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Fixed-time formation tracking for multiple nonholonomic wheeled mobile robots based on distributed observer

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Abstract—This paper studies the distributed fixed-time formation tracking problem of multiple nonholonomic wheeled mobile robots system over directed fixed and switching topologies. Through a classical nonlinear transformation, the formation control problem is transformed into a consensus problem. New control protocols based on a distributed observer are proposed. The communication topology between multiple nonholonomic wheeled mobile robots is directed, which can reduce the energy loss of communication. Some sufficient conditions of multiple robots achieving the expected formation pattern are given. All follower robots can track the leader's trajectory, form the desired formation shape within a fixed settling time, and make the leader in the geometric center of the formation. By adopting graph theory, Lyapunov stability method and fixed time theory, one can obtain an upper bound of the settling time, and the settling time is independent of the system's initial states. Finally, two examples are presented to illustrate the potential correctness of the main results.

Index Terms—fixed-time formation; multiple nonholonomic robots; distributed observer; Lyapunov stability method.

I. INTRODUCTION

Cooperative control of multiple robots has been greatly concerned over the past few years and nonholonomic wheeled mobile robots were widely used. The formation control of multi-robot systems can achieve many intricate tasks like target tracking, security and military operations, which can not be achieved by only one robot [1]–[4]. There are many ways to keep the formation shape, such as leader-follower [5]–[7], behaviour-based [8], [9], and virtual structure [10], [11].

Consensus-based control method has been adopted to achieve the formation tracking control of multi-robot systems. The convergence speed is a vital performance index for the formation tracking control. However, most prior studies did not consider the convergence speed, which indicated that the formation shape

is formed when time approaches infinity [12], [13]. A protocol that the convergence speed depends on the algebraic connectivity was proposed in [14]. Olfati-Saber presented some methods to improve the algebraic connectivity, but it was still only asymptotically convergent [15]. Then, the finite-time control technology is introduced to make the states converge quickly [16]–[19], and a distributed finite-time control law was proposed to solve the consensus tracking problem for nonlinear multi-agent systems. Zhao et al proposed a saturation protocol, which makes the consensus achieve for a second-order multi-agent system within a limited time [20]. In [21], [22], the consensus problem of the multi-robot system in the form of high-order chain structure is considered, and a finite-time cooperative control protocol is designed to ensure that the state consensus is reached within a limited time.

In most of the existing results, the settling time depends on the initial state of the system. However, the initial state usually cannot be accurate obtained. Sometimes, it is even unable to be obtained under certain circumstances. Therefore, if the initial state is unavailable, the result doesn't make much sense. In this case, the fixed time stability theory is introduced, which makes the settling time independent of the initial conditions [23], [24]. A fixed-time consensus control is designed for multi-agent systems with linear and nonlinear state measurement [25]. In addition, to resolve the problem of uncertain disturbances in multi-agent system, Hong et al proposed two fixed-time consensus controller for two cases: leaderless and leader-follower [26]. Chu et al studied the robust fixed-time consensus tracking problem of second-order multi-agent systems under undirected topology [27]. However, in the control protocols, they did not consider the angular velocity of the robot. In contrast to the existing works on fixed-time cooperative control [25]–[27] for multi-agent systems, in which the systems were regarded as linear systems, in this paper, each wheeled mobile robot is considered as a nonholonomic and nonlinear constrained system. Many previous studies about the formation tracking problem of multi-robots systems are based on undirected topologies [13], [19], [28]. However, in fact, two-way

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communication between two robots is unnecessary. Distributed fixed-time formation tracking protocols was studied in [32]. The mobile robots are nonholonomic, but the information of the leader is directly available to the followers. If the information of the leader is not directly available, the observer is very important.

As far as the author know, there are few results on the fixed-time control of nonholonomic constrained multi-robot systems based on an observer. So this paper studies the distributed fixed-time formation tracking problem for multiple wheeled mobile robots based on distributed observer. Information transmission between robots can be reduced, which can reduce the energy consumption, and improve the fault tolerance of a system.

II. PRELIMINARIES

In this paper, the considered robot system is over the directed communication topology. A directed graph G is used to describe the multi-robot system. Here, G is defined as a pair $\{Y, \xi, A\}$, where $Y = \{\nu_1, \dots, \nu_m\}$ represents the set of nodes, $\xi \subseteq Y \times Y$ is the set of edges, $A = (a_{ij})_{m \times m}$ represents a weighted adjacency matrix. The edge $(\nu_i, \nu_j) \in \xi$ means that the robot i is a neighbour of the robot j , and the robot i can send the state information to the robot j . The weighted adjacency matrix $A = [a_{ij}]_{m \times m}$, where $a_{ij} > 0$ if the robot j can receive the information from the robot i directly, otherwise $a_{ij} = 0$, and $a_{jj} = 0$. $N_j = \{\nu_j \in Y : a_{ji} \neq 0\}$ denotes the neighbour set of robot j . $D = \text{diag}\{d_1, d_2, \dots, d_m\}$ represents the degree matrix, and $d_j = \sum_{i=1}^m a_{ji}$. The Laplacian matrix of the graph G is $L = D - A$ with appropriate dimension. Let the diagonal matrix $B = \text{diag}\{b_1, b_2, \dots, b_m\}$ be the interconnection relationship between the leader and followers, where $b_j > 0$ if the information of the leader is accessible to the j th follower directly, otherwise $b_j = 0$. $\mathbf{1}_m, \mathbf{0}_m$ are m -dimensional column vectors, and $\mathbf{1}_m = [1, \dots, 1]^T$, $\mathbf{0}_m = [0, \dots, 0]^T$. Set $\text{sig}(z)^\gamma = |z|^\gamma \text{sign}(z)$, where $\text{sign}(z) \triangleq z/|z|$ for $z \neq 0$, and $\text{sign}(0) = 0$.

The following lemmas are introduced to develop our main results.

Lemma 1. [30] Define a matrix $H = L + B$, then H is a positive stable matrix. If the graph G has a directed spanning tree, the eigenvalues of H have positive real parts.

Lemma 2. [24] Consider the following system

$$\dot{z}(t) = g(z, t), \quad (1)$$

where $z \in \mathbb{R}^N$ and $g(z, t) : \mathbb{R}^N \times \mathbb{R}_+ \rightarrow \mathbb{R}^N$ is a nonlinear function. Assume that the origin is contained in the equilibrium point of system (1). If there is a continuous radial unbounded function $V(z) : \mathbb{R}^N \rightarrow \mathbb{R}_+ \cup \{0\}$ such that $V(z) = 0$ if and only if $z = 0$ and the inequality $D^*V(z(t)) \leq -c_1 V^{c_3}(z(t)) - c_2 V^{c_4}(z(t))$ for $c_1 > 0, c_2 > 0, 0 < c_3 < 1, c_4 > 1$ is satisfied. Then the origin is the global fixed-time stable equilibrium point of system (1) and the settling time is limited by

$$T = \frac{1}{c_1(1 - c_3)} + \frac{1}{c_2(c_4 - 1)}.$$

Fig. 1. Model of wheeled mobile robot.

Lemma 3. [31] If $p_1, p_2, \dots, p_v \geq 0$, then

$$\sum_{j=1}^v p_j^w \geq \left(\sum_{j=1}^v p_j \right)^w, \quad 0 < w \leq 1,$$

and

$$\sum_{j=1}^v p_j^w \geq v^{1-w} \left(\sum_{j=1}^v p_j \right)^w, \quad 1 < w \leq \infty.$$

Consider a system with m robots, which are nonholonomic wheeled mobile robots and have the same kinematics (see Fig. 1). The kinematics of the wheeled mobile robot j are described as following

$$\begin{aligned} \dot{x}_j(t) &= v_j(t) \cos \theta_j(t), \\ \dot{y}_j(t) &= v_j(t) \sin \theta_j(t), \\ \dot{\theta}_j(t) &= \omega_j(t). \end{aligned} \quad (2)$$

where x_j, y_j, θ_j denote the state of the horizontal axis, the state of longitudinal axis and the orientation of the wheeled mobile robot j , respectively. v_j and ω_j are two control inputs, denote the linear velocity and angular velocity of the robot j , respectively.

The control goal of this paper is to propose suitable velocity $v_j(t)$ and angular velocity $\omega_j(t)$, $j = 1, \dots, m$, such that the m followers achieve a desired formation shape F . Meanwhile, all the followers track the leader in a fixed-time T_{max} .

Use orthogonal coordinates (r_{jx}, r_{jy}) to define the required formation shape F , and (r_{0x}, r_{0y}) the leader's orthogonal coordinate. For simplicity, let $r_{0x} = 0, r_{0y} = 0$. The kinematics of the leader are described as following

$$\begin{aligned} \dot{x}_0(t) &= v_0(t) \cos \theta_0(t), \\ \dot{y}_0(t) &= v_0(t) \sin \theta_0(t), \\ \dot{\theta}_0(t) &= \omega_0(t). \end{aligned} \quad (3)$$

Definition 1. For the multi-robot system (2), if for any given bounded initial states, there exists a constant $T_{max} > 0$, such

that for any $j = 1, 2, \dots, m$, there is

$$\begin{aligned} \lim_{t \rightarrow T_{\max}} \begin{bmatrix} x_j(t) - x_i(t) \\ y_j(t) - y_i(t) \end{bmatrix} &= \begin{bmatrix} r_{jx} - r_{ix} \\ r_{jy} - r_{iy} \end{bmatrix}, \\ \lim_{t \rightarrow T_{\max}} (\theta_j(t) - \theta_0(t)) &= 0, \\ \lim_{t \rightarrow T_{\max}} \left(\sum_{j=1}^m \frac{x_j(t)}{m} - x_0(t) \right) &= 0, \\ \lim_{t \rightarrow T_{\max}} \left(\sum_{j=1}^m \frac{y_j(t)}{m} - y_0(t) \right) &= 0. \end{aligned} \quad (4)$$

then system (2) is said to accomplish the fixed-time formation tracking, and the settling time is $T_{\max}$.

The following assumptions are used throughout the paper.

Assumption 1. The orientation θ_j and the angular velocity ω_j of the robot j are bounded; and there exists a $k > 0$ such that $|\omega_j| \leq k, j = 1, 2, \dots, m$.

Assumption 2. The state of the leader is only available to some of the followers.

Assumption 3. There is a directed spanning tree in the communication topology G with the leader as the root node.

The wheeled mobile robot system is more complex than a linear system due to its nonholonomic characteristic. Therefore, the robot system needs to be partially linearized. There is a transformation in [31] as follows

$$\begin{aligned} p_{1j} &= \theta_j, \\ p_{2j} &= (x_j - r_{jx}) \sin \theta_j - (y_j - r_{jy}) \cos \theta_j, \\ p_{3j} &= (x_j - r_{jx}) \cos \theta_j + (y_j - r_{jy}) \sin \theta_j, \\ u_{1j} &= \omega_j, \quad u_{2j} = v_j - u_{1j} p_{2j}. \end{aligned} \quad (5)$$

After conversion, the inputs become u_{1j} and $u_{2j}, j = 0, 1, \dots, m$. The formation tracking control problem of system (2) becomes a state consensus problem. And system (2) can be denoted as

$$\begin{aligned} \dot{p}_{1j} &= u_{1j}, \\ \dot{p}_{2j} &= u_{1j} p_{3j}, \\ \dot{p}_{3j} &= u_{2j}. \end{aligned} \quad (6)$$

According to the transformation (5), the original control objective (4) can be transformed to the objective of system (6) as

$$\begin{aligned} \lim_{t \rightarrow T_{\max}} (p_{1j} - p_{10}) &= 0, \\ \lim_{t \rightarrow T_{\max}} (p_{2j} - p_{20}) &= 0, \\ \lim_{t \rightarrow T_{\max}} (p_{3j} - p_{30}) &= 0, \end{aligned} \quad (7)$$

for $j = 0, 1, \dots, m$.

Lemma 4. [32] If the equations in (7) hold for $j = 0, 1, \dots, m$, the original control objectives in (4) can be achieved in the fixed time $T_{\max}$. That is the m mobile robots form the desired formation shape F in the fixed time $T_{\max}$.

Assumption 4. Suppose that the leader's inputs w_0, v_0 are unknown to any follower; then $u_{10}, u_{10}p_{30}, u_{20}$ are also unknown to the followers and their upper bounds are denoted as $\max(u_{10}), \max(u_{10}p_{30}),$ and $\max(u_{20})$.

Remark 1. After the transformation (5), if the new control objective (7) of system (6) is achieved, then the control objective (4) of system (2) is achieved. Thus, next, distributed fixed-time consensus protocols for the system (6) will be designed.

III. MAIN RESULTS

This section gives the main results.

Denote $\hat{p}_{1j}, \hat{p}_{2j}, \hat{p}_{3j}$ the estimate of leader's state p_{10}, p_{20}, p_{30} by the j th follower respectively. Adopt the following fixed-time observer

$$\begin{aligned} \dot{\hat{p}}_{1j} &= kk_3 \text{sign} \left(\sum_{i=1}^N a_{ji} (\hat{p}_{1i} - \hat{p}_{1j}) + b_j (p_{10} - \hat{p}_{1j}) \right) \\ &\quad + kk_1 \left(\sum_{i=1}^N a_{ji} (\hat{p}_{1i} - \hat{p}_{1j}) + b_j (p_{10} - \hat{p}_{1j}) \right)^2, \end{aligned} \quad (8)$$

$$\begin{aligned} \dot{\hat{p}}_{2j} &= kk_4 \text{sign} \left(\sum_{i=1}^N a_{ji} (\hat{p}_{2i} - \hat{p}_{2j}) + b_j (p_{20} - \hat{p}_{2j}) \right) \\ &\quad + kk_1 \left(\sum_{i=1}^N a_{ji} (\hat{p}_{2i} - \hat{p}_{2j}) + b_j (p_{20} - \hat{p}_{2j}) \right)^2, \end{aligned} \quad (9)$$

$$\begin{aligned} \dot{\hat{p}}_{3j} &= kk_5 \text{sign} \left(\sum_{i=1}^N a_{ji} (\hat{p}_{3i} - \hat{p}_{3j}) + b_j (p_{30} - \hat{p}_{3j}) \right) \\ &\quad + kk_1 \left(\sum_{i=1}^N a_{ji} (\hat{p}_{3i} - \hat{p}_{3j}) + b_j (p_{30} - \hat{p}_{3j}) \right)^2, \end{aligned} \quad (10)$$

where $kk_1 = \frac{\varepsilon \sqrt{m}}{(2\lambda_{\min}(H))^{\frac{3}{2}}}, kk_2 = \varepsilon \sqrt{\frac{\lambda_{\max}(H)}{2\lambda_{\min}(H)}}, kk_3 = kk_2 + \max(u_{10}), kk_4 = kk_2 + \max(u_{10}p_{30}), kk_5 = kk_2 + \max(u_{20}), \lambda_{\min}(H), \lambda_{\max}(H)$ is the second and the largest eigenvalue of $H, \max(\cdot)$ is the upper bound of a function, and $\text{sign}(\cdot)$ is the sign function.

Lemma 5. [33] For any $\varepsilon > 0$, if Assumption 2 and Assumption 4 hold, then based on the distributed observers (8)-(10), the estimate of the leader's position state $\hat{p}_{1j}, \hat{p}_{2j}, \hat{p}_{3j}$ by agent j converges to the leader's real position state $p_{10}, p_{20}, p_{30}, i = 1, 2, \dots, m$, and the settling time $T_0 := \frac{\pi}{\varepsilon}$.

For the system (6), define the tracking errors between the robot j and the leader as

$$\tilde{p}_{1j} = p_{1j} - \hat{p}_{1j}, \tilde{p}_{2j} = p_{2j} - \hat{p}_{2j}, \tilde{p}_{3j} = p_{3j} - \hat{p}_{3j}. \quad (11)$$

Then, the local relative tracking errors of the robot $j(j =$

$1, 2, \dots, m)$ are defined as

$$\begin{aligned} e_{1j} &= \sum_{i \in N_j} a_{ji}(\hat{p}_{1j} - \hat{p}_{1i}) + b_j(p_{1j} - \hat{p}_{1j}), \\ e_{2j} &= \sum_{i \in N_j} a_{ji}(\hat{p}_{2j} - \hat{p}_{2i}) + b_j(p_{2j} - \hat{p}_{2j}), \\ e_{3j} &= \sum_{i \in N_j} a_{ji}(\hat{p}_{3j} - \hat{p}_{3i}) + b_j(p_{3j} - \hat{p}_{3j}). \end{aligned} \quad (12)$$

Let

$$\begin{aligned} p_1 &= [p_{11}, \dots, p_{1m}]^T, p_2 = [p_{21}, \dots, p_{2m}]^T, \\ p_3 &= [p_{31}, \dots, p_{3m}]^T, \tilde{p}_1 = [\tilde{p}_{11}, \dots, \tilde{p}_{1m}]^T, \\ \tilde{p}_2 &= [\tilde{p}_{21}, \dots, \tilde{p}_{2m}]^T, \tilde{p}_3 = [\tilde{p}_{31}, \dots, \tilde{p}_{3m}]^T, \\ \hat{p}_1 &= [\hat{p}_{11}, \dots, \hat{p}_{1m}]^T, \hat{p}_2 = [\hat{p}_{21}, \dots, \hat{p}_{2m}]^T, \\ \hat{p}_3 &= [\hat{p}_{31}, \dots, \hat{p}_{3m}]^T, e_1 = [e_{11}, \dots, e_{1m}]^T, \\ e_2 &= [e_{21}, \dots, e_{2m}]^T, e_3 = [e_{31}, \dots, e_{3m}]^T. \end{aligned}$$

Then (12) can be written into vector form as

$$e_1 = H\tilde{p}_1, \quad e_2 = H\tilde{p}_2, \quad e_3 = H\tilde{p}_3. \quad (13)$$

Define the auxiliary local relative tracking error as

$$\bar{e}_2 = e_2 - H \text{diag}(p_3)\tilde{p}_1,$$

where $\bar{e}_2 = [\bar{e}_{21}, \dots, \bar{e}_{2m}]^T$ and assume that

$$\eta = k_1 \bar{e}_2^T \text{sig}(\bar{e}_2)^{l_1} + k_3 \bar{e}_2^T \text{sig}(\bar{e}_2)^{l_2}.$$

For any $j = 0, 1, \dots, m$, design the following control inputs

$$\begin{aligned} u_{1j} &= \frac{1}{\sum_{i \in N_j} a_{ji} + b_j} (-k_1 \text{sig}(e_{1j})^{l_1} - k_3 \text{sig}(e_{1j})^{l_2} \\ &\quad + b_j \dot{\hat{p}}_{1j} + \sum_{i \in N_j} a_{ji} u_{1i} + u_{1j}^*), \end{aligned} \quad (14)$$

$$\begin{aligned} u_{2j} &= \frac{1}{\sum_{i \in N_j} a_{ji} + b_j} (-k_1 \text{sig}(e_{3j})^{l_1} - k_3 \text{sig}(e_{3j})^{l_2} \\ &\quad + b_j \dot{\hat{p}}_{3j} + \sum_{i \in N_j} a_{ji} u_{2i} - k_2 \text{sign}(e_{3j})|\bar{e}_{2j}|), \end{aligned} \quad (15)$$

and

$$\begin{aligned} u_{1j}^* &= \frac{1}{\sum_{i \in N_j} a_{ij} + b_j} \left(u_{2j} \left(\sum_{i \in N_j} a_{ij}(\bar{e}_{2j} - \bar{e}_{2i}) + b_j \bar{e}_{2j} \right) \right. \\ &\quad \left. + \sum_{i \in N_j} a_{ij} u_{1i}^* \right), \end{aligned} \quad (16)$$

where k_1, k_2, k_3, l_1 and l_2 are constant parameters of the controllers, and satisfy $k_1 > 0, k_2 > \kappa + \frac{\eta}{|e_3|^T |\bar{e}_2|}, k_3 > 0, 0 < l_1 < 1$ and $l_2 > 1$. From (14) one can get

$$\begin{aligned} \left(\sum_{i \in N_j} a_{ji} + b_j \right) u_{1j} &= -k_1 \text{sig}(e_{1j})^{l_1} - k_3 \text{sig}(e_{1j})^{l_2} \\ &\quad + b_j \dot{\hat{p}}_{1j} + \sum_{i \in N_j} a_{ji} u_{1i} + u_{1j}^*. \end{aligned} \quad (17)$$

And (17) can be rewritten as

$$\begin{aligned} &\sum_{i \in N_j} a_{ji}(u_{1j} - u_{1i}) + b_j u_{1j} \\ &= -k_1 \text{sig}(e_{1j})^{l_1} - k_3 \text{sig}(e_{1j})^{l_2} + b_j \dot{\hat{p}}_{1j} + u_{1j}^*. \end{aligned} \quad (18)$$

Equation (18) can be written in vector form as

$$Hu_1 = -k_1 \text{sig}(e_1)^{l_1} - k_3 \text{sig}(e_1)^{l_2} + B\dot{\hat{p}}_1 + u_1^*, \quad (19)$$

where $u_1^* = [u_{11}^*, u_{12}^*, \dots, u_{1m}^*]^T$. By multiplying both sides of (19) left by the matrix H^{-1} , one can obtain

$$u_1 = H^{-1}[-k_1 \text{sig}(e_1)^{l_1} - k_3 \text{sig}(e_1)^{l_2} + B\dot{\hat{p}}_1 + u_1^*].$$

Similarly, there is

$$\begin{aligned} u_2 &= H^{-1}[-k_1 \text{sig}(e_3)^{l_1} - k_3 \text{sig}(e_3)^{l_2} \\ &\quad + B\dot{\hat{p}}_3 - k_2 \text{diag}(\text{sign}(e_3))|\bar{e}_2|]. \end{aligned}$$

From (16) one can get

$$\begin{aligned} \left(\sum_{i \in N_j} a_{ij} + b_j \right) u_{1j}^* &= u_{2j} \left(\sum_{i \in N_j} a_{ij}(\bar{e}_{2j} - \bar{e}_{2i}) + b_j \bar{e}_{2j} \right) \\ &\quad + \sum_{i \in N_j} a_{ij} u_{1i}^*. \end{aligned} \quad (20)$$

Then (20) can be rewritten as

$$\begin{aligned} &\sum_{i \in N_j} a_{ij}(u_{1j}^* - u_{1i}^*) + b_j u_{1j}^* \\ &= u_{2j} \left(\sum_{i \in N_j} a_{ij}(\bar{e}_{2j} - \bar{e}_{2i}) + b_j \bar{e}_{2j} \right). \end{aligned} \quad (21)$$

And the vector form of (21) is

$$H^T u_1^* = \text{diag}(u_2) H^T \bar{e}_2. \quad (22)$$

Multiplying both sides of (22) left by the matrix $(H^T)^{-1}$ one can obtain

$$u_1^* = (H^T)^{-1} \text{diag}(u_2) H^T \bar{e}_2.$$

Based on the above analysis, control inputs (14) and (15) can be written in the vector form as

$$\begin{aligned} u_1 &= H^{-1}[-k_1 \text{sig}(e_1)^{l_1} - k_3 \text{sig}(e_1)^{l_2} + B\dot{\hat{p}}_1 \\ &\quad + (H^T)^{-1} \text{diag}(u_2) H^T \bar{e}_2], \end{aligned}$$

$$\begin{aligned} u_2 &= H^{-1}[-k_1 \text{sig}(e_3)^{l_1} - k_3 \text{sig}(e_3)^{l_2} + B\dot{\hat{p}}_3 \\ &\quad - k_2 \text{diag}(\text{sign}(e_3))|\bar{e}_2|], \end{aligned}$$

where $0 < l_1 < 1, l_2 > 1, k_1 > 0, k_2 > \kappa + \frac{\eta}{|e_3|^T |\bar{e}_2|}$, and $k_3 > 0$.

A. System over fixed topology

Theorem 1. Suppose that there is a directed spanning tree in the communication graph G , and Assumptions 2 and 4 hold. Then under the control inputs (14)-(15) with $0 < l_1 < 1, l_2 > 1, k_1 > 0, k_3 > 0$, there is systems (2) and (3) achieve the fixed-time formation tracking control. Particularly, $p_{1j} = p_{10}, p_{2j} = p_{20}, p_{3j} = p_{30}, j = 1, \dots, m$, for $T \geq T_{max}$. And the settling time T_{max} satisfies

$$T_{max} = T_0 + \frac{2}{2^{\frac{l_1+1}{2}} k_1 (1-l_1)} + \frac{2}{2^{\frac{l_2+1}{2}} k_3 m^{\frac{1-l_2}{2}} (l_2-1)},$$

where $T_0 = \frac{\pi}{\epsilon}$ as that in Lemma 5.

Proof: From Lemma 5, one can get that the estimate of the leader's position state $\hat{p}_{1j}, \hat{p}_{2j}, \hat{p}_{3j}$ by agent j converges to the leader's real position state p_{10}, p_{20}, p_{30} when $t \geq \frac{\pi}{\epsilon}$. That is $\hat{p}_{1j} = p_{10}, \hat{p}_{2j} = p_{20}, \hat{p}_{3j} = p_{30}$. Denote $\tilde{p}_{1j}^0, \tilde{p}_{2j}^0, \tilde{p}_{3j}^0$ the tracking errors between the robot j and the leader for $t \geq T_0$. That is

$$\begin{aligned}\tilde{p}_{1j}^0 &= p_{1j} - p_{10}, \\ \tilde{p}_{2j}^0 &= p_{2j} - p_{20}, \\ \tilde{p}_{3j}^0 &= p_{3j} - p_{30}.\end{aligned}\quad (23)$$

The tracking errors in (23) can be derived as

$$\begin{aligned}\dot{\tilde{p}}_{1j}^0 &= u_{1j} - u_{10}, \\ \dot{\tilde{p}}_{2j}^0 &= u_{10} \tilde{p}_{3j} + p_{3j} (u_{1j} - u_{10}), \\ \dot{\tilde{p}}_{3j}^0 &= u_{2j} - u_{20}.\end{aligned}\quad (24)$$

Then (24) can be written into vector form as

$$\begin{aligned}\dot{\tilde{p}}_1^0 &= u_1 - \mathbf{1}_m u_{10}, \\ \dot{\tilde{p}}_2^0 &= u_{10} \tilde{p}_3 + \text{diag}(p_3)(u_1 - \mathbf{1}_m u_{10}), \\ \dot{\tilde{p}}_3^0 &= u_2 - \mathbf{1}_m u_{20}.\end{aligned}\quad (25)$$

Similarly, the local relative tracking errors of the robot j for $t \geq T_0$ can be written as

$$\begin{aligned}e_{1j}^0 &= \sum_{i \in N_j} a_{ji} (p_{1j} - p_{1i}) + b_j (p_{1j} - p_{10}), \\ e_{2j}^0 &= \sum_{i \in N_j} a_{ji} (p_{2j} - p_{2i}) + b_j (p_{2j} - p_{20}), \\ e_{3j}^0 &= \sum_{i \in N_j} a_{ji} (p_{3j} - p_{3i}) + b_j (p_{3j} - p_{30}).\end{aligned}\quad (26)$$

Then (26) can be written into vector form as

$$e_1^0 = H \tilde{p}_1^0, \quad e_2^0 = H \tilde{p}_2^0, \quad e_3^0 = H \tilde{p}_3^0. \quad (27)$$

The local relative tracking errors (27) can be derived as

$$\begin{aligned}\dot{e}_1^0 &= H u_1 - B \mathbf{1}_m u_{10}, \\ \dot{e}_2^0 &= u_{10} e_3^0 + H \text{diag}(p_3)(u_1 - \mathbf{1}_m u_{10}), \\ \dot{e}_3^0 &= H u_2 - B \mathbf{1}_m u_{20}.\end{aligned}\quad (28)$$

The auxiliary local relative tracking error for $t \geq T_0$ can be written as

$$\tilde{e}_2^0 = e_2^0 - H \text{diag}(p_3) \tilde{p}_1^0. \quad (29)$$

And (29) can be derived as

$$\tilde{e}_2^0 = e_2^0 - H \text{diag}(p_3) \tilde{p}_1^0 - H \text{diag}(\dot{p}_3) \tilde{p}_1^0. \quad (30)$$

Substituting (6), (25) and (28) into (30), one can get

$$\tilde{e}_2^0 = u_{10} e_3^0 - H \text{diag}(u_2) \tilde{p}_1^0. \quad (31)$$

Choose the Lyapunov function candidate

$$V = \frac{1}{2} ((e_1^0)^T e_1^0 + (\tilde{e}_2^0)^T \tilde{e}_2^0 + (e_3^0)^T e_3^0).$$

The derivative of V along (28) and (31) is

$$\begin{aligned}\dot{V} &= (e_1^0)^T [-k_1 \text{sig}(e_1^0)^{l_1} - k_3 \text{sig}(e_1^0)^{l_2} \\ &\quad + (H^T)^{-1} \text{diag}(u_2) H^T \tilde{e}_2^0] \\ &\quad + (\tilde{e}_2^0)^T [u_{10} e_3^0 - H \text{diag}(u_2) \tilde{p}_1^0] \\ &\quad + (e_3^0)^T [-k_1 \text{sig}(e_3^0)^{l_1} - k_3 \text{sig}(e_3^0)^{l_2} \\ &\quad - k_2 \text{diag}(\text{sign}(e_3^0)) |\tilde{e}_2^0|] \\ &= -k_1 (e_1^0)^T \text{sig}(e_1^0)^{l_1} - k_3 (e_1^0)^T \text{sig}(e_1^0)^{l_2} \\ &\quad - k_1 (e_3^0)^T \text{sig}(e_3^0)^{l_1} - k_3 (e_3^0)^T \text{sig}(e_3^0)^{l_2} \\ &\quad + u_{10} (\tilde{e}_2^0)^T e_3^0 - k_2 |\tilde{e}_2^0|^T |\tilde{e}_2^0| - \tilde{e}_2^0 H \text{diag}(u_2) \tilde{p}_1^0 \\ &\quad + (\tilde{p}_1^0)^T \text{diag}(u_2) H^T \tilde{e}_2^0.\end{aligned}\quad (32)$$

By Assumption 1, one can get $|u_{10}| < \kappa$. Let $k_2 > \kappa + \frac{\eta}{|e_3^0|^T |\tilde{e}_2^0|}$, $k_3 > 0$, and $(\tilde{p}_1^0)^T \text{diag}(u_2) H^T \tilde{e}_2^0 = (\tilde{e}_2^0)^T H \text{diag}(u_2) \tilde{p}_1^0$. Then (32) can be simplified as

$$\begin{aligned}\dot{V} &\leq -k_1 (e_1^0)^T \text{sig}(e_1^0)^{l_1} - k_3 (e_1^0)^T \text{sig}(e_1^0)^{l_2} \\ &\quad - k_1 (e_3^0)^T \text{sig}(e_3^0)^{l_1} - k_3 (e_3^0)^T \text{sig}(e_3^0)^{l_2} \\ &\quad - k_1 (\tilde{e}_2^0)^T \text{sig}(\tilde{e}_2^0)^{l_1} - k_3 (\tilde{e}_2^0)^T \text{sig}(\tilde{e}_2^0)^{l_2} \\ &= -k_1 \sum_{j=1}^m (|e_{1j}^0|^2)^{\frac{l_1+1}{2}} - k_3 \sum_{j=1}^m (|e_{1j}^0|^2)^{\frac{l_2+1}{2}} \\ &\quad - k_1 \sum_{j=1}^m (|e_{3j}^0|^2)^{\frac{l_1+1}{2}} - k_3 \sum_{j=1}^m (|e_{3j}^0|^2)^{\frac{l_2+1}{2}} \\ &\quad - k_1 \sum_{j=1}^m (|\tilde{e}_{2j}^0|^2)^{\frac{l_1+1}{2}} - k_3 \sum_{j=1}^m (|\tilde{e}_{2j}^0|^2)^{\frac{l_2+1}{2}}.\end{aligned}\quad (33)$$

Note that $0 < \frac{l_1+1}{2} < 1, \frac{l_2+1}{2} > 1$. According to Lemma 3, (33) gives

$$\begin{aligned}\dot{V} &\leq -k_1 \left(\sum_{j=1}^m |e_{1j}^0|^2 + \sum_{j=1}^m |e_{3j}^0|^2 + \sum_{j=1}^m |\tilde{e}_{2j}^0|^2 \right)^{\frac{l_1+1}{2}} \\ &\quad - k_3 m^{\frac{1-l_2}{2}} \left(\sum_{j=1}^m |e_{1j}^0|^2 + \sum_{j=1}^m |e_{3j}^0|^2 + \sum_{j=1}^m |\tilde{e}_{2j}^0|^2 \right)^{\frac{l_2+1}{2}} \\ &\leq -k_1 (2V)^{\frac{l_1+1}{2}} - k_3 m^{\frac{1-l_2}{2}} (2V)^{\frac{l_2+1}{2}},\end{aligned}$$

which implies

$$\dot{V} \leq -2^{\frac{l_1+1}{2}} k_1 V^{\frac{l_1+1}{2}} - 2^{\frac{l_2+1}{2}} k_3 m^{\frac{1-l_2}{2}} V^{\frac{l_2+1}{2}}.$$

By Lemma 2, V will reach zero in a fixed time, and the settling time is bounded and satisfies

$$T_1 = \frac{2}{2^{\frac{l_1+1}{2}} k_1 (1-l_1)} + \frac{2}{2^{\frac{l_2+1}{2}} k_3 m^{\frac{1-l_2}{2}} (l_2-1)}.$$

Then $T_{max} = T_0 + T_1$, which implies that

$$\lim_{t \rightarrow T_{max}} e_1(t) = 0, \quad \lim_{t \rightarrow T_{max}} \bar{e}_2(t) = 0, \quad \lim_{t \rightarrow T_{max}} e_3(t) = 0.$$

Note that if $\lim_{t \rightarrow T_{max}} \bar{e}_2(t) = 0$, then $\lim_{t \rightarrow T_{max}} e_2(t) = 0$. If $\lim_{t \rightarrow T_{max}} e_h(t) = 0$, then $p_{hj} = p_{h0}$, $h = 1, 2, 3$. That is under distributed controllers (14) and (15), systems (2) and (3) complete the formation tracking with the given formation.

The proof is completed.

B. System over switching topologies

For a network of multiple mobile robots, the original communication connection may be broken due to obstacles between two robots. Similarly, since the robots reach an effective detection range between each other, some new links can be created between nearby robots. It is significant to study fixed-time formation tracking for systems over switching topologies. Let $G_c = \{Y, \xi, A_{\delta(t)}\}$ be a directed graph set. Define a set $\Upsilon = \{0, 1, 2, \dots, N\}$ and it represents the index set of the graph G_c . Let $\delta(t) : R^+ \rightarrow \Upsilon$ be a switching signal. Then let $t_0 = 0, t_1, \dots, t_r, \dots$, be a switching time sequence, at which the interaction topology changes. For any $t \in [t_r, t_{r+1})$, the topology $G_{\delta(t)} = G_r \in G_c$ is active. Simultaneously, the adjacency weight between agent j and i is a_{ji}^r , the Laplacian matrix is L^r . Then, there is the following result.

Theorem 2. Suppose that there is a directed spanning tree in any communication graph, and Assumptions 2 and 4 hold. Then under the control protocols (14) and (15) with $0 < l_1 < 1, l_2 > 1, k_1 > 0, k_3 > 0$, system (6) over directed switching topologies form the formation as (7) with the settling time T_{max} , and

$$T_{max} = T_0 + \frac{2}{2^{\frac{l_1+1}{2}} k_1 (1-l_1)} + \frac{2}{2^{\frac{l_2+1}{2}} k_3 m^{\frac{1-l_2}{2}} (l_2-1)},$$

where $T_0 = \frac{\pi}{\varepsilon}$ as that given in Lemma 5.

Proof: The fixed-time observer can be written as

$$\begin{aligned} \dot{\hat{p}}_{1j} &= kk_3 \text{sign} \left(\sum_{i=1}^N a_{ji}^r (\hat{p}_{1i} - \hat{p}_{1j}) + b_j (p_{10} - \hat{p}_{1j}) \right) \\ &\quad + kk_1 \left(\sum_{i=1}^N a_{ji}^r (\hat{p}_{1i} - \hat{p}_{1j}) + b_j (p_{10} - \hat{p}_{1j}) \right)^2, \\ \dot{\hat{p}}_{2j} &= kk_4 \text{sign} \left(\sum_{i=1}^N a_{ji}^r (\hat{p}_{2i} - \hat{p}_{2j}) + b_j (p_{20} - \hat{p}_{2j}) \right) \\ &\quad + kk_1 \left(\sum_{i=1}^N a_{ji}^r (\hat{p}_{2i} - \hat{p}_{2j}) + b_j (p_{20} - \hat{p}_{2j}) \right)^2, \\ \dot{\hat{p}}_{3j} &= kk_5 \text{sign} \left(\sum_{i=1}^N a_{ji}^r (\hat{p}_{3i} - \hat{p}_{3j}) + b_j (p_{30} - \hat{p}_{3j}) \right) \\ &\quad + kk_1 \left(\sum_{i=1}^N a_{ji}^r (\hat{p}_{3i} - \hat{p}_{3j}) + b_j (p_{30} - \hat{p}_{3j}) \right)^2, \end{aligned}$$

where $\forall t \in [t_r, t_{r+1}), r = 0, 1, \dots, i \in I$, and $kk_1 = \frac{\varepsilon \sqrt{m}}{(2\lambda_{\min}(L^r + B))^{\frac{3}{2}}}$, $kk_2 = \varepsilon \sqrt{\frac{\lambda_{\max}(L^r + B)}{2\lambda_{\min}(L^r + B)}}$, $kk_3 = kk_2 + \max(u_{10})$, $kk_4 = kk_2 + \max(u_{10} p_{30})$, $kk_5 = kk_2 + \max(u_{20})$.

The local relative tracking errors of the robot j ($j = 1, 2, \dots, m$) are

$$\begin{aligned} e_{1j} &= \sum_{i \in N_j} a_{ji}^r (\hat{p}_{1j} - \hat{p}_{1i}) + b_j (p_{1j} - \hat{p}_{1j}), \\ e_{2j} &= \sum_{i \in N_j} a_{ji}^r (\hat{p}_{2j} - \hat{p}_{2i}) + b_j (p_{2j} - \hat{p}_{2j}), \\ e_{3j} &= \sum_{i \in N_j} a_{ji}^r (\hat{p}_{3j} - \hat{p}_{3i}) + b_j (p_{3j} - \hat{p}_{3j}). \end{aligned} \quad (34)$$

Then (34) can be written into vector form as

$$\begin{aligned} e_1 &= (L^r + B) \tilde{p}_1, \\ e_2 &= (L^r + B) \tilde{p}_2, \\ e_3 &= (L^r + B) \tilde{p}_3. \end{aligned}$$

The auxiliary local relative tracking error can be written as

$$\bar{e}_2 = e_2 - (L^r + B) \text{diag}(p_3) \tilde{p}_1.$$

Then the distributed controllers are

$$\begin{aligned} u_{1j} &= \frac{1}{\sum_{i \in N_j} a_{ji}^r + b_j} (-k_1 \text{sig}(e_{1j})^{l_1} - k_3 \text{sig}(e_{1j})^{l_2} \\ &\quad + b_j \dot{\hat{p}}_{1j} + \sum_{i \in N_j} a_{ji}^r u_{1i} + u_{1j}^*), \\ u_{2j} &= \frac{1}{\sum_{i \in N_j} a_{ji}^r + b_j} (-k_1 \text{sig}(e_{2j})^{l_1} - k_3 \text{sig}(e_{2j})^{l_2} \\ &\quad + b_j \dot{\hat{p}}_{2j} + \sum_{i \in N_j} a_{ji}^r u_{2i} - k_2 \text{sign}(e_{3j}) |\bar{e}_{2j}|), \end{aligned}$$

where

$$\begin{aligned} u_{1j}^* &= \frac{1}{\sum_{i \in N_j} a_{ij}^r + b_j} \left(u_{2j} \left(\sum_{i \in N_j} a_{ij} (\bar{e}_{2j} - \bar{e}_{2i}) + b_j \bar{e}_{2j} \right) \right. \\ &\quad \left. + \sum_{i \in N_j} a_{ij}^r u_{1i}^* \right). \end{aligned}$$

Similarly, for $t \geq T_0$, the local relative tracking errors of the robot j can be written as

$$\begin{aligned} e_{1j}^0 &= \sum_{i \in N_j} a_{ji}^r (p_{1j} - p_{1i}) + b_j (p_{1j} - p_{10}), \\ e_{2j}^0 &= \sum_{i \in N_j} a_{ji}^r (p_{2j} - p_{2i}) + b_j (p_{2j} - p_{20}), \\ e_{3j}^0 &= \sum_{i \in N_j} a_{ji}^r (p_{3j} - p_{3i}) + b_j (p_{3j} - p_{30}). \end{aligned} \quad (35)$$

And (35) can be written as

$$\begin{aligned} e_1^0 &= (L^r + B) \tilde{p}_1^0, \\ e_2^0 &= (L^r + B) \tilde{p}_2^0, \\ e_3^0 &= (L^r + B) \tilde{p}_3^0. \end{aligned}$$

Fig. 2. Communication graph G_1 .

For $t \geq T_0$, the auxiliary local relative tracking error is

$$\bar{e}_2^0 = e_2^0 - (L^T + B) \text{diag}(p_3) \bar{p}_1^0. \quad (36)$$

Then (36) can be derived as

$$\dot{\bar{e}}_2^0 = u_{10} e_3^0 - (L^T + B) \text{diag}(u_2) \bar{p}_1^0.$$

Choose the Lyapunov function candidate

$$V = \frac{1}{2} ((e_1^0)^T e_1^0 + (\bar{e}_2^0)^T \bar{e}_2^0 + (e_3^0)^T e_3^0).$$

Next, similar to the proof of Theorem 1, from Lemma 2 one can get the T_{max} . This completes the proof.

IV. SIMULATION RESULT

In this section, two simulation examples are given to demonstrate the effectiveness of the fixed-time formation tracking control algorithm.

Consider a multiple mobile robots system contains six followers $F_i, i = 1, 2, \dots, 6$, and a leader L . For the system over the fixed directed topology as in Fig. 2, under the control inputs (14)-(15) with $\varepsilon = 1$, $k_1 = 8$, $k_2 = 26$, $k_3 = 4$, $l_1 = 0.9$ and $l_2 = 1.1$, one can calculate that $T_{max} = 7.0768s$. Choose the initial state $\theta(0) = [\pi/2, \pi/3, \pi/4, \pi/5, \pi/6, \pi/7]$, $x(0) = [13, 11, 8, 3, 6, 10]$, and $y(0) = [-15, -12, -18, -20, -23, -21]$.

The expected position between the followers and the leader L is shown in Fig. 3. The orthogonal coordinates of the followers are $(r_{1x}, r_{1y}) = (2, 0)$, $(r_{2x}, r_{2y}) = (1, \sqrt{3})$, $(r_{3x}, r_{3y}) = (-1, \sqrt{3})$, $(r_{4x}, r_{4y}) = (-2, 0)$, $(r_{5x}, r_{5y}) = (-1, -\sqrt{3})$, $(r_{6x}, r_{6y}) = (1, -\sqrt{3})$, and the orthogonal coordinates of the leader is $(r_{0x}, r_{0y}) = (0, 0)$.

Here, the leader's trajectory, i.e., the reference trajectory, is $x_0 = 10 \sin(2t + 1)$, $y_0 = -20 \cos(t + 1/2)$.

From Figs. 4-7, six followers constitute the desired formation shape under the distributed control protocols (14) and (15) over directed topology, and the trajectory of the leader L is also successfully tracked by the trajectory at the center of the formation shape of the six followers. E_{1j}, E_{2j}, E_{3j} denote actual errors between followers and the leader, and $E_{1j} = p_{1j} - p_{10}$, $E_{2j} = p_{2j} - p_{10}$, $E_{3j} = p_{3j} - p_{10}$. One can find from Figs. 8-10, E_{1j}, E_{2j}, E_{3j} converge to zero. The convergence time is about $1.3s$ and it is less than $7.0768s$, which illustrate that the control objects can be achieved in a fixed time.

Next, consider the system over the switching topologies. The initial topology is given in Fig 2. Then the topology was changed as shown in Fig. 11 after 0.5 second, and after 1 second the topology was changed again as in Fig. 12. The initial

Fig. 3. Desired shape of formation 1.

Fig. 4. Initial shape of formation 1.

Fig. 5. Shape of formation 1 in 0.88s and the trajectory of leader.

Fig. 6. Shape of formation 1 in 2s and the trajectory of leader.

Fig. 7. Shape of formation 1 in 7s and the trajectory of leader.

Fig. 8. Tracking errors between p_{1j} and p_{10} in formation 1.

Fig. 9. Tracking errors between p_{2j} and p_{20} in formation 1.

Fig. 10. Tracking errors between p_{3j} and p_{30} in formation 1.

Fig. 11. Communication graph 2.

Fig. 12. Communication graph 3.

state is $\theta(0) = [-\pi/2, \pi/3, -\pi/4, \pi/5, -\pi/6, \pi/7]$, $x(0) = [0, 3, 6, 10, 12, 8]$, and $y(0) = [-12, -20, -22, -10, -23, -15]$. And Fig. 13 shows the desired position between the followers and the leader. The orthogonal coordinates of the followers are $(r_{1x}, r_{1y}) = (-2, 2)$, $(r_{2x}, r_{2y}) = (-2, 0)$, $(r_{3x}, r_{3y}) = (-2, -2)$, $(r_{4x}, r_{4y}) = (2, 2)$, $(r_{5x}, r_{5y}) = (2, 0)$, $(r_{6x}, r_{6y}) = (2, -2)$, and the orthogonal coordinates of the leader is $(r_{0x}, r_{0y}) = (0, 0)$. Fig. 13 shows the initial position of followers. Figs. 14-17 provide the states of the followers and the trajectory of the leader. Fig. 18 shows the errors between p_{1j} and p_{10} , Fig. 19 gives the errors between p_{2j} and p_{20} , and Fig. 20 shows the errors between p_{3j} and p_{30} , $j = 1, \dots, 6$, respectively. From which one can see that the agents form a given formation over switching topologies, and the form time is less than $T_{\max}$, which verifies the efficiency of the methods.

V. CONCLUSION

This paper studies the distributed fixed-time formation tracking based on distributed observer for multiple nonholonomic wheeled mobile robots over directed topology. The stability of the distributed controllers is also proved by graph theory and Lyapunov theory. Some sufficient conditions of multiple robots

Fig. 13. Desired shape of formation 2.

Fig. 14. Initial shape of formation 2.

Fig. 18. Tracking errors between p_{1j} and p_{10} in formation 2.

Fig. 15. Shape of formation 2 in 0.88s and the trajectory of leader.

Fig. 19. Tracking errors between p_{2j} and p_{20} in formation 2.

Fig. 16. Shape of formation 2 in 2s and the trajectory of leader.

achieving the expected formation pattern are given. Several numerical simulations are presented to verify the efficiency of the adopting method. However, the communication delay between robots is ignored. Considering communication delay will be what we will do in future research. Using the event-triggered control technique to reduce the communication between robots also needs to be further studied.

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Fig. 17. Shape of formation 2 in 7s and the trajectory of leader.

Fig. 20. Tracking errors between p_{3j} and p_{30} in formation 2.

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Figures

Figure 1

Model of wheeled mobile robot.

Figure 2

Communication graph $G1$:

Figure 3

Desired shape of formation 1.

Figure 4

Initial shape of formation 1.

Figure 5

Shape of formation 1 in 0.88s and the trajectory of leader.

Figure 6

Shape of formation 1 in 2s and the trajectory of leader.

Figure 7

Shape of formation 1 in 7s and the trajectory of leader.

Figure 8

Tracking errors between p_{1j} and p_{10} in formation 1.

Figure 9

Tracking errors between p2j and p20 in formation 1.

Figure 10

Tracking errors between p3j and p30 in formation 1.

Figure 11

Communication graph 2.

Figure 12

Communication graph 3.

Figure 13

Desired shape of formation 2.

Figure 14

Initial shape of formation 2.

Figure 15

Shape of formation 2 in 0.88s and the trajectory of leader.

Figure 16

Shape of formation 2 in 2s and the trajectory of leader.

Figure 17

Shape of formation 2 in 7s and the trajectory of leader.

Figure 18

Tracking errors between p1j and p10 in formation 2.

Figure 19

Tracking errors between p2j and p20 in formation 2.

Figure 20

Tracking errors between p3j and p30 in formation 2.