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Entropy-Based Shear Stress Distribution in Open Channel for all types of Flow using Experimental Data

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Abstract: Korea's river design standards set general design standards for river and river-related projects in Korea, which systematize the technologies and methods involved in river-related projects. This includes measurement methods for parts necessary for river design, but do not include information on shear stress. Shear Stress is to one of the factors necessary for river design and operation. Shear stress is one of the most important hydraulic factors used in the fields of water especially for artificial channel design. Shear stress is calculated from the frictional force caused by viscosity and fluctuating fluid velocity. Current methods are based on past calculations, but factors such as boundary shear stress or energy gradient are difficult to actually measure or estimate. The point velocity throughout the entire cross section is needed to calculate the velocity gradient. In other words, the current Korea's river design standards use tractive force, critical tractive force instead of shear stress because it is more difficult to calculate the shear stress in the current method. However, it is difficult to calculate the exact value due to the limitations of the formula to obtain the river factor called the tractive force. In addition, tractive force has limitations that use empirically identified base value for use in practice. This paper focuses on the modeling of shear stress distribution in open channel turbulent flow using entropy theory. In addition, this study suggests shear stress distribution formula, which can be easily used in practice after calculating the river-specific factor T . and that the part of the tractive force and critical tractive force in the Korea's river design standards should be modified by the shear stress obtained by the proposed shear stress distribution method. The present study therefore focuses on the modeling of shear stress distribution in open channel turbulent flow using entropy theory. The shear stress distribution model is tested using a wide range of forty-two experimental runs collected from the literature. Then, an error analysis is performed to further evaluate the accuracy of the proposed model. The results revealed a correlation coefficient of approximately 0.95-0.99, indicating that the proposed method can estimate shear stress distribution accurately. Based on this, the results of the distribution of shear stress after calculating the river-specific factors show a correlation coefficient of about 0.86 to 0.98, which suggests that the equation can be applied in practice.

Keywords: Entropy, Shear stress distribution, Shannon's theory, Korea's river design standards

1. Introduction

Understanding fluid interaction is very important in almost all studies of open channel flows. Shear stress is used and applied in hydraulics, hydrology, fluid mechanics, and in various fields which is one of the most important mechanical factors [1]. It is a great challenge to the river engineers and researchers working in the field to estimate the distribution of bed shear stress in open channel flows [2].

Leighly [3] proposed that the bed shear stress can be balanced by the downstream component of the weight of water contained within the bounding orthogonal. Lundgren [4] modified the logarithmic law to a parabolic cross-sectional open channel and suggest

a method to estimate the velocity and shear stress distribution. Chiu [5-6] studied the complex interaction between primary and secondary flows, shear stress distribution, channel characteristics such as roughness, slope and geometry, and other related factors in open channel flows. However, velocity profile was required to estimate boundary shear stress.

Keulegan [7] and Johnson [8] contributed to the early development of shear stress, and Einstein's [9] hydraulic radius separation method is still used in various studies. Following this idea, Knight and his associates [10-16] proposed several empirical relations which are very helpful understanding open channel flows and sediment transport. Noutsopoulos [17], Hu [18], and Patel [19] have led to an improved understanding of the lateral distributions of wall shear stress in rectangular channels, prismatic channels and ducts.

Past literature shows that shear stress profile in open channel flows has been studied either experimentally or theoretically using deterministic approaches. Up to now, a probabilistic method of shear stress distribution in open channel flows using entropy concept has not been studied. For the past few decades, entropy theory has been applied in the field of river hydraulic geometry and fluvial hydraulics. Entropy concept was introduced in hydraulics by Chiu [20] with Shannon's entropy. Chiu studied the two-dimensional velocity distribution in an open channel [21-22]. Later, Choo [23] used Chiu's velocity equation to calculate the momentum and energy coefficient. Especially, Chiu [24] modified entropy concept to be applied in pipe flows. Here, Chiu compared with the Schlichting equation, and the relationship between frictional loss coefficient and entropy coefficient M but there was no study on the shear stress. Singh [25] had studied wide range of hydrology and water resource based on entropy theory.

Since then, various entropy-related research was studied. Chiu [26-27] applied the maximum velocity and regularity and a one-dimensional velocity distribution in an open channel with entropy concept. Singh [28] examined the one-dimensional velocity distribution in an open channel with entropy theory, where he used Shannon's entropy to derive the power law and logarithmic velocity distribution. Cui and Singh [29-31] studied velocity distribution and sediment concentration in open channels using Tsallis entropy. Singh and Cui [32] developed sediment concentration in debris flow by Tsallis entropy.

Shear stress have to be estimated to determine flow characteristic. Considering the importance in open channel, the complete evaluation is highly difficult due to the complexity of cross section and the various hydraulic parameters. The researches related to the shear stress are based on empirical outcome. The being so, the equations are difficult to be generally applied. Therefore, the current Korea's river design standards use simple to obtainable tractive force and critical tractive force that can be obtained through empirical method instead of shear stress.

The objective of this study therefore is to model shear stress distribution using entropy theory, verify the model using twenty-one experimental datasets obtained from Song's [33] experimental data and to prove the utility of the proposed equation. Based on the proven equation, it is then suggested that the tractive force and critical tractive force in the Korea's river design standards be revised to the shear stress obtained using the proposed shear stress distribution formula by presenting a method that can be easily used in the actual conditions.

2. Methodology

2.1. Entropy Theory

The variable x related to information $I(x)$ is shown as Equation (1) which provides the amount of information. Here, information $I(x)$ is the measurement of uncertainty related to a certain state as:

$$I(x) = \ln p(x) \quad (1)$$

Considering every state, the average value of information $I(x)$ can be expressed as Equation (2). Function is defined using Shannon's [34] entropy as:

$$H(x) = - \int_{-\infty}^{+\infty} p(x)I(x)dx = - \int_{-\infty}^{+\infty} p(x)\ln p(x)dx$$

where $p(x)$ and $\ln p(x)$ are dimensionless, but dx has a dimension thus $H(x)$ has the same dimension as dx . In Equation (2), a probability density function (PDF) $p(x)$ of a continuous variation state means maximizing the entropy of uncertainty x .

2.2 Constraint Conditions

The probability distribution of maximizing the entropy produces a bigger information from an already acquired basic knowledge. To solve PDF, i.e. the available information of variable x in Equation (3), the constraint conditions, such as average, variance, distortion etc., are applied as:

$$\int_a^b \Phi_i(x, p)dx \quad i = 1, 2, 3, \dots, n \quad (3)$$

Therefore, the PDF $p(x)$, which maximizes the entropy, can be obtained using the method from Lagrange in Equation (4), we have:

$$\frac{\partial I(x, p)}{\partial p} + \sum_{i=1}^n \lambda_i \frac{\partial \Phi_i(x, p)}{\partial p} = 0 \quad (4)$$

where λ_i is the Lagrange multipliers.

2.3 Entropy Maximization

Entropy concept can be applied to the shear stress using Equation (5) from Shannon's entropy as:

$$H(\tau) = - \int_0^{\tau_0} p(\tau)I(\tau)d\tau = - \int_0^{\tau_0} p(\tau)\ln p(\tau)d\tau \quad (5)$$

The available information for τ is using constraint conditions. First, the total probability must be satisfied for the PDF $p(\tau)$ as:

$$\int_0^{\tau_0} p(\tau)d\tau = 1 \quad (6)$$

which follows from the total probability rule.

Then, average information can be expressed as:

$$\int_0^{\tau_0} \tau \cdot p(\tau)d\tau = \bar{\tau} \quad (7)$$

2.4 Lagrange Method

Arranging the independent constraint conditions, can be given as:

$$\int_a^b \Phi_i(\tau, p)d\tau \quad i = 1, 2 \quad (8)$$

Therefore, PDF $p(\tau)$, which maximize the entropy can be obtained using method of Lagrange as:

$$\frac{\partial I(\tau, p)}{\partial p} + \sum_{i=1}^2 \lambda_i \frac{\partial \Phi_i(\tau, p)}{\partial p} = 0 \quad (9)$$

$$I(\tau, p) = p(\tau)\ln p(\tau) \quad (10)$$

where $\Phi_1(\tau, p) = p(\tau)$, $\Phi_2(\tau, p) = \tau \cdot p(\tau)$.

$$\frac{\partial \phi_1(\tau, p)}{\partial p} = 1, \frac{\partial \phi_2(\tau, p)}{\partial p} = \tau \quad (11)$$

Substituting Equation (9) into Equation (10) and (11) can be constructed as follows:

$$-1 - \ln p(\tau) + \lambda_1 + \lambda_2 \tau = 0 \quad (12)$$

where $\lambda_1 - 1 = a_1$, $\lambda_2 = a_2$ are the Lagrange multipliers and differentiating Equation (12) with respect to $p(\tau)$ results in the shear stress PDF as:

$$p(\tau) = \exp^{a_1 + a_2 \tau} \quad (13)$$

Applying the cumulative probability function to fluid flow use Equation (14), and Equation (13) will become Equation (14) by applying PDF as:

$$F(\tau) = \int_0^\tau p(\tau) d\tau = \int_0^\tau e^{a_1 + a_2 \tau} d\tau = 1 - \left[\frac{\xi - \xi_0}{\xi_{max} - \xi_0} \right] \quad (14)$$

2.5 Proposed Shear Stress Distribution Model

Solving Equation (14) can be written as:

$$\tau = \frac{1}{a_2} \ln \left[1 + \frac{a_2}{e^{a_1}} \left(1 - \left[\frac{\xi - \xi_0}{\xi_{max} - \xi_0} \right] \right) \right] \quad (15)$$

Open channel flow also uses the constraint conditions of Equation (6) and (7). Substitute Equation (13) into Equation (6) is given as:

$$\frac{a_2}{e^{a_1}} = (e^{a_2 \tau_0} - 1) \Rightarrow e^{a_1} = \frac{T}{(e^T - 1)\tau_0} \quad (16)$$

where, $T = a_2 \tau_0$ (normally called the entropy coefficient) to represent the model in a simple-to-use form and substituting Equation (13) to Equation (7) results can be expressed as:

$$\begin{aligned} \bar{\tau} &= \int_0^{\tau_0} \tau \cdot p(\tau) d\tau = \int_0^{\tau_0} \tau e^{a_1 + a_2 \tau} d\tau \\ \bar{\tau} &= \left[\frac{e^T}{e^T - 1} - \frac{1}{T} \right] \tau_0 = \phi(T) \tau_0 \end{aligned} \quad (17)$$

where, τ_0 is the boundary shear stress, T is the entropy coefficient, and $\phi(T)$ is a function of T . Equation (17) is the one of proposed shear stress distribution equation for open channel flows. Substituting Equation (17) to Equation (16) and arranging it to e^{a_1} term results as:

$$e^{a_1} = \frac{T}{(e^T - 1)\tau_0} = \frac{T \cdot \phi(T)}{(e^T - 1)\bar{\tau}} \quad (18)$$

Substituting Equation (18) into Equation (15) and rearranging it results in another proposed mean shear stress distribution formula as:

$$\tau = \frac{\bar{\tau}(e^T - 1)}{(T e^T - e^T + 1)} \ln \left[1 + (e^T - 1) \left(1 - \left[\frac{\xi - \xi_0}{\xi_{max} - \xi_0} \right] \right) \right] \quad (19)$$

By inserting Equation (16) into Equation (15) as:

$$\tau = \frac{\tau_0}{T} \ln \left[1 + (e^T - 1) \left(1 - \left[\frac{\xi - \xi_0}{\xi_{max} - \xi_0} \right] \right) \right] \quad (20)$$

Equation (20) is the last proposed model for boundary shear stress distribution. Generally, ξ_0 is close to 0, ξ_{max} is 1, and ξ for Equation (19) and (20) are the same which was formulated as follows:

$$\xi = \frac{y}{D - h} \exp \left(1 - \frac{y}{D - h} \right) \quad (21)$$

where D is the maximum depth, h is the depth where shear stress is 0 from the water surface (maximum velocity also occurs at this location), y is the vertical depth from the bed for given shear stress.

2.6 Shear Stress in Fluid

Normally when laminar and turbulent flow coincides, it can be written as:

$$\tau = \rho\nu \frac{du}{dy} - \rho\overline{u'v'} = \rho(\nu + \varepsilon) \frac{du}{dy} \quad (22)$$

where ρ is the fluid density, ν is the fluid viscosity, du/dy is the velocity gradient, $\overline{u'v'}$ is the Reynolds stress, and ε is the eddy viscosity. Reynolds stress is the shear stress caused by turbulent fluctuating velocity. The kinematic coefficient of viscosity, ν , is caused by the molecular motion of fluid, ε_y is caused by fluid particle mixing, which is much larger than molecular motion.

Boundary shear stress from Equation (22) can be expressed as when shear stress at the bed, $y = 0$, the velocity is 0, $u = 0$, as:

$$\tau_0 = \rho(\nu + \varepsilon) \left[\frac{du}{dy} \right]_{y=0} = \rho g R_h S_f \quad (23)$$

where g is the gravitational acceleration, R_h is the shape form of cross-section, and S_f is the energy gradient.

2.7 tractive force formula in Fluid

Tractive force means the running water force when the silt on the river bed is moved by water. The commonly used tractive force (τ_0) formula is:

$$\tau_0 = \omega R i \quad (24)$$

where ω is the unit weight of water, i is the bed slope, R is the hydraulic radius.

2.8 critical tractive force formula in Fluid

Critical tractive force means the tractive force at the beginning of the movement of the river bed silt due to the fact that the running water force is greater than the resistance of the river bed. The commonly used critical tractive fore formula (F_s) is :

$$F_s = \frac{\tau_0}{(\rho_s - \rho)gd} = \frac{u_{*c}^2}{\frac{1}{\rho}(\rho_s - \rho)gd} \quad (25)$$

where u_{*c} is critical friction velocity, ρ_s is density of silt particles, ρ is density of water, g is gravitational acceleration.

In terms of simplicity and convenience, Equation (22) and (23) has an advantage, but its accuracy is suspect. The reason is that energy gradient is actually a difficult factor to estimate. As it can be seen in Equation (22), measured point velocity of whole cross section is required for shear stress to get each gradient. In other words, the velocity gradient (du/dy), eddy viscosity coefficient (ε) and the energy gradient (S_f) in Equation (22) and (23) are factors that are very difficult to estimate. Also, as shown by the equation (24), it is difficult to accurately calculate the tractive force used by the Korea's river design standards due to the hard-to-find river factor such as the river bed. Critical tractive force is readily calculated, but is not certain in terms of accuracy because they are empirical formulas obtained from experiments. Therefore, this paper suggests equation that can express the shear stress distribution and boundary shear stress in open channel turbulent flow using the entropy-based modeling. This study demonstrates the utility of the proposed equation by using the Song data. It also proposes to revise the tractive force and critical tractive force of the Korea's river design standards to shear stress by presenting measures easily applicable in practice.

3. Experimental Data

The proposed model of shear stress distribution was validated with experimental observations available in the literature. To test the validity of the model, i.e., Equation (19) and (20) with a wide range of slope, discharge, and sediment flow conditions, experimental data from Song [33] were selected. Forty-six flows were used in this study, twenty-one uniform flow and twenty-one non-uniform flow and four unsteady flow according to four slope conditions. Out of twenty-one uniform flow, six runs were experimented under sediment conditions. For non-uniform flow, twelve accelerating flows and nine decelerating flows were tested. Four unsteady flow were tested according to four slope conditions. This study considered a various range of experimental runs for verification of the shear stress distribution.

4. Parameter Estimation and Comparison with Experimental Data

4.1 Parameter Estimation

Proposed shear stress distribution was used to estimate the entropy parameter T . First, estimate parameter T by inserting experimental point shear stress ($\tau_1, \tau_2, \dots, \tau_n$) and vertical depth from the bed for given shear stress ($y_1, y_2, \dots, y_n$) into Equation (19) and (20). Estimate the best boundary shear stress τ_0 and mean shear stress $\bar{\tau}$ value which has the least error for each run. Then use τ_0 and $\bar{\tau}$ from Equation (19) and (20), to estimate $\phi(T)$ from Equation (17). Lastly, calculate the shear stress distribution for given vertical depths ($y_1, y_2, \dots, y_n$).

4.2 Comparison with Experimental Data

Figures 1 to 4 compares the proposed model with the experimental data of Song [33] to determine if the estimated shear stress distribution fits well with the observed shear stress distribution. Each diagram shows shear stress at x-axis and y/D at y-axis with correlation coefficient of observed data and estimated shear stress.

Figure 1. Verification of the proposed shear stress distribution model with six uniform flows

For first (uniform), second (accelerating non-uniform), third (decelerating non-uniform) and fourth (unsteady) flow conditions (Figure 1 to 4), proposed model was applied to compare the estimated and observed values of shear stress distribution to see how well it is expressed.

In Figure 1, the proposed shear stress model showed a good agreement with experimental data despite the scattered nature of the data. As for the six sediment runs, correla-

tion coefficient was actually better in some cases compare to non-sediment runs. For uniform flows, with or without the sediment, entropy-based model seems to estimate shear stress accurately. In all uniform flows, correlation coefficient showed small range from 0.9375 to 0.9931.

Figure 2. Verification of the proposed shear stress distribution model with six accelerating non-uniform flows

From Figure 2, it is found that proposed model predicted well even in more complex flows. For accelerating non-uniform flows, entropy-based model seems to estimate shear stress accurately. In all accelerating non-uniform flows, correlation coefficient showed small range from 0.9522 to 0.9959. However, some flows have shown very scatter experimental data, especially in the bottom layer due to the difficulties in measuring.

Figure 3. Verification of the proposed shear stress distribution model with six decelerating non-uniform flows

From Figure 3, it can be seen that entropy-based model predicted very well. For decelerating non-uniform flows, proposed model seems to compute shear stress accurately. In all decelerating non-uniform flows, correlation coefficient showed small range from 0.9475 to 0.9822. Proposed model seems to express very accurate matching results.

Figure 4. Verification of the proposed shear stress distribution model with unsteady flow of one slope (S-25-931)

From Figure 4, it can be seen that entropy-based model predicted well. For unsteady flows, proposed model appears to compute shear stress well. In unsteady flow(S-25-931), the correlation coefficient showed between 0.8262 and 0.9843. And the results for the other three slopes are as follows. The correlation coefficient in S-60-933 showed between 0.9034 and 0.952, the correlation coefficient in S-10-31 showed between 0.8536 and 0.9899, and the correlation coefficient in S-30-932 showed between 0.712 and 0.9829. In four cases, the correlation coefficient is above 0.89 on average. It demonstrated the utility of the shear stress distribution equation using entropy.

4.3 Major Parameter Estimation Results

Table 5, 6 shows the result of major parameters from Figures 1 to 4. Entropy-based model showed 0.9375 to 0.9959 range of correlation coefficient in steady flow conditions and 0.712 to 0.9899 range of correlation coefficient in unsteady flow conditions. From these results, it seems that number of measured shear stress in one distribution does not have large effect on prediction. Entropy parameter T seems to have range of -1.441 to 6.405 in steady flow conditions and range of -5.576 to 4.6124 in unsteady flow conditions. And looking at correlation coefficient, decelerating non-uniform flow showed the least average, 0.9693, whereas accelerating non-uniform flow showed the best average, 0.9887 in steady flow conditions. For unsteady flow conditions, S30-932 showed the worst average, 0.89353, whereas S-10-932 showed the best average, 0.93602.

Table 1. Summary of major parameter result for unsteady flow conditions

Table 2. Summary of major parameter for steady flow conditions.

Data set	τ_0	$\bar{\tau}$	T	R^2
S-60-933(t1)	69.5773	12.2146	-5.5760	0.9491
S-60-933(t3)	58.2623	11.4943	-4.8793	0.9340
⋮	⋮	⋮	⋮	⋮
S-60-933(t57)	47.9180	13.5993	-2.9491	0.9512
S-60-933(t59)	51.2248	12.9439	-3.5383	0.9520
S-10-932(t2)	39.5775	23.7430	1.2288	0.9255
S-10-932(t6)	39.2120	25.3055	1.8404	0.9176
⋮	⋮	⋮	⋮	⋮
S-10-932(t54)	41.6155	16.2094	-1.3667	0.9867
S-10-932(t78)	37.9574	20.7072	0.5492	0.9267
S-25-931(t2)	28.0111	21.0477	3.6227	0.8393
S-25-931(t58)	37.6556	13.5880	-1.75	0.9800
⋮	⋮	⋮	⋮	⋮
S-25-931(t134)	32.2337	22.9028	2.8501	0.8262
S-25-931(t138)	30.1916	22.2638	3.3415	0.8382
S-30-932(t1)	39.6605	23.9780	1.2893	0.9098
S-30-932(t17)	44.3978	33.9348	3.9019	0.7628
⋮	⋮	⋮	⋮	⋮
S-30-932(t57)	44.8305	20.6737	-0.4679	0.9607
S-30-932(t59)	48.5676	17.8320	-1.6665	0.9461

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Data set	τ_0	$\bar{\tau}$	T	R^2
S25-Q31	20.531	14.458	2.742	0.9375
S25-Q40	21.519	16.134	3.588	0.9834
⋮	⋮	⋮	⋮	⋮
S150-Q40	102.850	58.927	0.887	0.9809
S150-Q50	123.535	78.079	1.656	0.9677
AS00-Q145	92.144	52.484	0.845	0.9957
AS00-Q100	71.481	41.464	0.976	0.9934
⋮	⋮	⋮	⋮	⋮
AS-93-Q100	41.036	23.846	0.989	0.9829
AS-93-Q80	38.194	19.674	0.182	0.9902
DS25-Q90	27.408	19.021	2.573	0.9769
DS25-Q70	23.983	17.553	3.236	0.9822
⋮	⋮	⋮	⋮	⋮
DS90-Q80	48.087	40.659	6.405	0.9734
DS90-Q70	33.711	27.751	5.532	0.9683

5. Proposal and verification of shear stress distribution method

5.1 Easily applied shear stress distribution formula for practice

If the main parameters of Tables 1 and 2 are used in expressions (17) to graph, the distribution of mean shear stress and floor shear stress, $\phi(T)$ can be obtained based on the slope values of the graph. Using the value of the obtained $\phi(T)$, it is possible to calculate the river-specific factor T' in equation (17).

$$\bar{\tau} = \left[\frac{e^T}{e^T - 1} - \frac{1}{T} \right] \tau_0 = \phi(T) \tau_0 \quad (17)$$

If the river-specific factor T' is calculated and the average shear stress is used, it is easy to calculate the boundary shear stress, which is an important hydraulic factor in river design. When the river-specific factor T' is used in the shear stress distribution Equation (19) using the boundary shear stress and the shear stress distribution Equation (20) using the average shear stress, the shear stress distribution formula can be easily used in practice.

$$\tau = \frac{\bar{\tau}(e^T - 1)}{(Te^T - e^T + 1)} \ln \left[1 + (e^T - 1) \left(1 - \left[\frac{\xi - \xi_0}{\xi_{max} - \xi_0} \right] \right) \right] \quad (19)$$

$$\tau = \frac{\tau_0}{T} \ln \left[1 + (e^T - 1) \left(1 - \left[\frac{\xi - \xi_0}{\xi_{max} - \xi_0} \right] \right) \right] \quad (20)$$

Song data was used to prove the utility of shear stress distribution when the river-specific factor T' was fixed. After the river-specific factor T' for each flow state is obtained by using the floor shear stress and average shear stress, which are the parameters of Figure 5, 6. Shear stress obtained by substituting T' for Equation (19) and Equation (20) and the actual measurement value of the Song data was compared and analyzed.

5.2 Estimation graph of river-specific factors T' in all flow conditions

Using the Equation (17) based on the data in Table 5-6, it is displayed graphically to calculate the river-specific factor for each flow state. Figure 5, 6 shows a graph for the calculating of river-specific factors in each flow state.

Figure 5. Equilibrium $\phi(T')$ in steady flow

Figure 5 is a graph of calculating equilibrium T' after calculating equilibrium $\phi(T')$ in uniform, non-uniform accelerating, and non-uniform decelerating flow. For each flow, equilibrium $\phi(T')$ is 0.6490 (uniform), 0.5344 (non-uniform accelerating), 0.7466 (non-uniform decelerating) and when equation (17) is calculated using equilibrium $\phi(T')$, equilibrium T' is calculated as 1.88 (uniform), 0.41 (non-uniform accelerating), 3.52 (non-uniform decelerating).

(a) S-60-933

(b) S-25-931

(c) S-10-932

(d) S30-932

Figure 6. Equilibrium $\phi(T')$ in unsteady flow

Unsteady flow was divided into four cases according to the gradient, and equilibrium T' was calculated after selecting equilibrium $\phi(T')$. Figure 6 is a graph that calculates equilibrium T' after obtaining equilibrium $\phi(T')$ from the unsteady flow. For each case in unsteady flow, equilibrium $\phi(T')$ is 0.3642 (S-60-933), 0.399 (S-25-931), 0.408 (S-10-932), 0.562 (S30-932) and when equation (17) is calculated using equilibrium $\phi(T')$, equilibrium T' is calculated as -1.70208 (S-60-933), 1.24195 (S-25-931), -1.1267 (S-10-932), 0.74953 (S30-932).

Put the equilibrium T' values in Equation 19 and Equation 20 and verify the utility of the equilibrium T' -fixed Equation 19 and Equation 20 through the Song data. If the effectiveness of the equation is verified, the T' -fixed method easily enables the calculation of the floor shear stress by average shear stress, and the distribution of shear stress can be easily calculated.

5.3 Shear Stress Distribution of fixed river-specific factors T' in all flow conditions

To verify the effectiveness of Equation 19 and Equation 20 when equilibrium T' was fixed, the equation was verified using the Song data for each flow state. Figure 7 is the distribution of shear stress with equilibrium T' fixed at 1.88 in uniform flow. The actual measurement value of Song data is expressed as a dot and the value obtained through the equation is expressed as a solid line.

Figure 7. Verification of the T' fixed shear stress distribution model with six uniform flows

Figure 8-13 is the distribution of shear stress with equilibrium T' fixed at 0.41 in non-uniform flow (accelerating & decelerating), -1.70208 in unsteady flow (S-60-933), -1.24195 in unsteady flow (S-25-931), -1.1267 in unsteady flow (S-10-932), 0.74953 in unsteady flows (S30-932).

In the uniform flow, the correlation coefficient was 0.9357-0.9927 and the average was 0.9708, so it was judged that the shear stress distribution formula using equilibrium T' would be available in practice.

Figure 8. Verification of the T' fixed shear stress distribution model with six non-uniform flows (accelerating)

In the non-uniform flow(accelerating), the correlation coefficient was 0.9373-0.9956 and the average was 0.9867, so it was judged that the shear stress distribution formula using equilibrium T' would be available in practice.

Figure 9. Verification of the T' fixed shear stress distribution model with six non-uniform flow (decelerating)

In the non-uniform flow(decelerating), the correlation coefficient was 0.8926-0.9821 and the average was 0.9867, so it was judged that the shear stress distribution formula using equilibrium T' would be available in practice.

Figure 10. Verification of the T' fixed shear stress distribution model with unsteady flow of one slope (S-60-933)

In the unsteady flow(S-60-933), the correlation coefficient was 0.8821-0.9494 and the average was 0.9128, so it was judged that the shear stress distribution formula using equilibrium T' would be available in practice.

Figure 11. Verification of the T' fixed shear stress distribution model with unsteady flow of one slope (S-25-931)

In the unsteady flow(S-25-931), the correlation coefficient was 0.7081-0.9822 and the average was 0.8604, so it was judged that the shear stress distribution formula using equilibrium T' would be available in practice.

Figure 12. Verification of the T' fixed shear stress distribution model with unsteady flow of one slope (S-10-932)

In the unsteady flow(S-10-932), the correlation coefficient was 0.7965-0.9895 and the average was 0.9212, so it was judged that the shear stress distribution formula using equilibrium T' would be available in practice.

Figure 13. Verification of the T' fixed shear stress distribution model with unsteady flow of one slope (S30-932)

In the unsteady flow(S30-932), the correlation coefficient was 0.6361-0.979 and the average was 0.8700, so it was judged that the shear stress distribution formula using equilibrium T' would be available in practice.

5.4 Result analysis

Table 3-5 shows the results of shear stress distribution when the equilibrium T' is fixed, and the correlation coefficient is the comparison value with the Song data. Table3 shows the correlation coefficient values of the shear stress distribution and boundary shear stress values, average shear stress values when the equilibrium T' is fixed in steady flow.

Table 3. Result of the T' fixed shear stress distribution model in steady flows

Data Set (uni- form)	Esti- mated $\bar{\tau}$	Esti- mated τ_0	Correla- tion with Measured Data (R^2)	Data Set (non-uni- form)	Estimated $\bar{\tau}$	Estimated τ_0	Correlation with Meas- ured Data (R^2)
S25-Q31	13.5938	20.9637	0.9357	AS00-Q145	50.8294	95.1794	0.995
S25-Q40	14.4964	22.3556	0.974	AS00-Q100	39.7844	74.4972	0.9919
S25-Q60	22.1253	34.1205	0.9765	AS00-Q80	32.0411	59.9977	0.9947
S25-Q90	25.464	39.2692	0.9906	AS-25- Q100	34.7822	65.1306	0.993
S50-Q50	31.811	49.0573	0.984	AS-25-Q80	27.1214	50.7855	0.9906
S50-Q70	38.355	59.149	0.9417	AS-25-Q60	25.0764	46.9562	0.9373
S50-Q90	37.9977	58.598	0.9927	AS-50- Q110	32.8282	61.4717	0.9956

S50-Q110	55.5911	85.7296	0.9739	AS-50-Q80	24.3288	45.5562	0.9877 ₃₄₁
S50-Q130	56.7324	87.4897	0.9755	AS-75-Q100	29.3979	55.0483	0.9915 ₃₄₃
S75-Q50	39.849	61.453	0.976	AS-75-Q80	24.0319	45.0004	0.9927 ₃₄₄
S75-Q65	44.8835	69.217	0.9761	AS-93-Q100	22.8274	42.7448	0.9801
S75-Q120	64.5273	99.5106	0.987	AS-93-Q80	19.8484	37.1666	0.9905
S90-Q60	51.0867	78.7831	0.952	Average	30.24	56.63	0.9867
S90-Q70	54.3894	83.8765	0.9766	DS25-Q90	19.567	26.514	0.974
S90-Q100	74.4912	114.8764	0.9538	DS25-Q70	17.623	23.88	0.9821
S100-Q55	62.1062	95.7768	0.9713	DS50-Q90	41.844	56.702	0.969
S100-Q70	61.0395	94.132	0.9509	DS50-Q70	32.044	43.422	0.953
S125-Q50	56.4195	87.0072	0.981	DS50-Q55	23.993	32.513	0.8926
S125-Q70	72.9085	112.4356	0.9733	DS75-Q80	20.509	27.791	0.9811
S150-Q40	63.1639	97.408	0.9769	DS75-Q60	21.543	29.193	0.9158
S150-Q50	79.4291	122.4913	0.9675	DS90-Q80	37.926	51.393	0.9421
Average	48.59	74.94	0.9708	DS90-Q70	25.868	35.054	0.9143
				Average	26.77	36.27	0.9471

Table 4 shows the correlation coefficient values of the shear stress distribution and boundary shear stress values, average shear stress values when the equilibrium T' is fixed in unsteady flow (S-60-933, S-10-932).

Table 4. Result of the T' fixed shear stress distribution model in unsteady flows (S-60-933, S-10-932)

Data Set (S-60-933)	Estimated $\bar{\tau}$	Estimated τ_0	Correlation with Measured Data (R^2)	Data Set (S-10-932)	Estimated $\bar{\tau}$	Estimated τ_0	Correlation with Measured Data (R^2)
1	13.3515	36.6228	0.942	2	18.6512	45.7097	0.9058
3	12.765	35.0139	0.927	6	18.8067	46.0907	0.8829
5	12.3861	33.9747	0.9077	10	18.164	44.5157	0.8463
7	12.26	33.6288	0.8967	14	17.5232	42.9452	0.8091

9	12.5924	34.5407	0.9018	18	17.6597	43.2796	0.7965
11	13.2115	36.2386	0.9068	22	19.102	46.8145	0.8234
13	14.0294	38.4821	0.9129	26	22.0095	53.9399	0.884
15	14.9653	41.0495	0.913	30	25.7913	63.2084	0.9293
17	15.8895	43.5845	0.9081	34	29.3796	72.0024	0.9658
19	16.6091	45.5583	0.915	38	30.5494	74.8694	0.9816
21	17.2244	47.246	0.8997	42	29.9873	73.4917	0.9857
23	17.3781	47.6677	0.895	46	26.4185	64.7454	0.9872
25	17.1052	46.919	0.8875	50	21.5012	52.6944	0.9895
27	16.5349	45.3548	0.8841	54	16.6244	40.7424	0.9863
29	15.5734	42.7173	0.8914	58	13.2981	32.5906	0.9851
31	14.5082	39.7955	0.8821	62	11.5639	28.3403	0.9663
33	13.3834	36.7104	0.8885	66	11.5722	28.3607	0.9378
35	12.3595	33.9018	0.8949	70	13.0318	31.9379	0.9226
37	11.556	31.6978	0.8995	74	15.2845	37.4586	0.92
39	11.0419	30.2876	0.9017	78	17.4394	42.7398	0.9188
41	10.8787	29.8398	0.905	Average	19.7179	48.32387	0.9212
43	11.0777	30.3859	0.9119				
45	11.5769	31.7551	0.9169				
47	12.3093	33.7641	0.9224				
49	13.1547	36.083	0.9372				
51	13.8743	38.0569	0.9428				
53	14.4074	39.5191	0.9461				
55	14.4849	39.7316	0.948				
57	14.2988	39.2212	0.9494				
59	13.8578	38.0114	0.9485				
Average	13.82151	37.912	0.912786667				

Table 5 shows the correlation coefficient values of the shear stress distribution and boundary shear stress values, average shear stress values when the equilibrium T' is fixed in unsteady flow (S-25-931, S30-932).

Table 5. Result of the T' fixed shear stress distribution model in unsteady flows (S-25-931, S30-932)

Data Set (uniform)	Estimated $\bar{\tau}$	Estimated τ_0	Correlation with Measured Data (R^2)	Data Set (non-uniform)	Estimated $\bar{\tau}$	Estimated τ_0	Correlation with Measured Data (R^2)
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2	14.4096	36.1079	0.7821	1	22.9458	40.8373	0.9086
6	13.3577	33.4721	0.7473	3	23.2016	41.2926	0.8722
10	12.3821	31.0273	0.7297	5	23.6078	42.0155	0.8071
14	11.5322	28.8977	0.7222	7	24.1684	43.0133	0.7344
18	10.9942	27.5495	0.7081	9	24.7573	44.0613	0.6753
22	10.7277	26.8818	0.7192	11	25.336	45.0912	0.643
26	10.7308	26.8895	0.7505	13	25.9224	46.1347	0.6361
30	10.9969	27.5563	0.7849	15	26.5118	47.1839	0.66
34	11.4958	28.8064	0.833	17	27.195	48.3998	0.7098
38	12.2102	30.5965	0.8816	19	28.0693	49.9558	0.7762
42	13.0574	32.7195	0.9249	21	29.5991	52.6783	0.8253
46	13.6674	34.2481	0.9482	23	30.7095	54.6545	0.8885
50	14.1156	35.3713	0.9632	25	32.4886	57.8208	0.9104
54	14.3411	35.9362	0.9719	27	34.4051	61.2317	0.9151
58	14.2989	35.8306	0.9788	29	36.3447	64.6836	0.9251
62	14.1939	35.5675	0.9796	31	37.7507	67.186	0.9277
66	13.3842	33.5385	0.9822	33	38.6511	68.7884	0.9304
70	12.6193	31.6216	0.9814	35	38.8448	69.1332	0.9302
74	11.7884	29.5397	0.9768	37	38.6861	68.8508	0.9299
78	11.0544	27.7004	0.9667	39	38.1294	67.86	0.9291
82	10.5681	26.4818	0.9535	41	36.8087	65.5095	0.9214
86	10.3358	25.8997	0.9282	43	34.5136	61.4247	0.9542
90	10.3225	25.8662	0.9003	45	33.3527	59.3586	0.9156
94	10.6904	26.7882	0.8743	47	31.4027	55.8882	0.9363
98	11.3414	28.4195	0.8539	49	29.2538	52.0639	0.9477
102	12.2715	30.7501	0.841	51	27.2581	48.5121	0.9634
106	13.3836	33.5369	0.8362	53	25.4296	45.2578	0.9754
110	14.6087	36.6068	0.8368	55	24.173	43.0214	0.979
114	15.7046	39.353	0.8304	57	23.2591	41.3949	0.9545
118	16.6228	41.6537	0.8311	59	22.7108	40.419	0.9164
122	16.857	42.2406	0.8328	61	22.6111	40.2416	0.8893
126	17.4177	43.6456	0.8287	63	22.5142	40.0691	0.8653
130	16.9344	42.4346	0.8235	65	22.7962	40.5711	0.884
134	16.4578	41.2404	0.8082	67	22.8309	40.6329	0.8987
138	15.5513	38.9689	0.8012	69	22.8384	40.6461	0.9128
Average	13.15501	32.96413	0.860354	Average	28.83078	51.31096	0.869954

Basically, the steady flow shows a high correlation coefficient value and proves the utility of the shear stress distribution equation. However, the unsteady flow shows a lower mean correlation value compared to the steady flow. Looking at the results of the unsteady flow in detail, notice that not all areas are observed with low correlation, but with high and low correlation. This indicates that the low correlation coefficient value from the unsteady flow is the error caused by the observation of the data. In practice, using point shear stress values calculated based on accurate observations can reduce error and have higher correlation coefficient values.

Based on the advantages of being able to express and easily obtain shear stress distribution by entropy shear stress distribution using equilibrium T' , it is suggested that the contents of tractive force and critical tractive force with unclear accuracy can be revised to shear stress in Korea's river design standards.

6. Discussion

Proposed equation has proved to show reasonable results. From Figures 1 to 3, in most cases, there is a good match between Song's observed data and shear stress values from proposed model. It was confirmed that shear stress distribution estimated are accurate showing average of 0.9780 correlation coefficient for forty-two types of steady open channel flows. And in four cases where the flow is unsteady, the mean value of the correlation coefficient was found to be 0.9084. The estimated values are not from empirical formula but from theoretical method which has a great meaning for open channel fields.

Using the estimated boundary shear stress and mean shear stress from all forty-two runs, entropy parameter T was analyzed as 1.629. Figure 4 was plotted using the relationship between bed shear stress and mean shear stress, Equation (17), which seems to have the tendency for equilibrium state. Equilibrium state of velocity distribution was studied previously by Chiu [35-36] which shows similar results.

The aforementioned discussion delineates that entropy-based model on shear stress, is able to describe the characteristics of shear stress from the frictional force caused by viscosity and fluctuating fluid velocity in open channel turbulent flow. Proposed equation has proved to show reasonable results.

Based on the parameters calculated in Tables 3 to 5, check the mean value of the correlation coefficient in the results of the shear stress distribution equation for each flow, 0.9708 for the uniform flow, 0.9867 for the non-uniform flow(accelerating), 0.9471 for the non-uniform flow(decelerating), and 0.8604-0.9212 in the unsteady flow for each gradient. Based on these results, it was confirmed that the shear stress distribution formula using river-specific factors can be available in practice.

Of course, part of the unsteady flow shows lower correlation coefficient values, but the correlation coefficient values are error generated from observations. When measuring the distribution of point shear stress for practical use, the conclusions based on accurate observations are determined to reduce the error further and obtain high accuracy shear stress distributions and results.

The estimated values are not from empirical formula but from theoretical method which has a great meaning for open channel fields. However, there are some minor limitations to this study. Some observed data, such as shear stress distribution and vertical depth from the bed, should be known in order to estimate the model. In other words, there needs to be some information in order to use this model. In addition, the basic shape of the model depends on PDF of shear stress and Equation (21). This is important because in complex flows such as unsteady flow, it can be difficult to show its distribution exactly, especially if the maximum velocity occurs below water surface. The model is based on probability and statistics which does not consider basic hydraulics in the beginning, but only depends on constraints and data. And because the accuracy of the observed data can affect the correlation value of the distribution equation, accurate point shear stress data must be obtained for use in practice.

7. Conclusion and Proposition

Although the tractive force and critical tractive force formulas used in the current Korea's river design standards are simple to obtain, the tractive force is a formula using a factor that is difficult to obtain, which is difficult to calculate the exact value. In addition, critical tractive force is a value obtained empirically, which also has the disadvantage of difficulty in calculating the exact value. However, in order to use the existing shear stress formula, shear stress can be calculated only with energy gradient, a factor very difficult to obtain, so Korea's river design standards use tractive force and critical tractive force instead of shear stress.

Therefore, using the entropy concept, this study proposes shear stress distribution and boundary layer shear stress, which can be applied in open channel flows. To determine how well the shear stress model fits with the observed data, Song's data was used. From this aspect, this paper validated with wide range of forty-two runs of experimental data published in the literature. Results show that the utility and reliability of Equation (19) and (20), in which the mean shear stress is considered, then using the qualified shear stress distribution, predicts the shear stress distribution over the whole flow depth of open channel turbulent flows.

Furthermore, based on equations proven utility and reliability, this paper proposes a method that can be easily used in practice by obtaining river-specific factor T , and suggest that the contents of tractive force and critical tractive forces with shortcomings in terms of accuracy in the Korea's river design standards are revised to the shear stress distribution method presented in this study. The results of the distribution of shear stress after the calculation of river-specific factors T were also considered to be highly correlated, and thus it was determined that it could be used in practice.

However, there is no best entropy parameter T value for every run. This means, we need to somewhat reach an agreement with solutions. To improve this, very precise methods must be applied in order to enhance the results. One of these methods can be optimum technique. Optimization techniques such as genetic algorithms or harmony search can be applied for searching better solutions to these kinds of problems, where we might even be getting closer to what we are seeking.

There have been very few studies about shear stress in open channel recently. It seems that there are only few models that can estimate shear stress distribution. Some of the models require very difficult parameters to calculate or even obtain or even have hypothesis which eventually makes the limit of its formula. And other models have many parameters to estimate, but the features of the proposed model have only two parameters, the entropy parameter (T) and unknown boundary shear stress or mean shear stress.

In addition, if the river-specific factors T are calculated for practical use based on the proposed model, the boundary shear stress, which is an important river factor for river design, can be calculated immediately, and shear stress distribution can be easily calculated. Proposed model can be used regardless of the shape or flow of the river (except for unsteady flow). Nevertheless, in the future research, our model has to be compared with some shear stress model.

Analysis has limitations but the results appear to be useful. If the point shear stress in the open channel and vertical depth from the bed are given, the shear stress distribution can be estimated simply from the model which will show high availability when designing or managing the open channel. In addition, the boundary shear stress can be estimated easily without the energy gradient when calculating the boundary shear stress in an open channel.

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Figures

Figure 1

Verification of the proposed shear stress distribution model with six uniform flows

Figure 2

Verification of the proposed shear stress distribution model with six accelerating non-uniform flows

Figure 3

Verification of the proposed shear stress distribution model with six decelerating non-uniform flows

Figure 4

Verification of the proposed shear stress distribution model with unsteady flow of one slope (S-25-931)

Figure 5

Equilibrium $\tau(\tau')$ in steady flow

(a) S-60-933

(b) S-25-931

(c) S-10-932

(d) S30-932

Figure 6

Equilibrium $\phi(T')$ in unsteady flow

Figure 7

Verification of the $\bar{\tau}$ fixed shear stress distribution model with six uniform flows

Figure 8

Verification of the τ^* fixed shear stress distribution model with six non-uniform flows (accelerating)

Figure 9

Verification of the τ^* fixed shear stress distribution model with six non-uniform flow (decelerating)

Figure 10

Verification of the τ^* fixed shear stress distribution model with unsteady flow of one slope (S-60-933)

Figure 11

Verification of the τ^* fixed shear stress distribution model with unsteady flow of one slope (S-25-931)

Figure 12

Verification of the ∇ fixed shear stress distribution model with unsteady flow of one slope (S-10-932)

Figure 13

Verification of the τ fixed shear stress distribution model with unsteady flow of one slope (S30-932)