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The structure of black holes

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Abstract: Contrary to the general belief that a black hole has only an event horizon and a singularity it is shown that it has a structure. The structure of a black hole is worked out using Schwarzschild interior solution. It is shown that there is an attraction towards the centre of the distribution when the radius r of the sphere is greater than $9m_0/4$ where $m_0 = GM_0/c^2$, M_0 being the mass of the distribution of matter. When the radius is between $9m_0/4$ and $2m_0$ the attraction is replaced by a repulsion first at the centre and then in a core gradually increasing in radius up to $2m_0$. The matter is repelled towards the surface ending up with a spherical shell of radius $2m_0$ and of infinite density, forming a black hole.

1. Introduction

In this paper we consider only non-rotating black holes. It is generally believed that such a black hole has an event horizon and a singularity. Consider the Schwarzschild metric in the form

$$ds^2 = \left(1 - \frac{2m}{r}\right)c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{2m}{r}\right)} - r^2 (d\theta^2 + \sin^2\theta d\phi^2) \quad (1.1)$$

The metric (1.1) is said to describe the external space-time due to a black hole, under certain conditions, with the event horizon at $r=2m$ and the singularity is at $r=0$. Here $m = GM/c^2$ (1.2) M being the mass of the black hole and it is said that a particle that comes from a large distance and crosses the event horizon would not come out. The event horizon is considered to be a coordinate singularity, and other coordinate systems have been formulated to overcome this difficulty.

It is generally assumed that there is neither matter nor radiation in a static state inside the event horizon as a particle that crosses the event horizon reaches the singularity at $r=0$. It is a million-dollar question to ask what happens to the particle having reached the singularity. In any event it is assumed that there is no static matter or radiation between the singularity and the event horizon.

However, one notices that when $r < 2m$, the coefficient of the dr^2 term in (1.1) is positive, making r a time-like coordinate. This implies that r has to increase, assuming of course, that time like coordinates always increase. In other words, for a particle that crosses the event

horizon r has to increase without decreasing. Thus, a particle that crosses the event horizon has to come back to the event horizon, without moving towards the singularity, if an event horizon has already been formed. It appears that the mass of a non-rotating black hole is concentrated in a spherical shell at the event horizon. This is different from the previous not with respect to the matter in between the event horizon and the singularity, but with respect to the end point of matter inside a black hole. In the previous it is assumed that matter ends up at the singularity but according to the present, matter is concentrated at the event horizon. We shall examine this further.

2. Schwarzschild interior solution

In this section we consider the Schwarzschild interior solution for matter in a static state. It is our intention to analyze equilibrium solutions for various radii r_0 of the Schwarzschild spheres, in the following sections. We follow Adler, Bazin and Schiffer [1] in presenting the Schwarzschild interior solution.

Consider the Schwarzschild interior metric in the form

$$ds^2 = \left[\frac{3}{2}(1-r_0^2/R^2)^{1/2} - \frac{1}{2}(1-r^2/R^2)^{1/2} \right]^2 c^2 dt^2 - (1-r^2/R^2)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta d\Phi^2) \quad (2.1)$$

where
$$R^2 = 3c^2/8\pi G\rho_0 \quad (2.2)$$

Let $M_0 = (4/3)\pi r_0^3 \rho_0$ (2.3) ρ_0 being a constant, r_0 being the radius of the sphere and M_0 being the mass of the spherical distribution of matter and radiation. From (1.2), (2.2) and (2.3) it is clear that $R^2 = r_0^3 / 2m_0$ (2.4), where $m_0 = GM_0 / c^2$.

From (2.4) it is clear that when $r_0 > 2m_0$, $R > 2m_0$, and when $r_0 < 2m_0$, $R < 2m_0$. When $r_0 = 2m_0$, $R = 2m_0$.

It should be noted that the metric (2.1) is defined only if $r < R$ and $r_0 < R$. As r is considered to be less than or equal to r_0 , the metric is defined only if r_0 is less than or equal to R , that is only if r_0 is greater than or equal to $2m_0$.

A noteworthy feature of the Schwarzschild interior solution is the presence of the term $(1-r^2/R^2)$ in the metric, which is similar in form to the term $(1 - \frac{1}{3}\Lambda r^2)$ in the de Sitter space-time with positive cosmological constant. This would give rise to a repulsion as seen below.

3. Acceleration of a particle in the Schwarzschild interior space – time

We assume that the Schwarzschild interior solution represents the space -time due to a spherical distribution of matter in equilibrium. We consider a particle at rest in the local inertial frame at (r, θ, ϕ) . The acceleration of the particle in the radial direction can be worked out by considering the Lagrange's Equations with respect to t and r , considering θ and ϕ to be constant. In writing down the equation for r we omit terms involving $\frac{dr}{ds}$ as the particle is considered to be at rest.

With the above restrictions the two equations can be written as

$$\frac{d}{ds} \left[\left\{ \frac{3}{2}(1-r_0^2/R^2)^{1/2} - \frac{1}{2}(1-r^2/R^2)^{1/2} \right\}^2 \frac{cdt}{ds} \right] = 0 \quad (3.1),$$

and

$$\frac{d}{ds} \left(\frac{dr}{ds} \right) + \frac{1}{2} \left[\frac{3}{2}(1-r_0^2/R^2)^{1/2} - \frac{1}{2}(1-r^2/R^2)^{1/2} \right] (1-r^2/R^2)^{1/2} \left(\frac{cdt}{ds} \right)^2 r/R^2 = 0 \quad (3.2)$$

where $R^2 = 3c^2/8\pi G\rho_0 = r_0^3/2m_0$ (2.2) and (2.4) .

The equation (3.1) yields $\left\{ \frac{3}{2}(1-r_0^2/R^2)^{1/2} - \frac{1}{2}(1-r^2/R^2)^{1/2} \right\}^2 \frac{cdt}{ds} = k$ (3.3)

Substituting (3.3) in the metric (2.1) at $r=r_1, \theta = \pi/2, \phi = 0$ one obtains

$$k = \left\{ \frac{3}{2}(1-r_0^2/R^2)^{1/2} - \frac{1}{2}(1-r_1^2/R^2)^{1/2} \right\} \quad (3.4) .$$

Hence from (3.2) the radial acceleration of a particle at rest at $r=r_1$ is given by

$$\frac{d}{ds} \left(\frac{dr}{ds} \right) = - (r_1/2R^2) (1-r_1^2/R^2)^{1/2} / \left\{ \frac{3}{2}(1-r_0^2/R^2)^{1/2} - \frac{1}{2}(1-r_1^2/R^2)^{1/2} \right\} \quad (3.5)$$

The acceleration at $r=r_1$ is negative, that is the acceleration is towards the centre $r=0$, if

$$\frac{3}{2}(1-r_0^2/R^2)^{1/2} - \frac{1}{2}(1-r_1^2/R^2)^{1/2} > 0$$

i.e. if $\frac{3}{2}(1-r_0^2/R^2)^{1/2} > \frac{1}{2}(1-r_1^2/R^2)^{1/2}$

i.e. if $r_1^2/R^2 > (9r_0^2/R^2) - 8 = (18m_0/r_0) - 8$ from (2.4)

i.e, As long as $(18m_0/r_0) - 8$ is negative, that is when $r_0 > 9m_0/4$, the above inequality is always satisfied and the acceleration of a particle at rest at $r=r_1$ is always towards the centre of the spherical distribution of matter. However, if $(18m_0/r_0) - 8$ is positive, that is when $r_0 < 9m_0/4$ there is a possibility that the above inequality is not satisfied for certain values of r_1 and thus giving a positive acceleration.

The acceleration of a particle at rest at $r=r_1$ is positive, that is towards the surface of the sphere, if $r_1^2/R^2 < (18m_0/r_0) - 8$, provided that $r_0 < 9m_0/4$. That is the acceleration of a particle at rest at $r=r_1$ is positive if $r_1^2 < 9r_0^2 - 4r_0^3/m_0$. In other words, inside the spherical surface defined by $r^2 = 9r_0^2 - 4r_0^3/m_0$, the acceleration of a particle at rest is towards the surface of the sphere when $r_0 < 9m_0/4$. In other words, a particle at rest in the sphere experiences a repulsion.

This repulsion inside the surface defined by $r^2 = 9 r_0^2 - 4 r_0^3/m_0$ changes to an attraction towards the centre at the surface given by $r^2 = 9 r_0^2 - 4 r_0^3/m_0$. (3.6)

This repulsion is due to the appearance of the term $(1-r^2/R^2)$ in the Schwarzschild interior solution, and may be due to the expanding properties of space – time.

The value of the radius of the spherical surface at which the repulsion inside changes to an attraction towards the centre can be calculated using (3.6) for different values of r_0 when $r_0 < 9m_0/4$.

When $r_0 = 9m_0/4$, from (3.6) the repulsion occurs only at $r = 0$.

When $r_0 = nm_0$, from (3.6) the repulsion occurs inside the sphere of radius r where

$$r^2 = \left(\frac{18}{n} - 8\right) r_0^3 / 2m_0. \quad (3.7)$$

This implies that $n \leq 9/4$.

Now as we have seen $r_0 \geq 2m_0$, $n \geq 2$. That is $2 \leq n \leq 9/4$.

When $n=2$, the repulsion occurs inside the sphere of radius r , where $r^2 = R^2 = r_0^3 / 2m_0$ from (3.7)

Since $r_0 = nm_0 = 2m_0$, the above implies that the repulsion of a particle at rest occurs inside the spherical surface $r = 2m_0$, which is the surface of the distribution for a black hole. In other words, inside the whole of a black hole there is repulsion, implying there is no matter inside the sphere. If there was matter it would have been repelled to the surface.

With the formation of the black hole all matter would have been repelled to the surface. It is implied that when $n=2$ the Schwarzschild interior solution represents a black hole.

In this connection we may also note from (3.6) that when r_0 is greater than $2m_0$, but less than $9m_0/4$ the repulsion of a particle at rest occurs in a sphere of radius less than $2m_0$ and from (3.5) the acceleration at the surface of separation is found to be infinite. However, when r_0 is equal to $2m_0$ the repulsion occurs in a sphere of radius $2m_0$ but from (3.5) the acceleration at the surface of separation is finite due to the presence of the term $(1-r_1^2/R^2)^{1/2}$ in the numerator of (3.5) that takes the value $(1-r_0^2/R^2)^{1/2}$ canceling the value of the denominator. Hence when $r_0 = 2m_0$, there is a finite repulsion at the surface $r_0 = 2m_0$ equal to $(r_0/2R^2) = m_0/r_0^2$ since $R^2 = r_0^3 / 2m_0$ (2.4).

The acceleration of a particle at rest at coordinate distance r_1 in the Schwarzschild exterior solution can also be worked out using (1.1) and at it is equal to m_0/r_0^2 at r_0 . Thus at the event horizon the repulsion from the interior balances the acceleration from the exterior, and particles do not cross the event horizon. The particles attracted to the black hole from outside will accumulate increasing the mass as well as the radius of the black hole.

As mentioned above it should be noted from (3.6) that at the surface of transmission from attraction to repulsion the magnitude of the acceleration becomes infinite, and this leads us to discuss the pressure and the density at the spherical surface of separation.

4. The density and pressure of the distribution

As Adler, Bazin and Schiffer [1] have mentioned, the fact that ρ_0 is a constant does not imply that the density is a constant, as the T^{00} component of the energy momentum tensor $T^{uv} = (\rho_0 + p/c^2) \frac{dx^u}{ds} \frac{dx^v}{ds} - (p/c^2) g^{uv}$ is a function of the coordinate distance and changes from one position to another. It can be shown that T^{00} takes the form

$$\rho_0 / [\frac{3}{2}(1-r_0^2/R^2)^{1/2} - \frac{1}{2}(1-r^2/R^2)^{1/2}]^2 \quad (4.1).$$

Going through the calculations indicated in Adler, Bazin and Schiffer [1], it can also be shown that the pressure p at a coordinate distance r is given by

$$p/c^2 = \rho_0 [(1-r^2/R^2)^{1/2} - (1-r_0^2/R^2)^{1/2}] / [3(1-r_0^2/R^2)^{1/2} - (1-r^2/R^2)^{1/2}]$$

and the radial component of pressure, T^{11} can be written as $\rho_0 (1-r^2/R^2) [(1-r^2/R^2)^{1/2} - (1-r_0^2/R^2)^{1/2}] / [3(1-r_0^2/R^2)^{1/2} - (1-r^2/R^2)^{1/2}] \quad (4.2).$

It is seen that at the points where there is a change from attraction to a repulsion both (4.1) and (4.2) tend to infinity. However, at $r = r_0$, T^{00} is finite when $r_0 > 2m_0$, but tends to infinity as r_0 tends to $2m_0$ since $R^2 = r_0^3 / 2m_0$ (2.4). In other words, the surface of a black hole is of infinite density.

On the other hand, from (4.2) we find that T^{11} is zero when $r = r_0$. This is to be expected as the Schwarzschild interior solution has been obtained with the boundary condition that the pressure is zero at the surface.

At the spherical surface of transmission from attraction to repulsion, the magnitude of the acceleration, the density and pressure tend to infinity indicating a qualitative change. The dominance of matter and radiation increases and becomes enormous applying a infinite thrust on space time. The space time reacts by inserting enormous thrust on matter and radiation exerting a repulsion.

This leads us to speculate that space time and matter are not separate mutually exclusive entities but are bound through the so - called cosmological constant.

In fact, this question has been addressed by Katugampala and de Silva [2,3] and by Hemantha and de Silva [4,5] in papers presented at the Annual Research Symposia of the University of Kelaniya, Sri Lanka and the effects of the cosmological constant has been studied by Jayakody [6]. Katugampala and de Silva are of the opinion that the so - called cosmological constant is not a constant as such but a parameter.

It would have been better if we had a Schwarzschild interior solution with the cosmological constant to compare with the Schwarzschild - de Sitter solution $ds^2 = (1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2) c^2 dt^2 - \frac{dr^2}{(1 - \frac{2m}{r} - \frac{1}{3}\Lambda r^2)} - r^2 (d\theta^2 + \sin^2\theta d\phi^2) .$

In the absence of such solution, we conclude that the distribution of matter represented by the Schwarzschild internal solution would end up as a spherical surface, after repulsive forces take over, with infinite density that could be identified as a non rotating spherical black hole. It is suggested that a similar study be carried out in respect of Kerr metric for rotating black holes.

5. Conclusion

In a spherical distribution of matter, there is an attraction towards the centre of the distribution when the radius r of the sphere is greater than $9m_0/4$ where $m_0 = GM_0/c^2$, M_0 being the mass of the distribution of matter. However, when the radius r of the sphere becomes less than or equal to $9m_0/4$, a qualitative change occurs due to the dominance of the term $(1-r^2/R^2)$ in the Schwarzschild interior metric. This term resembles the term $(1 - \frac{1}{3}\Lambda r^2)$ with a positive cosmological constant in the de – Sitter space time that accounts for the expansion of the space-time. When the radius of the spherical distribution is $9m_0/4$, there is a repulsion at the centre of the distribution. As the radius of the distribution of matter decreases from $9m_0/4$ to $2m_0$ a repulsive core is formed, whose radius gradually increases from zero to $2m_0$. The matter is gradually repelled from the centre to what becomes the event horizon as the radius of the distribution decreases from $9m_0/4$ to $2m_0$. When the radius of the distribution of matter is $2m_0$, the radius of the core of repulsion is also equal to $2m_0$ implying that in the case of a black hole all the matter is concentrated at the surface leaving an entire repulsive core of radius $2m_0$.

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