocky Mountain pollutant concentration distribution.

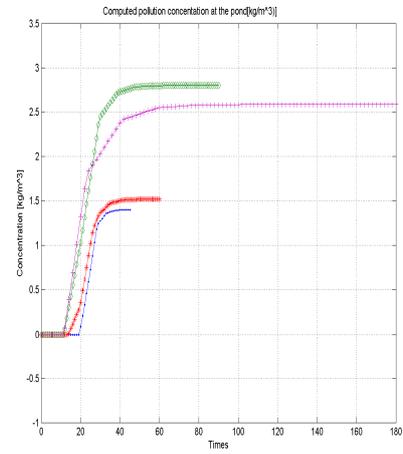


Fig. 6 Computed pollution concentration at the pond for different final times ($t_f = 45, t_f = 60, t_f = 90$ and $t_f = 180$ days).

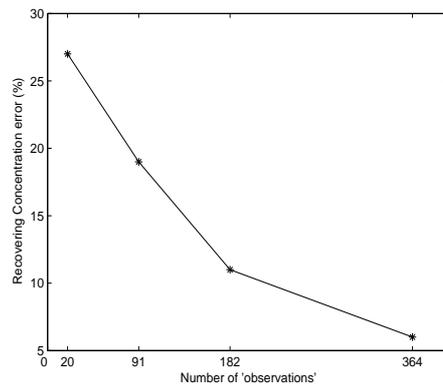


Fig. 7 Recovering concentration's error for different number of observations.

Figures

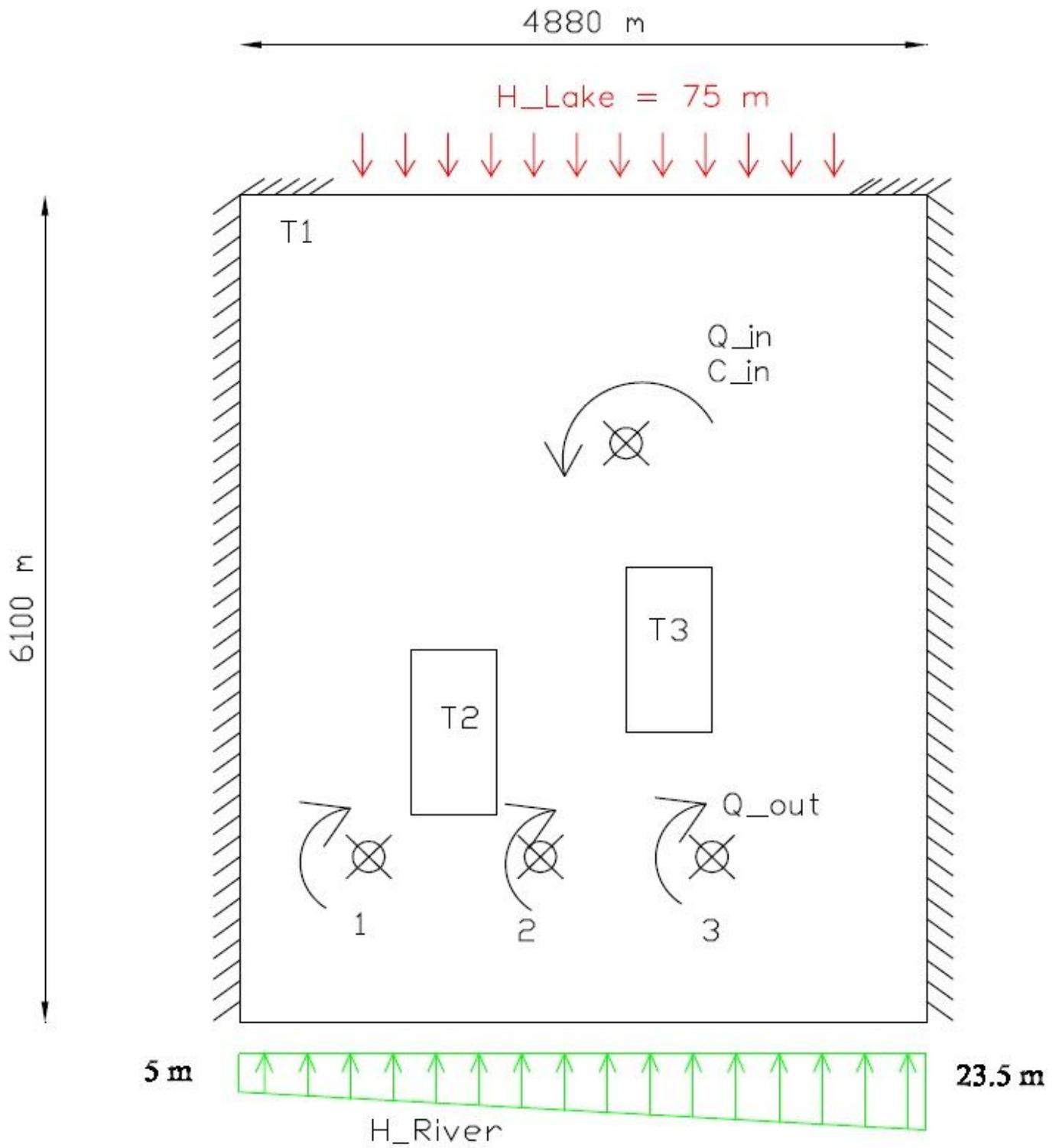


Figure 1

Rocky Mountain aquifer properties.

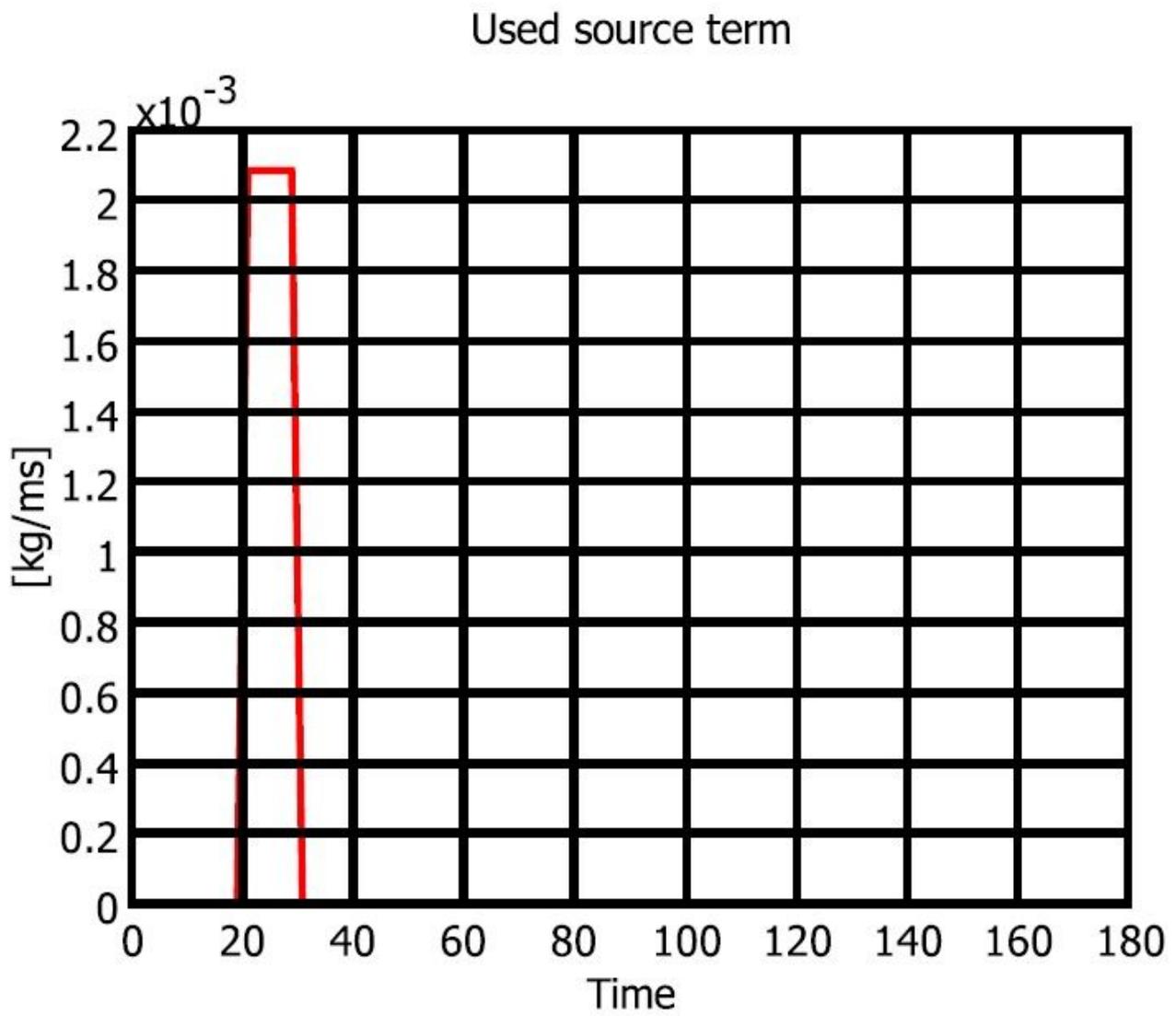


Figure 2

The used source term in equation (4).

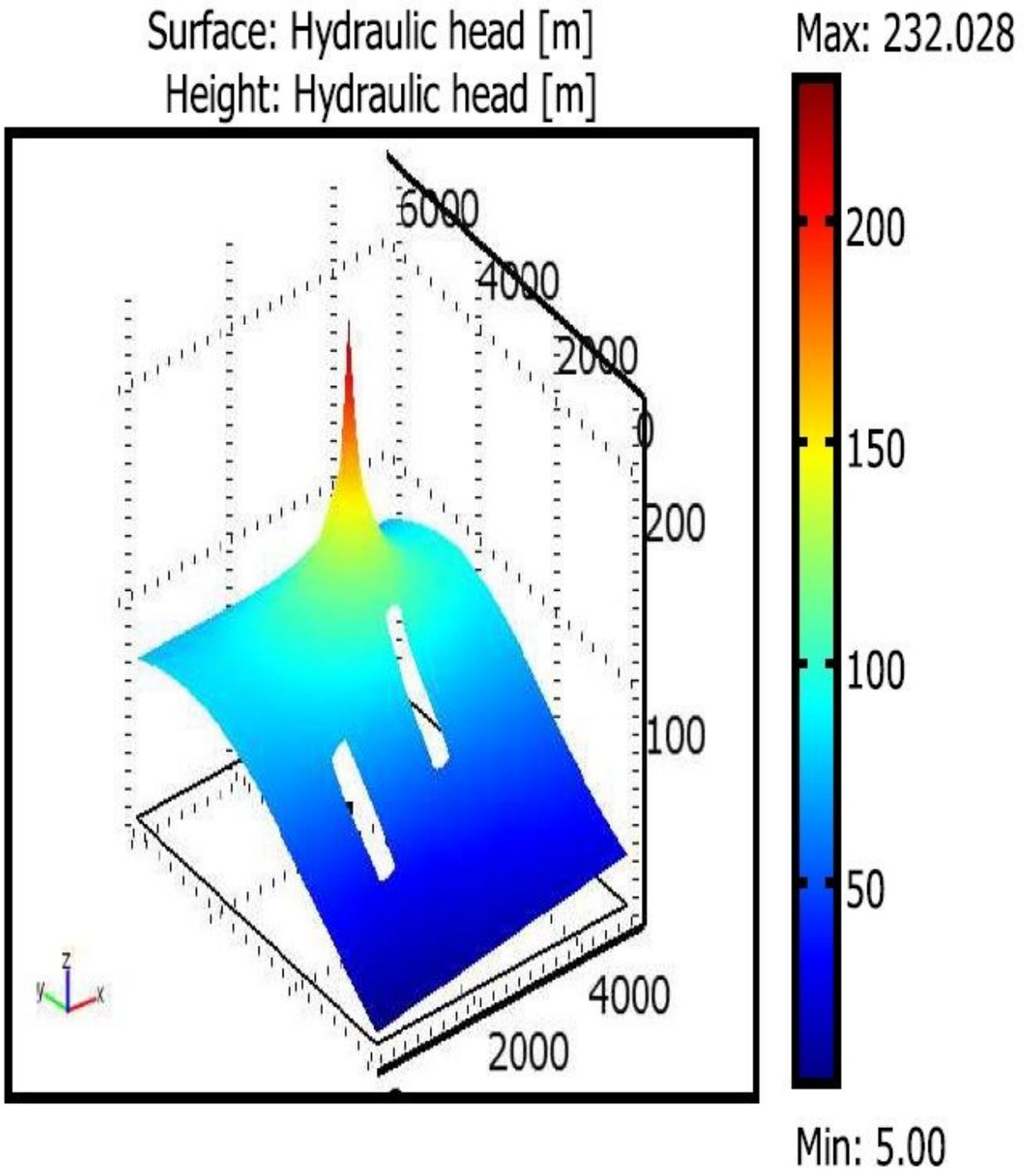


Figure 3

Rocky Mountain aquifer hydraulic head distribution.

Time=60 Surface: Concentration, c [kg/m^3]
Height: Concentration, c [kg/m^3]

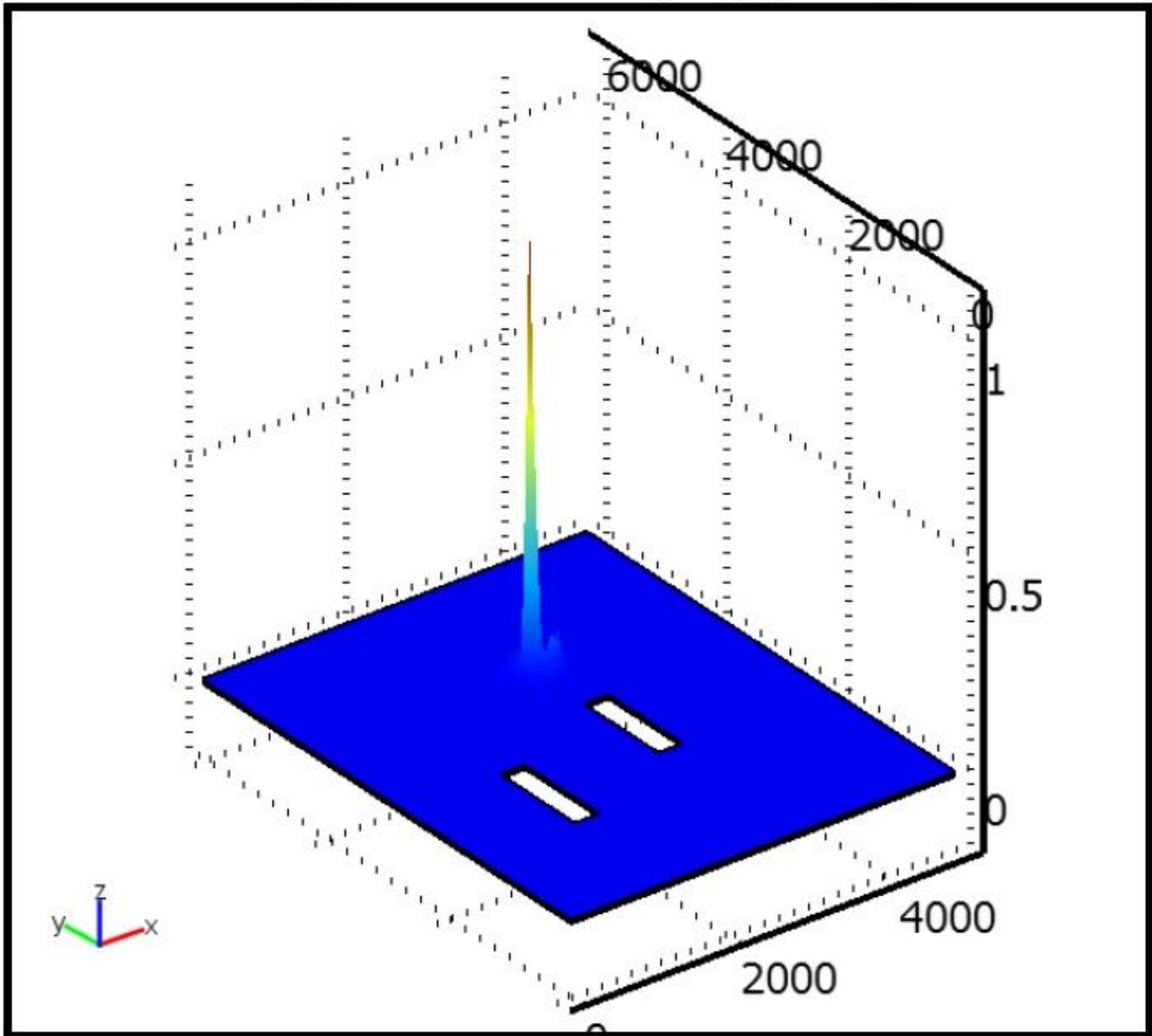


Figure 4

Rocky Mountain pollutant concentration distribution.

Total flux at the pond

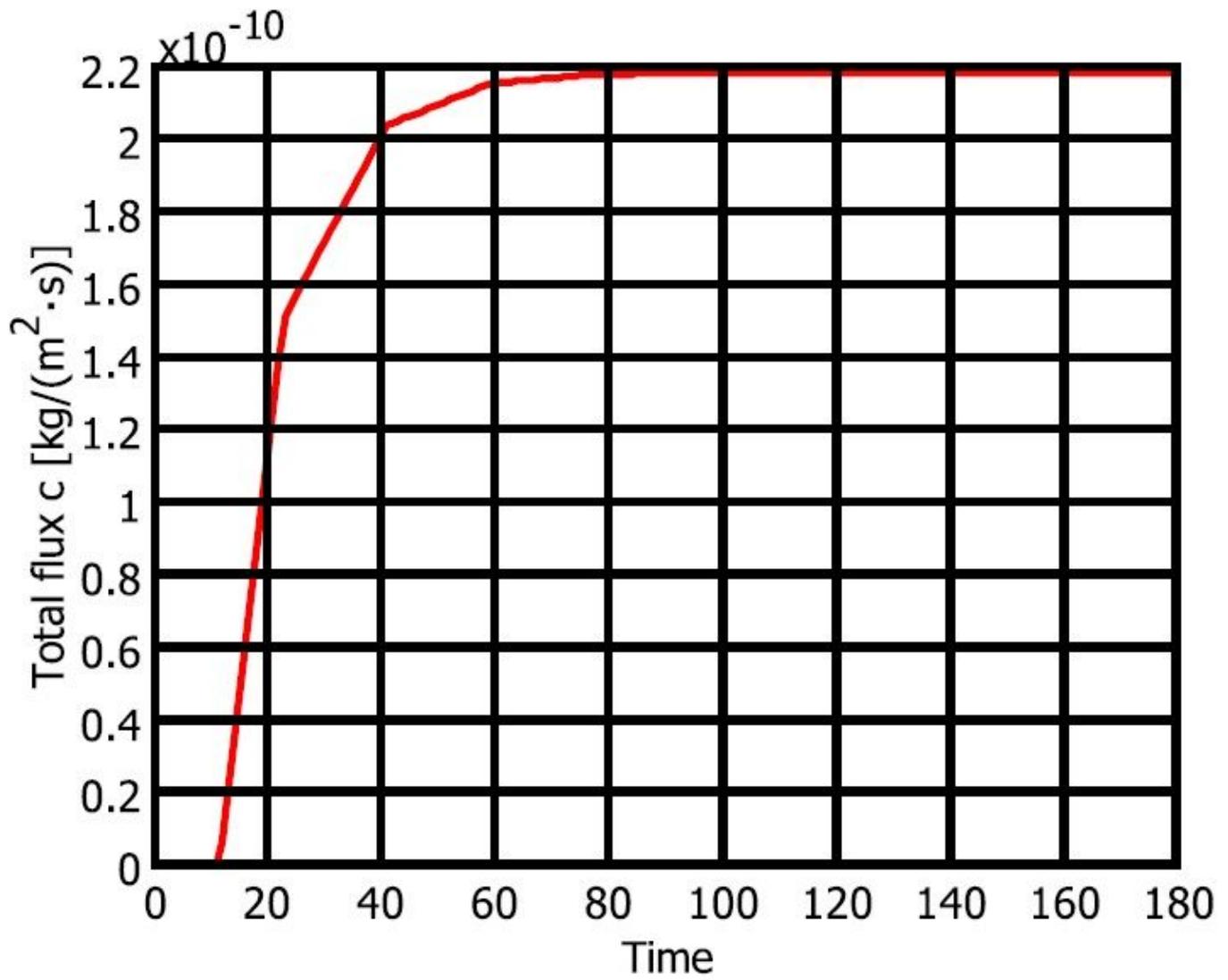


Figure 5

Total flux's pollute distribution at the pond.

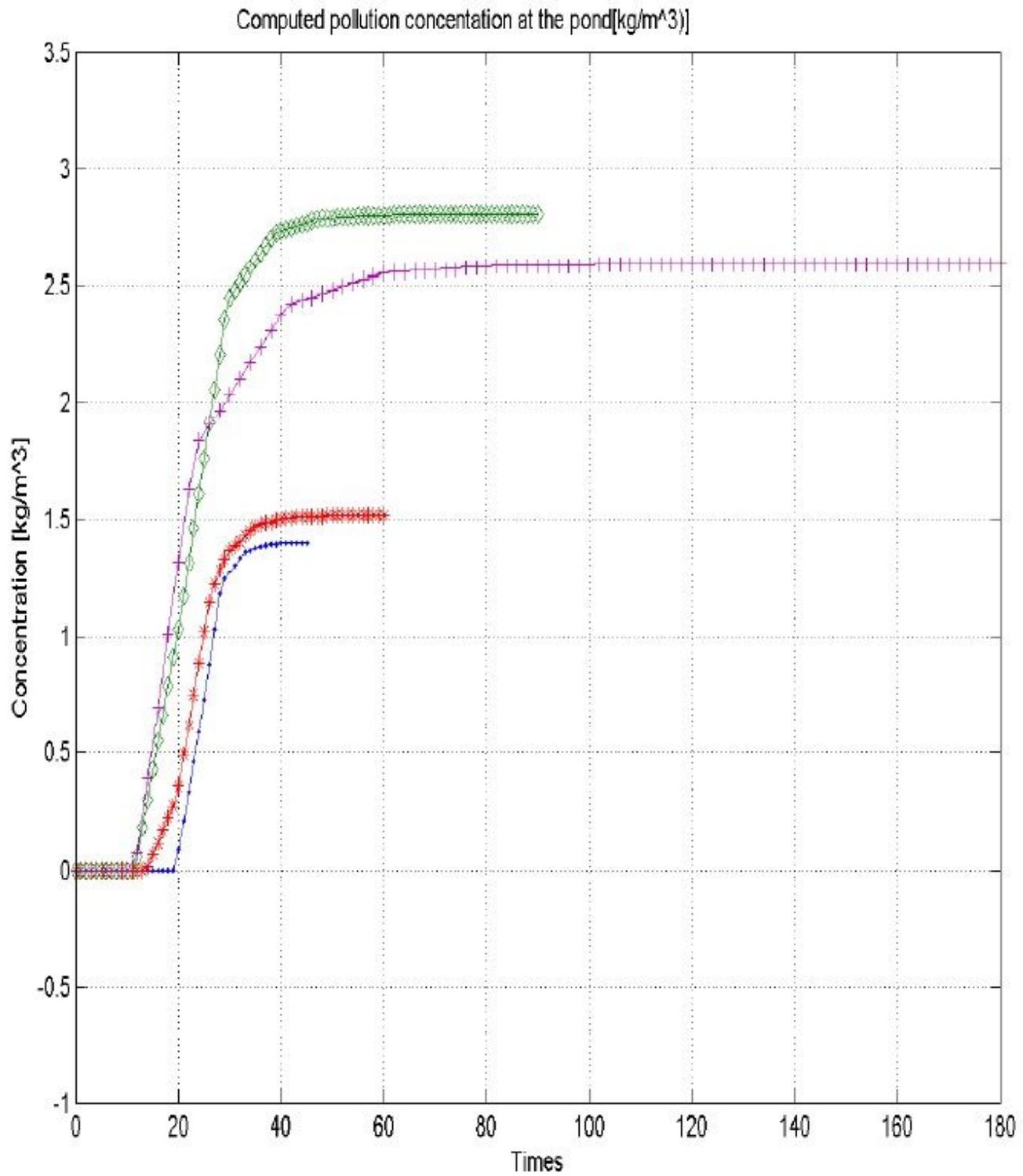


Figure 6

Computed pollution concentration at the pond for different final times ($t_f = 45$, $t_f = 60$, $t_f = 90$ and $t_f = 180$ days).

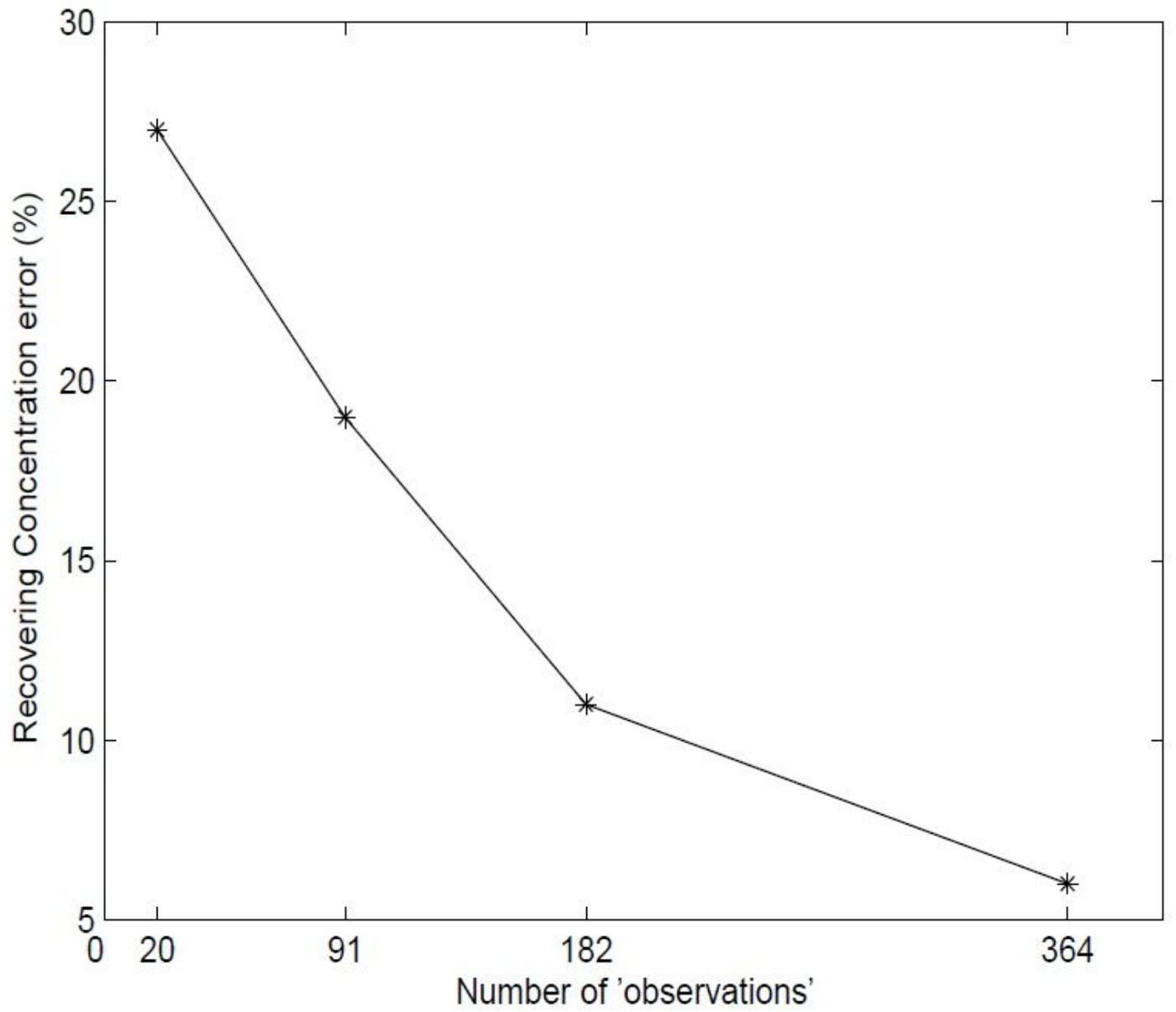


Figure 7

Recovering concentration's error for different number of observations.