Estimation of Roll Damping Coefficient from Roll and Trajectory Data Using Extended Kalman Filter

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Abstract—In this paper, a new method to estimate roll aerodynamic characteristics of a rolling projectiles is proposed. It is estimated from the measured roll rate and trajectory positional data using Extended Kalman Filter. Modified point mass model of trajectory modelling, in state space form, is used to represent system dynamics of Extended Kalman Filter. The roll and position data at every time step constitutes the measurement vector. Along with positions and velocities, roll damping coefficient is included as a state variable. As roll damping coefficient depends on projectile configurations and Mach number. Roll damping coefficients are estimated for two configurations viz. roll stabilized shell and fin stabilized rocket. The measurements are simulated for full flight regime to cover complete Mach regime. Estimated values are compared with known results for various Mach numbers. In both the cases estimation is in close agreement with known results.

Index Terms—Extended Kalman Filter, Modified Point Mass Model, Parameter estimation, Roll Damping Coefficient

I. INTRODUCTION

A projectile flight is directly influenced by launch velocity, atmospheric conditions, mass, center of gravity and inertia of projectile. Furthermore, the projectile flight highly depends on the aerodynamic coefficients. Aerodynamic coefficients are the parameters describing and connecting forces and moments acting on the projectiles [1].

The conventional projectile has its centre of gravity close to base and centre of pressure ahead of the centre of gravity. Such configuration is aerodynamically unstable and it is stabilized by rolling it about its longitudinal axis with sufficient high rate or by providing fins. A finned projectile is frequently imparted low roll to average out unavoidable manufacturing asymmetries during flight. When a projectile roll in a medium, due to viscous interaction of the medium and the projectile surface, roll damps down throughout the flight. Some projectiles have fins which impart a roll torque to the projectile such that the roll rate increases. Roll damping coefficient C_{l_p} is related to reduction in roll rate. In case of a finned projectile, a rolling moment can be induced by non-zero fin cant angle. The roll moment coefficient C_{l_δ} is associated with increase in the roll rate due fin cant angle δ .

The roll-damping coefficient is a necessary input to compute the rolling motion and the steady-state roll rate for projectiles and missiles. The accurate prediction of the roll characteristics of projectiles is important in projectile design.

Well-designed projectiles will have an equilibrium roll rate that avoids both the yawing frequency and their first natural frequency of vibration [2]. The roll rate is directly related to stability of projectile hence the roll damping coefficient is crucial input for trajectory modelling. It should be known during design and can be refined during development.

In order to design a projectile, the aerodynamic coefficients must be predicted to within engineering accuracy. Initial estimates of aerodynamic coefficients are usually obtained using semi-empirical engineering codes and computational methods. Analytical and semi-empirical methods are used in the literature to estimate C_{lp} and C_{ls} . Prediction of rolling moment coefficient is relatively easy and straightforward while only approximate methods are used for estimating roll damping coefficient. Difficulties arise because of the assumptions and limitations of each analysis or approximate method. Also, very often, only supersonic speed regimes are considered because of the relative ease in using the linearized supersonic theory [3]. Thus, estimation of roll damping coefficient in transonic and subsonic applications remain grey area.

Most of initial estimates of aerodynamic coefficients are refined further using wind tunnel tests and / or flight tests. However, the roll-damping moment coefficient is difficult to measure and requires a free rolling rig in association with a sting balance [4] for its measurement. As the magnitude of bearing friction of rolling rig under test conditions is difficult to estimate, friction effects on the measured aerodynamic characteristics are usually not adequately evaluated [5]. Thus, there is a need of estimating accurate roll-damping moment by other method.

An estimation of unknown aerodynamic coefficients from flight data has been of interest for many years. The coefficients are used to provide final verification flight [6]. Nowadays the position and roll during flight can be obtained using tracking radar or telemetry. Parameter estimation techniques allow an engineer to form an empirical model of a system using measured data. Filtering techniques are generally employed to remove noise and estimate unobserved state. Kalman Filter is an accepted and well-known technique for state estimation. The EKF estimation method can deal with both process and measurement noise. It gives an optimal state estimate even in the presence of noise and un-modelled dynamics. In Extended Kalman filter, nonlinearity of system is taken care by employing instantaneous linearization at each time step. It assumes Gaussian distribution for the uncertainties in system dynamics. Furthermore, unknown variables can be included as part of state variables and parameter estimation problem can be transformed into state estimation problem [7].

In [8], Kalman filtering technique is used for trajectory estimation of a maneuverable target utilizing the information acquired from ground-based radar. Use of different version of EKF for projectile identification and impact point prediction is also reported in [9]. Neural Kalman filter approach to target tracking is presented as a technique to improve the motion model of the target while it is being tracked in flight [10]. EKF is successfully used to estimate ballistics coefficient [11], drag coefficient [12-13] and atmospheric conditions [14] from trajectory data. Flight state parameters of guided projectile are obtained using Extended Kalman filter (EKF) and Linearized Kalman filter (LKF) [15].

The aim of this paper is to present an approach to extract information from flight test data using Extended Kalman Filter. An estimation approach is applied to predict the roll damping coefficients. Problem of estimation of roll damping coefficient is transformed into the state estimation problem. The roll damping coefficients of a projectile is obtained from the measured roll rate profile and positional trajectory data using Extended Kalman Filter. The simulation is carried out for roll stabilized and fin stabilized projectiles to cater different configuration and roll rate so that proposed method can be applied for real-world problem. There are limitations of methods of estimating roll damping coefficient during design. When trajectory data is available the method proposed in this paper will be a better tool to refine the estimates.

II. PROBLEM STATEMENT AND SYSTEM DESCRIPTION

A. Rolling Motion Equation

The equation of rolling motion about the body axis is usually written as

$$\left(C_{l_\delta} \delta + C_{l_p} \frac{p d}{2v} \right) q_\infty A_{\text{ref}} d = I_{xx} \ddot{\delta} \quad (1)$$

where

$p = \dot{\delta}$ is roll rate, d is reference diameter, v is velocity, q_∞ is dynamic pressure, A_{ref} is reference area and I_{xx} is axial moment of inertia and

For steady state roll motion ($p = p_s$), $\ddot{\delta} = 0$ and therefore

$$\frac{C_{l_p}}{C_{l_\delta}} = - \frac{2v\delta}{p_s d} \quad (2)$$

Various researchers have provided empirical relation between C_{l_p} and C_{l_δ} by assuming their ratio to be a constant during steady state motion.

For four-fin body in the plus (+) formation, i.e., cruciform fins, Adams and Dugan obtained the expression

$$\frac{C_{l_p}}{C_{l_\delta}} = - 0.627 \frac{d}{b_o} \quad (3)$$

where b_o is the total span of two-fin panels including the center body diameter. Eastman empirically wrote the following correlation in a form similar to the result of the analysis of Adams and Dugan

$$\frac{C_{l_p}}{C_{l_\delta}} = - 2.15 \frac{y_c}{d} \quad (4)$$

in which y_c is the distance between the rolling body axis and the area center of one fin panel. Eastman showed that his correlation was valid not only for supersonic speeds as Adams and Dugan's analysis implied but for all speed regimes. This approximation was extended to an arbitrary number of fins. This was further extended for curved and wraparound fins as follows

$$\frac{C_{l_p}}{C_{l_\delta} \frac{y_c}{d} \left(\frac{l}{h} \right)^m} = - 2.15 \quad (5)$$

Where $\left(\frac{l}{h} \right)$ is the ratio of the arc length of the curved fin chord to the distance between the two fin end points and m is constant based on Mach No.

It is known that C_{l_δ} is relatively easier to measure in wind tunnels or compute with analytic and empirical formulae. Estimating the roll-damping moment coefficient from wind tunnel is much more difficult. It requires a free rolling rig in association with a sting balance. Moreover, the magnitude of bearing friction of rolling rig under test conditions is difficult to estimate, so friction effects on the measured aerodynamic characteristics are usually not adequately evaluated [4-5]. The correlations in equations (3), (4) or (5) are used to find coefficient C_{l_p} using C_{l_δ} of specific projectile. It means

different projectile configurations like rolling, non-rolling and with different number of fins has different valid empirical relation.

Moreover, for constant $C_{lp}/C_{l\delta}$, steady state roll rate p_s must vary identically with the velocity along the trajectory. In general, velocity of projectile decreases over time in ballistics phase. Thus, p must also decrease to keep the ratio $C_{lp}/C_{l\delta}$ constant. Therefore, if p decreases and if the ratio of $C_{lp}/C_{l\delta}$ is constant along the trajectory, then this does not translate to a true steady state roll value [16]. Thus, none of these relations is direct, accurate, or valid for all speed regimes, and hence, each of these methods of estimating C_{lp} has limitations and regime of validity. Furthermore, these calculations are effective for a particular configuration. Thus, there is need to obtain accurate roll-damping moment coefficient by other practical methods.

B. Projectile Dynamics

If the projectile has good dynamic stability modified point mass (MPM) model can be used to approximate the motion of the projectile [13]. MPM model is conventional point mass model along with axial roll and instantaneous equilibrium yaw term added to it. The MPM takes into account accelerations due to drag, lift, Magnus force, Coriolis force and gravity. It estimates angle of yaw, drag, drift and Magnus force effects resulting from yaw of repose. Drag, lift, Magnus force coefficient and pitch and roll damping coefficients are aerodynamics inputs to the model [17]. It has been proved that modified point mass model accurately calculates trajectories of roll stabilized, slowly rolling and finned projectiles [18].

To define trajectory equations a right-handed, orthonormal, ground-fixed, Cartesian coordinate system as a frame of reference is considered and shown in Fig 1. The modified point mass model can be written in scalar form as follows

Fig 1. Reference frame for MPM

$$\frac{dx}{dt} = \dot{x} = u_x \quad (6)$$

$$\frac{dy}{dt} = \dot{y} = u_y \quad (7)$$

$$\frac{dz}{dt} = \dot{z} = u_z \quad (8)$$

$$\ddot{x} = \frac{d\dot{x}}{dt} = \frac{du_x}{dt} = \frac{\pi p d^2}{8m} \left[- \left(C_{D0} + C_{D_{a^2}} \alpha_r^2 \right) v v_x + C_{L_a} \alpha_x v^2 - \frac{d}{2} C_{\gamma_{pa}} p m_x \right] + c_x - g_x \quad (9)$$

$$\ddot{y} = \frac{dy}{dt} = \frac{du_y}{dt} = \frac{\pi \rho d^2}{8m} \left[- \left(C_{D0} + C_{D_{a^2}} \alpha_r^2 \right) v v_y + C_{L_a} \alpha_y v^2 - \frac{d}{2} C_{\gamma_{pa}} p m_y \right] + c_y - g_y \quad (10)$$

$$\ddot{z} = \frac{dz}{dt} = \frac{du_z}{dt} = \frac{\pi \rho d^2}{8m} \left[- \left(C_{D0} + C_{D_{a^2}} \alpha_r^2 \right) v v_z + C_{L_a} \alpha_z v^2 - \frac{d}{2} C_{\gamma_{pa}} p m_z \right] + c_z - g_z \quad (11)$$

$$\frac{dp}{dt} = \frac{1}{I_x} \frac{\rho V^2}{2} \frac{\pi d^2}{4} \left(C_{l_b} \delta + C_{l_p} \frac{pd}{2v} \right) d \quad (12)$$

where

$$c_x = 2\Omega [\sin(L_a) v_z - \cos(L_a) \sin(A_z) v_y]$$

$$c_y = -2\Omega [-\cos(L_a) \cos(A_z) v_z - \cos(L_a) \sin(A_z) v_x]$$

$$c_z = 2\Omega [-\cos(L_a) \cos(A_z) v_y - \sin(L_a) v_x]$$

$$g_x = g_o \frac{R^2}{r^3} (x-0)$$

$$g_y = g_o \frac{R^2}{r^3} (y-0)$$

$$g_z = g_o \frac{R^2}{r^3} (x+R)$$

$$g_0 = 9.80665 [1 - 0.0026373 \cos(2L_a) +$$

$$0.0000059 [\cos(2L_a)]^2]$$

$$\alpha_x = \frac{8pI_{xx}}{\pi \rho d^3 v^4 C_{M_a}} [v_y \ddot{z} - v_z \ddot{y}]$$

$$\alpha_y = \frac{8pI_x}{\pi \rho d^3 v^4 C_{M_a}} [v_x \ddot{z} - v_z \ddot{x}]$$

$$\alpha_z = \frac{8pI_x}{\pi \rho d^3 v^4 C_{M_a}} [v_x \ddot{y} - v_y \ddot{x}]$$

$$m_x = [\alpha_y v_z - \alpha_z v_y],$$

$$m_y = -[\alpha_x v_z - \alpha_z v_x],$$

$$m_z = [\alpha_x v_y - \alpha_y v_x]$$

$$v_x = u_x - w_x,$$

$$v_y = u_y - w_y,$$

$$v_z = u_z - w_z$$

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2},$$

$$\alpha_r = \sqrt{\alpha_x^2 + \alpha_y^2 + \alpha_z^2},$$

$$r = \sqrt{x^2 + y^2 + (x+R)^2}$$

d = diameter of projectile

m = mass of projectile

I_{xx} = axial moment of inertia

ρ = density of air

L_a = latitude

A_z = azimuth

Ω = angular speed of earth rotation (7.29×10^{-3} rad/ s)

p = roll rate

δ = cant angle

v = projectile speed

α = total angle of yaw

R = radius of earth

t = time

g_0 = gravity constant

c_x, c_y, c_z = Coriolis force components,

v_x, v_y, v_z = air relative velocity components,

u_x, u_y, u_z = projectile velocity components,

w_x, w_y, w_z = wind velocity components,

$\alpha_x, \alpha_y, \alpha_z$ = yaw of repose components,

$\ddot{x}, \ddot{y}, \ddot{z}$ = projectile acceleration components,

C_{D_0} = zero yaw drag force coefficient

$C_{D_{\alpha^2}}$ = yaw drag force coefficient

C_{L_α} = lift force coefficient

$C_{\gamma_{pa}}$ = Magnus force coefficients

C_{M_α} = pitching moments coefficient

C. Extended Kalman Filter

Kalman filtering is used to estimate unknown variables from a series of measurements containing statistical noise and other inaccuracies.

To apply Extended Kalman Filter (EKF) the set of non-linear differential equations representing the system must be expressed in a state space form as [11]

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{w} \quad (13)$$

where vector $\mathbf{x}$ represents the state of the system, $\mathbf{f}(\mathbf{x})$ is a nonlinear function of the state variables and $\mathbf{w}$ is a random, zero mean process noise. Presence of process noise indicates that model of real system is not precise. Covariance of process noise $\mathbf{w}$ is represented by a matrix $\mathbf{Q}$ and is called process noise matrix.

Let the relation between measurement and state vector be given by

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{v} \quad (14)$$

where $\mathbf{z}$ is the measurement vector, $\mathbf{h}(\mathbf{x})$ is the measurement equation and $\mathbf{v}$ is the zero mean measurement noise.

For a system in which measurements are discrete we can rewrite nonlinear measurement equations as

$$\mathbf{z}_k = \mathbf{h}(\mathbf{x}_k) + \mathbf{v}_k \quad (15)$$

where $\mathbf{z}_k$ is the measurement vector and $\mathbf{v}_k$ is noise vector at instance k . The discrete measurement noise matrix $\mathbf{R}_k$ consist of matrix of covariance between measurement noise source.

The system dynamics matrix $\mathbf{F}$ and the measurement matrix $\mathbf{H}$ at any instance are related to nonlinear system and measurement equations as

$$\mathbf{F} = \left. \frac{\partial \mathbf{f}(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x} = \hat{\mathbf{x}}} \quad (16)$$

$$\mathbf{H} = \left. \frac{\partial \mathbf{h}(\mathbf{x})}{\partial \mathbf{x}} \right|_{\mathbf{x} = \hat{\mathbf{x}}} \quad (17)$$

The fundamental matrix Φ_k required for propagating state forward can be approximated by Taylor series expansion for $\exp(\mathbf{F}T_s)$ and is given by

$$\Phi_k = I + FT_s + \frac{F^2 T_s^2}{2!} + \frac{F^3 T_s^3}{3!} + \dots$$

Often this series is approximated by only first two terms

$$\Phi_k \approx I + FT_s \quad (18)$$

Where T_s is the sampling time and I is the identity matrix. In EKF fundamental matrix is used for calculating Kalman gain.

Riccati equations, required for the computation of the Kalman gains are

$$M_k = \Phi_k P_{k-1} \Phi_k^T + Q_k \quad (19)$$

$$K_k = M_k H^T (H M_k H^T + R_k)^{-1} \quad (20)$$

$$P_k = (I - K_k H) M_k \quad (21)$$

where

P_k Covariance matrix representing errors in the state estimates after an update

M_k Covariance matrix representing errors in the state estimates before an update

Q_k Discrete process noise matrix

R_k Measurement noise matrix

K_k Kalman gain

Φ_k and H fundamental matrix and measurement matrix respectively

The discrete process noise matrix Q_k is calculated using equation (22)

$$Q_k = \int_0^{T_s} \Phi(t) Q \Phi^T(t) dt \quad (22)$$

With an extended Kalman filter the new state estimate is the old state estimate projected forward to the new measurement time plus gain times a residual

$$\hat{x}_k = \bar{x}_k + K_k [z_k - H \bar{x}_k] \quad (23)$$

where

$$\bar{x}_k = \hat{x}_{k-1} + f(\hat{x}_{k-1}) T_s \quad (24)$$

The time step for the integration was set to match the data rate of measurement and Runge-Kutta 4th order integration was carried out to predict the system states.

The system state variables are defined to include all the parameters of interest. The state is estimated at every measurement step and thus estimation of the parameters of interest is obtained.

III. METHODOLOGY

A. Formulation of Problem

This study includes both formulation of appropriate models and estimation of the parameters in these models. Estimation of parameters through the filtering method is an indirect process. Here, parameter estimation problem is transferred into a state estimation problem. Modified point mass model is chosen as system dynamics since it adequately describes projectile dynamics. The unknown parameter is defined as additional state variables in system state. In this problem, roll damping coefficient C_{lp} is considered as unknown variable and augmented to other seven state variables namely, three projectile positions, three velocities and roll. Total number of state variable will be eight and each will be estimated using EKF from four measured variables namely range, height, drift and roll of the projectile at every instance.

The projectile positions, roll, velocities and coefficient to be estimated is represented as state vector $\mathbf{x} = [x, y, z, sp, \dot{x}, \dot{y}, \dot{z}, \mu_{Clp}]^T$. Then MPM in state space form $\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{w}$ can be written as

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{p} \\ \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{\mu}_{Clp} \end{bmatrix} = \begin{bmatrix} u_x \\ u_y \\ u_z \\ \frac{1}{I_x} \frac{\rho V^2 \pi d^2}{2 \cdot 4} (C_{l\delta} \delta + C_{lp} \frac{pd}{2V}) d \\ \frac{\pi \rho d^2}{8m} \left[- (C_{D0} + C_{D_{a^2}} \alpha_r^2) v v_x + C_{L_a} \alpha_x v^2 - \frac{d}{2} C_{\gamma_{pa}} p m_x \right] + c_x - g_x \\ \frac{\pi \rho d^2}{8m} \left[- (C_{D0} + C_{D_{a^2}} \alpha_r^2) v v_y + C_{L_a} \alpha_y v^2 - \frac{d}{2} C_{\gamma_{pa}} p m_y \right] + c_y - g_y \\ \frac{\pi \rho d^2}{8m} \left[- (C_{D0} + C_{D_{a^2}} \alpha_r^2) v v_z + C_{L_a} \alpha_z v^2 - \frac{d}{2} C_{\gamma_{pa}} p m_z \right] + c_z - g_z \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ u_s \end{bmatrix} \quad (25)$$

We assume process noise u_s is present in the unknown state variable μ_{Clp} and its covariance defines the matrix $\mathbf{Q}$.

The measurement vector is $\mathbf{z} = [x, y, z, sp]^T$ and in state space form $\mathbf{z} = \mathbf{h}(\mathbf{x}) + \mathbf{v}$ can be written as

$$\begin{bmatrix} x \\ y \\ z \\ sp \end{bmatrix} = \begin{bmatrix} \tilde{x} \\ \tilde{y} \\ \tilde{z} \\ \tilde{sp} \end{bmatrix} + \begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ \delta_{sp} \end{bmatrix} \quad (26)$$

where $[\tilde{x}, \tilde{y}, \tilde{z}, \tilde{sp}]^T$ denotes actual positions and roll and $[\delta_x, \delta_y, \delta_z, \delta_{sp}]^T$ is measurement Gaussian noise in respective coordinates and matrix $\mathbf{R}$ covariance of measurement noise.

B. Extended Kalman Filter Design and Implementation

The simulation flow chart is shown in Fig 2. The MPM model i.e equation (6-12) is used to simulate the trajectory elements and suitable noise is added to it to reflect it as measurements as described in equation (26). These measurements

are passed to EKF model and all eight states variable are estimated for each time step.

Fig. 2: Concept of parameter estimation

Jacobi matrix $\mathbf{F}$ and $\mathbf{H}$ are calculated using equation (16) and (17). The simplified block diagram of the EKF estimation is shown in Fig 3. The initial conditions of different state variables are set using guess values. These guess values are passed to system dynamics. It consists of the modified point mass model i.e., equation (25) which predicts all state variables using equation (24). This is the predicted state at next time instance. The predicted state and measurements are used to compute Kalman gain for each variable and update error covariance using equation (18-21). Kalman gain is used to correct or update the estimates. The state update formulation is given in equation (23-24). These corrected estimates serve as the starting point for next iteration.

Fig. 3: Kalman filter prediction and correction

C. Filter Initialization

To begin the Kalman filtering procedure an initial estimate of both, the states and the covariance matrix, are essential. These can be initialized in different ways. For the unknown variable the most common way to do so is by a simple guess. Here initial value of unknown variable roll coefficient is obtained from empirical method. The other states can be initialized using first few measurements [19] and simple differentiation. The measurement noise covariance $\mathbf{R}$ is obtained from a model of actual radar noise characteristics. i.e., noise level added during measurement simulation. Large values are set as diagonal values of $\mathbf{P}$ to accommodate initialization error.

IV. SIMULATION AND RESULTS

A. *Simulation of Measurement Data*

Generally, projectiles are either roll stabilized or fin stabilized. To cover both type of configurations estimation of roll damping coefficients for roll and fin stabilized projectiles are carried out. In roll stabilized configuration there is no fin to projectile hence roll produced roll moment coefficient and fin cant angle value are zero. This configuration has very high roll rate. In fin stabilized configuration roll moment coefficients and fin cant angle are non-zero. Fin stabilized configuration is slowly rolling configuration. For simulation purpose it is assumed that these values are known.

Configurations and inputs considered are given in Table 1. The complete details on aerodynamics, mass and inertia for configuration A are referred from [20] and for configuration B from [2].

Table 1: Configuration details

Parameters	Unit	Configuration A	Configuration B
Stability	-	Roll	Fin
Configuration Type	-	Shell	Rocket
Diameter	m	0.155	0.122
Total Length	m	0.8787	2.870
In Flight Mass	kg	43.7	45.5
Axial Moment of Inertia	kgm ³	0.1444	0.1238
Lateral Moment of Inertia	kgm ³	1.7323	33.83
Muzzle Velocity	m/s	935	26.7
Roll rate at Muzzle	rps	1895	5.8
Peak Roll rate	rps	1895	190
Thrust/ Burn Time/	-	not applicable	applicable
Cant Angle	deg	0.0	1.0
Roll moment coefficient	-	absent	present

To simulate trajectory data for both configurations, modified point mass trajectory model equations (6-12) are used. The known roll damping coefficient and other aero coefficient are used while simulating the trajectory. Trajectory positional data and roll rate data are logged after every 0.1s. With assumption that the radar and launch point are collocated. The configuration A is of a shell which is roll stabilized. It does not have any fin and hence there is no cant angle and rolling moments. Whereas the configuration B is fin stabilized and has cant angle 1.0 deg and rolling moment. The Gaussian noise $\mathcal{N}(0, \sigma)$ is added in logged data of position and roll so that it represents the measurement obtained from onboard telemetry. Finally, $\mathbf{z} = [x, y, z, sp]^T$ is the measurement vector made available for estimation for every time step as defined in equation no (26).

B. *Estimation of Roll Damping Coefficient for Roll Stabilized Shell*

In this case, the measurements are simulated for muzzle velocity 935m/s and 50 deg elevation in standard atmospheric conditions for configuration A, i.e., roll stabilized shell described in Table 1. The measured data contains x, y, z, sp for every time step of 0.1s from launch to impact point. The maximum velocity is 935m/s and minimum velocity is 325m/s

during the flight it means Mach No 0.95 to 2.50 is covered.

The aim of simulations is to estimate the roll damping characteristic of the projectile. As it is roll stabilized configuration $C_{l\delta} = 0$ and $\delta = 0$ and characteristics of roll will depend only on C_{lp} . In EKF roll damping coefficient μ_{Clp} is included as unknown variable along with other state variables. The state $\mathbf{x} = [x, y, z, sp, \dot{x}, \dot{y}, \dot{z}, \mu_{Clp}]^T$ is estimated for each time step T_s . The initial conditions for EKF are appropriately set and process and measurement noise matrices are initialized based on noise level.

The velocity components namely horizontal ($\dot{x}=u_x$), vertical ($\dot{z}=u_z$) and lateral ($\dot{y}=u_y$) are estimated at every time step. The difference between estimated and actual is shown as error in estimation for all three velocities. These are plotted in Fig. 4. The standard deviation in error in estimate caused by noise is represented by square root of diagonal elements of covariance matrix $\mathbf{P}_k$ after an update and are used as the theoretical estimation levels. For simplicity theoretical bound of only horizontal velocity are plotted. Theoretical values for vertical and lateral velocities are in same order after convergence. It is observed that estimation errors are less than ± 1 m/s for all velocity components after initial few seconds.

Fig. 4: Estimation Error in velocity components u_x, u_y, u_z w.r.t time

The difference between actual and estimated roll rate is less than ± 2 rps and is given in Fig. 5. The errors in estimated roll damping coefficient and theoretical error for complete flight is shown in Fig 6. The percentage errors in estimation with respect to actual coefficient is of order of 5% and shown in Fig 7.

Fig. 5: Estimation Error in Roll rate w.r.t time

Fig. 6: Estimation Error in Roll Damping coefficient rate w.r.t time

Fig. 7: Percentage Error in Roll Damping Coefficient rate w.r.t time

C. Estimation of Roll Damping Coefficient for Fin Stabilized Rocket

Roll damping coefficient is successfully estimated for roll stabilized shell where $C_{l_5} = 0$ and $\delta = 0$. It is necessary to check same methodology for roll stabilized rockets where $C_{l_5} \neq 0$, $\delta \neq 0$ and characteristics of roll depends on C_{l_p} , C_{l_5} and δ . In this case rocket configuration B, as described in Table 1, is considered. An attempt is made to estimate roll damping coefficient in presence of roll moment coefficient and non-zero cant angle. In EKF roll damping coefficient μ_{Clp} is included as unknown variable along with other states variables. The state $\mathbf{x} = [x, y, z, sp, \dot{x}, \dot{y}, \dot{z}, \mu_{Clp}]^T$ is estimated for each time step T_s . The initial conditions for EKF are appropriately set and process and measurement noise matrices are initialized based on noise level.

This rocket has two phases power and unpowered phase. The trajectory data of only unpowered phase is used to estimate roll damping coefficient. Measured data contains x, y, z, sp for every time step of 0.01s from all burnt point of rocket to impact point. The maximum velocity is 680 m/s and minimum velocity is 305m/s during the flight. It means Mach No 2.0 to 0.90 is covered.

The convergence of three velocities are shown in Fig. 8. The difference between actual with estimated roll rate and errors in estimated roll damping coefficient with theoretical error shown in Fig. 9 and 10 respectively. The percentage

errors in estimation shown in in Fig. 11. In this case also percentage error is less than 5%.

Fig. 8: Estimation Error in velocity components u_x , u_y , u_z w.r.t time

Fig. 9: Estimation Error in Roll rate w.r.t time

Fig. 10: Estimation Error in Roll Damping coefficient w.r.t time

Fig. 11: Percentage Error in Roll Damping Coefficient w.r.t time

D. Mach -Roll Damping Coefficient Profile

The roll damping coefficient is estimated in absence and presence of roll moments with cant angel for configuration A and B respectively. The coefficient values are selected for Mach number experienced by flight for comparison purpose. In both cases the estimation of roll damping coefficient is within 5% after convergence. The estimated and known roll damping coefficient is plotted with respect to Mach number and shown in Fig. 12 and 13. The close agreement in estimation indicates practicability of proposed methods and mathematical model for estimation of roll damping coefficient.

Fig. 12: Roll Damping Coefficient Vs Mach no for Roll stabilized shell

Fig. 13: Roll Damping Coefficient Vs Mach no for Fin stabilized Rocket

V. CONCLUSION

The roll damping coefficient C_p is characterized by a motion generated by frictional or viscosity forces which damps the rotation of a projectile around its longitudinal axis. The roll damping coefficient is crucial input for checking stability and trajectory modelling of any projectile. This aerodynamic coefficient is difficult to obtain even experimentally, and there is little experimental data for reference. To get roll damping coefficient designer has to rely on computational methods, empirical or semi-empirical codes. Empirical method has limitations and Mach No regime of validity and calculations are effective for a particular configuration. To obtain roll-damping moment coefficient or to carry refinement of roll damping coefficient by other technique is difficult problem. Nowadays flight can be obtained using tracking radar or telemetry easily. An attempt is made to utilize this data to extract roll damping coefficients using EKF. A variable representing C_p is included in state variables. Fin stabilized and roll stabilized configurations are investigated in this study and it is observed that Mach – Roll damping coefficient profiles obtained using proposed method are in good agreement with known profiles. The results indicate that a proposed method is applicable for different configurations of projectiles and complete Mach regime. In view of this, estimation of roll damping coefficient from trajectory data using EKF is feasible and reliable method. Coefficients estimated from flight data can be used for refinement of preliminary calculated values. This method can be used on board and the estimated flight coefficients fed into control and guidance loop that can maximize the system performance.

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DECLARATIONS

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Figures

Figure 1

Reference frame for MPM

Figure 2

Concept of parameter estimation

Figure 3

Kalman filter prediction and correction

Figure 4

Estimation Error in velocity components u_x , u_y , u_z w.r.t time

Figure 5

Estimation Error in Roll rate w.r.t time

Figure 6

Estimation Error in Roll Damping coefficient rate w.r.t time

Figure 7

Percentage Error in Roll Damping Coefficient rate w.r.t time

Figure 8

Estimation Error in velocity components u_x , u_y , u_z w.r.t time

Figure 9

Estimation Error in Roll rate w.r.t time

Figure 10

Estimation Error in Roll Damping coefficient w.r.t time

Figure 11

Percentage Error in Roll Damping Coefficient w.r.t time

Figure 12

Roll Damping Coefficient Vs Mach no for Roll stabilized shell

Figure 13

Roll Damping Coefficient Vs Mach no for Fin stabilized Rocket