The Special theory of relativity in different media(Ⅱ)

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The Special theory of relativity in different media(I)

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Abstract

In this paper, the special theory of relativity in different media is established, based on the fundamental invariant of the space-time four-dimensional space $x^2 + y^2 + z^2 - c^2 t^2 = x'^2 + y'^2 + z'^2 - c'^2 t'^2$. First of all, the inertial coordinate system is strictly defined in mathematical language. The inertial coordinate system that uses the actual measured different speeds of light as the limit speed still retains its most basic characteristics as an inertial coordinate system. Then, the space-time coordinate transformation and velocity transformation formulas between inertial coordinate systems with different light velocity are derived. These results not only break through the limitation of "vacuum", but also all are exactly the same as the conclusions of the traditional special theory of relativity when $c' = c$; and when $c' \neq c$ give the new physical content. This all lifted the threat of the theory of relativity by the speed of light experiment, making $c' = c$ and $c' \neq c$ both inclusively under the basic point of view of the theory of relativity; which will inevitably broaden the way of using relativity to deal with physics problems and clarify many problems left over in the study of relativity. The article discusses the problem of relativistic kinematics involving the measurement of time and space, correctly interprets the effects of "ruler contraction" and "clock retardation", and uncovers and correctly answers the "clock paradox" that accompanied the birth of relativity. For two motion systems S and S', that are separated from each other by constant velocity, at any time and where, the product of the proper time elapsed evenly and uniformly and the speed of light in the respective system are equal, $c\tau = c'\tau'$; and the product of the coordinates time read out in observing and recognizing the other party's proper time and the speed of light in the respective system are also equal, $ct = c't'$. It is confirmed that the product of any moving individual's uniform disappearance proper time and its measured speed of light remain unchanged; and the proper time cannot be determined purely by the individual's subjective way. Deduced the uncertain relationship between the proper time and the coordinate time for an inertial coordinate system which was not noticed by the traditional special theory of relativity. Remind the practical astronomy workers who do the time measurement and the time service work to understand that it is impossible to equate practical scientific coordinate time and the proper time of ideal uniform disappearance (the so-called "Ephemeris Time"). Thereby pay attention to

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the impact of this uncertain relationship on the time measurement and the time service work, and propose ways to verify. Subsequent work will use this expanded special theory of relativity to conduct a comprehensive review of related physics, which will inevitably extend to issues that have not been or cannot be examined by traditional special theory of relativity.

Key words: Special relativity; Inertial coordinate system; Stick contraction and clock retardation effect; the uncertain relationship between the proper time and the coordinate time

1 Introduction

The special theory of relativity has been around for more than a century, it has been thought that it has a "light speed unchanged" basic assumption, many excellent experimental physicists arranged a variety of experiments to verify the correctness of this basic hypothesis. Many worry about whether one day such experiments will show a change in the speed of light, threatening to shake the foundations of special relativity [1] 、[2] 、[3]. D. C. Miller in 1933 [4] and W. Kanto in 1962 [5] were reported the opposite of these experiments separately, and there was tension and commotion in physics. Einstein was never believed that his theory of relativity was based on early optical experiments by Michelson-Morley and others, nor that the results of such optical experiments would one day truly threaten relativity. [6]

The core theory of relativity is to understand and master the laws of physics from the Covariance of the space-time four-dimensional geometric space. Applying four-dimensional Riemannian geometry to combine the physical quantity in the laws of physics, individually or in combination into scales, vectors, tensors, and high-order tensors in four-dimensional geometry, establish the relationship between scales, vectors and tensors in geometry based on the mathematical relationship between the quantities of physics established in physics. Geometry must rely on a coordinate system to deduce the argument, and the geometry of the deduction argument does not vary with the coordinate system chosen. In this way, the mathematical equations connecting these physical quantities have a very simple and symmetric covariant form. The most basic invariant scalar in this four-dimensional Riemannian geometric space is the distance of the space-time four-dimensional space, that is:

$$s^2 = x^2 + y^2 + z^2 + w^2 = x'^2 + y'^2 + z'^2 + w'^2 = s'^2 ; \quad w = ict , \quad w' = ict' .$$

Here the invariant constant " c " in relation to the coordinate of time t is the speed of light in the vacuum, which is a constant in the basic invariant degree of the four-dimensional Riemann space. It's also a source of constant speed of light! The linear transformation of space-time coordinates that Satisfies the fundamental invariant is the Lorentz transformation. The speed of light " c " in a vacuum is the speed at which electromagnetic waves travel in a vacuum. It's an important conclusion to the Maxwell equation group that summarizes all the classical electromagnetic theories; According to the theory of relativity, the mathematical form of any physical law should not vary

with the coordinate system. Therefore, the speed of light in a vacuum is constant " c " in any direction relative to any inertial system should be a basic theorem of relativity. It cannot be regarded as a basic hypothesis of the theory of relativity, nor can it be mistakenly thought that the theory of relativity is based on this basic hypothesis, and it has been luckily confirmed by existing experiments.

Although Einstein's special theory of relativity uses the speed of light in a vacuum as an invariant constant c , it is based on the viewpoint of the basic invariant of the four-dimensional space distance of Riemannian geometry in four-dimensional space, and it is not based on the "positive" results of the Michelson-Morley experiment that he never noticed; One day such optical experiments have detected changes in the speed of light c , which is enough to threaten the basis of relativity.

On the one hand, the theory of relativity has already achieved glorious achievements that pushed human civilization into the era of nuclear energy, and on the other hand, it may basically be threatened by optical experiments at any time. This is a very uncomfortable embarrassing situation. Is it possible to base the special theory of relativity on a broader and universal basis so that it can completely get rid of the threats from such optical experiments?

If you want to ask: Is there any unspecified basis and essential notes or conditions for the Lorentz transformation formula established using the speed of light c in vacuum as an invariant constant? For example, does this space-time coordinate transformation have to be valid for the inertial system in a vacuum, or does it not have this limit? Any relativist scholar would hesitate to answer no. Strangely, the inferences derived are actually different. If the Lorentz transformation formula can be applied to space-time coordinate transformation of various inertial systems in vacuum, it can also be applied to space-time coordinate transformation of inertial systems in vacuum to inertial systems in media. Then by doing this calculation, the speed of light in the medium is extrapolated, whether it is the phase speed or the energy propagation speed, it is not unchanged with the difference of the speed of the inertial coordinate system; it is clearly changed! And they all become sufficiently equivalent to the secondary value of $\frac{v}{c}$. [7] That is to say, in the inertial system S in the vacuum outside

the medium, to see the speed of light in the medium that travels at a uniform speed varies with the speed of travel. The so-called "constant speed of light" in any inertial coordinate system is limited to the absence of the medium. It is correct only in vacuum. On the other hand, in the S' system filled with static media to observe the speed of light in the vacuum system S outside the medium, although the result obtained by the Lorentz transformation is c , this is only the special speed addition of special relativity to ensure the limited maximum added numerical value; not the speed of light actually observed by the observer in the static medium S' system. The refractive index n of the light in the medium is greater than 1, so the speed of light that can be actually observed in the inertial system S' that is stationary with the medium is $\frac{c}{n}$, which is less than c . This clearly shows the actual difference: in the system S , c is the physical quantity that can be achieved in reality, and it is also the maximum speed value that

can be added but limited by the speed addition of special relativity; and in the S' system but otherwise, although c is also a finite maximum value that meets this special speed addition and subtraction, it is a physical quantity that is never realistic.

The actual finite maximum value is less than c , that is $\frac{c}{n}$. The speed of light in the S' system observed by the system S is different due to the speed of movement v of the S' system, but it can still achieve c when the v approaches c ; but the speed of light in the S system observed by the S' system can only be reached $\frac{c}{n}$. In any case, it can not reach c . So how should the threat that this realistic difference might pose to the special theory of relativity be lifted?

Considering the above-mentioned coordinate transformation that keeps the "four-dimensional space distance" unchanged, it can be used not only for space-time coordinate transformation between inertial systems in vacuum, but also for inertial systems S in vacuum to inertial systems S' in media. The S' system can be in the medium or under other special physical conditions. At this time, the speed of light measured in the S' system is no longer " c ", but another value " c' ". Can you replace $w' = ict'$ with $w' = ic't'$? The invariant of the linear transformation of space-time coordinates between S and S' becomes:

$$s^2 = x^2 + y^2 + z^2 + w^2 = x'^2 + y'^2 + z'^2 + w'^2 = s'^2; \quad w = ict; \quad w' = ic't'$$

The answer is yes.

First of all, from the view of Riemannian geometry, it can be considered that (x, y, z, t) space-time four-dimensional space is installed in a five-dimensional (s, x, y, z, t) or more multi-dimensional space, becoming a four-dimensional curved surface, $s^2 = x^2 + y^2 + z^2 - c^2 t^2$; taking $s = \text{constant}$, which is equal to taking the three-dimensional tangent plane of the four-dimensional curved surface at a fixed point on the s -axis, which is $x^2 + y^2 + z^2 - c^2 t^2 = \text{constant}$. In the real physical world, there is at least a five-dimensional space background. For example, "temperature" is another basic dimension of physics except space-time. Since there is a five-dimensional or more multi-dimensional space as the background, such four-dimensional surfaces can become "family" and more than one; taking three-dimensional tangent planes at the same fixed point on the s -axis is distinguished by the difference of c . This makes c a parameter that distinguishes individual surfaces in the family. The coordinate transformation of Einstein's special theory of relativity is the coordinate transformation between two points on a particular surface denoted by c in this family. Of course, it is possible to introduce coordinate transformation between two considerable points on two specific surfaces marked with c and c' in this family.

Secondly, the importance of the speed of light in the space-time coordinate system is because it is directly related to the time measurement in the coordinate system. The basis of the time measurement is the "simultaneity" of the two-site events, the standard clock placed at each point in the coordinate system must be adjusted in sync, forming the "proper time" of the coordinate system, and this adjustment synchronization is performed by the actual propagated optical signal. The inertial

system S in vacuum can use the speed of light in vacuum " c " to adjust the synchronization of each point standard clock to determine the "proper time" of the coordinate system. The inertial system in the medium S' can only be determined the "proper time" by its actually measured speed of light in the medium " c' ". If the measurement unit of time t' in the S' system is not mandatory (for example, it must be the same as the measurement unit of time t in the S system), that is, it is not mandatory that each inertial system has the same proper time, then " c ". It is of course possible to change to another value " c' ". Because the speed of light in the S system is " c ", and the speed of light in the S' system is another value " c' ", not only does it not violate the most basic principle of the theory of relativity; but also avoids the artificial forced unifying the proper time. Allowing the speed of light in the S system and the other different S' systems $c \neq c'$, the change and the changeless of the speed of light, that is, $c \neq c'$ and $c = c'$ are both Parallelism included under the basic theory of relativity, inevitable It will greatly broaden the way of dealing with physics problems from the point of view of relativity, and clarify many problems left over in the study of relativity.

This paper is the first part of "Special Relativity in Different Media". First of all, the inertial coordinate system is strictly defined in mathematical language. Then, use $x^2 + y^2 + z^2 - c^2 t^2 = x'^2 + y'^2 + z'^2 - c'^2 t'^2$ as the basic invariant of linear transformation to derive the transformation relationship of space-time coordinates between two inertial coordinate systems with different speeds of light as the limit speed and the foundation of "the special theory of relativity with variable speed of light" (or "the special theory of relativity in different Media") is established. Then, the relativistic kinematics issues involving time and space measurement were discussed in depth, correctly interpreted the effects of "ruler contraction" and "clock retardation", and uncovered and correctly answered the "clock paradox" that accompanied the birth of relativity. Deduced the uncertain relationship between the proper time and the coordinate time that the traditional special theory of relativity has never noticed; and proposed a basic method for verifying this relation in the time service of the National Astronomical Observatory. In the next article 《The Special Theory of Relativity in Different Media (II)》, the theory of this article will be used to deal with physics problems in different media, which can obviously overcome the difficulties and contradictions encountered by traditional special theory of relativity in dealing with these problems.

2 The inertial coordinate system

The fundamental difference between the special theory of relativity and the classical physics is that it negates the existence of a signal with infinite propagation speed, and chooses the speed of light in a vacuum as a signal to synchronize the standard clocks and calibrate time. If the limitation of vacuum is removed and the actual measured light speeds in different inertial coordinate systems are used as the maximum transmission signals in the respective coordinate systems to synchronize the standard clocks at various spatial points and calibrate time in the respective systems, then these inertial coordinate systems must have different proper Space-time metrical

unit. It is easy to prove that the inertial coordinate system with different proper space-time metrical units still retains its most basic characteristics as an inertial coordinate system.

Inertial coordinate system is a type of coordinate system with certain physical characteristics. It is necessary to understand the inertial coordinate system in essence, so as to give it a strict mathematical description.

[Definition 1] Each real physical individual or combination can describe a physical phenomenon with an accompanying space-time coordinate system, called a proper coordinate system, or coordinate system for short.

[Definition 2] Coordinate systems can be based on the numerical relationship of a specific number of physical quantities observed by each other, such as speed, to identify coordinate systems with certain physical properties and characteristics, such as inertial coordinate systems.

[Preparation Theorem] Among the various physical speeds of the real physical world that any coordinate system S can observe, there is a finite maximum value called the upper limit speed.

[Theorem 1] If the upper limit speeds observed by any two coordinate systems S and S' do not belong to the same real physical speed, the speed does not have a specific property that can mark the characteristics of the coordinate system; that is, there is no inertial coordinate system.

[Theorem 2] If an inertial coordinate system exists, there are at least two.

[Theorem 3] If S and S' are inertial coordinate systems, then the velocity values field observed in S and the velocity values field observed in S' must have two pairs of fixed values equivalent to each other (not necessarily equal). That is: (1) The upper limit c in S is equivalent to the upper limit c' in S' ; (2) The speed value v of S' measured by S is equivalent to the speed value v' of S measured by S' . c 、 c' 、 v 、 v' are fixed values, regardless of sign.

[Theorem 4] If the equivalent values of the two velocity values fields in S and S' are equal, then the two element merging rules of the field, namely addition and subtraction and multiplication and division, still make the combined element values equal. Therefore, the sufficient and necessary conditions for S and S' to be an inertial coordinate system in classical physics are: $c = c' \neq \infty$, $v < c$, $v' < c'$, $v = v'$.

[Inference 1] Retain the equivalence relationship of the addition and subtraction of the field (does not exclude equals), and adopt the equivalence relationship of the multiplication and division of the field, without damaging the essence of S and S' as the inertial coordinate system.

[Theorem 5] The sufficient and necessary conditions for S and S' to be a pair of inertial coordinate systems are: (1) There are equivalents for each, the values observed for the same real physical velocity are limited, maximum and fixed upper limits, namely c and c' , $c < \infty$, $c' < \infty$; (2) The observed speeds v and v' are equivalent, $v < c$, $v' < c'$; (3) $\frac{v}{c} = \frac{v'}{c'} = \text{constant}$; (4) The following [Theorem 6] is correct.

[Theorem 6] If S and S' are inertial coordinate systems and S'' and S''' are inertial coordinate systems, then S and S'' (or S''') are also inertial coordinate systems.

[Theorem 7] There is a set of inertial coordinate systems in the physical world that are pure and do not incorporate other properties.

[Inference 2] For the inertial coordinate system in classical physics determined by [Theorem 4], [Theorem 6] and [Theorem 7] are not applicable.

According to the above summary, the so-called inertia feature is not only that the speed (motion state) referred to in empirical consciousness is constant. This empirical consciousness comes from the appearance of the observed phenomenon, and its observed coordinate system must be compared with the fixed maximum finite speed observed by itself; of course, it is undeniable that the constant speed of the appearance is determined by the inherent characteristic nature. However, the characteristic of inertia should be more intrinsic basic invariance of "habitual" to actively observe physical phenomena, that is, keeping the space-time metrical unit unchanged. Being able to keep the space-time metrical unit of the observation coordinate system unchanged is its own inertia, and can correctly rely on it to observe the inertia displayed by the real physical world. This is why the inertial coordinate system must exist in pairs, and any two inertial coordinate systems that belong to different pairs can also be paired as a manifestation of existence. Each inertial coordinate system can maintain its inherently unchanged space-time metrical unit without being uniformly equal. This is inertia, and it is reflected in the objectively observed speed. When the speed changes, the internal metrical unit of space-time also changes, and vice versa. But it cannot be assumed that inertia is simply equal to constant speed.

The space-time metrical unit of any inertial coordinate system has the property of "proper", and the laws of physics that exist objectively and truly must be expressed in terms of the proper space-time metrical units of individuals or combinations that express this law. However, the recognition and determination of the law will inevitably be described by the proper space-time metrical unit of other inertial coordinate systems. For individuals or combinations that express this law, the latter is called the observation coordinate space-time metrical unit, as in general literature "Coordinate Time". The traditional special theory of relativity has been successfully established the relationship in cognition and being recognized between two inertial coordinate systems existing according to [Theorem 4] above, based on $s^2 = x^2 + y^2 + z^2 - c^2 t^2 = x'^2 + y'^2 + z'^2 - c'^2 t'^2 = s'^2$. According to the basic principles of relativity, of course, the space-time coordinates transformation relationship between the two inertial coordinate systems that exist according to [Theorem 5] above, could be established based on $s^2 = x^2 + y^2 + z^2 - c^2 t^2 = x'^2 + y'^2 + z'^2 - c'^2 t'^2 = s'^2$ ($c' \neq c$), so that it can also be ensure that the laws of physics derived from epistemology advance to the laws of physics that exist objectively and truly.

Einstein's special theory of relativity and classical physics both use the inertial coordinate systems existing according to [Theorem 4]. They both unify the proper space-time metrical unit of the physical world, thus narrowing the communication

channels between cognition and objective real existence. The difference between Newton and Einstein is that the former does not even distinguish the difference between the proper space-time metrical unit and the observation coordinate space-time metrical unit. It is determined that there are absolute space and absolute time; the latter seeks the relationship between two identical proper space-time metrical unit that cannot be consistent with each other in terms of mutual understanding and being recognized, and calls the one who actively recognizes the space-time metrical unit of observation coordinates.

Regardless of how strict the technology for scientists to unify the observation of space-time metrical units on the earth is, in fact, they can only unify the observation of space-time metrical units in any case, and it is not possible to force the unification of the proper space-time metrical unit of individual research subjects or combinations, because this is an inertia that cannot be forced to change. A special force field inside the nucleus should have a special proper space-time metrical unit, which should be significantly different from the proper space-time metrical unit within the outer "electron cloud" of the nucleus. ? Can there be the same proper space-time metrical unit as the ground in a laboratory on a spaceship carrying out interplanetary navigation? Therefore, to apply the theory of relativity in order to analyze the experimental data and search for the objective and real physical laws, it is best to consider the application of the inertial coordinate system in line with the above [Theorem 5]. In short, the theory of relativity established by $c = c'$ admits that there is absolutely the same proper space and proper time; while the theory of relativity established by $c \neq c'$ believes that there are different proper space and proper time. The former insists on unity of inertia, but can not remove the uniform doping within inertia; the latter always maintains the purity of inertia. Although the former may have some practical conveniences, it is easy to flow into empiricism, so that the physical laws of physics are all the products of empiricism, and it is easy to make people feel exhaustive; the latter can make people to continue chase the laws of physics that exist in the real physical world expressed in different proper space-time metrical unit continue and to be pursued incessantly.

3 Transformation of inertial coordinate systems with different speeds of light

Let S and S' be the two inertial coordinate systems that exist according to the previous section [Theorem 5]. In S , the observed speed of S' along the x-axis of S is v , and the speed of light is c ; and in S' , the observed speed of S along the x-axis of S' is v' , and the speed of light is c' . Let us, moreover, assume that the origin O' of S' coincides with the origin O of S at the time $t = t' = 0$. The linear transformation of space-time coordinates from S' to S can be written as:

$$\begin{aligned}
x' &= \alpha(x - vt) \\
y' &= y \\
z' &= z \\
t' &= \beta t + \gamma x
\end{aligned} \tag{1}$$

Coordinate transformation must meet:

$$s^2 = x^2 + y^2 + z^2 - ct^2 = x'^2 + y'^2 + z'^2 - c'^2 t'^2 = s'^2 \tag{2}$$

Substitute equation (1) into the right side of equation (2), and compare the coefficients of similar terms to solve α, β, γ ; and then substitute return to equation (1) to obtain the transformation of space-time coordinates from S' to S :

$$\begin{aligned}
x' &= \frac{x - vt}{\sqrt{1 - v^2/c^2}} \\
y' &= y \\
z' &= z \\
t' &= \frac{c}{c'} \frac{t - \frac{v}{c^2} x}{\sqrt{1 - v^2/c^2}}
\end{aligned} \tag{3}$$

Compared with the Lorentz transformation of the traditional special theory of relativity^[8], only the right side is multiplied by $\frac{c}{c'}$, and important key issues immediately appear. The inverse relations which are obtained by solving the four equations (3) with respect to the variables x, y, z, t are:

$$\begin{aligned}
x &= \frac{x' + \frac{c'}{c} vt'}{\sqrt{1 - v^2/c^2}} \\
y &= y' \\
z &= z' \\
t &= \frac{c'}{c} \frac{t' + \frac{v}{cc'} x'}{\sqrt{1 - v^2/c^2}}
\end{aligned} \tag{4}$$

In addition, the transformation of space-time coordinates from S to S' can also be obtained by the same method for equation (3):

$$\begin{aligned}
x &= \frac{x' - v't'}{\sqrt{1 - v'^2/c'^2}} \\
y &= y' \\
z &= z' \\
t &= \frac{c'}{c} \frac{t' - \frac{v'}{c'^2} x'}{\sqrt{1 - v'^2/c'^2}}
\end{aligned} \tag{5}$$

According to the relativistic relative relationship, the inverse solution (4) of the

formula (3) must be exactly the same as the formula (5) after the transposition of S and S' . Therefore, the only condition that the two sets of formulas in (4) and (5) are identical is that:

$$\frac{v'}{c'} = -\frac{v}{c} \quad (6)$$

Equations (3)-(6) are the space-time coordinate transformation relationship between two inertial systems with different speeds of light as the limit speed, or the coordinate transformation relationship of special relativity in different media. Obviously, when $c = c'$, it is the traditional Lorentz transformation. Therefore, it must contain and retain all the conclusions of the traditional special theory of relativity, and give new physical content when $c \neq c'$.

4 Transformation of particle velocities and the speed addition formula

Let the velocity of a moving particle relative to S and S' are $\bar{u}(u_x, u_y, u_z) = (\frac{dx}{dt}, \frac{dy}{dt}, \frac{dz}{dt})$ and $\bar{u}'(u'_x, u'_y, u'_z) = (\frac{dx'}{dt'}, \frac{dy'}{dt'}, \frac{dz'}{dt'})$ respectively.

Differentiate the two sides of formula (3), that is:

$$\begin{aligned} u'_x &= \frac{dx'}{dt'} = \frac{c'}{c} \frac{u_x - v}{1 - \frac{vu_x}{c^2}} = \frac{\frac{c'}{c}u_x + v'}{1 + \frac{u_x v'}{cc'}} \\ u'_y &= \frac{dy'}{dt'} = \frac{c'}{c} \frac{u_y \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{vu_x}{c^2}} = \frac{\frac{c'}{c}u_y \sqrt{1 - \frac{v'^2}{c'^2}}}{1 + \frac{u_x v'}{cc'}} \\ u'_z &= \frac{dz'}{dt'} = \frac{c'}{c} \frac{u_z \sqrt{1 - \frac{v^2}{c^2}}}{1 - \frac{vu_x}{c^2}} = \frac{\frac{c'}{c}u_z \sqrt{1 - \frac{v'^2}{c'^2}}}{1 + \frac{u_x v'}{cc'}} \end{aligned} \quad (7)$$

Equation (6) $\frac{v}{c} = -\frac{v'}{c'}$ has been applied here. Symmetrically, the two sides of (5) are differentiated to obtain:

$$\begin{aligned}
u_x &= \frac{dx}{dt} = \frac{c}{c'} \frac{u'_x - v'}{1 - \frac{v'u'_x}{c'^2}} = \frac{\frac{c}{c'}u'_x + v}{1 + \frac{u'_x v}{cc'}} \\
u_y &= \frac{dy}{dt} = \frac{c}{c'} \frac{u'_y \sqrt{1 - \frac{v'^2}{c'^2}}}{1 - \frac{vu'_x}{c^2}} = \frac{\frac{c}{c'}u'_y \sqrt{1 - \frac{v'^2}{c'^2}}}{1 + \frac{u'_x v}{cc'}} \\
u_z &= \frac{dz}{dt} = \frac{c}{c'} \frac{u'_z \sqrt{1 - \frac{v'^2}{c'^2}}}{1 - \frac{vu'_x}{c'^2}} = \frac{\frac{c}{c'}u'_z \sqrt{1 - \frac{v'^2}{c'^2}}}{1 + \frac{u'_x v}{cc'}}
\end{aligned} \tag{8}$$

This is the transformation of particle velocities in the special theory of relativity with variable speed of light. When $c = c'$, $v = -v'$, it becomes the transformation of particle velocities of traditional special theory of relativity.

If the particle motion is on the xy plane, and $\vec{u}$ and $\vec{u}'$ are all perpendicular to the z axis, and their angles to the x axis are θ and θ' , respectively; then, (7) can be expressed as:

$$u'_x = u' \cos \theta' = \frac{c'}{c} \frac{u \cos \theta - v}{1 - uv \cos \theta / c^2}, \quad u'_y = u' \sin \theta' = \frac{c'}{c} \frac{u \sin \theta \sqrt{1 - v^2 / c^2}}{1 - uv \cos \theta / c^2} \tag{7a}$$

$$u' = \sqrt{u'^2_x + u'^2_y} = u \left(\frac{c'}{c} \right) \frac{\sqrt{1 - 2v \cos \theta / u + v^2 / u^2 - v^2 \sin^2 \theta / c^2}}{1 - uv \cos \theta / c^2} \tag{7b}$$

$$\tan \theta' = \frac{\sin \theta \sqrt{1 - v^2 / c^2}}{\cos \theta - v / u} \tag{7c}$$

when $\theta = 90^\circ$, we obtain:

$$u'_x = u' \cos \theta' = -v \left(\frac{c'}{c} \right), \quad u'_y = u' \sin \theta' = \left(\frac{c'}{c} \right) u \sqrt{1 - v^2 / c^2} \tag{7d}$$

$$u' = \sqrt{u'^2_x + u'^2_y} = u \left(\frac{c'}{c} \right) \sqrt{1 + \frac{v^2}{u^2} - \frac{v^2}{c^2}} \tag{7e}$$

$$\tan \theta' = \frac{\sqrt{1 - v^2 / c^2}}{-v / u} \tag{7f}$$

When $\theta = 0$, we have $\theta' = 0$

$$u' = \left(\frac{c'}{c}\right) \frac{u - v}{1 - uv/c^2} = \frac{\frac{c'}{c}u + v'}{1 + \frac{uv'}{cc'}} \quad (7g)$$

Symmetrically:

$$u_x = u \cos \theta = \frac{c}{c'} \frac{u' \cos \theta' - v'}{1 - u'v' \cos \theta' / c'^2}, \quad u_y = u \sin \theta = \frac{c}{c'} \frac{u' \sin \theta' \sqrt{1 - v'^2 / c'^2}}{1 - u'v' \cos \theta' / c'^2} \quad (8a)$$

$$u = \sqrt{u_x^2 + u_y^2} = u' \left(\frac{c}{c'}\right) \frac{\sqrt{1 - 2v' \cos \theta' / u' + v'^2 / u'^2 - v'^2 \sin^2 \theta' / c'^2}}{1 - u'v' \cos \theta' / c'^2} \quad (8b)$$

$$\tan \theta = \frac{\sin \theta' \sqrt{1 - v'^2 / c'^2}}{\cos \theta' - v' / u'} \quad (8c)$$

when $\theta' = 90^\circ$, then:

$$u_x = u \cos \theta = -v' \left(\frac{c}{c'}\right), \quad u_y = u \sin \theta = \left(\frac{c}{c'}\right) u' \sqrt{1 - v'^2 / c'^2} \quad (8d)$$

$$u = \sqrt{u_x^2 + u_y^2} = u' \left(\frac{c}{c'}\right) \sqrt{1 + \frac{v'^2}{u'^2} - \frac{v'^2}{c'^2}} \quad (8e)$$

$$\tan \theta = \frac{\sqrt{1 - v'^2 / c'^2}}{-v' / u'} \quad (8f)$$

when $\theta' = 0$, we have $\theta = 0$;

$$u = \left(\frac{c}{c'}\right) \frac{|u' - v'|}{1 - u'v' / c'^2} = \frac{\frac{c}{c'}u' + v}{1 + \frac{u'v}{cc'}} \quad (8g)$$

It is easy to see that the above formulas are consistent with: when $u = c$, then $u' = c'$ (or when $u' = c'$, then $u = c$); when $v = c$, then $u' = c'$ (or when $v' = c'$, then $u = c$).

5 Time and space measurement

5.1 The effects of “ruler contraction” and “clock retardation”,

Classical physics denies that the observation and measurement of time and space is related to movement, regardless of the difference between the proper space-time and the observation space-time. The fact that it is difficult to accurately read and correct each other for two standard clocks or standard rulers that move at constant speeds have not been deeply noticed. The theory of relativity makes an important breakthrough in this. It seeks the relationship of mutual understanding and being recognized between two proper space-time coordinate. Call the active to cognize is the observation space-time coordinates.

Suppose that the two inertial frames S and S' along the straight line (x axis) leave each other at constant speed. In S , the observed speed of motion of S' is v , and the speed of light is c ; in S' , the observed speed of motion of S is v' , and the speed of light is c' . In S' , there is a stationary ruler, its length is $l'_0 = x'_2 - x'_1$; this is the true length of the ruler, that is, the proper length. Because both ruler and S' are traveling at a constant speed relative to S , the observer in S must measure the length of the ruler simultaneously ($t_2 = t_1$). In the first formula of (3), take $t_2 = t_1$, then

$$l = x_2 - x_1 = \sqrt{1 - v^2 / c^2} (x'_2 - x'_1) = l'_0 \sqrt{1 - v^2 / c^2} \quad (9a)$$

Symmetrically, if the ruler is stationary in S , its proper length is $l_0 = x_2 - x_1$, both ruler and S are traveling at a constant speed relative to S' . At this time, the observer in S' must measure the length of the ruler simultaneously ($t'_2 = t'_1$). In the first formula of (5), take $t'_2 = t'_1$, then

$$l' = x'_2 - x'_1 = \sqrt{1 - v'^2 / c'^2} (x_2 - x_1) = l_0 \sqrt{1 - v'^2 / c'^2} \quad (9b)$$

(9a) and (9b) are Lorentz contraction formulas. They symmetrically represent the relationship between the two objectively existing proper lengths l'_0, l_0 and the two observation lengths l, l' measured and read by each other. "Contraction" means that the observation length l or l' on the left side of the equation is less than the proper length l'_0 or l_0 on the right side. This is called the "ruler contraction" effect. Due to equation (6), the contraction of the length in two equations is the same, so the special theory of relativity with variable speed of light is still stick to conventions. It is worth noting that the proper length of the objective existence will not be elongated and shortened. The so-called contraction is the observed length when the observation coordinate system are restricted to "simultaneously" measure the amount of moving space length. The length was helplessly shortened.

The measurement of time is slightly more complicated. The relativistic space-time coordinate transformation is a linear transformation that keeps the four-dimensional distance unchanged. Let the space-time coordinates of the two points in the four-dimensional space in the two inertial systems S and S' be: $(x_1, y_1, z_1, t_1), (x_2, y_2, z_2, t_2)$ and $(x'_1, y'_1, z'_1, t'_1), (x'_2, y'_2, z'_2, t'_2)$ respectively, from (2), their constant four-dimensional distance can be written as:

$$\begin{aligned} s^2 &= (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - c^2(t_2 - t_1)^2 \\ &= (x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2 - c'^2(t'_2 - t'_1)^2 = s'^2 \end{aligned}$$

Suppose there is a particle traveling along the x -axis at a constant speed. An

infinitely long arm is installed in the direction perpendicular to its direction of advance, and an instantaneous reflector is installed at any point on the arm to reflect light in the direction of the arm, so light can reflect back to the particle at any time. Then, $\sqrt{-s^2}$ means the optical path that the light emitted by the moving particle at any previous instant (in S t_1 , in S' t'_1), and reflected to itself from the arm at any subsequent instant (in S t_2 , in S' t'_2). Optical path is the speed of light multiplied by time:

$$\begin{aligned} c^2 \tau^2 &= c^2 (t_2 - t_1)^2 - [(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2] \\ &= c'^2 (t'_2 - t'_1)^2 - [(x'_2 - x'_1)^2 + (y'_2 - y'_1)^2 + (z'_2 - z'_1)^2] = c'^2 \tau'^2 \end{aligned} \quad (10)$$

τ is the proper time of the moving particle determined in S , τ' is the proper time of the moving particle time determined in S' . Equation (10) indicates that using any space-time coordinates (x, y, z, t) in S or any other space-time coordinates (x', y', z', t') in S' to measure this distance (optical path) does not change, i.e. $c\tau = c'\tau'$. Of course, this particle can also use the coordinate system attached to it to determine its own proper time τ^0 , the special case is that the particle is regarded as the same position, that is $x_2^0 = x_1^0, y_2^0 = y_1^0, z_2^0 = z_1^0$; the proper time determined in this way is the time at which the stationary particle elapsed and disappeared evenly. Pay attention to the difference between τ 、 τ' in (10) and here τ^0 . In the traditional special theory of relativity, $c = c'$; therefore $\tau = \tau' = \tau^0$. That is to say: the space-time coordinates of any system can accurately measure the proper time of any particle τ^0 that objectively uniformly elapsed; that is, the proper time of any individual A and any other body B can be equally unified. Is this a convenient and convenient method of basic science, or is it the true side of truth? According to our formula (10) is $c\tau = c'\tau'$, not $\tau = \tau'$; that is to say, the proper time of each individuals that objectively uniformly elapsed cannot be determined. The value that can be measured is the product of the respective speed of light and the proper time. Individuals in different physical environments or motion states can have different proper times. Obviously, the traditional special theory of relativity's understanding of the so-called uniform elapsed time has been shoved to the end, and it can no longer be studied. While according to the special theory of relativity with different speeds of light, the value that can be measured and observed is the product of two physical quantities. This is a very important distinction.

For an infinitesimal distance in the four-dimensional space, equation (10) becomes:

$$c^2 d\tau^2 = c^2 dt^2 - [(dx)^2 + (dy)^2 + (dz)^2] = c'^2 dt'^2 - [(dx')^2 + (dy')^2 + (dz')^2] = c'^2 d\tau'^2$$

The above formula can be written as:

$$cd\tau = cdt\sqrt{1-v^2/c^2} = c'dt'\sqrt{1-v'^2/c'^2} = c'd\tau' \quad (11)$$

Here

$$v = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}, \quad v' = \sqrt{\left(\frac{dx'}{dt'}\right)^2 + \left(\frac{dy'}{dt'}\right)^2 + \left(\frac{dz'}{dt'}\right)^2},$$

They are the speeds of the particle measured by the space-time coordinate of S and S' respectively; rather than the speed of motion between S and S' measured each

other. Cannot misuse $\frac{v^2}{c^2} = \frac{v'^2}{c'^2}$ to deduce $cdt = c'dt'$. Equation (11) still only means

that the S system and the S' system respectively measure the proper time τ^0 of another constant velocity linear traveling particle, and the results are also limited to $cd\tau = c'd\tau'$.

If the particle is still attached to the system S , then equation (11) represents the relationship between the proper time $d\tau_0$ of system S and the coordinate time dt' observed by the own space-time coordinates of system S' . Then equation (11) should become:

$$cd\tau_0 = c'dt'\sqrt{1-v'^2/c'^2} = c'd\tau' \quad (12a)$$

The proper time $d\tau_0$ of the system S is the time difference $t_2 - t_1$ at the fixed point $x_2 = x_1, y_2 = y_1, z_2 = z_1$ in the system, which can be obtained immediately from the formula (3):

$$cd\tau_0 = c'(t'_2 - t'_1)\sqrt{1-v'^2/c'^2} = c'dt'\sqrt{1-v'^2/c'^2} \quad (12b)$$

Comparing (12a) and (12b), where v' and v are the speeds of mutual motion measured by each other between the system S and the system S' , so $\frac{v^2}{c^2} = \frac{v'^2}{c'^2}$, the

values of $cd\tau_0$ obtained by the two equations are equal. However, it should be

emphasized that equation (12a) means that the $cd\tau_0$ of S is purely determined by the observation of the system S' , so the relevant quantity is brought with ""; equation

(12b) means a mixed measurement of $cd\tau_0$, which is expressed in the speed of the

system S' actively observed by S and the coordinate time dt' of the system S' , that is, there are "v", "c" without "" (these are physical quantities in the system S), and "c'", "t'" with "" (these are physical quantities in the system S'). It should be interpreted as a uniform result that is purely objectively recognized and subjectively sought to be recognized. Because in order to determine the proper time of an individual, it must

first be determined that it is stationary, which necessarily involves the measurement of each other's speed; therefore, the value $cd\tau_0$ of any individual cannot be determined purely by subjective means! Not to mention the subjective and independent way to determine the proper time of even disappearance τ_0 ! This is an important difference from the traditional special theory of relativity.

Similarly, if the particle is still attached to the system S' , then equation (11) represents the relationship between the proper time $d\tau'_0$ of system S' and the coordinate time dt observed by the own space-time coordinates of system S . Then equation (11) should become:

$$c'd\tau'_0 = cdt\sqrt{1 - v^2/c^2} = cd\tau \quad (13a)$$

The proper time $d\tau'_0$ of the system S' is the time difference $t'_2 - t'_1$ at the fixed point $x'_2 = x'_1, y'_2 = y'_1, z'_2 = z'_1$ in the system, which can be obtained immediately from the formula (5):

$$c'd\tau'_0 = c(t_2 - t_1)\sqrt{1 - v'^2/c'^2} = cdt\sqrt{1 - v'^2/c'^2} \quad (13b)$$

The comparison and analysis of (13a) and (13b) are similar to the comparison and analysis of (12a) and (12b) above, and will not be repeated here.

It is worthy of further statement that the two ends of (11) or (12a) and (13a) express the simplest laws of physics $cd\tau_0 = c'd\tau'$, namely, "For all moving individuals, the product of its measurable speed of light and its evenly elapsed proper time remains unchanged", not just the simple proper time unchanged. This formula also shows that if the laws of physics containing several physical quantities have the same form of mathematical expression in any coordinate system, then the relevant physical quantities must be "proper quantities" with "proper" meaning. This of course refers to the objective existence of the laws of physics. If the quantities of observation coordinates were used to express the same law, its form is not necessarily the same, as in the first half of (12a) $cd\tau_0 = c'dt'\sqrt{1 - v'^2/c'^2}$. It is expressed in the coordinate

time of the system S' . And the first half of (13a) $c'd\tau'_0 = cdt\sqrt{1 - v^2/c^2}$, It is expressed in the coordinate time of the system S . The so-called "clock retardation" means that the $d\tau_0$ (or $d\tau'_0$) on the left side of the two equations is smaller than the dt' (or dt) on the right side. It is also worth noting that the proper time of objective existence does not slow down, and the effect of "clock retardation" is entirely epistemological relationship. The theory of relativity teaches people how to explore the objective existence of the physical world from subjective knowledge. The classical physics before

the theory of relativity must be developed under the guidance of the theory of relativity again. This development is quite laborious, and it is necessary to interleave the understanding of the transformation relationship (such as linear transformation relationship) of several (at least two) physical quantities in different coordinate systems. If (11) or (12a), (13a) can be said to be examples of "existentialism", (12b) and (13b) have more "epistemological" meanings. Care should be taken according to (12a), (12b), (13a), and (13b) to say that the clock becomes faster and slower due to movement.

In addition, when the equation (11) is written as (12a), the first equation in the equation (11) is omitted. Because if let $v = 0$ in the first equation mechanically we obtain $cd\tau_0 = cdt$, this will make people misunderstand that the coordinate time of the system S is its proper time, i.e. $t = \tau_0$. In fact, there is an indeterminate relationship

between the proper time τ_0 of the system S and the coordinate time t which is used to measure the proper time of other coordinate system. When the equation (11) is written as (13a), the second equation in the equation (11) is also omitted, and the reason is the same.

The above discussion on the measurement of space and time shows that the theory of relativity with variable speed of light further reveals the nature of space and time and its relationship with motion than the traditional theory of special relativity. The most important thing is that it reveals that there is an indeterminate relationship between the true proper time in any coordinate system and the coordinate time it uses, which is directly related to the so-called "twin paradox" or "clock paradox" associated with the birth of the theory of relativity. Discussing by the theory of relativity with variable speed of light, this mystery is not a mystery, but the correct evidence of the unfounded indeterminate relationship between τ_0 and t .

5.2 Uncover The "Two-Clock Mystery"

The "Two-clock mystery", also known as "clock paradox" or "twin paradox", is an issue that has been discussed among relativist scholars in various countries since the birth of the theory of relativity for many years. There are many ways to solve the problem, the answers are very different, and there are many references. [9]-[20]

The mystery says this: After two identical ideal standard clocks are adjusted in sync and proofread together, they move apart from each other at a constant speed (it is not necessary for a third person to decide which one to go and which one does not). According to the first half of the relations (12a) and (13a) obtained from the relativistic measurement of time between each other, the observers attached to the two clocks respectively thought that their own system was stationary, and found that the clock of the other party moving at a constant speed slowed down. However, when the relative constant velocity movement stopped, the two clocks returned to the same system. Which clock is really slow? If two clocks are compared to a pair of twin brothers, one

of whom travels into space by a spaceship near the speed of light, which one will become younger when they meet again? The reason why this problem becomes a mystery is that all relative motion must be relative to each other of the systems, not only in the coordinate systems of constant velocity motion of each other, even considering the two-clock stepping and setting up and even turning back, It is not easy to distinguish the essential acceleration from the actual man-made power, and to judge the difference between the host and the guest by the so-called "physical conditions". This involves the core of the general theory of relativity. According to the principle of equivalence, acceleration must also be a relative things between the two systems on the space-time stage. The gravitational field is equivalent to a closed and accelerated laboratory, so both clock can consider themselves to see each other in the "equivalent gravity field ". The special theory of relativity with variable speed of light guarantees the equivalence of any coordinate system. How do you think about me and what conclusion do you draw? I can also treat you likewise and draw the same conclusion. General relativity works and literature deviate from the above-mentioned mutual relative nature (and this is the essence of relativity), and study this problem one-sidedly (rather than symmetrically). Not only does it not help to solve this mystery, but it is easy to draw ridiculous conclusions such as "One day above the sky, the world has been a thousand years " from formal logic.

This mystery involves the fundamentals of the theory of relativity. Uncovering this mystery will inevitably contribute to a correct understanding of the theory of relativity.

The first half of formula (12a) means: The proper time interval of two events that occurred in succession on a fixed point ($x_2 = x_1$) in the system S , measured by the time $\Delta t'$ of the observation coordinate system S' as:

$$c\Delta\tau = (\sqrt{1 - v'^2 / c'^2})c'\Delta t' \quad (12)$$

Symmetrically, the first half of formula (13a) means: The proper time interval of two events that occurred in succession on a fixed point ($x'_2 = x'_1$) in the system S' , measured by the time Δt of the observation coordinate system S as:

$$c'\Delta\tau' = (\sqrt{1 - v^2 / c^2})c\Delta t \quad (13)$$

Equations (12) and (13) symmetrically give the relationship between two objectively existing proper time intervals $\Delta\tau$ (or $\Delta\tau'$) and the time intervals $\Delta t'$ (or Δt) of the two observation coordinate systems reading out each other. Although the time on the left and right sides of the above two equations is the metrical time in their respective coordinate systems, because the time on the right is used to read out as the observation coordinate system, it is called the coordinate time. "Moving clocks move slower than stationary clocks", meaning that the left $\Delta\tau$ (or $\Delta\tau'$) of the equation is smaller than the right $\Delta t'$ (or Δt). This is correct, but it is a rather vague statement. The proper time of the experience of things cannot be changed faster or slower, and it exists objectively and uniformly disappears continuously. The so-called "slowing down" expression is that the static one can only see the objective existing "past time"

of the other moving party, at most the "present time". That is to say, the observers beside the two clocks call to proofread each other, then everyone will find, for example, I saw your clock being at two o'clock at my three o'clock. However, when the two clocks stop relative motion and return to the same system, the two clocks' time should be the proper time of their respective objective existence $\Delta\tau$ and $\Delta\tau'$. Before the relative movement stops, the time recognized by the two parties mutual measured and read $\Delta t'$ and Δt is both the "past time". Each thinks that the other party's clock is slower and the other party's younger is just the relationship between mutual understanding, which shows the helpless deficiencies of human subjective understanding when reflecting the objective real existence. The root of the mystery is: for the above four time quantities $\Delta\tau$, $\Delta\tau'$, Δt , and $\Delta t'$, the theory of relativity only gives two cross-relationships, namely (12) and (13), from which we cannot clearly see the relationship between two proper time quantities $\Delta\tau$ and $\Delta\tau'$, and the two coordinate time quantities $\Delta t'$ and Δt of measuring each other. It cannot be simply assumed that a clock's amount of objective proper time $\Delta\tau$ (or $\Delta\tau'$) at any moment must be the same as the amount of coordinate time Δt (or $\Delta t'$) subjective to measure readout of the other party. Because if one determines $\Delta\tau = \Delta t$, then from (12) and (13) can deduce

$$\Delta\tau' = \sqrt{(1 - \frac{v^2}{c^2})(1 - \frac{v'^2}{c'^2})} \Delta t', \text{ it contradicts to } \Delta\tau' = \Delta t'. \text{ Similarly, if one determines}$$

$$\Delta\tau' = \Delta t' \text{ then from (13) and (12) can deduce } \Delta\tau = \sqrt{(1 - \frac{v^2}{c^2})(1 - \frac{v'^2}{c'^2})} \Delta t, \text{ it contradicts}$$

to $\Delta\tau = \Delta t$. This is the mystery

For space, the two formulas (9a) and (9b) also symmetrically give the relationship between the two objectively existing proper space quantities $\Delta l'$ (or Δl) and the two observation coordinate space quantities Δx (or $\Delta x'$) reading out each other :

$$\Delta l' = \frac{\Delta x}{\sqrt{1 - v^2 / c^2}} \quad (14)$$

and

$$\Delta l = \frac{\Delta x'}{\sqrt{1 - v'^2 / c'^2}} \quad (15)$$

"ruler contraction" means that the right Δx (or $\Delta x'$) of the two formulas is smaller than the left $\Delta l'$ (or Δl). It should be noted that the proper space will not be elongated and shortened. The so-called "contraction" is to use the metrical amount of space of the observation coordinate system and restraint "simultaneously" to measure the space amount of moving space, so it is helplessly shortened

Same reason, you cannot casually think that the value of the objective true length of a ruler at any time must be equal to the value of the other party's length measured and read out subjectively by the ruler. Because if $\Delta l' = \Delta x'$ and $\Delta l = \Delta x$ are identified,

$$\text{then from (14) and (15), } \Delta l' = \frac{\Delta x'}{\sqrt{(1 - v^2 / c^2)(1 - v'^2 / c'^2)}} \quad \text{and}$$

$$\Delta l = \frac{\Delta x}{\sqrt{(1 - v^2/c^2)(1 - v'^2/c'^2)}} \quad \text{can be mechanically derived and contradict them.}$$

This is also a "ruler paradox" similar to the "clock paradox", but why has no one ever proposed the "ruler paradox"? Obviously, people's understanding of space is more specific than time. The ruler is a real thing, and the shortening of mutual recognition and be recognized are easy to accept. When the relative movement between the two systems S and S' stops and returns to the same system, no one doubts that the ruler of an entity has really shortened due to a history of movement. The time that the clock walked out and the spent time of people passed away evenly, it was ideal disappearing evenly and not like a ruler that can be checked repeatedly. Therefore, it is easy to suspect that length of time of rest and movement could be extend and shorten, and the checks are not correct each other. In order to completely uncover the "two-clock mystery", we continue to analyze in depth to find the relationship between the two objective existences proper time $\Delta\tau$ and $\Delta\tau'$ of the two mutual movement clocks, and the relationship between the two coordinate time $\Delta t'$ and Δt to measure and read out the other party.

The theory of relativity teaches people that we study the laws of physics should Step forward from the epistemological point of view, that is, to grasp the objective existence of the laws of physics from cognition. Like time and space, the speed of two coordinate systems separated from each other should have four values: the speed of objective existence V and V' , and the speed of known to each other v and v' . The v and v' are the speeds of recognized out by each other in the self-perceived static system S and S' respectively, which satisfy the equation (6). If only the value is considered, it is:

$$\frac{v}{c} = \frac{v'}{c'} \quad (16)$$

The proper speed value V' of the objective real existence of the system S' corresponding v is the proper space value $\Delta l'$ of the system S' divided by the proper time value $\Delta\tau'$:

$$V' = \frac{\Delta l'}{\Delta\tau'} \quad (17)$$

From equations (13) and (14), the relationship between it and the speed v of the system S' measured out by the space-time coordinates metrical values in the static observation coordinate system S is:

$$\frac{V'}{c'} = \frac{1}{1 - v^2/c^2} \frac{v}{c} \quad (17a)$$

Symmetrically, the proper speed value V of the objective real existence of the system S corresponding v' is the proper space value Δl of the system S divided by the proper time value $\Delta\tau$:

$$V = \frac{\Delta l}{\Delta\tau} \quad (18)$$

From equations (12) and (15), the relationship between it and the speed v' of the system S measured out by the space-time coordinates metrical values in the static

observation coordinate system S' is:

$$\frac{V}{c} = \frac{1}{1 - v'^2/c'^2} \frac{v'}{c'} \quad (18a)$$

According to Eq. (16), it can be proved that the speeds at which two systems S and S' separated from each other at a constant speed V and V' exist objectively also satisfy:

$$\frac{V}{c} = \frac{V'}{c'} \quad (19)$$

There are also four values for the distance separated from each other, but from an axiom point of view, the actual distance separated from each other is only one after all, so the two objective real proper values of this distance must be equal, namely:

$$\Delta l = \Delta l' \quad (20)$$

And the two mutually recognized values can be proved by (20) and then applying (14) and (15):

$$\Delta x = \Delta x' \quad (21)$$

Now according to (17), (18), (19) and (20), it is easy to prove:

$$c\Delta\tau = c'\Delta\tau' \quad (22)$$

Then according to (12), (13) and (16), we can get:

$$c\Delta t = c'\Delta t' \quad (23)$$

The two formulas (22) and (23) are the determined relations satisfied between the two objectively existing proper times and between the two coordinate times measured and read out the other party's proper time. That is to say: that the proper times of two clock uniformly elapsed at any time anywhere of S and S' multiply by the speed of light in the respective system are equal; and that the two coordinate times observed and recognized the other party's proper time multiply by the speed of light in the respective system are also equal. If the two systems are in the same physical environment, $c = c'$, then there must be $\Delta\tau = \Delta\tau'$, $\Delta t = \Delta t'$. This cannot always be guessed from the two relations (12) and (13). On the contrary, it's easy to hold a clock for the values of "Proper Time of Objective Real Existing" must be equal to "Coordinate Time of Subjectively Recognizing the Other Party", i.e. $\Delta\tau = \Delta t$ (or $\Delta\tau' = \Delta t'$), and get $\Delta\tau' \neq \Delta t'$ (or $\Delta\tau \neq \Delta t$). So, the "mystery" was established, and it was the "mystery", that's all! In fact, there is no definite relationship between the proper time τ elapsed objectively truly and uniformly in any coordinate system and its coordinate time t observed and recognized of external things. The next section will further analyze this problem.

The last thing to note is that when two clocks stop moving with each other and return to the same system, the time of the two clocks should be the proper time of the objective real existence, and the relationship is: $c\Delta\tau = c'\Delta\tau'$; The time recognized by both parties through measuring and reading out is "past time", the relationship is: $c\Delta t = c'\Delta t'$; both parties have a time period that is not recognized by the measuring, and the relationship is: $c(\Delta\tau - \Delta t) = c'(\Delta\tau' - \Delta t')$.

5.3 Uncertain relationship between proper time and coordinate time

In a coordinate system there is an indeterminate relationship between the proper time τ_0 elapsed objectively truly and uniformly, and the coordinate time t observed and recognized of external things. This uncertain relationship must be derived from the mutual understanding relationship between at least two coordinate systems. Now we choose S, S' and S'' three inertial coordinate systems, assuming that their $x (x', x'')$ axes coincide, the relative constant velocity linear motions of each other are parallel to the $x (x', x'')$ axis direction, and the origins at the time $t = t' = t'' = 0$ coincide. The speeds of light observed in their respective coordinate systems are c, c', c'' . The speeds of S' and S'' observed by S are v and u respectively; The speeds of S and S'' observed by S' are v' and w' respectively; The speeds of S and S' observed by S'' are u'' and w'' respectively. Taking S' as an object, let the relationship between coordinate time t' and proper time τ'_0 be: $t' = \phi \tau'_0$.

Applying (12a) and (13a) to S' and S'' , S' and S respectively,

$$c'' d\tau_0'' = c' dt' \sqrt{1 - w'^2 / c'^2} = \phi c' d\tau_0' \sqrt{1 - w'^2 / c'^2} = \phi c dt \sqrt{1 - w'^2 / c'^2} \sqrt{1 - v^2 / c^2}$$

Expand the root of the above formula and keep to the quadratic term of $\frac{v}{c}, \frac{w'}{c'}$, then:

$$c'' d\tau_0'' = \phi c dt \left(1 - \frac{w'^2}{2c'^2} - \frac{v^2}{2c^2}\right) \quad (24)$$

Then apply (12a) to S and S'' , and taken the movement speed u of S'' observed in as a combination of the movement speed w' of S'' relative to S' and the movement speed v of S' relative to S , apply the speed addition formula (8), and omit the higher order terms $(\frac{vw'}{c^2 c'^2})^2$:

$$c'' d\tau_0'' = c dt \sqrt{1 - u^2 / c^2} = c dt \sqrt{1 - \left(\frac{v}{c} + \frac{w'}{c'}\right)^2 \left(1 - \frac{vw'}{cc'}\right)^2}$$

Similarly, expand the root of the above formula and retain to the quadratic term of $\frac{v}{c}, \frac{w'}{c'}$, then:

$$c'' d\tau_0'' = c dt \left(1 - \frac{v^2}{2c^2} - \frac{w'^2}{2c'^2} - \frac{vw'}{cc'}\right) \quad (25)$$

Comparing the two formulas (24) and (25), we have:

$$\phi = 1 - \frac{vw'}{cc'} \left(1 - \frac{w'^2}{2c'^2} - \frac{v^2}{2c^2}\right)^{-1}$$

Expanding $\left(1 - \frac{w'^2}{2c'^2} - \frac{v^2}{2c^2}\right)^{-1}$ and retaining to the quadratic term of $\frac{v}{c}, \frac{w'}{c'}$, we finally

get:

$$\phi = 1 - \frac{v}{c} \frac{w'}{c'} = 1 - \frac{v}{c} \frac{w''}{c''} = 1 - \frac{v'}{c'} \frac{w'}{c'} \quad (26)$$

This is an important formula not seen in the traditional special theory of relativity. It indicates that in S' system the uncertain relationship between the proper time τ'_0 and the coordinate time t' must involve two other coordinate systems S and S'' of two observations to determine. These two observations may be the active observations of S' to the speed v' of S and to the speed w' of S'' ; It may also be the speeds v and w'' of S' passively observed by S and S'' respectively; Maybe one is the active observations of S' to the speed w' of S'' and the other is the speeds v of S' passively observed by S . The most important finding is that the proper time of S' recognized within the quadratic range of $\frac{v}{c}$, $\frac{w'}{c'}$ or $\frac{w''}{c''}$ etc. is that the coordinate time is a free relationship of unit selection, rather than the correct relationship. This degree of freedom is difficult to determine and must be determined by the other two coordinate systems S and S'' ; and when it happens to depend on another S_1 and S_2 , its degree of certainty changes again. In short, this shows that the proper time of any system S objectively and uniformly elapsed cannot be directly communicated with the outside world; There must be a correlation between the three in order to transform the evenly elapsed proper time into coordinate time that can be measured by the outside world, and it must also retain a second-level uncertain relationship. More importantly: the laws of physics involving any individual must rely on the proper time of the body's objective static and uniform elapsed to express its invariant form. This is an objective existence that does not depend on the observation coordinate system. How to discover and express this existence is an epistemological endeavor, and we must start with the understanding of the basic difference between the proper time and the coordinate time. This is an indispensable contribution to the theory of relativity! The traditional special theory of relativity of "the speed of light is constant" is always too hot to be perfect.

The National Astronomical Observatory's time measurement and time service work is to determine the coordinate time of the Earth's coordinate system. The goal of the effort is to hope that it can be closer to the ideal state of uniform disappearance. Astronomers have not generally possessed the theory of relativity to understand the proper time, nor the name of the "proper time"; "Ephemeris Time" was once considered to be the time that an ideal evenly elapsed and was used as the time definition of the earth system. In principle, any celestial body in the solar system can be used to specify the ephemeris time as long as it accurately grasps its movement law. It is generally not considered that there is a basic problem when the time measured according to this target is used as the coordinate time of the earth. However, if we can recognize and pay attention to the uncertain relationship between the proper time and the coordinate time proposed in this paper, then we should find that the influence of (26) in this paper does exist in this type of time measurement work. According to

the theory of this article, these problems are not discoverable by the astronomical spherical triangle deduction of the stellar timing through zenith. The instantaneous lateral angular velocity of a star passing through the zenith obviously conforms to a law of change in the average refractive index due to the atmosphere; the medium close to the star has the same law of change as it passes through sky space. According to the distance between it and the earth, the lateral angular velocity is converted into a lateral linear velocity, the relationship between the two speeds observed with each other by the coordinate system of the star and the coordinate system of the earth at the moment of passing the zenith is the relationship between v of S and v' of S' , They should satisfy the formula (6) in this article, not $v = -v'$. The speed of light in each other's coordinate system is also not $c = c'$. These differences are also significantly strengthened by the difference between the gravity field of the Earth and the gravity field around the star.

Modern photography time measurement technology has the conditions to strive to achieve the accuracy of measuring one star up to ± 0.001 seconds. Select two pairs of stars measured on the same film, and calculate with the film of each pair for one time measurement, equation (26) must reflect a fixed constant error. There must be a fixed difference between the two measured constant errors. This difference can also be planned to be strengthened by using star pairs. For example, one pair is large in proper motion and the other is small; one pair is far away and the other is close; or one pair is the white dwarf with a large gravity field and the other is red giant star with a small gravity field. Because the arrangement device for photogrammetry has eliminated any instrument errors and human error, it does not matter if the two pairs of stars are photographed on different films. There is no other theoretical source for the constant difference measured in this way, such as small changes in the rotation speed of the earth etc. . If it is suspected that the ephemeris coordinates of these two pairs of stars have system constants with different roots, it can be eliminated by used a self-defined "average ephemeris position" in mutual proofreading of multiple measurements of these two pairs of stars. This will be the method to directly verify formula (26).

The general photogrammetric surveying work is in the first half of the night, and there are about ten stars on the film every night, and the limitation of star selection is far more inconvenient than that of the transit method by Transit Instrument. Therefore, the seasonal selection of star group transformation is obviously systematic. The effect of measuring and calculating the film about ten stars on formula (26) must still be expressed as an average constant error, which must also vary with the seasonal selection of star groups. It is a very meaningful encouragement to further verify the formula (26) by using this method if the photogrammetric surveying data collected for many years can screen out the average constant error, and suspect that the previously explained reasons are not necessarily valid.

The accuracy of the transit method of time measurement by Transit Instrument is far inferior to that in photography time measurement. It is difficult to say with certainty that the measured accuracy of a single star can be reach at 0.01 seconds. But it may also be useful to use the above paragraph to explain the constant errors of time

measurement that are noticed due to different star groups selection. In addition, the influence of formula (26) can also be suspected to be one of the reasons why the moon experience formula or Brown's authoritative moon formula has not yet been perfect.

The meaning of formula (26) also exists in the traditional special theory of relativity. The reason why it has never been noticed is probably because the special theory of relativity has been entangled in the "two-clock mystery" since its birth no one tried to make it clear word by word basically.

6 Conclusion

Summarize the above analysis and discussion as follows:

(1) This article establishes the main foundation of the special theory of relativity in different media (or with variable speed of light), the obtained space-time coordinate transformation and velocity transformation formulas and the basic relationship $\frac{v}{c} = -\frac{v'}{c'}$, $c\tau = c'\tau'$, breaks through the limitation of "vacuum"; both $c = c'$ and $c \neq c'$ are included in parallel with the basic point of view of the theory of relativity, so that real physics can be objectively observed and understood in more leeway.

(2) The special theory of relativity with variable speed of light further reveals the essence of space and time and its relationship with motion than the traditional special theory of relativity. For the two systems S and S' that are separated from each other at constant speed, the "ruler contraction" and "clock retardation" effects are the metrical amount of two proper space-time and the metrical amount of two coordinate space-time measured out each other, namely $(\Delta l, \Delta \tau)$ and $(\Delta x', \Delta t')$, or $(\Delta l', \Delta \tau')$ and $(\Delta x, \Delta t)$, no consistent relationship between mutual understanding and being recognized.

The proper space and proper time of the existence of things cannot be shortened or slowed down. It can be strictly proved: At anytime and anywhere, the two separated proper space distances of S and S' must be equal $\Delta l = \Delta l'$, and the values of the space distance of mutual recognition read out by the two coordinate space metrical amount are also equal $\Delta x = \Delta x'$; Anytime and anywhere, the product of the proper time of uniform elapsed and the speed of light in the respective system is equal, i.e. $c\tau = c'\tau'$; The product of the coordinate time of observing and recognizing the other party's proper time and the speed of light in the respective system is also equal, i.e. $ct = c't'$.

(3) This article reveals that there is an indeterminate relationship between the proper time τ_0 of uniform elapsed of the coordinate system and the coordinate time t

it uses $\tau_0 = \phi t$, $\phi = 1 - \frac{v}{c} \frac{w'}{c'} = 1 - \frac{v}{c} \frac{w''}{c''} = 1 - \frac{v'}{c'} \frac{w'}{c'}$. Remind the practical astronomers who do the time measurement and the time service work to understand that the practical scientific coordinate time and the proper time of uniform disappearance (that is, the so-called " Ephemeris Time") can not be completely identical unified, thereby pay attention to the impact of this uncertain relationship on the time measurement and the time service work, and propose ways to verify.

Subsequent work will use this expanded special theory of relativity to conduct a comprehensive review of the relevant physics, which will inevitably extend to issues

that have not been or cannot be examined by traditional special theory of relativity.

Data Availability Statement

The data that supports the findings of this study are available within the article [and its supplementary material].

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