

Title: Does the energy content of a body depend upon its inertia?

Author: Sunny Kumar

Address: Aryabhatasya Academy of Mathematics, Near Jagdamba Gas Agency/Opposite Devrishi apartment, Village-Danada, IT Park, Sahastradhara Road, Dehradun, 248013, Uttarakhand (India)

E-mail: sunnykumar40@gmail.com

Abstract: In this article, the mass-energy equivalence is analyzed based on the total energy content & inertia of a body. The analysis included quantum energy formalization with relativity. The finding suggested that photons have absolute mass to justify their momentum.

Keywords: inertial mass-energy, *vis-viva*, *vis-mortua*, quantum mass-energy factor

1. Introduction

Albert Einstein rise the question that does the inertia of a body depends upon its energy content?^[1] And he concluded that *if a body gives off the energy E in the form of radiation, its mass diminished by E/c^2* (where c is the speed of light). But here we ask the question again in its reversal form that does the energy content of a body depend upon its inertia? It follows because of my recent work on de Broglie's hypothesis on the matter-waves,^[2] in which I derived an equation of total energy of a particle having rest (absolute) mass m and relative velocity v and a frequency f because of matter-wave observed by an observer.

If $\beta = \frac{v}{c}$ & $\alpha = \frac{1}{\gamma} = \sqrt{1 - \beta^2}$ then

$$\alpha mc^2 + (\frac{1}{2}mv^2 + hf) = mc^2$$

$$\sum E = mc^2 \tag{1}$$

Now we know that Einstein never considers a quantum-energy in his theory of relativity. So how can it appear in his theory of special relativity? To analyze this, let have a look at his above paper in which he gives the change in kinetic energy by ΔT ,

$$\Delta T = (\gamma - 1)mc^2 \tag{2}$$

If we expand the above in binomial terms, we get

$$\Delta T = mc^2(\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{126}\beta^8 + \dots \dots \dots)$$

Now if we see his paper, he smartly hides the quantum energy by neglecting the fourth and higher orders and approximate the result for low velocity as

$$\Delta T \approx \frac{\beta^2}{2} mc^2 = \frac{1}{2} mv^2$$

Let the sum of all binomial terms (excluding first) is Q , then

$$\Delta T = \frac{\beta^2}{2} mc^2 + Q$$

If $v \ll c$, then Einstein approximation holds good. But for velocities near c the quantity Q grows fast and demands explanations because the theory of special relativity is the study of high velocities. And that is the point where I build my argument in my recent work on the de Broglie hypothesis. If we denote inertial energy by E_α , kinetic energy by E_k and quantum-energy of matter-wave by E_q , then work-done (by the force, which is the reason behind the motion) in the center of mass frame is given by the change in inertial energy,

$$(1 - \alpha)mc^2 = E_k + E_q$$

And the work done (by the same force as stated above) in the relative frame is given by the change in kinetic energy by equation (2), which can be expressed as

$$(\gamma - 1)mc^2 = \gamma(E_k + E_q)$$

As we can see that both are an equivalent statement.

2. *Vis-Viva & Vis-Mortua*^[3]

The competition between two physical quantities (momentum & kinetic energy) to describe the changes in nature arise in late 1650 by two sides of philosophers. One side led by German mathematician Gottfried Leibniz with Emilie du Chatelet for *Vis-Viva (energy)*. And the defendants of momentum led by Rene Descartes of France & Newton himself. Descartes belief that God, the general cause of all motion in the universe, preserves the same quantity of motion and rest put into the world at the time of creation. The measurement of this quantity is mv . While Leibniz objected and argue that the quantity which remains absolute and indestructible is not momentum but *vis-viva*. Now, these two principles are understood as complementary together. But we can see that the 'living force' (*vis-viva*) gets more attention from philosophers over the 'dead force' (*vis-mortua*). The dead force or as I like to call it *sleeping force/energy* is the concept, which not got eyes. But I find it useful in my analysis. It can be interpreted as inertial mass/energy. On the other hand, the *living force* gets limited to kinetic energy. If we see the equation (1) then, we can compare the term (αmc^2) as *the sleeping energy* & the sum of the remaining two terms $(\frac{1}{2}mv^2 + hf)$ as *the living/awaken energy*. And whenever there is an increment in relative motion then the sleeping thing gets awoken and becomes a living thing and vice-versa. Now we can ask that what is Newton's idea of force?

"It is an agent, which produces a change in nature". The reason by which energy flows in an isolated system. The force can only change the inertia of a body and can't change the absolute or overall energy when applied to that body.

3. Factors for energy in special relativity

The factor for inertial energy is α and for kinetic energy is $\frac{\beta^2}{2}$.

We can derive the factor of equivalence for a quantum energy E_q by equation (1)

$$E_q = \left(1 - \alpha - \frac{\beta^2}{2}\right) mc^2$$

$$E_q = Amc^2$$

The table for values of these factors is given in the appendix,^[1] as relative velocity increases 0 to c . In this table, we can see that how the quantum energy of matter-wave in a moving body evolves with kinetic energy and create a catastrophe in the relative energy for photons.

4. Photons

The relative energy is given by the expression,

$$E_\gamma = \gamma mc^2$$

If we manipulate this equation, then

$$(E_\gamma)^2 = (mc^2)^2 + (P_\gamma c)^2$$

Where $P_\gamma = \gamma mv$

$$(\gamma mc^2)^2 = (mc^2)^2 + (\gamma mvc)^2 \quad (3)$$

On dividing γ^2 , we can get

$$E^2 = (mc^2)^2 = (\alpha mc^2)^2 + (mvc)^2 \quad (4)$$

And it is not mathematically right to derive, the total energy E of photons by equation (3), because it is relative energy not the total energy of the photon. The total energy of a photon can be measure by equation (4) only. Obtaining momentum from equation (3) by ignoring γ term is a serious mathematical mistake, which is defiantly an obstacle for quantum energy to get formalize with relativity. It's just like saying that *infinity or zero is equal to some real number*. A photon has all its inertial (sleeping) energy in the awaken (vis-viva) state and contains an absolute mass, which causes momentum. Rest mass notation m_0 for photons create confusion and destroy the beauty of relativity.^[4]

5. Conclusion:

From equation (1) it directly follows that: If a body absorbs any radiation of frequency f (*in meters*), then its absolute mass get increases by $2hf/c^2$ or $(1.4745 \times 10^{-50} f) kg$. The fact that mass-energy withdraws from a body becomes the energy of radiation makes no difference, so we can conclude that the inertia of a body is the measure of its energy content and vice-versa.

Declaration: The author has no conflict of interest.

References:

[1] Albert Einstein, *Does the Inertia of a Body depend upon its energy content?* (Annalen der Physik), **18:639**, (1905) [Link to article](#)

[2] Sunny Kumar, *A mathematical analysis of de Broglie hypothesis and general solution for quantization to linear momentum and mass*, Aryabhatasya Academy of Mathematics, (2020) [Link to article](#)

[3] Carolyn Iltis, *Leibniz and the vis-viva controversy*, (The History of Science Society, Chicago), **vol. 62, No.1** pp. 21-35, (Spring 1971), [Link to article](#)

[4] L. B. Okun, *The concept of mass (mass, energy, relativity)*, (Institute of Theoretical and Experimental Physics, Moscow), Usp. Fiz. Nauk **158**, pp. 511-530 (1989) [Link to article](#)

Appendix

[1] Table for factors of energy in relativity,

v	β	γ	α	$\frac{\beta^2}{2}$	A
0	0	1	1	0	0
29979245.8	0.1	1.0050378152	0.9949874371	0.0050000000	0.0000125629
59958491.6	0.2	1.0206207261	0.9797958971	0.0200000000	0.0002041029
89937737.4	0.3	1.0482848367	0.9539392014	0.0450000000	0.0010607986
119916983.2	0.4	1.0910894511	0.9165151389	0.0800000000	0.0034848611
149896229	0.5	1.1547005383	0.8660254037	0.1250000000	0.0089745963
179875474.8	0.6	1.2500000000	0.8000000000	0.1800000000	0.0200000000
209854720.6	0.7	1.4002800840	0.7141428428	0.2450000000	0.0408571572
239833966.4	0.8	1.6666666667	0.6000000000	0.3200000000	0.0800000000
269813212.2	0.9	2.2941573387	0.4358898943	0.4050000000	0.1591101057
c	1	∞	0	0.5000000000	0.5000000000