Optimized Space Station CV Method and Its Differences from GNSS CV

Yinhua Liu (✉ liuyh@ntsc.ac.cn)

National Time Service Center <https://orcid.org/0000-0003-1442-1677>

Xiaohui Li

National Time Service Center

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3 Liu Yinhua, National Time Service Center, the Chinese Academy of Sciences, Xi'an,

4 China¹, University of Chinese Academy of Sciences, Beijing, China², liuyh@ntsc.ac.cn

5 Li Xiaohui, National Time Service Center, the Chinese Academy of Sciences, Xi'an,

6 China¹, University of Chinese Academy of Sciences, Beijing, China²,

7 xiaohui@ntsc.ac.cn

8 **Liu Yinhua is the corresponding author.**

9

10 **Abstract**

11 There will be better atomic clock system and micro-wave time comparison link in the
12 near earth space station, like Chinese Space Station and European ACES(Atomic Clock
13 Ensemble in Space) system, than those in the GNSS(Global Navigation satellite
14 System) satellites. Therefore, the space station common-view (CV) will realize more
15 accurate time comparison than GNSS CV in theory. But due to the orbit characteristic
16 of the space station, there are some limitations if traditional GNSS CV time comparison
17 method is applied to the space station. In order to solve these problems, the GNSS CV
18 method is optimized and the method that is appropriate for the space station is proposed.
19 First, the basic CV principle is analyzed, and the delay items which are needed to be
20 considered for GNSS and space station CV are compared and analyzed. Then, the
21 differences between GNSS and space station CV are studied, and the influences of orbit
22 error on these two CV methods are analyzed in detail. The GNSS CV method is
23 optimized to be fit for the space station next. Finally, the performance of the optimized
24 method is validated by simulated experiments. The simulation results show that the
25 space station time comparison accuracy of several tens of picoseconds can be obtained

26 by the optimized method. Furthermore, the problem of CV blind area is solved by the
27 optimized method effectively.

28 **Keywords**

29 CV Time Comparison, Space Station, GNSS, Orbit Error

30 **Main Text**

31 **1.Introduction**

32 The CV technology is wildly applied for time comparison at present. The magnitude of
33 nanosecond time comparison has been achieved by GPS CV. With the development of
34 GNSS systems, the accuracy of GNSS CV is continuously being improved. Its accuracy
35 is about one to five nanoseconds at present (Wang 2017), and it is the main method for
36 high accuracy time comparison over long distances. But with the expanded application,
37 nanosecond time comparison accuracy cannot satisfy many requirements, such as the
38 measurement of VLBI(Very Long Baseline Interferometry), the research of
39 gravitational red shift, and the deep space exploration. Bai (2012) and Miao (2015) all
40 stated the significance of high accurate time comparison technology in their PhD
41 dissertations. Matsakis et al. (2014) analyzed and compared the modern time transfer

technologies, and revealed that more accurate time comparison techniques are needed.

The stable atomic clock systems with better performance than the clocks on the ground due to the microgravity environment will be carried on Chinese Space Station

and European ISS(International Space Station), where the equipment of ACES project will be loaded. Hess M et al.(2012) and Yang et al. (2016) all pointed out this

characteristics of the atomic clocks in the space station. Because the main

characteristics related to time comparison of the Chinese Space Station and European

ACES project are nearly the same as each other, like their atomic clock performances,

time compare links and the orbit characteristics, we call them “space stations”

collectively for convenience. The analysis results below are all for these two space

stations. The clock ensemble carried in the space station will be the most stable time

system that will cover almost the whole earth periodically (Hobiger et al. 2013). Besides,

the space-to-ground micro-wave time comparison link also has good performance.

Duchayne et al. (2008) pointed out that the specification of the micro-wave link which

is used to perform space-to-ground time comparisons requested the time comparison

stability of 0.3 picoseconds at the interval of 300 seconds and 7 picoseconds at one

day's interval. Cacciapuoti et al. (2009) and Much et al. (2009) all stated that the uncertainty of space station clock synchronization is in the level of 100 picoseconds, so more accurate time comparison would be realized by the CV method. Delva et al. (2013) and Meynadier et al. (2018) validated the performance of the micro-wave link by simulation experiments, and these analysis results showed that the micro-wave link satisfied the requirement stated above.

Therefore, several tens of picoseconds CV time comparison accuracy will be achieved by the micro-wave link from the space station to the ground. This accuracy is two orders of magnitude higher than what is achieved with current GNSS CV techniques.

The space station orbit height is about 400 kilometers. The orbit of the space station is far lower than that of the GNSS satellite. Therefore, the orbit characteristics of the space station differ greatly from that of the GNSS satellites. The orbit error of space station affects the CV time comparison more than that of the GNSS satellite, which limits the space station time comparison accuracy to several hundreds of picoseconds level. Also, a low orbit causes smaller coverage areas. There are CV blind areas for the

74 space station. Thus, the GNSS CV method cannot be used to the space station directly.

75 It is necessary to study the CV time comparison method that is suitable for the space
76 station aiming at several tens of picoseconds accuracy.

77 The basic CV principle is analyzed, and the delay items which are needed to be
78 considered for GNSS CV and space station CV are compared and analyzed at first.

79 Then, the differences between GNSS CV and space station CV are studied. The
80 limitations are analyzed according to simulation, which will occur if the GNSS CV
81 method is used to the space station time comparison directly. The GNSS CV method is
82 optimized in order to solve these limitations to obtain better space station CV
83 performance. Finally, the performance of the optimized method is validated by some
84 simulated experiments. Several tens of picoseconds time comparison accuracy is
85 realized in the simulation experiments.

86 **2. Principle of CV time comparison**

87 The target accuracy of GNSS CV is nanosecond, but our target of the space station CV
88 accuracy is several tens of picoseconds. In order to satisfy these two accuracy targets,
89 the mathematical models are established based on the high-accurate general relativity

theory (Liu 2004; Petit et al. 1994). Hobiger et al. (2013) and Duchayne et al. (2008) all introduced the mathematical formulas of the space station based on the general relativity theory. Some errors less than 1 nanosecond must be considered, which are ignored in the traditional GNSS CV method. The GCRS(Geocentric Celestial Reference System) is used as the basic coordinate system of CV time comparison.

If ground stations A and B receive the time comparison signal send by the spatial aircraft, such as the space station or GNSS satellite, at t_0 time, the clock biases between the ground station and the aircraft can be expressed as follows (Meynadier et al. 2018; Duchayne et al. 2008):

$$VT_{AS} = P_A - \left(\frac{\rho_{SA}}{c} + \tau_{SA}^{ion} + \tau_{SA}^{tro} + \frac{\rho_{SA} \cdot \dot{\rho}_{SA}}{c^2} + \frac{\rho_{SA}}{2c^3} (\dot{\rho}_{SA}^2 + \dot{\rho}_{SA} \cdot \ddot{\rho}_{SA} + \frac{(\dot{\rho}_{SA} \cdot \dot{\rho}_{SA})^2}{\rho_{SA}^2}) + \frac{2GM_E}{c^3} \ln \frac{r_S + r_A + \rho_{SA}}{r_S + r_A - \rho_{SA}} + \tau_{SA}^{trans} \right) \quad (1)$$

$$VT_{BS} = P_B - \left(\frac{\rho_{SB}}{c} + \tau_{SB}^{ion} + \tau_{SB}^{tro} + \frac{\rho_{SB} \cdot \dot{\rho}_{SB}}{c^2} + \frac{\rho_{SB}}{2c^3} (\dot{\rho}_{SB}^2 + \dot{\rho}_{SB} \cdot \ddot{\rho}_{SB} + \frac{(\dot{\rho}_{SB} \cdot \dot{\rho}_{SB})^2}{\rho_{SB}^2}) + \frac{2GM_E}{c^3} \ln \frac{r_S + r_B + \rho_{SB}}{r_S + r_B - \rho_{SB}} + \tau_{SB}^{trans} \right) \quad (2)$$

where VT_{AS} denotes the clock bias between station A and the s aircraft, VT_{BS} denotes the clock bias between station B and the spatial object, P_A and P_B denote the pseudo range of station A and B respectively, $\dot{\rho}_{SA}$ and $\dot{\rho}_{SB}$ denote the distance vector from the aircraft to ground station A and B respectively, c denotes the light speed, $\dot{\rho}_{SA}$ and $\ddot{\rho}_{SA}$

107 denote the speed and acceleration vector of station A respectively, v_B and a_B denote the
 108 speed and acceleration vector of station B respectively, τ_{SA}^{ion} and τ_{SB}^{ion} denote the
 109 ionospheric delay, τ_{SA}^{tro} and τ_{SB}^{tro} denote the tropospheric delay, and other terms are the
 110 delay components related to the relativity theory. $\frac{\rho_{SA} \cdot v_A}{c^2}$ and $\frac{\rho_{SB} \cdot v_B}{c^2}$ denote the Sagnac
 111 delay, they are in the magnitude of ten nanoseconds for the space station, but they are in
 112 the magnitude of hundred nanoseconds for GNSS satellites. $\frac{\rho_{SA}}{2c^3}(v_A^2 + \rho_{SA} \cdot a_A + \frac{(\rho_{SA} \cdot v_A)^2}{\rho_{SA}^2})$ and
 113 $\frac{\rho_{SB}}{2c^3}(v_B^2 + \rho_{SB} \cdot a_B + \frac{(\rho_{SB} \cdot v_B)^2}{\rho_{SB}^2})$ are related to the ground station acceleration and the square of
 114 its velocity, and their orders of magnitude are about 0.1 picoseconds for the space
 115 station and hundreds of picoseconds for GNSS satellites. The last but one on the right of
 116 (1) and (2) are the Shapiro delay terms, in which G denotes Newton's gravitational
 117 constant, M_E denotes the earth mass, r_s denotes the distance between the center of the
 118 earth to the space object, r_A denotes the distance between the center of the earth to
 119 ground station A, and r_B denotes the distance between the center of the earth to
 120 ground station B. The effect of Shapiro delay is about ten picoseconds for the space
 121 station and more than one hundred picoseconds for GNSS satellites.

122 The last terms on the right of (1) and (2) are converting delay from coordinate time to

the ground station proper time, which should be considered in general relativity theory.

According to the relationship between time-space interval and metric tensor in general relativity theory, the relationship between the ground station proper time and coordinate time can be expressed as follows:

$$\frac{d\tau_A}{dt} = 1 - \left(\frac{GM_E}{r_A c^2} + \frac{v_A^2(t)}{2c^2} \right) \quad (3)$$

$$\frac{d\tau_B}{dt} = 1 - \left(\frac{GM_E}{r_B c^2} + \frac{v_B^2(t)}{2c^2} \right) \quad (4)$$

where τ_A and τ_B denote the proper time, t denotes the coordinate time.

Therefore, τ_{SA}^{trans} and τ_{SB}^{trans} can be expressed as follows:

$$\tau_{SA}^{trans} = - \left(\frac{GM_E}{r_A c^2} + \frac{v_A^2(t)}{2c^2} \right) T_{SA} \quad (5)$$

$$\tau_{SB}^{trans} = - \left(\frac{GM_E}{r_B c^2} + \frac{v_B^2(t)}{2c^2} \right) T_{SB} \quad (6)$$

where T_{SA} and T_{SB} denote the signal propagating delay from the aircraft to the two ground stations, which contain the delay items in the bracket on the right side of (1) and (2) except for τ_{SA}^{trans} and τ_{SB}^{trans} . Their effects of τ_{SA}^{trans} and τ_{SB}^{trans} are about several picoseconds for the space station and ten picoseconds for GNSS satellites.

The items less than one picosecond can be neglected in (1) and (2) for space station

CV. Differencing (1) and (2), the following equation can be obtained:

$$\begin{aligned}
139 \quad VT_{AB} = & (P_A - P_B) - \left(\frac{\rho_{SA} - \rho_{SB}}{c} \right) - (\tau_{SA}^{ion} - \tau_{SB}^{ion}) - (\tau_{SA}^{tro} - \tau_{SB}^{tro}) - \left(\frac{\dot{\rho}_{SA} \cdot \dot{V}_A}{c^2} - \frac{\dot{\rho}_{SB} \cdot \dot{V}_B}{c^2} \right) \\
140 \quad & - \frac{2GM_E}{c^3} \left(\ln \frac{r_S + r_A + \rho_{SA}}{r_S + r_A - \rho_{SA}} - \ln \frac{r_S + r_B + \rho_{SB}}{r_S + r_B - \rho_{SB}} \right) - (\tau_{SA}^{trans} - \tau_{SB}^{trans})
\end{aligned} \tag{7}$$

141 where VT_{AB} denotes the time bias between ground station A and B.

142 The items less than one nanosecond can be neglected in (1) and (2) for GNSS CV,

143 thus VT_{AB} for GNSS CV can be expressed as

$$144 \quad VT_{AB} = (P_A - P_B) - \left(\frac{\rho_{SA} - \rho_{SB}}{c} \right) - (\tau_{SA}^{ion} - \tau_{SB}^{ion}) - (\tau_{SA}^{tro} - \tau_{SB}^{tro}) - \left(\frac{\dot{\rho}_{SA} \cdot \dot{V}_A}{c^2} - \frac{\dot{\rho}_{SB} \cdot \dot{V}_B}{c^2} \right) \tag{8}$$

145 where there are no terms for the Shapiro delay and converting delay from the coordinate
146 time to the proper time.

147 Equations (7) and (8) show that differencing operation is the essence of CV principle.

148 The common errors can be canceled by the differencing operation, so the time
149 comparison accuracy is improved by CV method. But the CV time comparison requires
150 that two ground stations observe the spatial aircraft at the same time to compute their
151 clock bias by using the atomic clocks in the aircraft as a medium. By contrasting (7) and
152 (8), we can see that the Shapiro delay and the converting delay from coordinate time to
153 the proper time are computed in space station CV but not in GNSS CV. These two
154 items are greater than one picosecond but less than one nanosecond. So, these delay

155 items cannot be neglected if we want to get several tens of picoseconds accuracy. But
156 they can be neglected in GNSS CV which aims at nanosecond time comparison
157 accuracy.

158 **3. Comparising space station and GNSS CV**

159 As mentioned above, due to the target accuracy between the space station CV and
160 GNSS CV is different, the delay items to be considered are also different. Furthermore,
161 there are several differences between them, such as the orbit characteristic and effect,
162 space atomic clock performance and down-link signal structure, and so on. These
163 differences limit the application of the traditional GNSS CV method to the space
164 station.

165 **3.1 System design differences between space station and GNSS CV techniques**

166 The system design differences between the space station and GNSS satellite mainly
167 contain three aspects: the orbit characteristics, space atomic clock system and signal
168 structure of the time comparison down-link.

169 First, the orbit characteristics of these two systems are compared. The space station
170 has low orbit, and the average orbit height is only about 400 kilometers. The speed of

171 the space station is far faster than that of GNSS satellites. The average orbital period is
172 about 1.5 hours, and there are about nine visible flight periods in one day. The average
173 visible time interval per one orbit period for the ground station is also very short and
174 about 400 seconds. These characteristics of the space station are mentioned by Yang et
175 al. (2016), Duchayne et al. (2008) and Föckersperger et al. (2004). Thus, the space
176 station CV time comparison can only be operated in a few minutes per day. When the
177 ground stations are in the blind areas, the traditional CV operation cannot be carried out.
178 The space station visibility of one day to several representative cities in China is shown
179 in Table 1. In this table, the cut-off elevation is ten degree, and the data in the table
180 denotes the cumulative minutes for one day. The table shows that every visible interval
181 of each city is very short and its mean value is about four minutes. Mohe is the
182 northernmost city of China, there is only one valid observation time interval which is
183 about four minutes. The other observation time for other cities is longer than that of
184 Mohe. Table 1 also shows that some cities cannot observe the space station at the same
185 time, thus traditional GNSS CV computation cannot be done. In other words, there are
186 blind areas if GNSS CV method is applied to the space station. For example, if one

187 station is located in Kashi and another station is located in Sanya, GNSS CV method is
188 invalid.

189 But there are many GNSS satellites, which can supply uninterrupted CV time
190 comparison service (Chen et al. 2014; Zhang et al. 2017; Kou 2012). For example, there
191 are more than twenty MEO satellites in GPS, their orbit heights are about 20000 km,
192 and their orbit periods are about 11 hours and 58 minutes. Thus, the coverage capability
193 of GPS satellite is the whole earth. Taking Beidou system for example, there are GEO,
194 IGSO and MEO satellites, and Beidou satellites also can be used as continuous CV
195 objects.

196 Then, the atomic clock carried by the space station is compared with that carried by
197 the GNSS satellite. The space station atomic clock has better performance. First, the
198 laser cooling techniques and the space microgravity environment will be combined to
199 improve the stability of the atomic clock in the space station. Although the GNSS
200 satellites are also in the space microgravity environment, there are no cooled atomic
201 clocks carried on the GNSS satellites. Second, the performance level of the space
202 station atomic clock is higher than that of the GNSS satellite clock. There will be two

203 atomic clocks carried in the ISS for the ACES project. One is a hydrogen maser whose
204 frequency stability is better than $1.5\text{E-}13$ when the averaging time interval τ is equal
205 to 1 second and is better than $1.5\text{E-}15$ when τ is equal to 86400 seconds. The other is a
206 cold atom cesium clock, whose frequency stability should be better than $1\text{E-}13/\sqrt{\tau}$.
207 These two clocks will be integrated to output better time and frequency signals, so the
208 frequency stability of ACES integrated signal is better than $1\text{E-}13$ when τ is equal to 1
209 second and is better than $1\text{E-}15$ when τ is equal to 86400 seconds. The payload of
210 Chinese Space Station not only contains ordinary atomic clocks, but also contains a
211 strontium atom light clock with the performance at least an order of magnitude higher
212 than that of ACES clocks. Most of clocks carried by GNSS satellites are rubidium or
213 cesium atomic clocks. Although there are some passive hydrogen atomic clocks in
214 Galileo satellites, the frequency stability of integrated signal is an order of magnitude
215 worse than the ACES signal. The atomic clocks of the space station and GNSS satellites
216 are the CV references. Although the clock errors have been canceled by the CV
217 algorithm, excellent clock reference is helpful to optimize the space station CV method
218 that will be mentioned latter.

219 Finally, the signal structure differences between these two systems are explained. The
220 differences related to time comparison performance consist of the code chip rate and the
221 signal frequency. The down link code chip rate of the space station is 100 Mchip/s,
222 which is nearly ten times the GPS P-code. Thus, the downlink signal pseudo code
223 ranging accuracy of the space station is higher than that of the GNSS satellite. The
224 signal frequency of GNSS is commonly in L band, but that of ACES is in the Ku band,
225 and that of the Chinese Space Station is higher and in Ku and Ka bands. The
226 ionospheric delay is inversely proportional to the square of the signal frequency. The
227 ionospheric delay of the space station is far less than that of GNSS. For GNSS CV, the
228 ionospheric delay is the main error source. But for space station CV, the ionospheric
229 delay can be calculated by the pseudorange combination of two or three carriers in
230 different frequencies with the accuracy of picosecond magnitude. Therefore, the time
231 comparison performance of the space station down link is better than that of GNSS, and
232 higher CV time comparison accuracy can be achieved by this link.

233 **3.2 Effect differences of orbit error on space station and GNSS CV**

234 Because the target accuracy of the space station CV is two orders magnitude higher than

235 that of GNSS CV, their error correcting methods are different. The atmospheric delay,
 236 the earth rotation, and the gravity delay are not explained here, and only the influence of
 237 orbital error on the space station CV and GNSS CV is analyzed in detail.

238 From the equations (7) and (8), the differential coefficient of the aircraft position
 239 error can be gotten by the following equation:

$$240 \quad dVT_{AB} = \frac{1}{c} \left(\frac{\dot{\rho}_{SB}}{\rho_{SB}} - \frac{\dot{\rho}_{SA}}{\rho_{SA}} \right) dX_s^v \quad (9)$$

241 where X_s^v denotes the position error of the aircraft. Equation (9) can be rewritten as:

$$242 \quad dVT_{AB} = \frac{1}{c} \left(\frac{\rho_{SA} \dot{\rho}_{SB} - \rho_{SB} \dot{\rho}_{SA}}{\rho_{SB} \rho_{SA}} \right) dX_s^v \quad (10)$$

243 Assuming that $\rho_{SA} = \rho_{SB} + \rho$, we can get the following equation:

$$244 \quad dVT_{AB} = \frac{1}{c} \left(\frac{\rho_{SB} \dot{\rho}_{BA} + \Delta \rho \dot{\rho}_{SB}}{\rho_{SB} \rho_{SA}} \right) dX_s^v \quad (11)$$

245 where ρ_{BA}^v denotes the distance vector from station B to A. After some simple
 246 transformation of (11), the following inequality is obtained:

$$247 \quad |dVT_{AB}| \leq \frac{1}{c} \left(\frac{|\rho_{BA}^v| + |\Delta \rho|}{\rho_{SA}} \right) |dX_s^v| \quad (12)$$

248 which shows that the effect of orbit error on CV time comparison is related to the
 249 distance between the aircraft and the ground station, the length of the CV baseline and
 250 the orbit error itself.

251 Generally. GNSS satellite orbit height is usually about 20000 kilometers, so the
 252 following relation expression can be obtained for GNSS:

$$253 \quad \frac{|\Delta\rho|}{\rho_{SA}} = 1 \quad (13)$$

254 equation (12) can be rewritten approximately as for GNSS CV:

$$255 \quad |dVT_{AB}| \leq \frac{1}{c} \frac{|\dot{\rho}_{BA}^V|}{\rho_{SA}} |dX_s^V| \quad (14)$$

256 GNSS CV baseline is up to thousand kilometers, which is much less than ρ_{SA} . So the
 257 effect of GNSS satellite position error is reduced. Taking the CV baseline as 2000
 258 kilometers and GNSS orbit error as one meter for example, the maximum influence
 259 magnitude of the orbit error to GNSS CV is about 300 picoseconds according to (14).
 260 This effect can be ignored for a 3 to 5 nanoseconds time comparison accuracy.
 261 Additional orbital improvements need not to be made for GNSS CV, and the CV
 262 algorithm itself is enough.

263 The orbit height of space station is about 400 kilometers. But the distance between
 264 the space station and the ground station ranges from several hundred to several thousand
 265 of kilometers. Thus, the following express is sometimes correct for the space station:

$$266 \quad \frac{|\Delta\rho|}{\rho_{SA}} > 1 \quad (15)$$

thus the effect of space station orbit error may be magnified.

On the other hand, the following express is also correct for the space station at most times:

$$\frac{|\rho_{BA}^v|}{\rho_{SA}} > 1 \quad (16)$$

So, if the CV baseline length is longer than the distance between the space station and the ground station, the effect of orbit error is probably enlarged. But the CV baseline length is often longer than several hundred kilometers. The space station position data is provided by a GNSS receiver. With the development of GNSS technology, the positioning accuracy of GNSS receiver reaches decimeter or even centimeter at present (Martin et al. 2012; Cerri et al. 2010; Lemoine et al. 2010). If the baseline is 2000 kilometers and the orbit error is 10 centimeters, the effect of orbit error on the space station CV is up to one nanosecond. This impact is significant for the target accuracy of several tens of picoseconds. Thus, GNSS CV method cannot be applied to the space station CV time comparison directly. This method must be optimized to meet the high accurate requirement.

3.3 Application limitations when using GNSS CV method to the space station

283 From the analysis results above, it is known that the traditional GNSS CV method
284 requires two ground stations observing the space station simultaneously. Due to the low
285 orbit and fast orbital speed of the space station, the ground stations in some areas cannot
286 observe the space station simultaneously all the time. Therefore, GNSS CV method
287 cannot be used for this case. This is the first application limitation.

288 The other limitation relates to CV time comparison accuracy. As mentioned above,
289 the GNSS CV method will result in great time comparison errors for the space station
290 CV. This limitation is validated by simulation experiments in which the GNSS CV
291 method is applied to the space station.

292 In the simulation, the orbit error is divided into three components, the radial (R),
293 tangential (T) and the normal (N) vector components. Each component is set to 0.1
294 meters in R, T and N. The mean value of observation noise is set to zero, and its
295 standard deviation is set to 1 picosecond. The ionospheric delay data is generated based
296 on the VTEC (Vertical Total Electronic Content) data provided by IGS. The
297 tropospheric delay is modeled by the Saastamoinen model. Minimum elevation of the
298 space station is taken as 10° . The temperature is set to 298 K, the pressure is set to 1 bar,

299 and the water vapor pressure is set to 0.5 bar. The two ground stations are located in Xi'
300 an and Sanya respectively, so the baseline length is about 2000 kilometers. The
301 frequency stability of the Xi'an atomic clock when τ is equal to 1 second is set to $1\text{E-}13$,
302 and the frequency stability when τ is equal to 86400 seconds is set to $1\text{E-}15$. The
303 frequency stability of the Sanya atomic clock when τ is equal to 1 second is set to $5\text{E-}13$,
304 and the frequency stability when τ is equal to 86400 seconds is set to $1\text{E-}14$.
305 Xi'an and Sanya space station CV time comparison errors are shown in Figure 1. The
306 traditional GNSS CV method is applied in this figure. The data displayed in the abscissa
307 indicates the cumulative seconds of one day, and the data displayed in the ordinate
308 indicates the value of time comparison error. It can be seen from the figure that there are
309 two CV intervals for Xi'an and Sanya to do space station CV time comparison for one
310 day, and each interval lasts for about three minutes. It also can be seen that the
311 maximum time comparison error is close to 800 picoseconds, and the error fluctuation
312 range is about 600 picoseconds. Thus, GNSS CV method cannot be directly used to the
313 space station CV time comparison, when the time comparison target accuracy is better
314 than several tens of picoseconds.

315 Figure 2 shows the effect of the space station orbit error on time comparison when
316 GNSS CV time comparison method is applied. The data of Figure 2 is obtained by the
317 method of geometrical computation, and the computation formula will be introduced in
318 the next section in detail. Comparing Figures 1 and 2, we find that the error
319 distributions of these two figures are virtually identical. Therefore, we know that the
320 orbit error is the main reason that results in the low accuracy if GNSS CV method is
321 used to the space station CV time comparison.

322 In order to verify the correctness of the above analysis result, other simulation
323 experiments also have been done, in which the ground stations are located in Shanghai
324 and Kunming or Xi'an and Changchun. Consistent conclusions have been obtained. So,
325 in order to realize the CV accuracy of several tens of picoseconds, the traditional GNSS
326 CV method must be optimized.

327 **4. Optimization of space station CV method**

328 It is known that the traditional GNSS CV method cannot be used to the space station
329 directly for two aspects of reasons. One is the existence of the CV blind area. The other
330 is the time comparison accuracy is limited to hundreds of picoseconds. In order to solve

331 these problems, the GNSS CV method is optimized.

332 The effect of orbit error on time comparison can also be expressed by the flowing
333 equation:

$$334 \quad c \cdot |dVT_{AB}| = \left| dX_S^R (\cos \alpha_{SA} - \cos \alpha_{SB}) + dX_S^T (\cos \beta_{SA} - \cos \beta_{SB}) + dX_S^N (\cos \gamma_{SA} - \cos \gamma_{SB}) \right|$$

335 (17)

336 where dX_S^R , dX_S^T and dX_S^N denote the R, T and N orbit error component respectively,
337 α_{SA} , β_{SA} and γ_{SA} denotes the angle between the vector from the space station to
338 ground station A and the R, T and N vector respectively, α_{SB} , β_{SB} and γ_{SB} denotes the
339 angle between the vector from the space station to ground station B and the R, T and N
340 vector respectively. The data of Figure 2 is computed by (17).

341 If the cosine of α_{SA} , β_{SA} and γ_{SA} is approximately equal to the cosine of α_{SB} , β_{SB}
342 and γ_{SB} respectively, the calculation result of the right side of (17) is close to zero. The
343 orbit error has little effect on time comparison at this time. Therefore, the relative
344 position relationship between the two ground stations and the space station is very
345 important for cancelling the effect of the orbit error. The relative position relationship
346 satisfying the condition that the cosine of α_{SA} , β_{SA} and γ_{SA} is close to the cosine of α_{SB} ,

347 β_{SB} and γ_{SB} respectively need to be found. The CV time comparison computation is
 348 done based on this position relationship and most influence of the orbit error can be
 349 removed. The following equation is used as a judgment condition to find this position
 350 relationship:

$$351 \quad flag = |\cos \alpha_{SA} - \cos \alpha_{SB}| + |\cos \beta_{SA} - \cos \beta_{SB}| + |\cos \gamma_{SA} - \cos \gamma_{SB}| \leq Thod \quad (18)$$

352 where $flag$ denotes the decision factor, and $Thod$ denotes the decision threshold.
 353 Combining equation (17) and (18), the relationship between $Thod$ and $|dVT_{AB}|$ can be
 354 expressed as:

$$355 \quad c \cdot |dVT_{AB}| \leq flag \times \{|dX_S^R|, |dX_S^T|, |dX_S^N|\}_{\max} \leq Thod \times \{|dX_S^R|, |dX_S^T|, |dX_S^N|\}_{\max} \quad (19)$$

356 where $\{|dX_S^R|, |dX_S^T|, |dX_S^N|\}_{\max}$ denotes the maximum of $|dX_S^R|$, $|dX_S^T|$ and $|dX_S^N|$.

357 Therefore, the influence of orbit error on CV time comparison can be adjusted by the
 358 decision threshold $Thod$. For example, if the orbit errors in R, T and N are all less than
 359 0.1 meters, and the decision threshold $Thod$ is set to be 0.03, the time comparison error
 360 caused by the orbit error will be less than ten picoseconds. If $Thod$ is set to be 0.06, the
 361 time comparison error caused by the orbit error will be up to twenty picoseconds. But it
 362 is important to note that the decision threshold does not follow the rule less the better.

363 The less the decision threshold $Thod$ is, the less position combination that meet the
364 judgment condition (18) will be found. In a general way, $Thod$ in the range from 0.03 to
365 0.05 is ok. But for some long baseline CV, $Thod$ need to be magnified properly. For
366 example, the baseline length of Changchun and Kunming is about 3500 km, $Thod$ need
367 to be set as 0.06. If $Thod$ is set to be 0.05, the condition (18) can not be satisfied.

368 Furthermore, two ground stations that meet the judgment condition of (18) do not
369 need to observe the space station simultaneously. For example, ground station A collects
370 observation data at t_1 , ground station B collects observation data at t_2 , the relative
371 position of ground station A and the space station at t_1 and the relative position of
372 ground station B and the space station at t_2 satisfy the judgment condition of (18). Thus,
373 the orbit error component in the calculated clock bias between station A and the space
374 station at t_1 , $VT_{AS}(t_1)$, is close to the orbit error component in the calculated clock bias
375 between station B and the space station at t_2 , $VT_{BS}(t_2)$. $VT_{AS}(t_1)$ and $VT_{BS}(t_2)$ is
376 computed by the one-way time comparison method. After the differencing operation of
377 $VT_{AS}(t_1)$ and $VT_{BS}(t_2)$, the remnants of orbit error are almost removed.

378 Because the relative frequency bias between two atomic clocks makes the phase bias

379 change with time. $VT_{AS}(t_1)$ cannot be minus $VT_{BS}(t_2)$ directly to calculate the clock
380 bias of stations A and B. It is necessary to get the clock biases $VT_{AS}(t)$ and $VT_{BS}(t)$ at the
381 same time to calculate the clock bias between two ground stations. This is the
382 fundamental idea of CV time comparison method.

383 The atomic clock of the space station has the stability higher than that of the ground
384 atomic clock. The frequency stability of the space clock when τ is equal to 1 second is
385 better than $1E-13$, and the frequency stability when τ is equal to 86400 seconds is better
386 than $1E-15$. The ground clocks that need several tens of picoseconds time comparison
387 accuracy also have good stability and little frequency drift. Thus, we can use the linear
388 polynomial to model the clock bias between the ground station and the space station.
389 The modeling target is to compute the relative frequency deviation of these two clocks.
390 The clock bias model is shown as follows:

391
$$VT_{AS}(t) = a + b \times (t - t_0) \quad (20)$$

392 where a denotes the constant term, b denotes the first-order coefficient of the
393 polynomial, and t_0 denotes the model starting time.

394 The following equation will be used to obtain the clock bias between station A and

395 the space station at t_2 based on the extrapolation principle.

396
$$\text{VT}'_{\text{AS}}(t_2) = \text{VT}_{\text{AS}}(t_1) + b \times (t_2 - t_1) \quad (21)$$

397 where $\text{VT}'_{\text{AS}}(t_2)$ denotes the clock bias between station A and the space station. Then,

398 we can compute the clock bias of station A and B by the following equation:

399
$$\text{VT}_{\text{AB}}(t_2) = \text{VT}'_{\text{AS}}(t_2) - \text{VT}_{\text{BS}}(t_2) = (\text{VT}_{\text{AS}}(t_1) - \text{VT}_{\text{BS}}(t_2)) + b \times (t_2 - t_1) \quad (22)$$

400 because the orbit error component in $\text{VT}_{\text{AS}}(t_1)$ and $\text{VT}_{\text{BS}}(t_2)$ is nearly equal to each other,

401 the orbit error is almost removed in (22). So, the GNSS CV method has been optimized

402 and new space station CV method has been proposed.

403 It can be seen from (20) that the modeling data source is the clock bias that is

404 calculated based on one-way time comparison method. So, the data source contains the

405 orbit error, and some modeling error must be applied to (22).

406 The error of $\text{VT}_{\text{AB}}(t_2)$ can be expressed by the following equation:

407
$$\varepsilon(\text{VT}_{\text{AB}}(t_2)) = (\varepsilon_{\text{orbit}}(\text{VT}_{\text{AS}}(t_1)) - \varepsilon_{\text{orbit}}(\text{VT}_{\text{BS}}(t_2))) + (\varepsilon_{\text{else}}(\text{VT}_{\text{AS}}(t_1)) - \varepsilon_{\text{else}}(\text{VT}_{\text{BS}}(t_2))) + \varepsilon_{\text{mod}}$$

408 (23)

409 where $\varepsilon_{\text{orbit}}(\text{VT}_{\text{AS}}(t_1))$ and $\varepsilon_{\text{orbit}}(\text{VT}_{\text{BS}}(t_2))$ denote the orbit error in $\text{VT}_{\text{AS}}(t_1)$ and $\text{VT}_{\text{BS}}(t_2)$

410 respectively, $\varepsilon_{\text{else}}$ denotes other errors except the orbit error, ε_{mod} denotes the modeling

411 error of (20). Because the relative position of ground station A and the space station at t_1
412 and the relative position of ground station B and the space station at t_2 satisfy the
413 judgment condition of (18), the first error term of the right side of (23) is cancelled. The
414 orbit error is canceled by CV principle, but the modeling error is introduced. This
415 method need to obtain the linearized parameter b of clock bias through modeling.
416 Although the orbit error in the modeling data source is large, the time comparison error
417 caused by modeling is much less than the original orbit error. The simulation
418 experiments will be used to validate this method later.

419 From above optimizing procedure, it is known that (18) presents the basic principle
420 of the optimized method. Two ground stations must obtain the space-ground clock bias
421 of the same time to calculate the clock bias between these two ground stations based on
422 CV method. But the synchronous space-ground clock biases of the two ground stations
423 are not all calculated by the observing data. One is calculated by the observing data. The
424 other is obtained by the method of modeling and extrapolating. The optimized method
425 makes use of the high stability of the space and ground atomic clocks to model the
426 space-ground clock bias. By modeling and extrapolating, the space-ground clock biases

427 at the same time are obtained, and the orbit errors are cancelled at that time. After
428 calibrating other errors of (7), several tens of picoseconds CV time comparison
429 accuracy can be achieved.

430 Moreover, the optimized method does not require that two ground stations observe
431 the space station at the same time, so this method can be applied to the blind areas
432 where GNSS CV method is invalid.

433 **5. Simulation verification of optimized method**

434 In order to verify the validity of the optimized space station CV time comparison
435 method mentioned above, we selected two ground station groups as examples to do the
436 simulation experiments. Xi'an and Sanya stations form the first group and Kashi and
437 Sanya stations form the second group which faces the problem of GNSS CV blind area.

438 **5.1 Analysis of Xi'an and Sanya Simulation**

439 The simulation condition of the optimized method is almost the same as that of GNSS
440 CV method. The decision threshold of (12) is set to be 0.03. The optimized method is
441 used to realize the CV time comparison of Xi'an and Sanya stations, the decision factor
442 is shown in Figure 3, and time comparison error is shown in Figure 4. These two figures

443 are all in three-dimensional coordinate format. The data of X and Y axis indicates the
444 cumulative seconds of one day for Xi'an and Sanya, respectively. The data of Z axis of
445 Figure 3 indicates the decision factor, and that of Figure 4 indicates the time comparison
446 error.

447 It can be seen from Figure 3 that the decision factor is calculated based on
448 un-simultaneous space station position for Xi'an and Sanya. Thus, the space-ground
449 clock biases of these two stations are not derived from the synchronous observation
450 data.

451 Figure 4 shows that the absolute time comparison errors are less than 30 picoseconds
452 based on the optimized space station CV method.

453 It is noteworthy that the stability of Xi'an atomic clock is better than that of Sanya, so
454 Xi'an space-ground clock bias is modeled and extrapolated. By this means, less
455 modeling error is introduced, and higher accuracy is obtained.

456 **5.2 Analysis of Kashi and Sanya Simulation**

457 It can be seen from Table 1 that traditional GNSS CV method is invalid if one ground
458 station is in Kashi, and the other station is in Sanya. Because these two ground stations

459 cannot observe the space station simultaneously. But the optimized method is valid to
460 implement the CV time comparison of these ground stations.

461 The baseline length of Kashi and Sanya is longer than 5000 kilometers, and the
462 simulation condition is essentially the same as that of Xi'an and Sanya simulation
463 except for the atomic clock parameters of Kashi. The frequency stability of the Kashi
464 clock when τ is equal to 1 second is set to $5\text{E-}13$, and the frequency stability when τ is
465 equal to 86400 seconds is set to $1\text{E-}14$. The decision factor of Kashi and Sanya stations
466 is shown in Figure 5, and the time comparison error is shown in Figure 6.

467 Although the baseline length of Kashi-Sanya is almost two times that of Xi'an-Sanya,
468 and the atomic stability of Kashi is worse than that of Xi'an, but it can be seen from
469 Figure 6 that the equivalent CV time comparison accuracy is obtained. Figure 6 shows
470 that the absolute time comparison errors of Kashi and Sanya are less than 35
471 picoseconds. It is known from this simulation experiment that the optimized method
472 solves the problem of the blind area and also is effective to realize super high accurate
473 time comparison.

474 It can be seen from Figure 5 that there are not many observation time groups of

475 Sanya and Kashi when the decision threshold is set to 0.03. But the time comparison
476 error of Figure 6 is little. Therefore, the threshold can be appropriately magnified to
477 obtain more times of CV time comparison.

478 **6. Conclusion**

479 The traditional GNSS CV method is optimized to be more effective for the space station.
480 First, the CV time comparison theory has been introduced. The difference between the
481 space station CV and GNSS CV were pointed out. Then, the application limitations of
482 GNSS CV method for the space station were analyzed based on simulation. After that,
483 the traditional CV method was optimized, and the limitations were improved. Finally,
484 the performance of the optimized method was validated by simulation experiments. The
485 following conclusions are drawn:

486 (1) There are three differences between the space station and the GNSS satellite, the
487 orbit characteristics, downlink signal structure and the atomic clock performance. The
488 space station advantages regarding downlink structure and clock performance are the
489 premise for achieving high accurate CV time comparison. But the space station orbit
490 error cannot be canceled substantially by the traditional CV principle, and additional

491 orbit error calibrating method must be designed and used.

492 (2)There are two limitations for traditional GNSS CV method to be applied to the
493 space station. On the one hand, there are GNSS CV blind areas. On the other hand,
494 GNSS CV method is not good in dealing with the orbit error and it is only useful for
495 several hundreds of picoseconds space station CV accuracy.

496 (3)The optimized method has excellent performance. First, several tens of
497 picoseconds CV time comparison accuracy can be realized by the optimized method,
498 and the CV baseline length can be up to several thousands of kilometers. Second, the
499 optimized method does not need the ground stations observe the space station
500 simultaneously, so it solves the problem of CV blind area handily.

501 **Abbreviations**

502 CV: Common-view; GNSS: Global Navigation satellite System; ACES: Atomic Clock
503 Ensemble in Space; VLBI: Very Long Baseline Interferometry; ISS: International Space
504 Station; GCRS: Geocentric Celestial Reference System; RTN: the radial (R), tangential
505 (T) and the normal (N) vector; VTEC: Vertical Total Electronic Content.

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507 Not applicable.

508 **Authors' contributions**

509 Doctor Liu Yinhua designed the research , implemented the algorithm and simulated
510 software of the optimized CV method and did the simulation experiments. Doctor Li
511 Xiaohui proposed the original thought of the space station optimized CV method. All
512 authors read and approved the final manuscript.

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517 **Availability of data and materials**

518 The datasets used and/or analyzed during the current study are available from the
519 corresponding author on reasonable request.

520 **Competing interests**

521 The authors declare that they have no competing interests.

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575

576 **Figure Legend**

Figure 1: Errors of Xi'an and Sanya traditional CV time comparison. The data displayed in the abscissa indicates the cumulative seconds of one day, and the data displayed in the ordinate indicates the value of time comparison error between Xi'an and Sanya using traditional CV method.

Figure 2: Xi'an and Sanya CV time comparison error caused by orbit error. The data displayed in the ordinate indicates the orbit error contained in time comparison error between Xi'an and Sanya using traditional CV method.

Figure 3: Flag of Xi'an and Sanya CV time comparison based on optimized method. The data of X and Y axis indicates the cumulative seconds of one day for Xi'an and Sanya, respectively. The data of Z axis indicates the decision factor.

Figure 4: Errors of Xi'an and Sanya CV time comparison based on optimized method. The data of X and Y axis indicates the cumulative seconds of one day for Xi'an and Sanya, respectively. The data of Z axis indicates the value of time comparison error

590 between Xi'an and Sanya using optimized CV method.

591 Figure 5: Flag of Kashi and Sanya CV time comparison based on optimized method.

592 The data of X and Y axis indicates the cumulative seconds of one day for Kashi and

593 Sanya, respectively. The data of Z axis indicates the decision factor.

594 Figure 6: Errors of Kashi and Sanya CV time comparison based on optimized method.

595 The data of X and Y axis indicates the cumulative seconds of one day for Kashi and

596 Sanya, respectively. The data of Z axis indicates the value of time comparison error

597 between Kashi and Sanya using optimized CV method.

598

599 **Table List**

600 Table 1: Space station visibility to several representative cities of China. The data in the

601 table indicates the cumulative minutes of one day, the unit of that is min.

Figures

Figure 8

Figure 1

Errors of Xi'an and Sanya traditional CV time comparison. The data displayed in the abscissa indicates the cumulative seconds of one day, and the data displayed in the ordinate indicates the value of time comparison error between Xi'an and Sanya using traditional CV method.

Figure 9

Figure 2

Xi'an and Sanya CV time comparison error caused by orbit error. The data displayed in the ordinate indicates the orbit error contained in time comparison error between Xi'an and Sanya using traditional CV method.

Figure 3

Flag of Xi'an and Sanya CV time comparison based on optimized method. The data of X and Y axis indicates the cumulative seconds of one day for Xi'an and Sanya, respectively. The data of Z axis indicates the decision factor.

Figure 4

Errors of Xi'an and Sanya CV time comparison based on optimized method. The data of X and Y axis indicates the cumulative seconds of one day for Xi'an and Sanya, respectively. The data of Z axis indicates the value of time comparison error between Xi'an and Sanya using optimized CV method.

Figure 5

Flag of Kashi and Sanya CV time comparison based on optimized method. The data of X and Y axis indicates the cumulative seconds of one day for Kashi and Sanya, respectively. The data of Z axis indicates the decision factor.

Figure 6

Errors of Kashi and Sanya CV time comparison based on optimized method. The data of X and Y axis indicates the cumulative seconds of one day for Kashi and Sanya, respectively. The data of Z axis indicates the value of time comparison error between Kashi and Sanya using optimized CV method.

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