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A New Explainable Robust High Order Intuitionistic Fuzzy Time Series Method

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Abstract

Fuzzy time series forecasting methods based on type-1 fuzzy sets continue to have largely proposed in the literature. These methods use only membership values in determining fuzzy relations. However, Intuitionistic fuzzy time series models basically use both membership values and non-membership values. So, it can be considered that the using of intuitionistic fuzzy time forecasting models will be able to increase the forecasting performance in the fuzzy time series analyses because of the fact that more information is used. Therefore, Intuitionistic fuzzy time series models have started to use for solving the real-life series in the fuzzy time series literature since 2013. In this study, a new explainable robust high order intuitionistic fuzzy time series forecasting method are proposed based on new defined model. In the proposed method, the algorithm of intuitionistic fuzzy c-means is used for fuzzification of observations and a robust regression method is used to determine fuzzy relations. Because the robust regression is employed to define fuzzy relation, all inputs of the method can be explainable and they can be statistically tested and commented. Applications of this study have been made by using energy data of Primary Energy Consumption (PEC) between 1965 and 2016 for 23 countries in the region of Europe&Eurasia. Forecasting performances obtained from these applications by using the proposed method have been compared with performances of some other time series method in the literature and the results have been discussed.

Keywords: Intuitionistic fuzzy time series, Intuitionistic fuzzy sets, forecasting, robust regression, intuitionistic fuzzy c-means, explainable artificial intelligence, principal component analysis, energy data forecasting.

1. Introduction

Classical time series methods continue to be largely used in the time series analysis at the present time. The most fundamental method of classical time series is the autoregressive integrated moving average (ARIMA) [1]. The approach of Box-Jenkins [2] is frequently used as a basic approach in the literature for obtaining the best ARIMA model. In addition, exponential smoothing methods are also preferred in the literature for time series forecasting. Indeed, exponential smoothing models are special cases of ARIMA models. In classical time series, exponential smoothing methods are generally described under three headings as the simple exponential smoothing, Holt's exponential smoothing and Winter's exponential smoothing [1]. However, there are also new exponential smoothing methods proposed in the literature. One of these methods is the ATA exponential smoothing method proposed by Yapar [3]. ATA exponential smoothing method [3,4,5] is an updated and effective time series analysis method proposed as an alternative to exponential smoothing methods in the literature.

Classical time series methods necessitate that observations of the time series have some statistical assumptions like the normality, the stability and the reversibility because of the fact that these methods are principally the probabilistic methods. However, many time series in a real-life do not satisfy any assumptions and so these time series need to be solved via using some non-probabilistic methods in the literature. In the literature, non-probabilistic time series forecasting methods are generally separated into three parts as the methods of fuzzy time series, artificial neural network and other computational methods. Additionally, these non-probabilistic methods could be used together. For instance, the solving algorithm of a fuzzy time series method could include an artificial neural network or a computational method. In this regard, the content of this study is generally a new non-probabilistic method.

The first fuzzy time series approach was proposed by Song and Chissom [6]. According to Song and Chissom [6], the solving of a fuzzy time series consists of generally three phases. These phases are the fuzzification, the identification of fuzzy relation and the defuzzification respectively. In the fuzzification phase, dividing the universe of discourse proposed by Song and Chissom [6,7] is used largely in literature. Additionally, there are some studies with regard to be selected the optimal interval length in the literature, because determining the interval length effects forecasting performance significantly in the fuzzification phase. In the literature, the first study of these studies is Huarng's approach [8] to determine the optimal interval length. Huarng [8] proposed two different approaches based on distribution and average. Huarng [9] also proposed a ratio-based method for selecting the optimal interval length. The other some studies for determining the optimal intervals are given as Yolcu et al. [10], Egrioglu et al. [11] and Egrioglu et al. [12]. In the fuzzy time series literature, the centralization technique is mainly used in the defuzzification phase that is the final phase to solve the fuzzy time series.

The most important stage is the identification of fuzzy relation which is the second stage in the forecasting algorithm of the fuzzy time series. Reason for this that this stage is largely effective in increasing forecasting performance of any fuzzy time series method. The Song and Chissom [7] used the complex matrix operations to determine fuzzy relations as being based on the fuzzy set theory of Zadeh [13]. Then, Chen [14] proposed a new method which is based on fuzzy relation tables instead of the complex matrix operations of Song and Chissom [7]. Thereby, many fuzzy time series methods based on fuzzy relation tables have been proposed so far. In the fuzzy time series literature, some methods that use fuzzy relation tables for defining fuzzy relations are given as Chen [15], Chen and Chen [16], Chen and Tanuwijaya [17], Kocak [18], Cheng et al. [19] and Kocak [20]. Artificial neural networks (ANNs) are intensely used in algorithms of the fuzzy time series. Aladag et al. [21] used the ANN to identify fuzzy relations for the high order fuzzy time series model in the fuzzy time series literature. Egrioglu et al. [22] also used the ANNs for the high order multivariate time series method. Yu and Huarng [23] proposed a new method based on the ANNs neural network for defining fuzzy relations. In the fuzzy time series literature, some methods that use ANNs for defining fuzzy relations are given as Egrioglu et al. [24], Aladag et al. [25], Aladag [26], Kocak [27], Bas et al.[28] Kocak et al. [29] and Bas et al.[30]. Also, Many fuzzy time series methods have been proposed based on particle swarm optimization (PSO). In the fuzzy time series literature, Kuo et al. [31] developed a new method named hybrid particle swarm optimization (MPSO) for solving the historical data of enrollments of the University of Alabama. Similarly, Hsu et al [32] modified the PSO method and proposed a new method named modified turbulent particle swarm optimization (MTPSO) for forecasting the fuzzy time series of temperature. There are other many fuzzy time-series studies based on PSO in the fuzzy time series literature. Some methods that use PSO are given as Kuo et al. [33], Park et al [34], Huarng et al. [35], Aladag et al. [36], Chen and Kao [37], Singh and Borah [38] and Cagcag Yolcu and Lam [39]. Besides, there are some studies using fuzzy C means (FCM) for especially calculating membership values of the fuzzy time series in the literature. Egrioglu et al. [40] and Cagcag Yolcu and Alpaslan [41] proposed different models based on FCM and PSO.

There are also non-probabilistic and non-parametric time series forecasting methods based on methods of machine learning, the artificial neural network, the artificial intelligence besides of fuzzy time series methods. Some of these studies are given as Yolcu et al. [42], Aladag et al. [43], Zhou et al. [44] and Egrioglu et al. [45]. Yolcu et al. [42] proposed a new artificial neural network based on PSO for solving linear and nonlinear time series. Aladag et al. [43] proposed a new artificial neural network time series forecasting method based on median neuron model. Zhou et al. [44] predicted some financial time series by using a dendritic neuron model. Egrioglu et al. [45] proposed a new median Pi ANN model in solving time series.

Fuzzy time series uses only membership values for solving fuzzy time series. However, Intuitionistic fuzzy time series based on intuitionistic fuzzy sets use both membership and non-membership values. Therefore, more information is used during forecasting the fuzzy time series. The first article with respect intuitionistic fuzzy time series was made by Zheng et al. [46] in the year 2013. For this reason, the subject of IFTS can be thought a new and an update field for time series literature. Fundamental definitions of IFTS were initially given by Zheng et al. [46]. Furthermore, Zheng et al.[47] used initially intuitionistic fuzzy c-means algorithm proposed by Chaira [48] in order to determine membership and non-membership values of the intuitionistic fuzzy time series. Thereby, the use of intuitionistic fuzzy c-means algorithm has still continued to commonly use in applications of intuitionistic fuzzy time series. Wang et al. [49] proposed a high order intuitionistic fuzzy time series model by using unequal partition the universe of discourse based on the fuzzy clustering algorithm. Kumar and Gangwar [50] determined intuitionistic fuzzy relations to solve the intuitionistic fuzzy time series. Fan et al. [51] proposed a long term intuitionistic fuzzy time series forecasting model. Egrioglu et al. [52] introduced a new intuitionistic fuzzy time series forecasting method based on pi-sigma artificial neural networks and artificial bee colony. Also, some other studies interested in intuitionistic fuzzy time series can be given as Zheng et al. [53, 54], Wang et al. [55], Joshi et al. [56], Hu et al. [57], Fan et al. [58] and Abhishekh et al. [59].

In this study, a new explainable robust high order intuitionistic fuzzy time series method based on new model definiton is proposed. In the proposed method, intuitionistic fuzzy c-means [48] is used in the fuzzification phase and robust regression is used in determining fuzzy relations. After the fuzzification phase of the proposed method, principal component analysis is used. In this way, dimension reduction is made and multicollinearity problems are eliminated. Moreover, inputs of the model can be explainable and they can be statistically tested and commented. In the second section, the intuitionistic fuzzy c-means algorithm is given step by step. Robust regression is summarized in the third section. Some definitions of intuitionistic fuzzy time series are given in the fourth section. The algorithm of the proposed method is intrıduced with step by step algorithm in the fifth section. Applications of the proposed method and other methods in the literature are given in section six. And finally, conclusions and discussions about statistical results are given in section seven.

2. Intuitionistic fuzzy time series definitions

Many definitions of fuzzy time series are based on Song and Chissom's definition that is the first definition of fuzzy time series in the literature. However, in recent years, New definitions of fuzzy time series have been made and many researchers have been prefered to use new definitions in their studies. Because, some fuzzy time series forecasting models proposed in the decade are based on using membership values via fuzzy c-means, intuitionistic fuzzy c-means, or other fuzzy clustering methods instead of defining fuzzy set. New fuzzy time series and intuitionistic fuzzy time series definitions were broadly explained in the study of Egrioglu et al. [52]. Some of intuitionistic fuzzy time series definitions of Egrioglu et al. [52] are given below:

Definition 1. (Intuitionistic Fuzzy Time Series)

Let X_t is a time series with real observations. $A_1, A_2, \dots, A_c$ are intuitionistic fuzzy sets on universal set. Intuitionistic fuzzy time series (IF_t) is a multivariate time series with real observations, membership and non-membership values for each fuzzy sets.

$$IF_t = \left\{X_t, \mu_{A_1}(t), \mu_{A_2}(t), \dots, \mu_{A_c}(t), \nu_{A_1}(t), \nu_{A_2}(t), \dots, \nu_{A_c}(t)\right\} \tag{1}$$

Where $\mu_{A_j}(t)$, $\nu_{A_j}(t)$ are membership and non-membership values of t^{th} observation to j^{th} intuitionistic fuzzy set and these values can be obtained intuitionistic fuzzy c-means or other intuitionistic fuzzy clustering methods.

Definition 2. (High Order Single Variable Intuitionistic Fuzzy Time Series Forecasting Model)

Let IF_t be an intuitionistic fuzzy time series. $A_1, A_2, \dots, A_c$ are intuitionistic fuzzy sets on universal set. $\mu_{A_j}(t)$, $\nu_{A_j}(t)$ are membership and non-membership values of t^{th} observation to j^{th} intuitionistic fuzzy set. The high order single variable intuitionistic fuzzy time series model is given below:

$$X_t = G\Big(X_{t-1}, \dots, X_{t-p}, \mu_{A_1}(t-1), \mu_{A_2}(t-1), \dots, \mu_{A_c}(t-1), \dots, \mu_{A_1}(t-p), \mu_{A_2}(t-p), \dots, \mu_{A_c}(t-p), \nu_{A_1}(t-1), \nu_{A_2}(t-1), \dots, \nu_{A_c}(t-1), \dots, \nu_{A_1}(t-p), \nu_{A_2}(t-p), \dots, \nu_{A_c}(t-p)\Big) + \varepsilon_t \tag{2}$$

In equation (2), G is a linear or non-linear function, ε_t is an error term with zero mean, $X_{t-1}, \dots, X_{t-p}$ are lagged variables of X_t, $\mu_{A_1}(t-1), \dots, \mu_{A_c}(t-p)$ and $\nu_{A_2}(t-1), \dots, \nu_{A_c}(t-p)$ are lagged membership values and non-membership values, respectively, obtained from $X_{t-1}, \dots, X_{t-p}$.

Equation (2) expresses that any times series is affected lagged time series, lagged membership values and non-membership values. However, membership values and non-membership values are actually obtained from the lagged time series. Therefore, in this paper, it is thought that it will be more suitable to reduce input variables and to decrease analysis time if lagged time series are not involved in equation (2). So, in this study, two new definitions are given below, respectively.

Definition 3. (New Definition: High Order Single Variable Intuitionistic Fuzzy Time Series Forecasting Model)

Let IF_t be an intuitionistic fuzzy time series. $A_1, A_2, \dots, A_c$ are intuitionistic fuzzy sets on universal set. $\mu_{A_j}(t)$, $\nu_{A_j}(t)$ are membership and non-membership values of t^{th} observation to j^{th} intuitionistic fuzzy set. The high order single variable intuitionistic fuzzy time series model is given below:

$X_t = G\left(\mu_{A_1}(t-1), \mu_{A_2}(t-1) \ldots, \mu_{A_c}(t-1), \ldots, \mu_{A_1}(t-p), \mu_{A_2}(t-p), \ldots, \mu_{A_c}(t-p), \nu_{A_1}(t-1), \nu_{A_2}(t-1), \ldots, \nu_{A_c}(t-1), \ldots, \nu_{A_1}(t-p), \nu_{A_2}(t-p), \ldots, \nu_{A_c}(t-p)\right) + \varepsilon_t$ (3)

In equation (2), G is a linear or non-linear function, ε_t is an error term with zero mean, $\mu_{A_1}(t-1), \ldots, \mu_{A_c}(t-p)$ and $\nu_{A_2}(t-1), \ldots, \nu_{A_c}(t-p)$ are lagged membership values and non-membership values, respectively, obtained from $X_{t-1}, \ldots, X_{t-p}$. Equation (3) expresses that any times series is affected by lagged membership values and non-membership values.

Definition 4. (New Definition: High Order Single Variable Intuitionistic Fuzzy Time Series Forecasting Model based on the principal components analysis and the robust regression analysis)

Let IF_t be an intuitionistic fuzzy time series. $A_1, A_2, \ldots, A_c$ are intuitionistic fuzzy sets on universal set. $\mu_{A_j}(t)$, $\nu_{A_j}(t)$ are membership and non-membership values of t^{th} observation to j^{th} intuitionistic fuzzy set for j=1,2,…,c. The high order single variable intuitionistic fuzzy time series model based on the principal components analysis is given below:

$X_t = \beta_0 + \beta_1 z_1 + \beta_2 z_2 + \cdots \beta_q z_q + \varepsilon_t \quad for\ q \leq cp$ (4)

Where $z_i, i = 1,2,..,q$ are principal components.

$z_i = f_i(\mu_{A_1}(t-1), \mu_{A_2}(t-1) \ldots, \mu_{A_c}(t-1), \ldots, \mu_{A_1}(t-p), \mu_{A_2}(t-p), \ldots, \mu_{A_c}(t-p), \nu_{A_1}(t-1), \nu_{A_2}(t-1), \ldots, \nu_{A_c}(t-1), \ldots, \nu_{A_1}(t-p), \nu_{A_2}(t-p), \ldots, \nu_{A_c}(t-p)) \quad i = 1,2,,..,q$

In equation (4), p is the number of lagged variables, $\mu_{A_1}(t-1), \ldots, \mu_{A_c}(t-p)$ and $\nu_{A_2}(t-1), \ldots, \nu_{A_c}(t-p)$ are lagged membership values and non-membership values obtained from $X_{t-1}, \ldots, X_{t-p}$, q is the number of the important principal component for $q \leq cp$, f_i is i^{th} linear function obtained by using the correlation matrix of the principal component analysis, z_i is the i^{th} z-score calculated via the principal component analysis, $\beta_0, \beta_1, \beta_2 \ldots \beta_q$ are regression coefficients obtained via the robust regression method and ε_t is the error term with zero means. In definition 4, Equation (4) is a special situation of equation (3) given definition 3.

3. Intuitionistic fuzzy c-means algorithm (IFCM)

Intuitionistic fuzzy c-means clustering algorithm (IFCM) proposed Claira [48] has been commonly used Intuitionistic fuzzy time-series studies [46,47,52] in the literature. Also, In this study, the intuitionistic fuzzy c-means algorithm [43] is used in the fuzzification of the algorithm of the proposed method. intuitionistic fuzzy c-means clustering algorithm proposed by Chaira [43] is given below:

Algorithm 1. The intuitionistic fuzzy c-means clustering algorithm

Step 1. Calculate the membership values ($u_{ik}, i = 1,2, \ldots, c; k = 1,2, \ldots, n$) by using equation (5).

$u_{ik} = \frac{r_{ik}}{\sum_{i=1}^{c} r_{ik}}$ (5)

Suppose x_k $(k = 1,2, \dots, n)$ is a time series with n observations. In equation (5), c is clustering number determined by researcher and r_{ik} $(i = 1,2, \dots, c; k = 1,2, \dots, n)$ are random values generated from a uniform distribution that has the parameters $(0,1)$.

Step 2. Calculate hesitation degrees $(\pi_{ik}, i = 1,2, \dots, c; k = 1,2, \dots, n)$ and intuitionistic fuzzy membership values $(u_{ik}^*, i = 1,2, \dots, c; k = 1,2, \dots, n)$ by using equations (6) and (7), respectively. Thereafter, save the intuitionistic membership values (u_{ik}^*) into a matrix named U_{old}.

$$\pi_{ik} = 1 - u_{ik} - (1 - u_{ik}{}^{\alpha})^{1/\alpha}, \ \ \alpha > 0 \tag{6}$$

$$u_{ik}^* = u_{ik} + \pi_{ik} \tag{7}$$

Step 3. Calculate centres of the clusters $(v_i^*, \ i = 1,2, \dots, c)$ of the intuitionistic membership values (u_{ik}^*) by using equation (8).

$$v_i^* = \frac{\sum_{k=1}^{n} (u_{ik}^*)^f x_k}{\sum_{k=1}^{n} (u_{ik}^*)^f} \ ; i = 1,2, \dots, c \tag{8}$$

In equation (7), x_k $(k = 1,2, \dots, n)$ are observations of the time series the and f is the fuzziness index.

Step 4. Modify the membership values $(u_{ik}, i = 1,2, \dots, c; k = 1,2, \dots, n)$ by using equation (9).

$$u_{ik} = \frac{1}{\sum_{j=1}^{c} \left(\frac{d_{ik}}{d_{jk}}\right)^{2/(f-1)}} \ \ i = 1,2, \dots, c; k = 1,2, \dots, n \tag{9}$$

$$d_{ik} = \sqrt{(x_k - v_i^*)^2} \tag{10}$$

In equation (10), d_{ik} $(i = 1,2, \dots, c; k = 1,2, \dots, n)$ is the Euclidean distance measure between i^{th} cluster centre and k^{th} observation.

Step 5. Go to Step 2 and modify hesitation degrees $(\pi_{ik}, i = 1,2, \dots, c; k = 1,2, \dots, n)$ and fuzzy membership values $(u_{ik}^*, i = 1,2, \dots, c; k = 1,2, \dots, n)$ by using u_{ik} values obtained in Step 4. Thereafter, save the new intuitionistic membership values (u_{ik}^*) into a matrix named U_{new}.

Step 6. Calculate non-membership values $(v_{ik}, \ i = 1,2, \dots, c; k = 1,2, \dots, n)$ by using equation (11). Thereafter, save the new intuitionistic membership values (u_{ik}^*) into a matrix named V.

$$u_{ik}^* + \pi_{ik} + v_{ik} = 1 \tag{11}$$

In equation (10), u_{ik}^* and π_{ik} are intuitionistic membership values and hesitation degrees obtained Step 5.

Step 7. Check Stopping criteria given in equation (12). If the condition is satisfied, stop the algorithm, otherwise make $U_{old} = U_{new}$ and go to Step 3. ε is a small positive number and $\|.\|_2$ is the L_2 norm.

$$\|U_{new} - U_{old}\|_2 < \varepsilon \tag{12}$$

In equation (11), ε is a small positive number and $\|.\|_2$ is the L_2 norm.

4. The Robust Regression

Robust regression is commonly used instead of the least square method in the literature because of the fact that robust estimators are not affected from outliers too much. Fundamental robust regression techniques can be given as L1 technique of Edgeworth [60], M-Estimates of Mallows [61], and S-Estimates of Rousseeuw [62]. Algorithms of robust regression based on these fundamental robust techniques have been proposed in the literature. One of these algorithms is the iteratively reweighted least-squares technique [63, 64] that is an M-estimate approach. In this study, robust regression estimates are made by using iteratively reweighted least-squares technique in defining fuzzy relation phase. Accordingly, the algorithm of the iteratively reweighted least squares is given in Algorithm 2.

Algorithm 2. Iteratively reweighted least squares algorithm

Step 1. Calculate initial regression coefficient estimates $\hat{\beta}$ of β via the least square method for linear regression model given in equation (13) and calculate standard deviation estimate $\hat{\sigma}$ of σ by using equation (14).

$$Y = X_{n*(p+1)}\beta_{(p+1)*1} + \mathcal{E}_{n*1} \tag{13}$$

$$\hat{\sigma} = \frac{Median\ (|e_i|)}{0.6745},\ \text{i=1,2,…,n} \tag{14}$$

In equation (2), n is observation number, p is the independent variable number, Y is dependent variable vector, X is the independent variable matrix that values of its first column are 1, Ɛ is error parameter vector, e_i is an estimate of $\mathcal{E}_i$ for i[th] observation obtained via the least and 0.6745 is the constant used to make estimates for normal distribution.

Step 2. Calculate residuals (e_i; i=1,2,…,n) by using equation (15)

$$e_i = y_i - x_i'\hat{\beta} \tag{15}$$

In equation (3), y_i is i[th] observation value of the dependent variable and x_i' is a row vector that is the transpose of the first column of X matrix given Step 1.

Step 3. Calculate weight values ($w_i; i = 1,2,..n$) via equation (16) by using bisquare weight function (W) given in equation (17).

$$w_i = W(\tfrac{e_i}{\hat{\sigma}}) \tag{16}$$

$$W(e) = \left\{\begin{matrix} \left[1 - \left(\frac{e}{4.685}\right)^2\right]^2 & ,for\ |e| \le 4.685 \\ 0 & ,for\ |e| > 4.685 \end{matrix}\right\} \tag{17}$$

In equations (15) and (16), $\hat{\sigma}$ is standard deviation estimate calculated in Step 1 and 4.685 is the tuning constant for the bisquare distribution.

Step 4. Calculate $\hat{\beta}_{new}$ by using equation (18).

$$\sum_{i=1}^{n} w_i\, x_i (y_i - x_i' \hat{\beta}_{new}) = 0 \tag{18}$$

Step5 .Calculate $e_{i(new)}$ $(i = 1,2, \dots n)$ by using equation (19).

$$e_{i(new)} = y_i - x_i' \hat{\beta}_{new} \tag{19}$$

Step 6. Check Stopping criteria given in equation (20). If the condition is satisfied, stop the algorithm, otherwise make $\hat{\beta} = \hat{\beta}_{new}$ and go to Step 2.

$$max_i\left(\left|e_i - e_{i(new)}\right|\right) < \varepsilon \tag{20}$$

In equation (20), ε is a small positive number and $|.|$ is the absolute value.

5. The Proposed Method

In this study, a new intuitionistic high order fuzzy time series forecasting method is proposed. The proposed method generally uses equations (3) given in Definition 3 and completely uses equation (4) given in Definition (4). Thereby, both lagged memberships and lagged non-memberships are used in accordance with intuitionistic fuzzy time series approach, unlike the fuzzy time series approach that uses only non-membership values. In the proposed method, the intuitionistic fuzzy c-means clustering method is used given in Section (3) to obtain lagged memberships and lagged non-memberships in the fuzzification phase. Furthermore, the principal component analysis is used to eliminate the multicollinearity problem and reduce dimension because there are many inputs before the phase of determining fuzzy relationships of the proposed method. The proposed method is based on robust regression. For this reason, Iteratively reweighted least squares technique that is a Robust regression method given in Section 4 is used to define functional relationships in the stage of determining fuzzy relationships. Also, the proposed method does not need the defuzzification step because the outputs of the proposed method are real forecasts. The flow chart and Algorithm of the proposed method is given in Figure 1 and Algorithm 3, respectively.

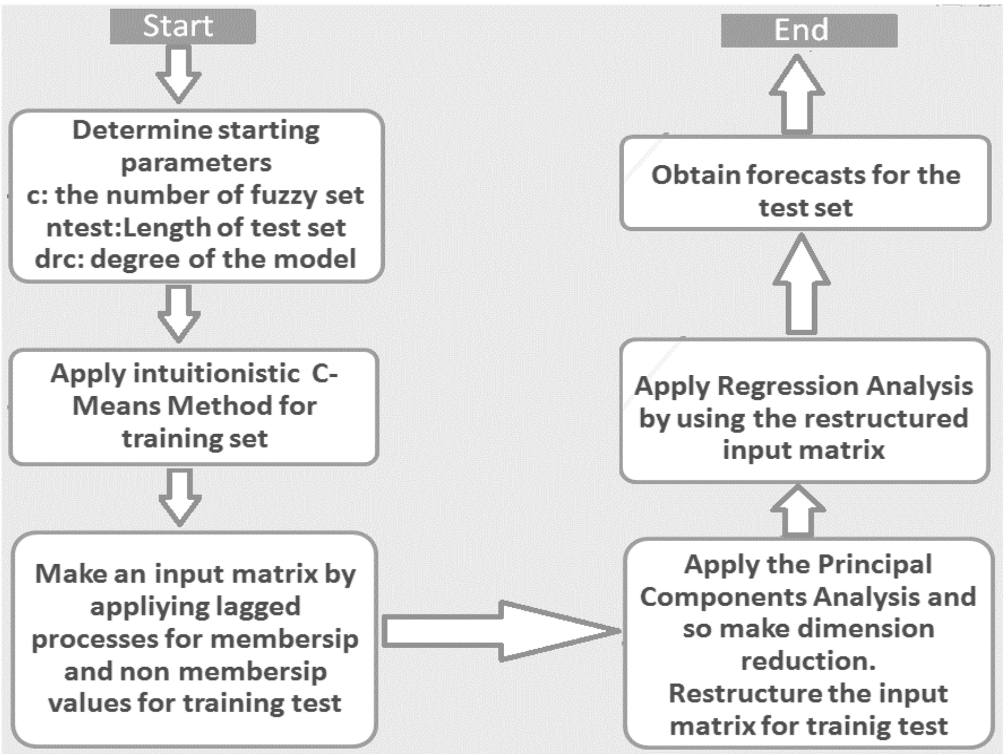

Figure 1. Flowchart of Proposed Method

Algorithm 3. Algorithm of the proposed Method.

Step 1. Determine X_t, X_t^{test},X_t^{train}, n, n_{test}, n_{train}, c and m (1,2,3,…) that are data and parameters used in the application of the proposed method. Here, X_t is the time series, X_t^{test} is the test set of X_t, $X_t^{train}$is the training test of X_t,, n is the length of X_t , n_{test} (the last part of %5, %10 or %15 of n) is the length of X_t^{test}, n_{train} is the length of X_t^{train}, c (2,3,…) is the number of fuzzy sets and m (1,2,3,…) is the order of the model.

Step 2. Calculate memberships and non-memberships for the training time series (X_t^{train}) by using Intuitionistic fuzzy c-means (IFCM) clustering method via Algorithm 1 given in Section 3.

Here, the matrix of memberships named U ($U_{ij}; i = 1,2, \dots, n_{train}$; $j = 1,2, \dots, c$), the matrix of non-memberships named V $\left(V_{ij};\ i = 1,2, \dots, n_{train};\ j = 1,2, \dots, c\right)$ and the vector of centres of clusters named v^* ($v_i^*; i = 1,2, \dots, c$)) are obtained. Also, the intuitionistic fuzzy time series matrix (IF_t; $i = 1,2, \dots, c; j = 1,2, \dots, (2c + 1)$) is constituted given in equation (1) by combining of X_t^{train}, U and V, respectively.

For example, Let $X_t = \{8,10,11,12,16,13,14\}$, $X_t^{test} = \{14\}$, $X_t^{train} = \{8,10,11,12,16,13\}$, c=3, m=2 and n_{test}=1 and n_{train}=6. Also, let $\mu_{A_i}(t)$, $\nu_{A_i}(t)$ are membership and non-membership values of t^{th} observation to i^{th} intuitionistic fuzzy set. U, V, v^* and IF_t calculated via *Algorithm 1* are given below for $X_t^{train} = \{8,10,11,12,16,13\}$.

$$U = \{\, \mu_{A_1}(t), \quad \mu_{A_2}(t), \quad \mu_{A_3}(t)\} = \begin{bmatrix} 0.0314 & 0.9630 & 0.1021 \\ 0.0482 & 0.8408 & 0.3144 \\ 0.0574 & 0.2494 & 0.8778 \\ 0.0017 & 0.0025 & 0.9988 \\ 0.9985 & 0.0015 & 0.0037 \\ 0.1878 & 0.1056 & 0.8907 \end{bmatrix} \tag{21}$$

$$V = \{\, v_{A_1}(t), \quad v_{A_2}(t), \quad v_{A_3}(t)\} = \begin{bmatrix} 0.9513 & 0.0030 & 0.8527 \\ 0.9270 & 0.0765 & 0.5897 \\ 0.9139 & 0.6664 & 0.0504 \\ 0.9971 & 0.9957 & 0.0012 \\ 0.0015 & 0.9974 & 0.9937 \\ 0.7420 & 0.8480 & 0.0419 \end{bmatrix} \tag{22}$$

$$v^* = \begin{bmatrix} 15.8612 \\ 8.9699 \\ 11.9189 \end{bmatrix} \tag{23}$$

$$IF_t = \{U, V\} = \begin{bmatrix} 0.0314 & 0.9630 & 0.1021 & 0.9513 & 0.0030 & 0.8527 \\ 0.0482 & 0.8408 & 0.3144 & 0.9270 & 0.0765 & 0.5897 \\ 0.0574 & 0.2494 & 0.8778 & 0.9139 & 0.6664 & 0.0504 \\ 0.0017 & 0.0025 & 0.9988 & 0.9971 & 0.9957 & 0.0012 \\ 0.9985 & 0.0015 & 0.0037 & 0.0015 & 0.9974 & 0.9937 \\ 0.1878 & 0.1056 & 0.8907 & 0.7420 & 0.8480 & 0.0419 \end{bmatrix} \tag{24}$$

Step 3. Apply lag process for U and V, separately. And so, obtain input matrix (X_{input}) that includes lagged variables of memberships and non-memberships in accordance with equation (4) given in Definition 4.

For example, X_{input} matrix obtained by using U and V in (21) and (22) is given below:

$$X_{input} = \begin{bmatrix} 0.0482 & 0.8408 & 0.3144 & 0.0314 & 0.9630 & 0.1021 & 0.9270 & 0.0765 & 0.5897 & 0.9513 & 0.0030 & 0.8527 \\ 0.0574 & 0.2494 & 0.8778 & 0.0482 & 0.8408 & 0.3144 & 0.9139 & 0.6664 & 0.0504 & 0.9270 & 0.0765 & 0.5897 \\ 0.0017 & 0.0025 & 0.9988 & 0.0574 & 0.2494 & 0.8778 & 0.9971 & 0.9957 & 0.0012 & 0.9139 & 0.6664 & 0.0504 \\ 0.9985 & 0.0015 & 0.0037 & 0.0017 & 0.0025 & 0.9988 & 0.0015 & 0.9974 & 0.9937 & 0.9971 & 0.9957 & 0.0012 \end{bmatrix} \tag{25}$$

Step 4. Apply principal component analysis (PCA) to X_{input} matrix obtained in *Step 3.* And so, obtain the matrix of principal component coefficients (PCC), the matrix of principal component (PC), the matrix of important principal component (IPC) and the new input matrix (P_{input}), respectively.

Firstly, elements of X_{input} matrix is transformed to the standard normal matrix named Z_{input} that means and standard deviations of each column of Z_{input} are 0 and 1. Then, PCA is applied to Z_{input}. And so, PCC and PC matrix are calculated. Also, important principal components are selected from principal components by taking into consideration the criterion that the explained variance ratio needs to be at least 0.85. In this way, IPC matrix is obtained. Lastly, the new input matrix named P_{input} is obtained by putting a vector that consists of 1 values to the first column of IPC.

For example, PCC, PC, IPC and P_{input} matrices obtained by applying PCA to X_{input} in (26) are given below.

$$PCC^{T} = \begin{bmatrix} 0.3408 & -0.2341 & -0.2191 & -0.2722 & -0.3262 & 0.3051 & -0.3375 & 0.2447 & 0.2465 & 0.2765 & 0.3362 & -0.2889 \\ -0.1459 & -0.3597 & 0.3900 & 0.3239 & -0.1962 & 0.2564 & 0.1551 & 0.3544 & -0.3568 & -0.3169 & 0.1599 & -0.2918 \\ -0.3145 & 0.4443 & -0.1921 & 0.1181 & -0.3364 & 0.2708 & 0.3473 & -0.3256 & 0.2305 & -0.1198 & 0.3505 & -0.2223 \end{bmatrix} \tag{26}$$

$$PC = \begin{bmatrix} -2.0114 & -2.2301 & 0.3612 \\ -1.7560 & 0.4749 & -0.6783 \\ -0.2017 & 2.5459 & 0.4131 \\ 3.9691 & -0.7907 & -0.0960 \end{bmatrix} \tag{27}$$

$$IPC = \begin{bmatrix} -2.0114 & -2.2301 \\ -1.7560 & 0.4749 \\ -0.2017 & 2.5459 \\ 3.9691 & -0.7907 \end{bmatrix} \tag{28}$$

$$P_{input} = \begin{bmatrix} 1 & -2.0114 & -2.2301 \\ 1 & -1.7560 & 0.4749 \\ 1 & -0.2017 & 2.5459 \\ 1 & 3.9691 & -0.7907 \end{bmatrix} \tag{29}$$

In (26), "^{T}" indicates transpose operation. The explained variance ratio of two important principal components in (28) is % 97.86. So, it can be said that the aims of dimension reduction and obtaining orthogonal inputs are accomplished via PCA.

Step 5. Define fuzzy relations by using Robust Regression.

Robust regression coefficient estimates ($\hat{\beta}$) is calculated via iteratively reweighted least squares algorithm given *in Algorithm 2* by taking X=P_{input} and Y=X_t^{train} in step 1 of *Algorithm 2.* Then, predictions of the training set ($\hat{X}_t^{train}$) can be calculated by using (30) if the aim of any researcher is to predict.

$$\hat{X}_t^{train} = P_{input}\hat{\beta} \tag{30}$$

For example, $\hat{\beta}$ obtained via *Algorithm 2* by taking X=P_{input} in (28) and Y=$X_t^{train} = [-,-,11,12,16,13]$ is given below.

$$\hat{\beta} = \begin{bmatrix} 13.000 \\ 0.2257 \\ 0.9445 \end{bmatrix} \tag{31}$$

Step 6. Apply determining membership values for time series (X_t) in order to obtain new memberships (U_{new}) and new non-memberships (V_{new}) of X_t by using centres of clusters (v^*) obtained in *Step 1.*

For example, U_{new} and V_{new} for $X_t = \{8,10,11,12,16,13,14\}$ by using v^* in (23) calculated for $X_t^{train} = \{8,10,11,12,16,13\}$ are given below.

$$U_{new} = \begin{bmatrix} 0.0314 & 0.9630 & 0.1021 \\ 0.0482 & 0.8408 & 0.3144 \\ 0.0574 & 0.2494 & 0.8778 \\ 0.0017 & 0.0025 & 0.9988 \\ 0.9985 & 0.0015 & 0.0037 \\ 0.1878 & 0.1056 & 0.8907 \\ 0.6269 & 0.1226 & 0.5278 \end{bmatrix} \tag{32}$$

$$V_{new} = \begin{bmatrix} 0.9513 & 0.0030 & 0.8527 \\ 0.9270 & 0.0765 & 0.5897 \\ 0.9139 & 0.6664 & 0.0504 \\ 0.9971 & 0.9957 & 0.0012 \\ 0.0015 & 0.9974 & 0.9937 \\ 0.7420 & 0.8480 & 0.0419 \\ 0.2570 & 0.8256 & 0.3573 \end{bmatrix} \tag{33}$$

Step 7. Apply lag process for U_{new} and V_{new} obtained in Step 6 and obtain new input matrix (X_{input}^{new}), the new standardized input matrix (Z_{input}^{new}) and standardized input matrix of the test set (Z_{input}^{test}) that includes z scores of lagged variables of memberships and non-memberships.

Firstly, the new input matrix (X_{input}^{new}) that includes lagged variables of memberships and non-memberships is obtained by using U_{new} and V_{new}. Then, elements of X_{input}^{new} matrix are transformed to standard normal values and so, Z_{input}^{new} matrix is obtained that means and standard deviations of each column of Z_{input}^{new} are 0 and 1. Lastly, Z_{input}^{test} that includes the last some rows of Z_{input}^{new} is obtained.

For example, X_{input}^{new} and matrix obtained by using U_{new} and $V_{\ new}$ obtained for $X_t = \{8,10,11,12,16,13,14\}$,c=3 and m=2 is given below:

$$X_{input}^{new} = \begin{bmatrix} 0.0482 & 0.8408 & 0.3144 & 0.0314 & 0.9630 & 0.1021 & 0.9270 & 0.0765 & 0.5897 & 0.9513 & 0.0030 & 0.8527 \\ 0.0574 & 0.2494 & 0.8778 & 0.0482 & 0.8408 & 0.3144 & 0.9139 & 0.6664 & 0.0504 & 0.9270 & 0.0765 & 0.5897 \\ 0.0017 & 0.0025 & 0.9988 & 0.0574 & 0.2494 & 0.8778 & 0.9971 & 0.9957 & 0.0012 & 0.9139 & 0.6664 & 0.0504 \\ 0.9985 & 0.0015 & 0.0037 & 0.0017 & 0.0025 & 0.9988 & 0.0015 & 0.9974 & 0.9937 & 0.9971 & 0.9957 & 0.0012 \\ 0.1878 & 0.1056 & 0.8907 & 0.9985 & 0.0015 & 0.0037 & 0.7420 & 0.8480 & 0.0419 & 0.0015 & 0.9974 & 0.9937 \end{bmatrix} \tag{34}$$

$$Z_{input}^{new} = \begin{bmatrix} -0.502 & 1.712 & -0.696 & -0.454 & 1.196 & -0.788 & 0.513 & -1.673 & 0.577 & 0.455 & -1.126 & 0.781 \\ -0.480 & 0.027 & 0.600 & -0.415 & 0.931 & -0.320 & 0.482 & -0.132 & -0.674 & 0.398 & -0.974 & 0.203 \\ -0.613 & -0.677 & 0.878 & -0.394 & -0.352 & 0.923 & 0.684 & 0.729 & -0.758 & 0.367 & 0.245 & -0.983 \\ 1.764 & -0.680 & -1.411 & -0.523 & -0.887 & 1.190 & -1.742 & 0.733 & 1.494 & 0.563 & 0.926 & -1.092 \\ -0.169 & -0.383 & 0.629 & 1.787 & -0.889 & -1.005 & 0.063 & 0.343 & -0.666 & -1.784 & 0.930 & 1.091 \end{bmatrix} \tag{35}$$

$$Z_{input}^{test} = [-0.169 \quad -0.383 \quad 0.629 \quad 1.787 \quad -0.889 \quad -1.005 \quad 0.063 \quad 0.343 \quad -0.666 \quad -1.784 \quad 0.930 \quad 1.091] \tag{36}$$

In (36), $\mathrm{Z}_{\mathrm{input}}^{\mathrm{test}}$ is the last row of Z_{input}^{new} given in (35) because $n_{test} = 1$.

Step 8. Calculate the matrix of principal component (PC_{test}), the matrix of important principal component (IPC_{test}) and the new input matrix (P_{input}^{test}) for X_t^{test} by using $\mathrm{Z}_{\mathrm{input}}^{\mathrm{test}}$ obtained in Step 7.

Firstly, PC_{test} is calculated by using equation (36). In the equation (36), PCC is the matrix of principal component coefficients calculated in *Step 4*. Also, important principal components are selected from principal components by taking into consideration the criterion that the explained variance ratio needs to be at least 0.85. In this way, IPC_{test} matrix is obtained. Lastly, the new input matrix (P_{input}^{test}) is obtained by putting a vector that consists of 1 values to the first column of IPC.

$$PC_{test}=Z_{input}^{test} * PCC \tag{37}$$

For example, PC_{test}, IPC_{test} and P_{input}^{test} calculated by using PCC and Z_{input}^{test} obtained in (26) and (36) are given below for $X_t^{test} = \{14\}$.

$$PC_{test} = [-1.2061 \quad 1.6675 \quad 0.0540] \tag{38}$$

$$IPC_{test} = [-1.2061 \quad 1.6675] \tag{39}$$

$$P_{input}^{test} = [1 \quad -1.2061 \quad 1.6675] \tag{40}$$

Step 9. Calculate the forecasts of X_t^{test} ($\hat{X}_t^{test}$) by using $\hat{\beta}$ and P_{input}^{test} obtained *Step 5* and *Step 8* via equation (41).

$$\hat{X}_t^{test} = P_{input}^{test} * \hat{\beta} \tag{41}$$

For example, $\hat{X}_t^{test}$calculated by using $\hat{\beta}$ and P_{input}^{test} obtained in (31) and (40) are given below for $X_t^{test} = \{14\}$.

$$\hat{X}_t^{test} = [14.3028] \tag{42}$$

Step 10. Root of mean square error (RMSE) and mean absolute percentage error (MAPE) values are calculated by using $\hat{X}_t^{test}$obtained in *Step 9* via the equations (43) and (44).

$$RMSE = \sqrt{\frac{1}{ntest}\sum_{t=1}^{ntest}\left(X_t^{test} - \hat{X}_t^{test}\right)^2} \tag{43}$$

$$MAPE = \frac{1}{ntest}\sum_{t=1}^{ntest}\left|\frac{X_t^{test} - \hat{X}_t^{test}}{X_t^{test}}\right| \tag{44}$$

For example, $RMSE$ and $MAPE$ calculated for $X_t^{test} = \{14\}$ by using $\hat{X}_t^{test}$ obtained in (42) via equation (43) and equation (44) are given below.

$$RMSE = 0.3028 \; and \; MAPE = 0.0216 \tag{45}$$

6. Applications

Energy data taken the ministry of energy and natural resources of Turkey consists of 69 variables of Primary Energy Consumption (PEC). Each of 69 PEC time series has 52 yearly observations between the years 1965 and 2016 for each country in the world. These 69 countries are universally divided 6 continents called North America, South&Central America, Europe&Eurasia, Middle East, Africa, and the Asia Pacific, respectively.

Initially, the energy data of the Europe&Eurasia continent that consists of PEC variables of 32 countries were selected to make applications. However, 23 variables of these 32 variables have complete data that includes 52 observations, because of the fact that PEC variables of the other 9 countries have some missing observations. For this reason, the proposed method and some other forecasting methods in the literature are applied to the 24 PEC variables that include PEC variables of 24 countries in Europe&Eurasia and the PEC variable of total Europe&Eurasia continent. So, it is aimed to compare the forecasting performances of the proposed method with some alternative forecasting methods for the entire Europe&Eurasia continent. The names of 24 countries are Austria, Belgium, Bulgaria, Czech Republic, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Netherland, Norway, Poland, Portugal, Romania, Slovakia, Spain, Sweden, Switzerland, Turkey, United Kingdom and other Europe & Eurasia.

Some of the distributions of the 25 PEC time series used in the application are similar and the others are different. When the 24-time series graphs are viewed in general, it can be seen that the 24-time series may have characteristics such as linear increasing trend, curvilinear increasing trend, stationary or random fluctuations. Some graphs of 24 PEC time series that differ from each other are given in Figure 2.

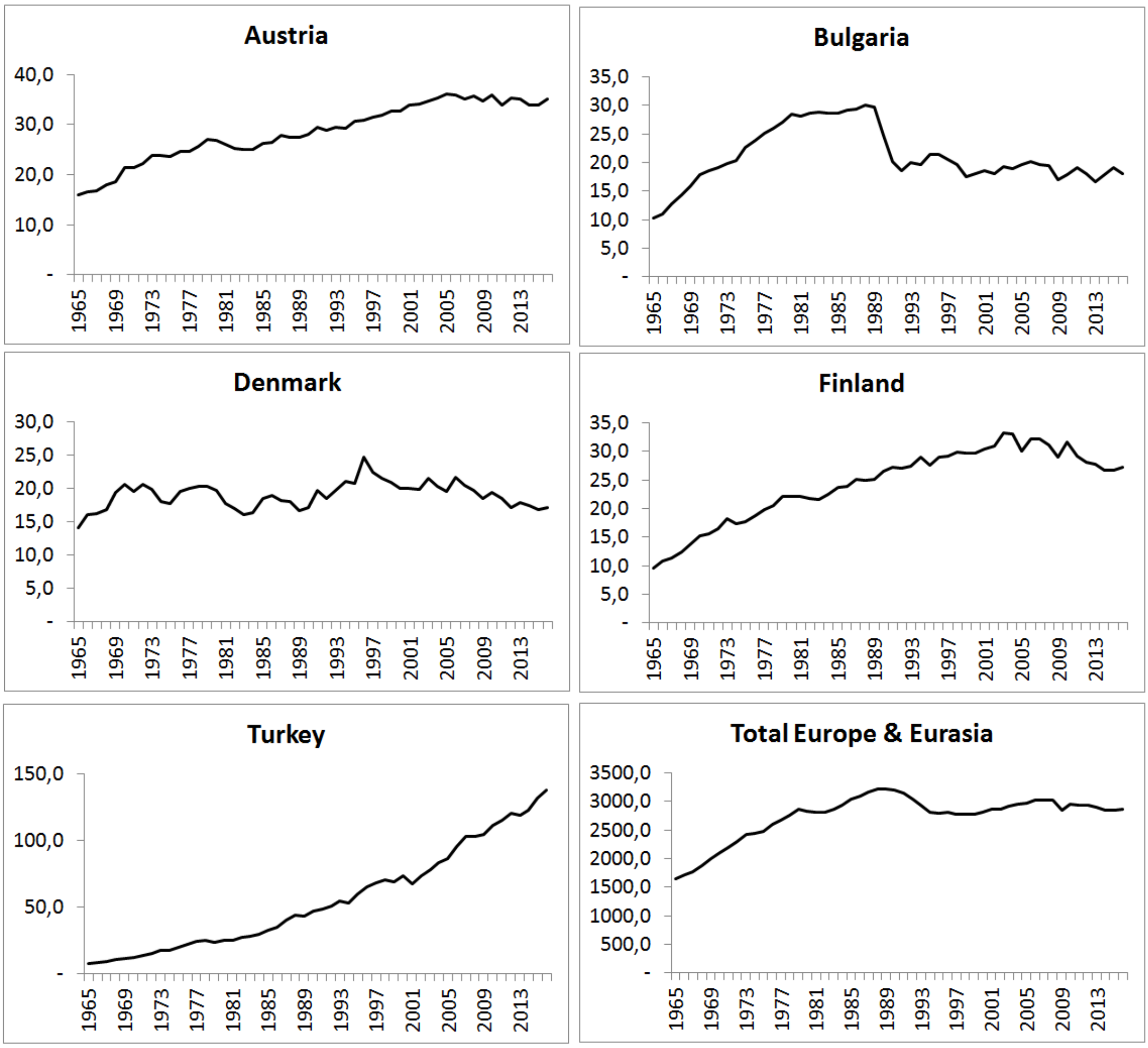

Figure 2. Yearly PEC observations of some countries in the million tonnes of oil equivalent.

In the applications, the alternative time series methods compared with the proposed method are given below.:

DN-PSO Method: The Dendritic neuron (DN) method is an artificial intelligence model proposed by Zhou et al. [44] to predict financial time series. Particle swarm optimization (PSO) is used in the training algorithm of the DN model and a Matlab m-file" is prepared for solving PEC time series with DN-PSO method.

ANN-PSO Method: This method is a Linear&Nonlinear Artificial Neural Network (ANN) Model based on PSO proposed by Yolcu et.al [42] for time series forecasting. A Matlab m-file" is prepared for solving PEC time series with ANN-PSO method.

Median ANN Method: This method is a multilayer artificial neural network (ANN) model based on Median Neuron Model proposed by Aladag et.al [43] for time series forecasting. A Matlab m-file" is prepared for solving PEC time series with Median ANN method.

Median-Pi-ANN Method: The Median-Pi Artificial Neural Network (Median-Pi-ANN) is a time series forecasting method proposed by Egrioglu et.al [45] A Matlab m-file" is prepared for solving PEC time series with Median-Pi-ANN method.

FTS-N: This method is a fuzzy time series network (FTS-N) model proposed Bas et.al. [28] to forecast the linear and nonlinear time series. A Matlab m-file" is prepared for solving PEC time series with FTS-N method.

SC-FTS: Song and Chissom (SC) [6] method is the first fuzzy time series (FTS) forecasting model in the fuzzy time series literature. A Matlab m-file" is prepared for solving PEC time series with SC-FTS method.

C-FTS: This method is a fundamental fuzzy time series (FTS) model based on fuzzy logic relations proposed Chen (C) [15]. A Matlab m-file" is prepared for solving PEC time series with C-FTS method.

ARIMA Method: Autoregressive integrated moving average (ARIMA) method is a classic time series model that is often preferred by many researchers for time series analysis [1]. Box Jenkins's approach [2] is an effective method that provides to be obtained the best ARIMA model. A Matlab m-file" is prepared for solving PEC time series with Box Jenkıns's approach [2].

H-ES Method: Holt's exponential Smoothing (H-ES) method is one of the exponential smoothing methods for time series analysis [1]. A Matlab m-file" is prepared for solving PEC time series with H-ES method.

A-ES Method: Ata's exponential Smoothing (A-ES) method proposed Yapar [3] is a new exponential smoothing method in the time series literature. A Matlab m-file" is prepared for solving PEC time series with A-ES method.

How applications are made with the proposed method is given below.

- Each of 24 PEC time series has 52 observations. The first 47 and the last 5 observations of the time series are used for the training set and the test set, respectively. In other words, Length of the test set is determined as 5 for all applications
- The number of fuzzy sets (c) is tried between 3 and 10 increasing 1 unit.
- Degree of the model (m) is tried between 2 and 5 increasing 1 unit.
- The architecture that has minimum RMSE value is selected as the best architecture. And so, the best forecasts are obtained for the test set.
- 30 runs are executed via using random values (rn) and so, some statistics are calculated for RMSE and MAPE values.

How applications are made with alternative methods is given below.

- The methods of DN-PSO [44], ANNPSO [42], Median ANN [43], Median-Pi-ANN [45] and FTS-N [28] are applied like those applications of the proposed method abovementioned.
- In applications of SC-FTS method [6] and C-FTS method [15], the number of fuzzy sets is tried between 3 and 10 increasing 1 unit.
- The methods of the ARIMA method [2], Holt's Exponential Smoothing [1] and ATA optimization [3] method are applied as classic time series methods.

In the applications, DN-PSO method [44], ANN-PSO method [42], Median ANN method [43], Median-Pi-ANN method, FTS-N method and the proposed method are methods that can be solved by trying different random initial parameters. For this reason, for each of 25 PEC time series, these methods were tried 30 times by using 30 different random seed values and so 30 different RMSE and MAPE values were obtained for each time series in the applications of this study. Means and medians of these RMSE values are given in Table 1. When Table 1 is examined, it is seen that the RMSE statistics of the proposed method are smaller than the RMSE statistics of the other methods for the vast majority of 25 PEC time series solved. Therefore, it can be said that the proposed method is largely better than other methods in forecasting the future.

In the applications, for each of 25 PEC time series, minimum RMSE values of all methods used in this study are given in Table 2. When Table 2 is examined, it is seen that minimum RMSE values of the proposed method are smaller than minimum RMSE values of the other methods for the vast majority of 25 PEC time series solved. Therefore, it can be said that the proposed method has largely better forecasting performance than other methods.

Table 1. Means and medians of RMSE values of the methods for PEC variables of the countries in the Europe&Eurasia continent

Methods	DN-PSO Method [44]		ANN-PSO Method [42]		Median ANN Method [43]		Median-Pi-ANN Method [45]		FTS-N Method [28]		Proposed Method	
Countries	Mean	Median	Mean	Median	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Austria	3.386	3.774	0.929	0.894	1.906	1.109	**0.889**	**0.830**	1.339	1.322	1.103	1.157
Belgium	4.199	4.223	3.904	3.895	**2.419**	2.432	2.420	**2.430**	3.989	3.815	3.050	2.929
Bulgaria	1.625	1.471	1.377	1.358	2.312	2.503	1.080	1.083	1.638	1.695	**0.744**	**0.681**
Czech Republic	1.273	1.085	1.273	1.085	1.692	1.180	1.212	1.206	0.940	0.881	**0.555**	**0.555**
Denmark	1.241	1.201	**0.870**	0.857	1.345	1.373	1.121	1.046	1.273	1.312	0.942	**0.830**
Finland	2.042	2.007	1.694	1.675	**0.536**	**0.499**	1.249	1.232	1.476	1.543	1.449	1.470
France	6.501	5.792	9.903	10.33	4.651	4.687	8.097	7.863	8.044	7.160	**3.405**	**3.108**
Germany	9.665	9.205	9.639	8.804	9.010	7.862	**8.141**	8.191	8.775	7.616	9.702	**5.568**
Greece	2.502	2.134	4.155	4.440	**1.030**	**0.977**	2.464	2.326	1.449	1.206	1.580	1.540
Hungary	2.469	2.466	1.387	1.257	1.190	1.057	1.020	0.877	0.952	0.954	**0.731**	**0.706**
Ireland	0.741	0.699	1.185	1.226	0.772	0.635	1.626	0.718	1.210	1.176	**0.339**	**0.339**
Italy	**6.145**	**5.500**	11.87	11.80	6.597	6.889	9.876	9.225	8.003	8.268	6.390	6.210
Netherland	3.894	3.936	4.116	3.985	3.514	3.128	3.602	3.603	4.574	3.839	**3.317**	**3.114**
Norway	4.742	4.721	3.961	4.047	5.825	5.225	3.068	3.103	**1.950**	**1.732**	4.507	4.839
Poland	5.421	5.536	4.998	4.537	4.434	3.184	2.437	2.388	3.680	3.628	**2.048**	**1.923**
Portugal	1.417	1.269	1.238	1.232	2.202	1.477	1.293	1.261	1.865	1.791	**0.928**	**0.793**
Romania	5.144	4.547	2.796	2.819	3.261	2.334	2.059	**1.394**	**1.955**	1.951	3.668	4.035
Slovakia	1.104	1.081	1.070	1.015	1.345	1.373	0.839	0.834	**0.678**	**0.743**	1.021	1.014
Spain	11.38	9.169	5.883	6.087	8.795	6.758	7.124	6.297	6.478	6.233	**4.159**	**4.300**
Sweden	2.468	2.641	1.729	1.734	1.472	1.478	1.405	1.439	2.013	1.859	**1.196**	**1.294**
Switzerland	1.937	1.992	1.106	1.109	1.156	1.157	1.135	1.102	1.527	1.573	**0.982**	**1.011**
Turkey	36.03	33.58	24.13	24.36	37.05	37.02	21.93	23.92	**4.865**	**4.972**	39.27	36.27
United Kingdom	11.37	10.76	7.866	7.702	12.56	12.85	12.64	12.51	8.074	7.909	**5.039**	**5.040**
Other	3.923	2.633	4.678	4.002	4.708	2.989	3.184	2.767	3.427	3.174	**2.648**	**2.648**
Europe& Eurasia Continent	44.66	45.72	57.77	59.36	36.94	35.53	56.24	56.44	66.89	70.60	**12.58**	**12.58**

Table 2. Minimum RMSE values of the methods for PEC variables of the countries in the Europe &Eurasia continent

	DN-PSO Method [44]	ANN-PSO Method [42]	Median ANN Method [43]	Median-Pi-ANN Method [45]	FTS-N Method [28]	SC-FTS Method [6]	C-FTS Method [15]	ARIMA Method [1,2]	H-ES Method [1]	A-ES Method [3]	Proposed Method
Austria	0.966	0.680	1.226	**0.671**	0.942	0.872	0.982	1.054	1.008	1.015	0.673
Belgium	2.283	2.643	**2.063**	2.39	2.778	2.981	5.284	3.188	3.184	3.672	2.929
Bulgaria	0.712	0.857	0.911	1.006	1.230	0.988	1.165	1.057	1.179	1.092	**0.665**
Czech Republic	0.584	0.584	0.570	0.721	0.562	0.883	0.996	0.730	0.776	0.615	**0.555**
Denmark	0.636	**0.550**	0.920	0.828	0.846	0.699	0.699	0.664	0.812	1.102	0.828
Finland	0.749	**0.259**	0.381	1.201	0.665	1.241	2.083	0.824	1.012	1.607	1.360
France	3.549	4.174	2.976	3.771	3.245	4.400	4.400	5.445	6.136	5.051	**2.947**
Germany	5.451	4.884	7.514	4.478	**4.654**	5.504	5.504	8.202	8.139	7.154	5.415
Greece	0.820	1.282	0.854	1.470	**0.684**	2.085	3.493	0.868	0.783	1.129	1.380
Hungary	1.251	0.827	0.924	0.756	0.834	1.292	2.331	0.845	1.067	1.226	**0.678**
Ireland	0.508	0.764	0.460	0.348	0.560	0.423	0.791	0.395	0.446	0.470	**0.331**
Italy	**3.477**	7.304	3.669	7.537	5.109	6.707	22.31	5.090	6.831	5.978	5.433
Netherland	2.209	2.819	**2.045**	2.142	2.731	3.221	3.229	2.505	3.699	4.352	3.066
Norway	3.667	3.385	3.427	2.234	**0.790**	3.038	4.878	1.862	1.944	1.530	2.971
Poland	1.794	2.834	2.199	2.098	2.350	1.659	1.659	2.631	2.638	2.651	**1.136**
Portugal	1.206	1.179	1.163	1.168	1.374	1.179	1.153	1.297	1.575	1.497	**0.782**
Romania	0.989	**0.901**	0.923	0.921	1.335	2.645	4.775	1.571	1.238	1.321	2.961
Slovakia	0.585	0.574	0.920	0.743	**0.209**	0.792	1.340	0.389	0.732	0.592	0.865
Spain	3.490	3.054	3.082	5.365	2.959	8.699	15.11	3.471	4.262	4.087	**1.975**
Sweden	1.369	1.229	1.244	1.077	1.178	1.174	1.174	1.443	1.904	1.639	**0.832**
Switzerland	1.062	0.984	1.067	0.918	1.089	1.008	1.276	1.207	1.193	1.311	**0.812**
Turkey	19.98	20.80	20.44	13.62	3.547	8.758	7.584	**2.989**	4.165	3.960	27.38
United Kingdom	9.043	5.566	9.76	11.44	**2.526**	6.878	6.690	5.808	5.920	5.809	5.033
Other	1.926	2.519	2.008	2.050	2.240	3.451	**1.482**	2.238	2.881	3.550	2.648
Europe& Eurasia Continent	37.65	26.98	29.25	40.04	30.67	53.03	53.03	30.91	43.89	38.37	**12.58**

In addition, success rates obtained by examining Table 1 and Table 2 are given in Table 3. When Table is examined, it is seen that the proposed method has the highest success rates with 56%, 64% and 44% for minimum RMSE means, minimum RMSE medians and minimum RMSE values, respectively. For instance, According to Table 5, 16 RMSE medians (16/25 = 64%) of 25 RMSE medians of the proposed method are smaller than RMSE medians of the other methods. Other methods have quite lower success rates than the success rates of the proposed method. Therefore, it can be said that the proposed method has largely better forecasting performance than other methods according to RMSE measurement. Similarly, the proposed method has also the highest success rates with 44%, 44% and 32% for minimum MAPE means, minimum MAPE medians and minimum MAPE values, respectively. So, it can be said that the proposed method has largely better forecasting performance than other methods according to MAPE measurement. However, the success rates with scores of 36% and 32% of ANN-PSO method [42] for MAPE means and median means, respectively and the success rate 24% of FTS-N method [28] for minimum MAPE values are remarkable high values. Thereby, as a result, it can be said that the proposed method is the best method, ANN-PSO method [42] is the second-best method and FTS-N method [28] is the third-best method according to MAPE in forecasting the future.

Table 3. Success Rates of the methods

	RMSE			MAPE		
Mehods	Mean	Median	Minimum	Mean	Median	Minimum
DN-PSO Method [44]	1(% 4)	1(% 4)	1(% 4)	0 (% 0)	0(% 0)	1(% 4)
ANN-PSO Method [42]	1(% 4)	0(% 0)	3 (% 12)	9(% 36)	8(% 32)	2(% 8)
Median ANN Method [43]	3(% 12)	2 (% 8)	2(% 8)	3(% 12)	4 (% 16)	3(% 12)
Median-Pi-ANN Method [45]	2(% 8)	3(% 12)	1(% 4)	3 (% 12)	1 (% 4)	3 (% 12)
FTS-N Method [28]	4(% 16)	3 (% 12)	5(% 20)	2 (% 8)	2(% 8)	6 (% 24)
SC-FTS Method [6]	-	-	0 (% 0)	-	-	0(% 0)
C-FTS Method [15]	-	-	1 (% 4)	-	-	1(% 4)
ARIMA Method [1,2]	-	-	1 (% 4)	-	-	2(% 8)
H-ES Method [1]	-	-	0 (% 0)	-	-	0(% 0)
A-ES Method [3]	-	-	0 (% 0)	-	-	0(% 0)
Proposed Method	14(% 56)	16(% 64)	11 (% 44)	11(% 44)	11(% 44)	8(% 32)

The optimal parameter values of the best method for each series are given in a Table in the supplementary file.

7. Explainable Results of the proposed method

The inputs in the model can be tested and commented. The inputs of the models are scores which are obtained from principal component analysis. Contribution of each score to forecasting model can be tested and commented. An example for the Turkey time series is given below by using tables and graphs.

Table 4. Explainable Results for Turkey Time Series

No	Coefficients	Std. Dev.	t-statistics	Prob.
1	49,72	0,74	66,81	0,000000
2	-10,55	0,29	-36,74	0,000000
3	1,05	0,31	3,35	0,001790
4	-4,28	0,34	-12,78	0,000000
5	-3,62	0,35	-10,38	0,000000
6	-2,59	0,43	-6,07	0,000000

In the forecasting model of Turkey time series, six PCA score variables are employed as inputs. It is clearly seen that all inputs are significant in the model. The importance of the inputs can be determined according coefficients. The importance graph is given in Figure 3.

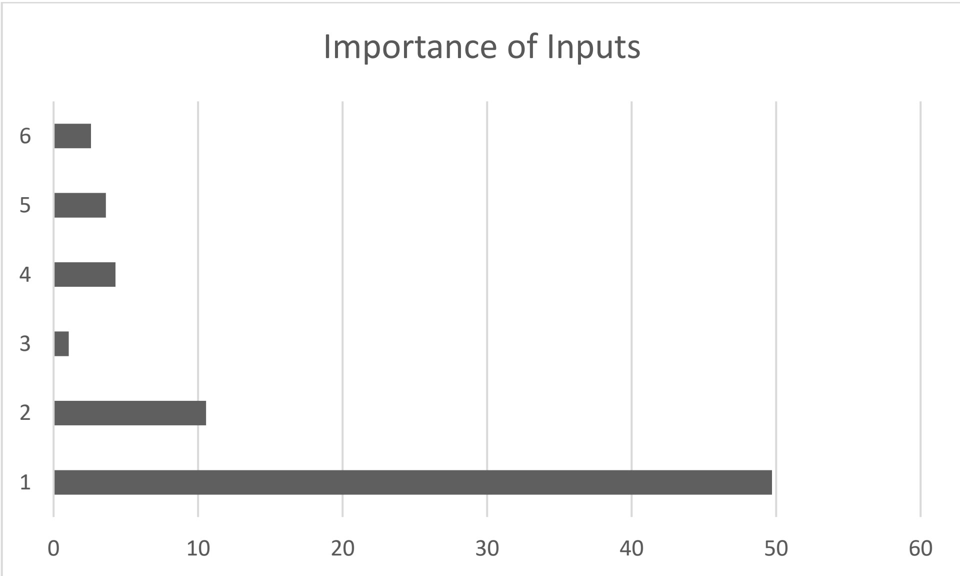

Figure 3. The importance graph of model inputs

Importance of inputs can be determined absolute values of model coefficients. The graph of these values is given in Figure 3. According to figure 3, the most important input is the first input. This input is also the most contributed PCA component. The second input is the second importance order to explain forecasts.

8. Conclusions and Discussion

The high order definition of intuitionistic fuzzy time series definition of Egrioglu et al. [42] given in Definition 2 means that any time series is affected by lagged time series, lagged membership values and non-membership values. However, indeed, membership and non-membership values are obtained from the lagged time series. So, it can be thought that membership and non-membership values include largely the information of lagged time series.

From this viewpoint, in this study, a new high order fuzzy intuitionistic time series definition given definition 3 is introduced. It is explained that any time series is affected by lagged membership values and non-membership values in Definition 3. For this reason, it can be said that the advantage of the proposed method that uses Definition 3 is simplifying the model since the number of inputs in the model is reduced. In the literature, intuitionistic fuzzy time series models have been proposed in recent years since the definitions of classical fuzzy time series in literature can be insufficient for much real lifetime series. Reason for this is fuzzy time series approaches use only membership values for forecasting of any time series. However, both membership and non-membership values are used in the intuitionistic fuzzy time series methods. Thus, the intuitionistic fuzzy time series methods are the approaches to increase the forecasting performance because of being used more information. Also in this study, similarly, it is been shown that the forecasting performance of the proposed method that has a new intuitionistic fuzzy time series approach has much better forecasting performance than the classical time series and the classical fuzzy time series like the methods of SC-FTS [6], C-FTS [15], ARIMA method [1,2], H-ES [1] and method A-ES [3]. Therefore, in future studies, it could be suggested that researchers prefer to use the intuitionistic fuzzy time series approaches instead of the classical time series and the classical fuzzy time series approaches.

In the analysis of time series, the using of artificial intelligence methods such as artificial neural networks and particle swarm optimization has effects to increase the forecasting performance. However, any artificial intelligence method is not used in the proposed method. In this way, it is aimed to show that the forecasting performance can be greatly improved by using only an intuitionistic fuzzy time series approach without using any artificial intelligence method. In this study, a new high order intuitionistic fuzzy time series forecasting algorithm is proposed and it is shown that the proposed method has better forecasting performance with the best success rates between 32% and 64% then many time series models in the literature. In the proposed method, the principal component analysis method was used to make the dimension reduction and to eliminate the relationship structure between the input variables. Then, the robust regression is used in the stage of determination relationships of the proposed method. Therefore, it can be suggested that some regression methods or statistical methods are used instead of using some artificial intelligence methods in the stage of determination relationship of the intuitionistic fuzzy time series that will be made in the future. The other advantage of this study is that time series analyzes is broadly made for an entire continent. In this study, the forecasting performances of 11 different time series forecasting methods are compared with each other. Then, for each PEC time series in the Europe&Eurasia continent, the best methods and the parameters of the best methods are given in Table 6. So, researches that have worked in the energy fields could decide which methods they will select by looking to Table 6 when they would like to solve any PEC time series in the Europe&Eurasia continent. The proposed method is an explainable artificial intelligence method. It is possible to test, order and comment of all inputs in the model. The proposed method is the first explainable fuzzy time series method in the literature. The studies about explainable fuzzy time series methods will be increased in the near future.

Compliance with ethical standards

Ethical approval This article does not contain any studies with human participants or animals performed by any of the authors.

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Author contributions

Cem Kocak contributed to writing original draft, creating algorithm of the proposed method and data analysis, Erol Egrioglu and Eren Bas contributed to creating algorithm of the proposed method, writing program codes, and data analysis.

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Figures

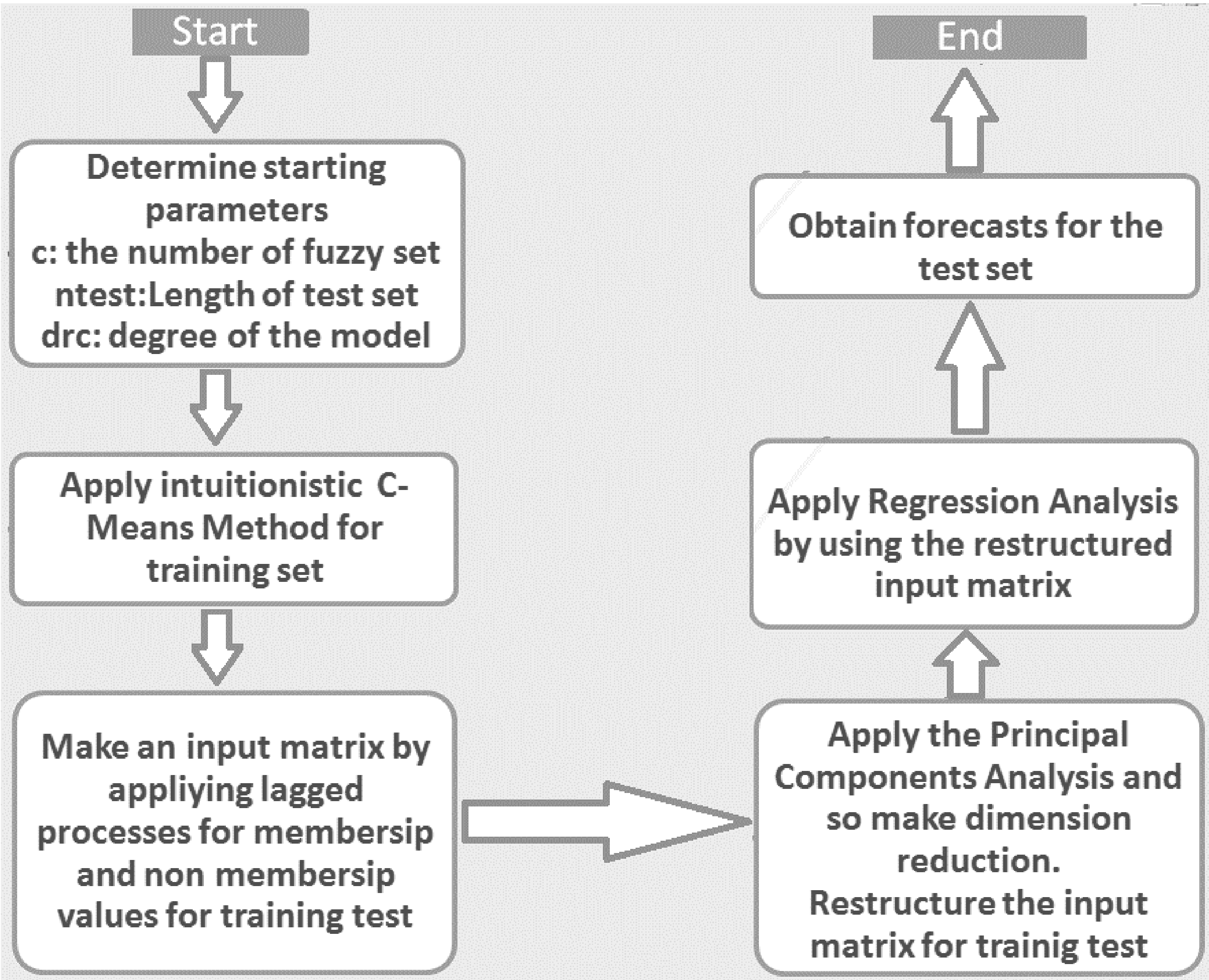

Figure 1

Flowchart of Proposed Method

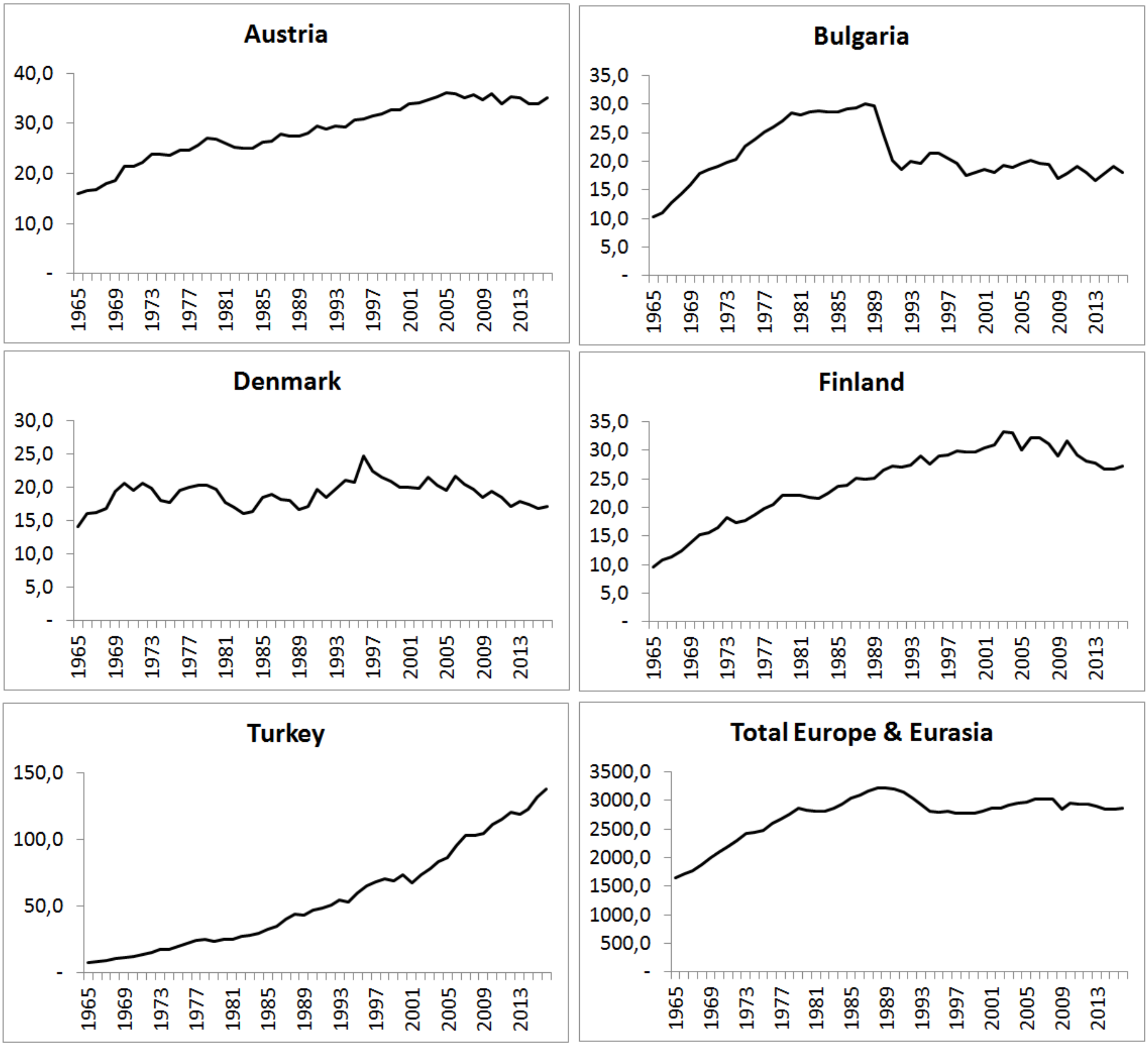

Figure 2

Yearly PEC observations of some countries in the million tonnes of oil equivalent.

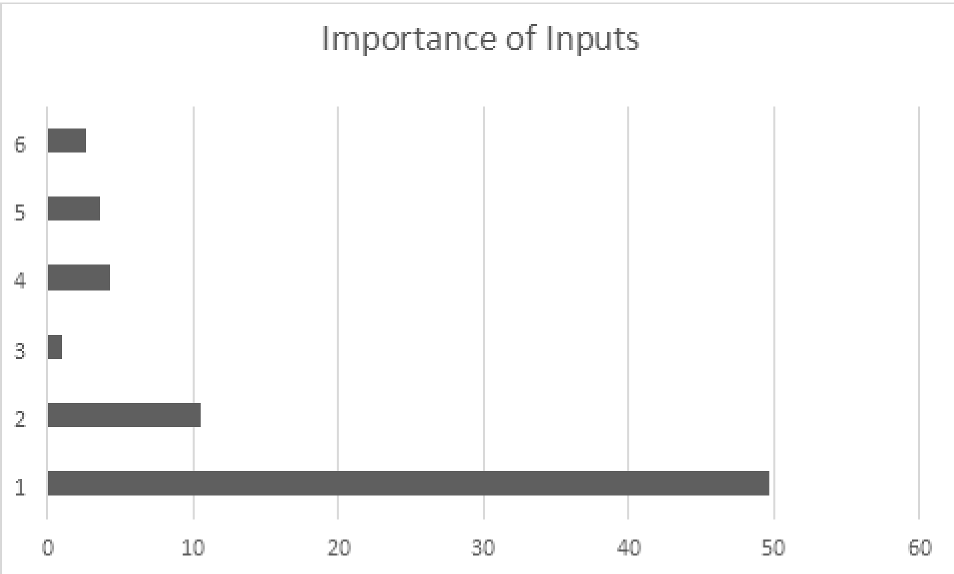

Figure 3

The importance graph of model inputs

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