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Weighted k -domination problem in fuzzy networks

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Abstract

In real-life scenarios, both the vertex weight and edge weight in a network are hard to define exactly. We can incorporate the fuzziness into a network to handle this type of uncertain situation. Here, we use triangular fuzzy number to describe the vertex weight and edge weight of a fuzzy network G . In this paper, we consider weighted k -domination problem in fuzzy networks. The weighted k -domination (WKD) problem is to find a k dominating set D which minimizes the cost $f(D) := \sum_{u \in D} w(u) + \sum_{v \in V \setminus D} \min\{\sum_{u \in S} w(uv) | S \subseteq N(v) \cap D, |S| = k\}$. First, we put forward an integer linear programming model with a polynomial number of constraints for the WKD problem. If G is a cycle, we design a dynamic algorithm to determine its exact weighted 2-domination number. If G is a tree, we give a label algorithm to determine its exact weighted 2-domination number. Combining a primal-dual method and a greedy method, we put forward an approximation algorithm for general fuzzy network on the WKD problem. Finally, we describe an application of the WKD problem to police camp problems.

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Keywords and Phrases. Fuzzy network; Triangular fuzzy number; Weighted k -domination; Algorithm.

1 Problem description

Given a network/graph G and a positive integer k , a k -dominating set in G is a set $D \subseteq V(G)$ such that every vertex $v \in V(G) \setminus D$ has at least k neighbors in D . The minimum k -domination problem aim to find the minimum size of a k -dominating set in a given graph. The notion of k -domination was introduced by Fink and Jacobson [4], and has been studied by a number of authors including [1, 2, 3]. If G is a weight network, then the vertex weighted k -domination (VWKD) problem is to find a k -dominating set D which minimizes the cost

$$f(D) := \sum_{u \in D} w(u).$$

The minimum k -domination problem and vertex weighted k -domination problem are useful in various applications, for example in forming sets of representatives or in resource allocation in distributed computing systems. However, these problems are known to be NP-hard and also hard to approximate.

Ma et al. [5] consider the weighted total domination problem. The weighted total domination problem is to find a total dominating set D which minimizes the cost

$$f(D) := \sum_{u \in D} w(u) + \sum_{e \in E[D]} w(e) + \sum_{v \in V \setminus D} \min\{w(uv) | u \in N(v) \cap D\}.$$

They formulate the problem into three integer linear programming models, and present numerical results implemented on random graphs.

Motivated from uncertain situations, we consider the weights of the vertices and edges are fuzzy numbers instead of crisp numbers. That is, the network considered here is a fuzzy network. The fuzzy networks are flexible. In this article, the weights of vertices and edges are taken as a triangular fuzzy number which is one of the widely used numbers in a fuzzy setting.

Now we define the triangular fuzzy number (TFN) and ranking operations on fuzzy numbers.

Definition 1 A triangular fuzzy number is represented by a triplet $\tilde{N} = \langle \underline{n}, n_0, \bar{n} \rangle$ with the membership function

$$\mu(x) = \begin{cases} \frac{x - \underline{n}}{n_0 - \underline{n}}, & \text{if } \underline{n} \leq x \leq n_0 \\ \frac{\bar{n} - x}{\bar{n} - n_0}, & \text{if } n_0 < x \leq \bar{n} \\ 0, & \text{otherwise} \end{cases}$$

Here, $\underline{n}$ is the lowest possible value, n_0 is the primary possible value, and $\bar{n}$ is the maximum possible value.

Definition 2 The addition of two TFNs $\tilde{N} = \langle \underline{n}, n_0, \bar{n} \rangle$ and $\tilde{M} = \langle \underline{m}, m_0, \bar{m} \rangle$ is given by $\tilde{N} + \tilde{M} = \langle \underline{n} + \underline{m}, n_0 + m_0, \bar{n} + \bar{m} \rangle$.

Ranking is the most significant part in any decision-making in a fuzzy optimization problem. The ranking operation of fuzzy number is done by a defuzzification technique which transfers the fuzzy number into a real number. This real value is used to find the rank of n fuzzy numbers. There exist several ranking methods. In this paper, we use the canonical form of a triangular fuzzy number [6] in our proposed algorithm.

Definition 3 Let $\tilde{N} = \langle \underline{n}, n_0, \bar{n} \rangle$ be a TFN. The graded mean of $\tilde{N}$ is calculated as $P(\tilde{N}) = \frac{1}{6}(\underline{n} + 4n_0 + \bar{n})$.

The comparison between two fuzzy number $\tilde{N} = \langle \underline{n}, n_0, \bar{n} \rangle$ and $\tilde{M} = \langle \underline{m}, m_0, \bar{m} \rangle$ can be described as follows:

- (a) The value of $\tilde{N}$ is equal to the value of $\tilde{M}$ if $P(\tilde{N}) = P(\tilde{M})$.
- (b) The value of $\tilde{N}$ is greater than the value of $\tilde{M}$ if $P(\tilde{N}) > P(\tilde{M})$.
- (c) The value of $\tilde{N}$ is less than the value of $\tilde{M}$ if $P(\tilde{N}) < P(\tilde{M})$.

In this paper, we study the weighted k -domination problem in fuzzy networks. The weighted k -domination (WKD) problem is to find a k -dominating set D which minimizes the cost

$$f(D) := \sum_{u \in D} w(u) + \sum_{v \in V \setminus D} \min \{ \sum_{u \in S} w(uv) | S \subseteq N(v) \cap D, |S| = k \}.$$

In other words, the cost function consists of two parts:

- (1) the total weight of vertices in D ;
- (2) the total weight of the minimum-weight edges connecting every vertex v in $V \setminus D$ to the exactly k vertices located in D .

Remark. Suppose that the weights of vertex and edge are crisp numbers. Note that if we set $w(e) = 0$ for each $e \in E$ and $w(v) = 1$ for each $v \in V$, then the WKD problem is reduced to the minimum k -domination problem. Moreover, if we only set $w(e) = 0$ for each $e \in E$, the WKD problem is equivalent to the vertex weighted k -domination problem. Therefore, the WKD problem considered in this paper is an extension of the previous problems.

The manuscript is organized as follows. In this section, we define the triangular fuzzy number and describe the WKD problem. Section 2 presents the mathematical model for the WKD problem in a fuzzy network. In Section 3, we study the weighted 2-domination problem for two types of fuzzy networks. If G is a cycle with a fuzzy weight, we design a dynamic algorithm to determine its exact weighted 2-domination number. If G is a tree with a fuzzy weight, we give a label algorithm to determine its exact weighted 2-domination number. In Section 4, combining a primal-dual method and a greedy method, we present an approximate algorithm to determine the fuzzy constrained weighted k -dominating set in the fuzzy network. In the last section, we describe an application of the WKD problem to police camp problems.

2 Mathematical model for weighted k -domination problem in fuzzy networks

In this section, we mainly investigate the WKD problem in a fuzzy network from the point of integer linear programming. A fuzzy mathematical

programming method is introduced to model the problem.

Let $G = (V, E)$ be a connected fuzzy network consisting of n vertices and m edges. Let $E(v) = \{e \in E(G) \mid \text{vertex } v \text{ is incident to edge } e\}$, and let Δ be the maximum degree of G . For any edge $e = uv$, the triangular fuzzy number $w(e)$ is used to represent the fuzzy cost of the edge. For any vertex $v \in V$, the triangular fuzzy number $w(v)$ is used to represent the fuzzy cost of the vertex. The fuzzy mathematical programming of the problem is described as follows

Firstly, we use binary variables x_v to indicate whether vertex v is chosen for the solution or not, and use binary variable y_e to indicate whether edge e is chosen for the solution or not. Suppose that D is a k -dominating set of G . Let

$$f(D) = \sum_{v \in D} w(v) + \sum_{v \in V \setminus D} \min\{\sum_{u \in S} w(uv) \mid S \subseteq N(v) \cap D, |S| = k\}.$$

D is called as fuzzy optimal weighted k -dominating set in G if and only if $f(D) \leq f(D')$ for any k -dominating set D' of G . Let

$$\gamma_k(G) = \min\{f(D) \mid D \text{ is a } k\text{-dominating set}\}$$

and $\gamma_k(G)$ is called the weighted k -domination number of G .

The parameter $\gamma_k(G)$ equals the optimum value of the following integer linear program

$$\min \sum_{v \in V(G)} w(v)x_v + \sum_{e \in E} w(e)y_e \quad (1)$$

$$s.t. \sum_{u \in N(v)} x_u + kx_v \geq k, \quad \forall v \in V(G) \quad (2)$$

$$\sum_{e \in E(v)} y_e + kx_v \geq k, \quad \forall v \in V(G) \quad (3)$$

$$\sum_{e \in E(v)} y_e \leq k + \Delta x_v, \quad \forall v \in V(G) \quad (4)$$

$$x_v \in \{0, 1\}, \forall v \in V(G) \quad (5)$$

$$y_e \in \{0, 1\}, \forall e \in E(G) \quad (6)$$

In this problem, our main objective is to select a k -dominating set with the minimum fuzzy

cost. In the model, the objective function value is given by (1). Constrains (2) are the k -dominating set constrains which ensure that each non-selected vertex has at least k neighbors which are selected. Constrains (3) and (4) ensure that each non-selected vertex $v \in V$ is incident to exactly k edges which are in the solution. We set the decision variables x_v and y_e to be binary for constrains (5) and (6).

The WKD problem is NP-hard, since the minimum k -domination problem, a particular case of WKD, has been proven to be NP-hard even for chordal bipartite graphs.

3 Weighted 2-domination problem for two types of fuzzy networks

In this section, we study the weighted 2-domination problem for two types of fuzzy networks. The networks are a cycle and a tree, respectively. If G is a cycle with a fuzzy weight, we design a dynamic programming algorithm to determine its exact weighted 2-domination number. If G is a tree with a fuzzy weight, we give a label algorithm to determine its exact weighted 2-domination number. Let $C_n = v_1 v_2 \cdots v_n$ be a fuzzy weighted cycle with n vertices. Let $\gamma'_2(P_{ij}) = \min\{f(D) \mid D \text{ is a 2-dominating set of path } P_{ij} = v_i v_{i+1} \cdots v_j, \text{ such that } v_i, v_j \in D\}$.

Lemma 1 *Let $P_{ij} = v_i v_{i+1} \cdots v_j$ be a fuzzy weighted path with $i + 3 \leq j$. Then*

$$\gamma'_2(P_{ij}) = \min\{w(v_i) + \gamma'_2(P_{(i+1)j}), w(v_i) + w(v_i v_{i+1}) + w(v_{i+1} v_{i+2}) + \gamma'_2(P_{(i+2)j})\}.$$

Lemma 2 *Let $C_n = v_1 v_2 \cdots v_n$ be a fuzzy weighted cycle with n vertices. Then*

$$\gamma_2(C_n) = \min\{\gamma'_2(P_{1(n-1)}) + w(v_1 v_n) + w(v_{n-1} v_n), \gamma'_2(P_{1n}), \gamma'_2(P_{2n}) + w(v_1 v_n) + w(v_1 v_2)\}.$$

Proof. Let D be a fuzzy optimal weighted 2-dominating set in C_n . That is $\gamma_2(C_n) = f(D)$. Our discussion can be divided into the

following three cases.

Case 1 $v_n \notin D$. Then $v_1, v_{n-1} \in D$. Hence $D \cap V(P_{1(n-1)})$ is a 2-dominating set of $P_{1(n-1)}$ such that $v_1, v_{n-1} \in D$.

So

$$\begin{aligned} \gamma'_2(P_{1(n-1)}) &\leq f(D \cap V(P_{1(n-1)})) \\ &= f(D) - w(v_1 v_n) - w(v_{n-1} v_n) \\ &= \gamma_2(C_n) - w(v_1 v_n) - w(v_{n-1} v_n). \end{aligned}$$

Case 2 $v_n \in D$ and $v_1 \in D$. Then D is a 2-dominating set of P_{1n} such that $v_1, v_n \in D$.

So

$$\gamma'_2(P_{1n}) \leq f(D) = \gamma_2(C_n).$$

Case 3 $v_n \in D$ and $v_1 \notin D$. Then $v_2 \in D$. Hence $D \cap V(P_{2n})$ is a 2-dominating set of P_{2n} such that $v_2, v_n \in D$.

So

$$\begin{aligned} \gamma'_2(P_{2n}) &\leq f(D \cap V(P_{2n})) \\ &= f(D) - w(v_1 v_n) - w(v_1 v_2) \\ &= \gamma_2(C_n) - w(v_1 v_n) - w(v_1 v_2). \end{aligned}$$

By Case 1, Case 2 and Case 3, it follows that

$$\gamma_2(C_n) \geq \min\{\gamma'_2(P_{1(n-1)}) + w(v_1 v_n) + w(v_{n-1} v_n), \gamma'_2(P_{1n}), \gamma'_2(P_{2n}) + w(v_1 v_n) + w(v_1 v_2)\}.$$

It is obvious that

$$\gamma_2(C_n) \leq \min\{\gamma'_2(P_{1(n-1)}) + w(v_1 v_n) + w(v_{n-1} v_n), \gamma'_2(P_{1n}), \gamma'_2(P_{2n}) + w(v_1 v_n) + w(v_1 v_2)\}.$$

Hence,

$$\gamma_2(C_n) = \min\{\gamma'_2(P_{1(n-1)}) + w(v_1 v_n) + w(v_{n-1} v_n), \gamma'_2(P_{1n}), \gamma'_2(P_{2n}) + w(v_1 v_n) + w(v_1 v_2)\}.$$

■

Algorithm 2.1: (Dynamic programming algorithm for weighted 2-domination problem in fuzzy weighted cycle)

Input: Fuzzy weighted cycle

$C_n = (\{v_1, v_2, \dots, v_n\}, E)$, and vertex weight $w(v)$ and edge weight $w(e)$ for any $v \in V$ and $e \in E$.

- (1) **For** $j \leftarrow 1$ to $n - 1$ **do**
 $\gamma'_2(P_{j(j+1)}) \leftarrow w(v_j) + w(v_{j+1}).$
- (2) **For** $j \leftarrow 1$ to $n - 2$ **do**
 $\gamma'_2(P_{j(j+2)}) \leftarrow \min\{w(v_j) + w(v_{j+1}) + w(v_{j+2}), w(v_j) + w(v_j v_{j+1}) + w(v_{j+1} v_{j+2}) + w(v_{j+2})\}.$
- (3) **For** $i \leftarrow 3$ to $n - 1$ **do**
For $j \leftarrow 1$ to $n - i$ **do**
 $\gamma'_2(P_{j(j+i)}) \leftarrow \min\{w(v_j) + \gamma'_2(P_{(j+1)(j+i)}), w(v_j) + w(v_j v_{j+1}) + w(v_{j+1} v_{j+2}) + \gamma'_2(P_{(j+2)(j+i)})\}.$
- (4) $\gamma_2(C_n) \leftarrow \min\{\gamma'_2(P_{1(n-1)}) + w(v_1 v_n) + w(v_{n-1} v_n), \gamma'_2(P_{1n}), \gamma'_2(P_{2n}) + w(v_1 v_n) + w(v_1 v_2)\}.$
- (5) **Output:** $\gamma_2(C_n).$

In the following, we will give a labeling algorithm to obtain the fuzzy optimal weighted 2-domination number of a tree T . First root the tree T at a vertex r with degree at least two. The height of T is the maximum distance between r and all other vertices. Let h be the height of T . The i -th level A_i ($0 \leq i \leq h$) be the set of vertices of T which are at distance i from the root. For such a rooted tree T with order n we can label the vertices of T with $v_1, v_2, \dots, v_n$ as follows. We go on every level starting with level h to level 0. For each i ($0 \leq i \leq h$), we traverse the vertices in arbitrary order, say from left to right. Finally, we list out the fathers of all vertices of T (we mean that the vertex v_n has no father by writing $father(v_n) = 0$), and thus we can represent T by a data structure called a vertex father array.

For any vertex $v \in V(T)$, let T_v be the subtree of T rooted at vertex v . Assume the father vertex of v in T is the vertex v' .

$\alpha(T_v) = \min\{f(D) | D \text{ is a 2-dominating set of } T_v \text{ such that } v \in D\}.$

$\beta(T_v) = \min\{f(D) - w(v') | D \text{ is a 2-dominating set of the subtree induced by } V(T_v) \cup \{v'\}$

such that $|N_{T_v}(v) \cap D| = 1, v \notin D, v' \in D\}.$

$\kappa(T_v) = \min\{f(D) | D \text{ is a 2-dominating set of } T_v \text{ such that for any 2-dominating set } D' \text{ of } T \text{ with } D \subseteq D', v, v' \notin D'\}.$

$\lambda(T_v) = \min\{f(D) - w(v') | D \text{ is a 2-dominating set of the subtree induced by } V(T_v) \cup \{v'\} \text{ such that } |N_{T_v}(v) \cap D| \geq 2, v \notin D, v' \in D\}.$

If vertex v is a leaf, it is obvious that v belongs to every 2-dominating set of T . Hence we define $\beta(T_v) = \kappa(T_v) = \lambda(T_v) = \infty$ and $\alpha(T_v) = w(v).$

Lemma 3 *Let T_v be a subtree of T , and v' be the father vertex of v . Then*

- (1) $\alpha(T_v) = w(v) + \sum_{u \in N_{T_v}(v)} \min\{\alpha(T_u), \beta(T_u), \lambda(T_u)\}.$
- (2) $\beta(T_v) = \min_{u \in N_{T_v}(v)} \{\alpha(T_u) + w(vu) + w(vv') + \sum_{w \in N_{T_v}(v) \setminus \{u\}} \kappa(T_w)\}.$
- (3) $\kappa(T_v) = \min_{S \subseteq N_{T_v}(v)} \{\sum_{u \in S} \alpha(T_u) + \min\{w(vw) + w(vu) | u, w \in S\} + \sum_{u \in N_{T_v}(v) \setminus S} \kappa(T_u) | |S| \geq 2\}.$
- (4) $\lambda(T_v) = \min_{S \subseteq N_{T_v}(v)} \{\sum_{u \in S} \alpha(T_u) + \min\{w(vw) + w(vu) | u, w \in S \cup \{v'\}\} + \sum_{u \in N_{T_v}(v) \setminus S} \kappa(T_u) | |S| \geq 2\}.$

Proof. We only prove the Case (1).

The proves of other cases are similar.

Let D be a 2-dominating set of T_v such that $v \in D$ and $f(D) = \alpha(T_v).$

For each $u \in N_{T_v}(v)$, let

$$D_u = (D \cap V(T_u)) \cup \{v\}.$$

Then D_u is a 2-dominating set of the subtree induced by

$$V(T_u) \cup \{v\}.$$

Then

$$f(D) = \sum_{u \in N_{T_v}(v)} (f(D_u) - w(v)) + w(v).$$

Our discussion can be divided into the following three cases.

Case 1 $u \in D_u.$

Then $D_u \setminus \{v\}$ is a 2-dominating set of T_u such that $u \in D_u \setminus \{v\}$. So

$$\alpha(T_u) \leq f(D_u) - w(v).$$

Case 2 $u \notin D_u$ and $|N_{T_u}(u) \cap D_u| = 1$.
 Say, $N_{T_u}(u) \cap D_u = \{w\}$.
 Then D_u is a 2-dominating set of $T_u \cup \{v\}$
 such that $u \notin D_u, v \in D_u$
 and

$$|N_{T_u}(u) \cap D_u| = 1.$$

So

$$\beta(T_u) \leq f(D_u) - w(v).$$

Case 3 $u \notin D_u$ and $|N_{T_u}(u) \cap D_u| \geq 2$.
 Then D_u is a 2-dominating set of $T_u \cup \{v\}$ such
 that $u \notin D_u, v \in D_u$
 and

$$|N_{T_u}(u) \cap D_u| \geq 2.$$

So

$$\lambda(T_u) \leq f(D_u) - w(v).$$

By Case 1, Case 2 and Case 3, it follows that

$$\min\{\alpha(T_u), \beta(T_u), \lambda(T_u)\} \leq f(D_u) - w(v).$$

Hence,

$$\begin{aligned} & \sum_{u \in N_{T_v}(v)} \min\{\alpha(T_u), \beta(T_u), \lambda(T_u)\} \\ & \leq \sum_{u \in N_{T_v}(v)} (f(D_u) - w(v)) \\ & = f(D) - w(v) \\ & = \alpha(T_v) - w(v). \end{aligned}$$

It is easy to prove that

$$\sum_{u \in N_{T_v}(v)} \min\{\alpha(T_u), \beta(T_u), \lambda(T_u)\} + w(v) \geq \alpha(T_v).$$

Hence,

$$\alpha(T_v) = \sum_{u \in N_{T_v}(v)} \min\{\alpha(T_u), \beta(T_u), \lambda(T_u)\} + w(v).$$

■

Algorithm 2.2: (A labeling algorithm for weighted 2-domination problem in a fuzzy weighted tree)

Input: rooted tree T represented by its vertex father array, vertex weight $w(v)$ and edge weight $w(e)$ for any $v \in V$ and $e \in E$.

(1) **For** $j \leftarrow 1$ to n **do**

If v_j is a leaf **then**

$$\beta(T_v) \leftarrow \infty; \kappa(T_v) \leftarrow \infty;$$

$$\lambda(T_v) \leftarrow \infty; \alpha(T_v) \leftarrow w(v).$$

else if $|N_{T_v}(v)| = 1$, **then**

$$\kappa(T_v) \leftarrow \infty; \lambda(T_v) \leftarrow \infty.$$

else

$$\alpha(T_v) \leftarrow w(v) + \sum_{u \in N_{T_v}(v)} \min\{\alpha(T_u), \beta(T_u), \lambda(T_u)\}.$$

$$\beta(T_v) \leftarrow \min_{u \in N_{T_v}(v)} \{\alpha(T_u) + \sum_{w \in N_{T_v}(v) \setminus \{u\}} \kappa(T_w) + w(vu) + w(vv')\}.$$

$$\kappa(T_v) \leftarrow \min_{S \subseteq N_{T_v}(v)} \{\sum_{u \in S} \alpha(T_u) + \min\{w(vw) + w(vu) | u, w \in S\} + \sum_{u \in N_{T_v}(v) \setminus S} \kappa(T_u) | |S| \geq 2\}.$$

$$\begin{aligned} \lambda(T_v) \leftarrow & \min_{S \subseteq N_{T_v}(v)} \{\sum_{u \in S} \alpha(T_u) + \min\{w(vw) + w(vu) | u, w \in S \cup \{v'\}\} \\ & + \sum_{u \in N_{T_v}(v) \setminus S} \kappa(T_u) | |S| \geq 2\}. \end{aligned}$$

$$(2) \gamma_2(T) \leftarrow \min\{\alpha(T_{v_n}), \kappa(T_{v_n})\}.$$

$$(3) \text{ Output } \gamma_2(T).$$

4 Approximate algorithm for weighted k -domination problem

In the section, combining a primal-dual method and a greedy method, we present an approximate algorithm to determine the fuzzy constrained weighted k -dominating set in the fuzzy networks. First, we consider the following fuzzy vertex weighted k -dominating problem.

$$\min \sum_{v \in V(G)} w(v)x_v \quad (1)$$

$$s.t. \sum_{u \in N(v)} x_u + kx_v \geq k, \quad \forall v \in V(G) \quad (2)$$

$$x_v \in \{0, 1\}, \forall v \in V(G) \quad (3)$$

Then we relax it to the following linear program:

$$\min \sum_{v \in V(G)} w(v)x_v \quad (1)$$

$$s.t. \sum_{u \in N(v)} x_u + kx_v \geq k, \quad \forall v \in V(G) \quad (2)$$

$$x_v \geq 0, \forall v \in V(G) \quad (3)$$

Its dual linear program is

$$\max \sum_{v \in V(G)} ky_v \quad (1)$$

$$s.t. \sum_{u \in N(v)} y_u + ky_v \leq w(v), \quad \forall v \in V(G) \quad (2)$$

$$y_v \geq 0, \forall v \in V(G) \quad (3)$$

Let $A = (a_{ij})$ be the adjacent matrix of graph G , where $a_{ii} = k$ for $1 \leq i \leq n$, $a_{ij} = 1$ if $v_i v_j \in E$, and $a_{ij} = 0$ if $v_i v_j \notin E$ for $i \neq j$. Using a primal-dual method, we give an algorithm to give a dominating set with weight as little as possible.

Algorithm 3.1 (vertex weighted dominating set in a fuzzy network)

Input: Fuzzy graph $G = (\{v_1, v_2, \dots, v_n\}, E)$, the adjacent matrix A , vertex weight $w(v)$ for any $v \in V$.

(1) Set $\vec{x}^0 \leftarrow 0$; $\vec{y}^0 \leftarrow 0$; $k \leftarrow 0$; $D \leftarrow \emptyset$;
 $I_0 \leftarrow \{i | 1 \leq i \leq n, \sum_{j=1}^n a_{ij}x_j^0 = 0\}$.

(2) **While** $I_k \neq \emptyset$ **do**
 $J_k \leftarrow \{j | 1 \leq j \leq n, x_j^k = 0\}$.

Choose $r \in J_k$ such that

$$\frac{w(v_r) - \sum_{i=1}^n a_{ir}y_i^k}{\sum_{i \in I_k} a_{ir}} = \min_{j \in J_k} \frac{w(v_j) - \sum_{i=1}^n a_{ij}y_i^k}{\sum_{i \in I_k} a_{ij}} = \beta.$$

For $i \leftarrow 1$ **to** n **do**

$$y_i^{k+1} = \begin{cases} y_i^k + \beta, & \text{if } i \in I_k \\ y_i^k, & \text{if } i \notin I_k \end{cases}$$

For $i \leftarrow 1$ **to** n **do**

$$x_i^{k+1} = \begin{cases} 1, & \text{if } i = r \\ x_i^k, & \text{otherwise} \end{cases}$$

$D \leftarrow D \cup \{v_r\}$; $I_{k+1} \leftarrow \{i | 1 \leq i \leq n, \sum_{j=1}^n a_{ij}x_j^{k+1} = 0\}$.
 $k \leftarrow k + 1$

(3) **Output** D

Algorithm 3.2 (Approximation Algorithm for WKD)

Input: Fuzzy graph $G = (\{v_1, v_2, \dots, v_n\}, E)$, the adjacent matrix A , vertex weight $w(v)$ and edge weight $w(e)$ for any $v \in V$ and $e \in E$.

(1) Set $\overline{D} \leftarrow \emptyset$; $g(\overline{D}) \leftarrow \langle 0, 0, 0 \rangle$.

(2) **For** $i = 1$ **to** k **do**

Apply algorithm 3.1 to G .

$\overline{D} \leftarrow \overline{D} \cup D$

$G \leftarrow G - D$.

(3) **For** $v \in V - \overline{D}$ **do**

$g(\overline{D}) \leftarrow g(\overline{D}) + \min\{\sum_{u \in S} w(uv) | S \subseteq N_G(v) \cap \overline{D}, |S| = k\}$.

(4) $f(\overline{D}) \leftarrow \sum_{v \in \overline{D}} w(v) + g(\overline{D})$.

(5) **Output** $f(\overline{D})$.

It is easy to prove that $\overline{D}$ is a k -dominating set of G and $f(\overline{D})$ is an approximation on the weighted k -domination number in fuzzy network G .

5 An application on weighted k -domination problem

Suppose there are a thousand police camps in a country to maintain the country's own law and order. It is natural that if a place is connected

to a camp by an edge, then it is fully under the control of the camp. If a sudden event occurs in any place, the adjacent camp have to send policemen to it.

To maintain law and order, the government generally decides to choose the places to position the camp such that the following two conditions are satisfied. (1) each place having no camp is adjacent to at least k camp. (2) the total cost is minimum, where the total cost consists of the cost of positioning the camp and the cost of the potential deployments.

In practice, a proper decision is prone to a high degree of indeterminacy, mostly raised by unpredictable events that do not obey the rules and prerequisites of the probability theory. Hence, the cost are not certainly defined. Suppose that every place is represent by a vertex. The weight of the vertex v denotes the cost of positioning the camp at the corresponding place. The weight of the edge uv denotes the cost of potential deployments from the corresponding place of u to the corresponding place of v . Hence the police camp problems can be modeled as a weighted k -domination problem in fuzzy networks.

6 Conclusions

This study introduced weighted k -domination of fuzzy networks and described a major property of weighted k -domination of fuzzy networks. We define the triangular fuzzy number and describe the WKD problem. We present the mathematical model for the WKD problem in fuzzy networks. We study the weighted 2-domination problem for two types of fuzzy networks. If G is a cycle with a fuzzy weight, we design a dynamic algorithm to determine its exact weighted 2-domination number. If G is a tree with a fuzzy weight, we give a label algorithm to determine its exact weighted 2-domination number. Combining a primal-dual method and a greedy method, we present an approximate algorithm to determine the fuzzy constrained weighted k -

dominating set in the fuzzy networks. Finally, we describe an application of the WKD problem to police camp problems.

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