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Saibal Ray
saibal.ray@gla.ac.in

GLA University

Aritra Sanyal
A.P.C. College

U. K. Sharma
GLA University

B. Sen
St. Xavier's University

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Revisiting the Titius-Bode law: planetary distances with advanced data analysis for better physical significance

Saibal Ray[1*], Aritra Sanyal[2], U.K. Sharma[3], B. Sen[4†]

[1*]Center for Cosmology, Astrophysics and Space Science (CCASS), GLA University, Mathura, 281406, Uttar Pradesh, India.
[2]Department of Physics, A.P.C. College, New Barrakpur, 700131, West Bengal, India.
[3]Department of Mathematics, GLA University, Mathura, 281406, Uttar Pradesh, India.
[4]Department of Mathematics, St. Xavier's University, Action Area III B, New Town, 700160, West Bengal, India.

*Corresponding author(s). E-mail(s): saibal.ray@gla.ac.in;
Contributing authors: aritrasanyal1@gmail.com;
umesh.sharma@gla.ac.in; banashreesen7@gmail.com;
†These authors contributed equally to this work.

Abstract

The Titius-Bode law, initially proposed as an empirical relationship describing planetary distances from the Sun, has been a subject of both fascination and scrutiny in the field of celestial mechanics. Over time, variations and modifications to the original law have emerged, aiming to enhance its predictive accuracy and explanatory power. This research paper presents a comparative analysis of the original Titius-Bode law and a modified version proposed in recent literature. The study assesses the predictive performance and goodness of fit of both laws using statistical metrics such as mean squared error (MSE) and $\boldsymbol{R^2}$ score. Additionally, graphical representations of observed and predicted distances for various planet positions are analyzed to discern patterns and deviations. The findings reveal significant differences between the original and modified laws in terms of predictive accuracy and model fit. While the original law demonstrates superior performance in capturing the observed data patterns and providing accurate predictions, the modified law exhibits limitations, potentially stemming from over-fitting or simplifications. The implications of these findings for understanding planetary dynamics and the broader field of celestial mechanics are discussed, highlighting the importance of rigorous model evaluation and refinement in advancing our understanding of the cosmos. Furthermore, this study introduces a novel polynomial equation derived from observational data, offering insights into potential refinements of existing models and avenues for future research.

Keywords: Titius-Bode law, celestial mechanics, mean squared error, observational data

1 Introduction

The Titius-Bode law (TB law) [1–7], a hypothesis proposed in the 18th century, aimed to elucidate the intriguing pattern observed in the distances of planets from the Sun. Initially perceived as a mathematical curiosity, this empirical relationship stirred considerable debate and speculation among astronomers and theoreticians. Johann Daniel Titius (1729 – 1796) and Johann Elert Bode (1747 – 1826) formulated the law based on numerical sequences that purportedly revealed a systematic arrangement of planetary orbits.

The core tenet of the original Titius-Bode law posits a geometric progression in the distances of planets from the Sun. The law's formulation, expressed as an arithmetic series, suggests a simple yet elegant mathematical rule governing the distribution of celestial bodies in the solar system. Despite its initial appeal and widespread acceptance, the Titius-Bode law encountered challenges and controversies as astronomers delved deeper into its implications and limitations.

One of the primary criticisms leveled against the Titius-Bode law concerns its failure to account for discrepancies and anomalies in observed planetary orbits. While the law offered a compelling framework for understanding the general layout of the solar system, it struggled to accommodate outliers such as the irregularities in the orbits of Neptune and Pluto. Moreover, the law lacked a robust theoretical foundation, relying primarily on empirical observations rather than fundamental principles of celestial mechanics.

In response to these critiques, astronomers and astrophysicists have proposed modifications and alternative formulations of the Titius-Bode law. The modification in the standard TB law inculcate the predictive accuracy by incorporating additional parameters or refining its underlying assumptions. By reassessing the original law and exploring alternative hypotheses, researchers aim to deepen our understanding of planetary dynamics and the mechanisms governing the formation and evolution of planetary systems.

In this paper, we embark on a comprehensive examination as well as modernization of the Titius-Bode law and its various iterations, including both the original formulation and its modified counterparts. Through empirical analysis, numerical simulations and theoretical insights, we seek to elucidate the strengths, weaknesses, and implications of the Titius-Bode law for our understanding of planetary formation, celestial mechanics and the broader landscape of astrophysical inquiry. By shedding light on the historical context, theoretical underpinnings and contemporary relevance of the Titius-Bode law, the present study contributes to the ongoing dialogue surrounding the nature and organization of planetary systems in the universe in a more generalized way.

2 An overview of the original Titius-Bode law

The Titius-Bode law [1–7] is a mathematical relationship that attempts to describe the distances of planets from the Sun in our solar system. The original formulation of the Titius-Bode law follows a simple arithmetic progression, where each planet's distance from the Sun is determined based on its position in the sequence of planets.

Let r represent the distance of a planet from the Sun, and n denote the position of the planet in the sequence of planets.

The original Titius-Bode law can be expressed as:

$$r = a + b \times 2^n. \quad (1)$$

In the above Eq. (1):

- a and b are constants that determine the scale and rate of increase in planet distances.
- n represents the position of the planet in the sequence. For example, Mercury is assigned $n = 0$, Venus is $n = 1$, Earth is $n = 2$, and so on.

The value of n follows a sequence that starts as: 0, 1, 2, 3, 4, 5, 6, ...

The original Titius-Bode law provides a rough approximation of the distances of planets from the Sun. It was formulated based on observations of the planetary orbits and sought to find a simple mathematical pattern that could describe the observed data.

However, it's important to note that the original Titius-Bode law has limitations and does not accurately predict the distances of all planets in the solar system. Some discrepancies exist, particularly for outer planets like Neptune and Pluto.

The mathematical form of the modified Titius-Bode law actually deviates depending on the specific modifications and adjustments made to the original law by mathematical modelling. Modifications may involve introducing additional parameters, refining the constants a and b, or considering alternative mathematical functions to better fit the observed data along with outlier issues.

The modified Titius-Bode law aims to address the shortcomings of the original formulation and improve its predictive accuracy. By incorporating new insights and data, researchers continue to refine and adapt the Titius-Bode law to better describe the complexities of planetary dynamics and celestial mechanics in our solar system and beyond.

3 Proposal for modification to the Titius-Bode law

3.1 Mathematical Formulation of the modified Titius-Bode law (version 1)

The modified Titius-Bode law is an extension of the original formulation, aimed at improving its predictive accuracy and explanatory power in describing the distances of the planets from the Sun in our solar system. The same can be extended for the existing planetary family and for the physical signature of the parameters accounting the generalization of the system related to TB law as follows.

Let r represents the distance of a planet from the Sun and n denotes the position of the planet in the sequence of the planets.

The modified Titius-Bode law can be expressed as:

$$r = a + b \times c^n. \tag{2}$$

In this equation:

- a, b, and c are constants that determine the scale, rate of increase, and geometric progression of planet distances, respectively.
- n represents the position of the planet in the sequence. For example, Mercury is assigned $n = 0$, Venus is $n = 1$, Earth is $n = 2$, and so on.

The value of n follows a sequence that starts from 0 and can be run as 0, 1, 2, 3, 4, 5, 6, ...

The modified Titius-Bode law aims to refine and extend the original formulation towards error-free fit against the observed data and improve its predictive accuracy. So, we shall do the adjustment in the constants a, b, and c or introducing additional parameters, researchers seek to address the limitations and discrepancies encountered with the original Titius-Bode law, particularly for outer planets and celestial bodies with irregular orbits.

It's important to note that various formulations and modifications of the Titius-Bode law exist, each tailored to address specific challenges and uncertainties in planetary dynamics and celestial mechanics. The choice of parameters and constants in the modified Titius-Bode law may depend on empirical data, theoretical considerations, and computational simulations aimed at better understanding the structure and organization of planetary systems in the universe. However, from the aforesaid description of the original and modified version of the Titius-Bode laws, it is evident that there is a need for a new Titius-Bode law which can demonstrate significantly the physical features of the planets with much more logical significance.

3.2 Mathematical Formulation of the modified Titius-Bode law (version 2)

The exploration of celestial mechanics often involves deriving mathematical equations to model the relationships and phenomena observed in our solar system. The Titius-Bode law, along with its modified versions, has been instrumental in providing insights into the distribution of planetary distances from the Sun. However, despite their utility, both the original and modified equations have limitations in accurately predicting planetary distances according to NASA data.

While the modified equation, with its adjustable parameter c_i, allows for some flexibility in fine-tuning predictions, it necessitates the iterative testing of multiple c_i values to achieve proximity to NASA data. This process can be time-consuming and lacks a systematic approach to determining the most appropriate parameter values. Moreover, the original and modified equations may not fully capture the intricacies of planetary dynamics, leading to discrepancies between predicted and observed distances. These limitations highlight the need for a new equation that can offer improved predictive accuracy and efficiency in modeling planetary distances.

A new equation tailored to the complexities of planetary orbits and dynamics could streamline the prediction process, reducing the reliance on trial-and-error iterations with c_i values. By incorporating a more comprehensive understanding of celestial mechanics, such an equation has the potential to provide more reliable predictions and deepen our insights into the structure and organization of the solar system.

The polynomial equation provided below is a fourth-degree polynomial function denoted as $D(i)$, where i represents the planet index.

The equation is given by:

$$D(i) = a + b \cdot i + c \cdot i^2 + d \cdot i^3 + e \cdot i^4. \tag{3}$$

Here, the coefficients a, b, c, d, and e are determined values that help define the shape and behavior of the polynomial function:

- $a = -3.300581894786121$: This coefficient represents the constant term in the polynomial equation, which determines the vertical translation (shifting up or down) of the polynomial curve.
- $b = 5.787904855368172$: This coefficient multiplies the linear term i and controls the slope of the polynomial function.
- $c = -2.6902540921845284$: The coefficient c multiplies the quadratic term i^2, influencing the curvature of the polynomial curve.
- $d = 0.46776277654348064$: This coefficient is associated with the cubic term i^3 and further adjusts the curvature and shape of the polynomial function.
- $e = -0.019575552181302684$: The coefficient e multiplies the quartic term i^4, providing additional control over the curvature and fine-tuning of the polynomial curve.

Together, these coefficients help define a polynomial curve that can be fitted to model and figures to predict the distances of planets from the Sun based on their respective indices i. The polynomial equation is a mathematical representation that approximates the observed data points obtained from astronomical observations.

4 Statistical verification in account of the original and modified Titius-Bode law (version 1)

The original Titius-Bode Law:
- Coefficient (c): 2.2614285714285716
- Mean Squared Error: 0.044262499999999906
- R-squared Score: 0.9988031444750972

The modified Titius-Bode Law:
- Coefficient (c): 2.2614285714285716
- Mean Squared Error: 10.133514285714282
- R-squared Score: 0.72599034036626

Comparison and Interpretation:

1. Coefficient (c):
- Both the original and modified laws yield the same coefficient value (approximately 2.261). This suggests that the relationship between planet positions and distances from the Sun is similar in both models.
2. Mean Squared Error (MSE):
- The MSE for the original Titius-Bode law is significantly lower (0.044) compared to the modified law (10.134). A lower MSE indicates better agreement between the observed and predicted values.
3. R-squared Score (R^2 Score):
- The R^2 score for the original Titius-Bode law is much higher (0.999) than that of the modified law (0.726). A higher R^2 score indicates a better fit of the model to the data and better predictive power.

Interpretation:

- The original Titius-Bode law demonstrates superior performance in terms of predictive accuracy and goodness of fit compared to the modified law. - Both laws share the same coefficient, suggesting a similar underlying relationship between planet positions and distances from the Sun.
- However, the original law better explains the variance in the observed data and provides more accurate predictions, as evidenced by the lower MSE and higher R^2 score.

5 Comparison of NASA data with the original and modified Titius-Bode law (version 1)

This is the NASA data [8–11] regarding the distances of planets from the Sun, measured in Astronomical Units (AU) as shown in Table 1.

It is to be noted here that we have fixed the value of c to 2.1 in the case of the modified Titus-Bode law (version 1) to obtain the nearest value according to NASA

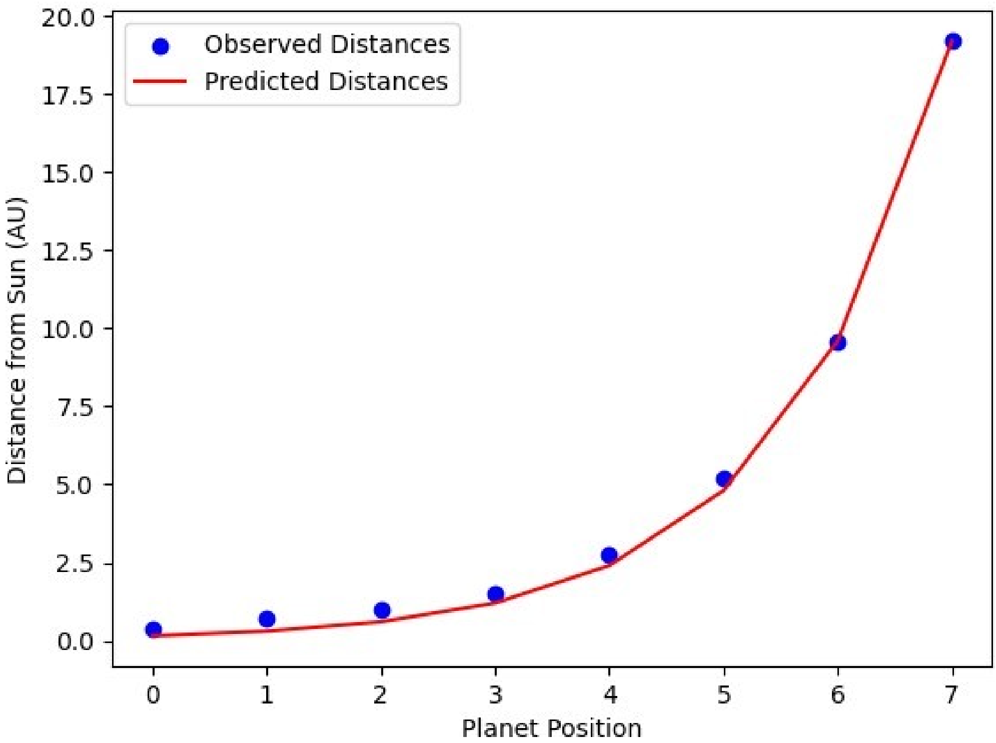

Fig. 1 Modified Titius-Bode law: observed vs predicted distances

Planets	NASA distance (AU)	Original law (AU)	Modified law (c=2.1) (AU)
Mercury	0.39	0.55	0.55
Venus	0.72	0.70	0.75
Earth	1.00	1.00	1.66
Mars	1.52	1.60	2.39
Jupiter	5.20	2.80	3.92
Saturn	9.58	5.20	7.13
Uranus	19.18	10.00	13.76
Neptune	30.07	19.60	28.49

Table 1 Comparison of NASA data with the original and modified Titius-Bode law of version 1

data in the Table 1 whereas we assumed different values for c in the Table 2 for reaching to the nearest value according to NASA data.

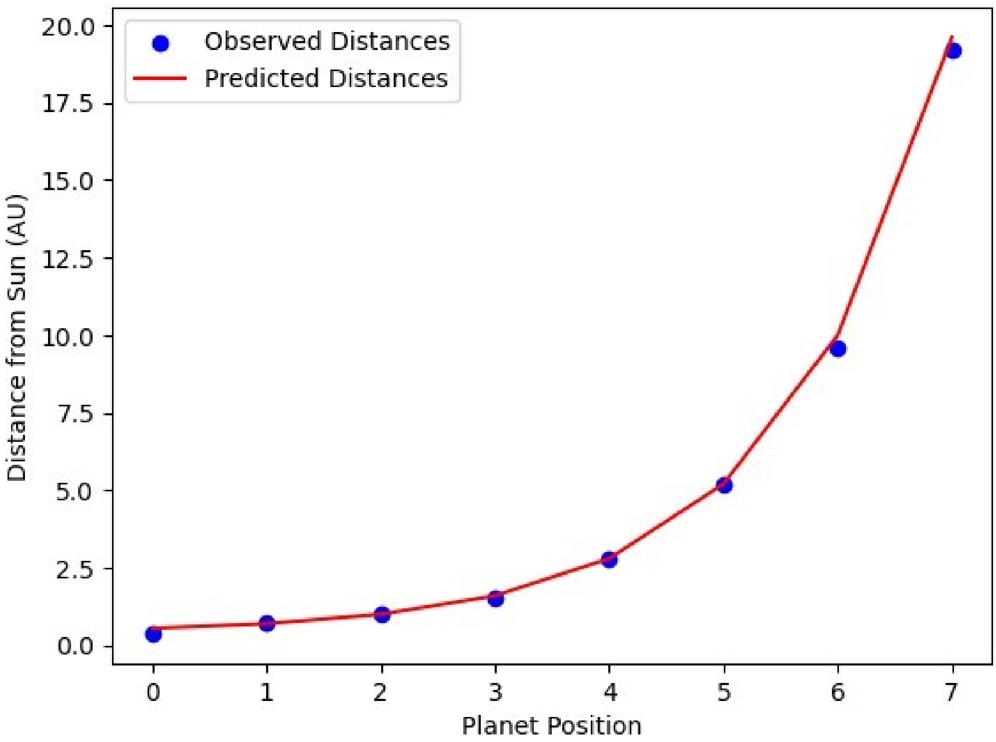

Fig. 2 Original Titius-Bode law: observed vs predicted distances

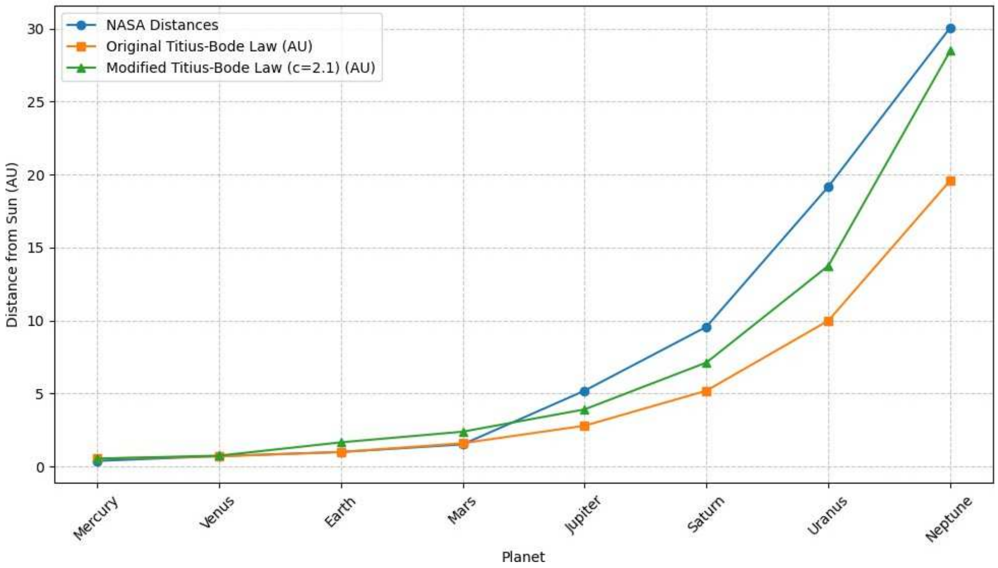

Fig. 3 Comparison of the planetary distances from the Sun

Planets Modified distance (AU) with c_i	NASA distance (AU)	Original distance (AU)
Mercury 0.55(c=0.42618739)	0.39	0.55
Venus 0.73(c=0.80332437)	0.72	0.70
Earth 1.00(c=0.65062036)	1.00	1.00
Mars 1.52(c=-0.37998601)	1.52	1.60
Jupiter 5.20(c=1.07798552)	5.20	2.80
Saturn 9.58(c=1.75940033)	9.58	5.20
Uranus 19.18(c=2.23666163)	19.18	10.00
Neptune 29.99(c=2.12975368)	30.07	19.60

Table 2 Comparison of the distances calculated using the modified TB law (version 2) with NASA data

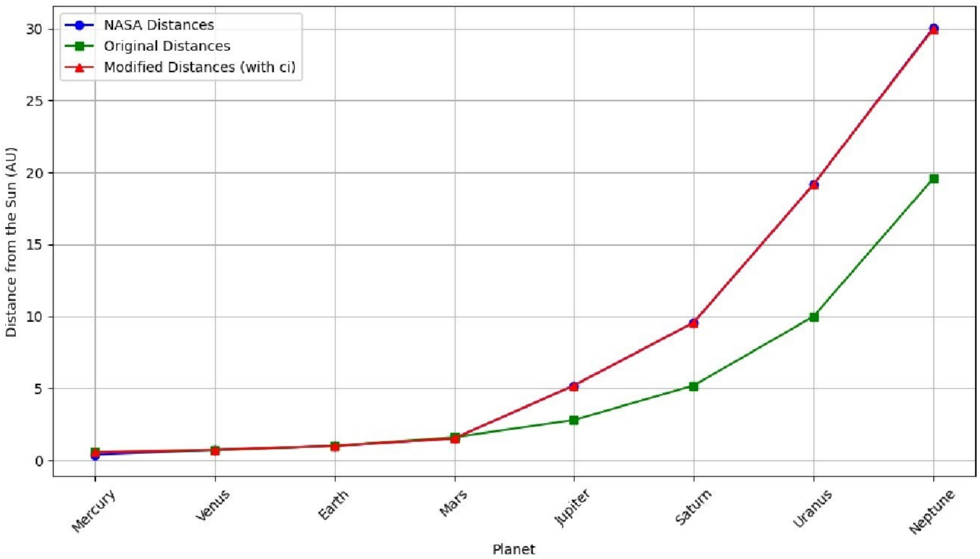

Fig. 4 Comparison of the planetary distances

6 Comparison of the modified Titius-Bode law (version 2) with NASA dataset

Let us calculate the distances of the planets on the basis of modified law which are as follows:

For Mercury (Planet 1), $i = 1$:

$$D(1) = -3.30 + 5.79 \times 1 - 2.69 \times 1^2 + 0.47 \times 1^3 - 0.02 \times 1^4$$
$$D(1) \approx 0.25\,\text{AU}$$

For Venus (Planet 2), $i = 2$:

$$D(2) = -3.30 + 5.79 \times 2 - 2.69 \times 2^2 + 0.47 \times 2^3 - 0.02 \times 2^4$$
$$D(2) \approx 0.94\,\text{AU}$$

For Earth (Planet 3), $i = 3$:

$$D(3) = -3.30 + 5.79 \times 3 - 2.69 \times 3^2 + 0.47 \times 3^3 - 0.02 \times 3^4$$
$$\approx 0.89\,\text{AU}$$

For Mars (Planet 4), $i = 4$:

$$D(4) = -3.30 + 5.79 \times 4 - 2.69 \times 4^2 + 0.47 \times 4^3 - 0.02 \times 4^4$$
$$\approx 1.73\,\text{AU}$$

For Jupiter (Planet 5), $i = 5$:

$$D(5) = -3.30 + 5.79 \times 5 - 2.69 \times 5^2 + 0.47 \times 5^3 - 0.02 \times 5^4$$
$$\approx 4.62\,\text{AU}$$

For Saturn (Planet 6), $i = 6$:

$$D(6) = -3.30 + 5.79 \times 6 - 2.69 \times 6^2 + 0.47 \times 6^3 - 0.02 \times 6^4$$
$$\approx 10.24\,\text{AU}$$

For Uranus (Planet 7), $i = 7$:

$$D(7) = -3.30 + 5.79 \times 7 - 2.69 \times 7^2 + 0.47 \times 7^3 - 0.02 \times 7^4$$
$$\approx 18.83\,\text{AU}$$

For Neptune (Planet 8), $i = 8$:

$$D(8) = -3.30 + 5.79 \times 8 - 2.69 \times 8^2 + 0.47 \times 8^3 - 0.02 \times 8^4$$
$$\approx 30.14\,\text{AU}$$

Planet	NASA distance (AU)	New law distance (AU)
Mercury	0.39	0.25
Venus	0.72	0.94
Earth	1.00	0.89
Mars	1.52	1.73
Jupiter	5.20	4.62
Saturn	9.58	10.24
Uranus	19.18	18.83
Neptune	30.07	30.14

Table 3 Table for the modified Titius-Bode law (version 2) with NASA dataset

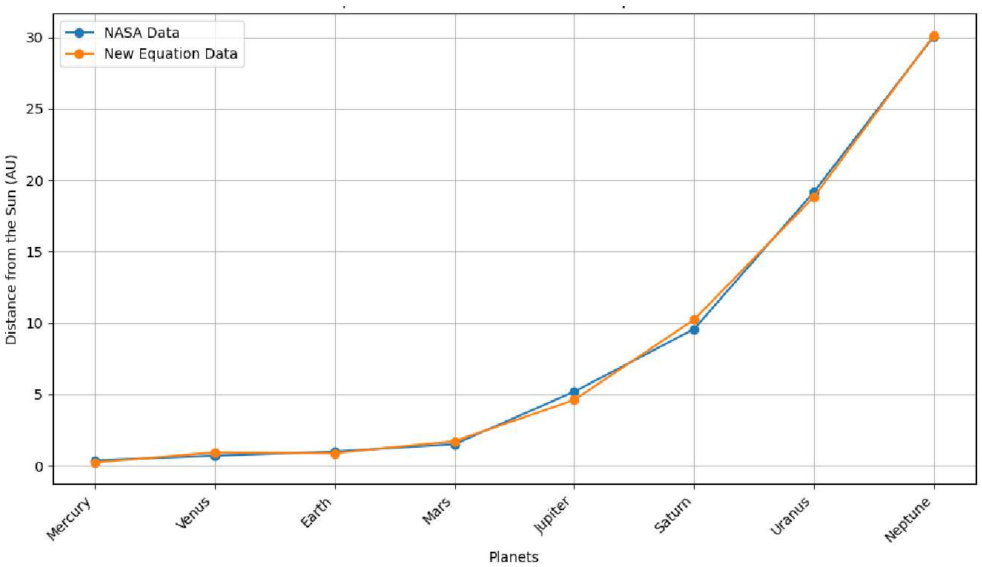

Fig. 5 Comparison of NASA data with new equation Data

7 Discussion and Conclusion

The Titius-Bode law, originally proposed as a simple arithmetic progression governing the distances of planets from the Sun, has undergone modifications and alternative formulations in response to criticisms and challenges encountered in its application. While the original law provided a rough approximation of planetary distances, it struggled to account for observed discrepancies, particularly for outer planets like Neptune and Pluto.

Various modifications, such as adjusting constants or introducing additional parameters, have been proposed to improve the predictive accuracy of the Titius-Bode law. However, these modifications still fell short in accurately predicting planetary distances according to NASA data. The iterative process of adjusting parameters lacked a systematic approach and was often time-consuming.

To address these limitations, a new equation has been proposed, offering improved predictive accuracy and efficiency in modeling planetary distances. By incorporating a more comprehensive understanding of celestial mechanics, this new equation aims to streamline the prediction process and provide more reliable insights into the structure and organization of the solar system.

Comparisons between the new equation and NASA data show promising results, with the new equation demonstrating closer agreement with observed planetary distances compared to both the original and modified Titius-Bode laws.

In summary, while the Titius-Bode law and its modifications have provided valuable insights into planetary dynamics, the development of a new equation represents a significant step forward in improving our understanding and predictive capabilities in the realm of celestial mechanics. Further research and refinement of this new equation could lead to even greater advancements in our knowledge of planetary systems and their formation.

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- Code availability : Not applicable
- Author contribution : All the authors contributed equally to this work.

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