Adjustable Robust Optimization for Multi-Period Water Allocation in Droughts under Uncertainty

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Adjustable robust optimization for multi-period water allocation in droughts under uncertainty

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Abstract

Optimal, rational water resource allocation can go some way to overcoming water deficiencies; however, its achievement is complex due to conflicting hierarchies and uncertainties, such as water availability (WA) and water demand (WD). This study developed a robust water withdrawal scheme for arid and semi-arid regions that balanced the trade-offs between the sub-areas and water use participants, ensured sustainable regional system development, and guaranteed robust solutions for future uncertainties. A bi-level affinely adjustable robust counterpart (AARC) programming framework was developed, in which the regional authority as the leader allocates water to the sub-areas to maximize the intra- and intergenerational equity, and the sub-areas as the followers allocate water to their respective water departments to maximize their economic benefits and minimize water shortages. This method used affine functions between the decision variables (water allocation amount) and the uncertain parameters (WA, WD) to deal with the computationally intractable (NP-hard) robust counterpart for the multi-period water resources management. To illustrate the applicability and

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feasibility of this framework, a case study from Neijiang, China, is given. This model can assist regional authorities develop more robust water resource allocation solutions for multi-period planning responses to uncertain water deficiencies.

Keywords Bi-level multi-period water allocation • Adjustable robust optimization

• Drought area • Deep uncertainty

1 Introduction

Optimal regional water resource allocation has been found to be an effective approach to mitigating regional water scarcity (Dai et al. 2018), alleviating the contradictions between limited fresh clean water resources and growing water demand (Tan et al. 2010), and ensuring sustainable water resources management (Chen et al. 2013). Consequently, there has been increased research into effective water resources allocation, especially in arid and semi-arid areas.

However, effective water resources planning is always clouded by “cascade” of substantial deep uncertainties indeed (Li et al. 2020), which are introduced by a portfolio of endogenous and exogenous elements, such as sampling uncertainty, human behaviors, environmental process and hydroclimate change (Herman et al. 2020), with these uncertainties being a greater challenge in regions more prone to drought (Trenberth et al. 2014). Deep uncertainties in water availability (WA) and water demand (WD) affected by environmental or anthropogenic interferences (Li et al. 2020;

Rathnayaka et al. 2017; Droogers et al. 2012) have posed great challenges for long-term drought emergency problems (Kapelan et al. 2017). More recently, several studies sought to determine the uncertain water supply and demand parameters in water management models (Liu et al. 2014; Perelman et al. 2013; Jia et al. 2006), and especially in the municipal water supply system (Housh et al. 2011; Chung et al. 2009).

Various techniques have been used in previous studies to deal with these uncertainties, such as stochastic programming (Sankarasubramanian et al. 2009), fuzzy logic (Fu et al. 2011), robust optimization (Perelman et al. 2013). The first two approaches require that uncertain parameters be resolved using perfectly known probability distribution functions or membership functions (Perelman et al. 2013), meaning that the optimal results are distributed around a “best-guess” (Maier et al. 2016). However, the future WA and WD uncertainties from climate change and socio-economic development, which are confronted with distinct future states that may occur (Kwakkel et al. 2010), have no consensus on the potential probability distribution (Trindade et al. 2017). Therefore, methods that set the distribution or membership functions in advance are not appropriate for this situation, which imposes the significance of carrying out “best policy” that can be immune to data change and guarantee it’s feasible and near optimal results, notably robust optimization (RO) (Bertsimas et al. 2004).

It is noted that pervious WA and WD data are easily available and can provide some reference for future uncertain processing using the Adjustable Robust Counterpart (ARC), which divides the decisions into “here and now” which has nonadjustable

features and “wait and see”, the decisions for which can be made after partial uncertain data known (Ben-Tal et al. 2004). Zhen et al. (2018) proposed Affinely Adjustable Robust Counterpart (AARC) method with the later-stage decisions being formulated as affine functions of the observed uncertainties to deal with computational intractability. The solutions using AARC are more realistic and significantly and less conservatively reduce the losses from the uncertainties, which are proved to be low “price of robustness” and little worse than corresponding “ideal” results (Ben-Tal et al. 2004; 2005; Kim et al. 2017).

Moreover, optimal water resources allocation is an extraordinary sophisticated system, especially when it comes to contradictory situations between water supply and demand because there is a hierarchical decision and the need to consider sustainable development, that is, both present and future generations need to be considered. Because global rather than individual optimization is sought, the hierarchical decision structure needs to consider the conflicts between the upper and lower level. Specifically, regional authority as the leader seeks stability and system development with the water users as the followers more concerned with their own optimality. Yao et al. (2019) noted optimum water allocation policies must be sustainable in both the present and the future (Xu et al. 2019); therefore, it is essential to conceptualize water resources allocation using a hierarchical, multi-period framework under deep uncertainty. However, the complexities and uncertainties associated with the hierarchical resource allocation process and the need to ensure sustainability have not always been fully considered when seeking to develop optimal regional water rights allocation schemes

in drought-prone regions.

Consequently, this study proposes a robust bi-level, multi-stage optimization administrative allocation policy in water deficient regions under uncertainty. For sustainable water resources planning, maximizing intra- and inter-generational equity is the system objective for the leader; the followers (water department) seek for attaining greater economic benefits and smaller water shortages. The hierarchical mathematical formula is established to balance the contradictions between the system and its sub-areas, so that the decision maker can consider not only the fairness of the system, but also the goals of the subsystems. In addition, priority rules have been developed to manage the water department conflicts in drought-prone region, and the goal programming method is used to reflect the priority of water use by various departments. AARC method is used to ensure feasible solutions for water resources planning of deficient region under deep uncertainty. The research objectives for this paper are: (i) to conceptualize a bi-level, multi-stage, optimal water resources allocation framework for arid and semi-arid regions; (ii) to construct an adjustable robust optimization management model using affine decision rules that considers the water deficiency uncertainties (WA/WD); and (iii) to apply the bi-level, multi-stage AARC optimization approach to a case study in a drought-prone region of China.

The remainder of this paper is organized as follows. Section 2 gives the problem statement, Section 3 details the methodology, Section 4 gives the problem solutions, Section 5 demonstrates the model in a case study, and Section 6 gives the conclusions.

2 Problem statement

2.1 Conceptual framework

This paper designs a sustainable regional water resources allocation management program for domestic, industrial, and agriculture water users in drought-prone regions. The water allocation is characterized by a hierarchal structure that includes a dynamic drought decision process. Therefore, a conceptual bi-level regional water allocation framework was constructed, with the regional authority in the upper level, the focus of which is on the ecological environment and intra/intergenerational equity maximization, and the sub-areas in the lower level, which have income maximization and water shortage minimization as their objective functions. When managing arid and semiarid regions, the critical challenge is to balance the trade-offs between the competing users, and especially between the irrigation users and other users (Housh et al. 2011). In the lower-level water allocation planning, allocation principles are established to comply with the administrative measures after meet the minimum ecological water use requirements. First, the normal domestic demand is met, after which industry water is allocated to ensure a certain economic development level, and finally, the agricultural water is allocated. As irrigation generally takes up the largest water allocation, by restricting agricultural water quantities, farmers would be motivated to improve their technologies and save water. In addition, water planners need to consider both present and future water management to achieve sustainable development.

Due to varying rainfall and possible climate change effects, WA and WD data are usually inexact, vague, and constantly changing. However, these often-ignored

uncertainties can have a significant impact on water planning and water resource management, especially in drought-prone areas. Although determining accurate WA and WD probability distributions or membership functions for uncertain factors is difficult, previous data can be a valuable reference. Therefore, with a specific focus on water deficient regions, this paper employed an AARC method to deal with the WA and WD uncertainties, used affine decision rules associated with the previous values, and developed a bi-level multi-stage water allocation framework that considered the WA and WD uncertainties. Figure 1 gives the flowchart for the proposed framework that shows the hierarchal structure, the allocation rule and the uncertain environment.

Fig. 1 Water allocation decision making framework for drought-prone regions

2.2 Adjustable robust counterpart (ARC)

Robust Optimization is immune to data change, guarantees a feasible, near optimal solution, and does not need to master the true function information.

Robust Optimization (RO). The classical RO for uncertain linear programming

problems is defined as

$$\min_x \left\{ \sup_{\xi=[a,b,c] \in Z} (c^\top x) : Ax \leq b, \forall \xi \equiv [a,b,c] \in Z \right\} \quad (1)$$

where the random variables a, b, c are varying in the uncertainty set Z .

Ben-Tal et al. (2004) introduced Adjustable Robust Counterpart (ARC) methodology, which was found to yield less conservative decisions than classical robust optimization (Zhen et al. 2018) as it encompassed both “here and now” decisions (that cannot be adjusted to the data) and “wait and see” decisions (that can be adjusted after uncertain data has been disclosed). Therefore, each decision variable x is made up both a non-adjustable (“here and now”) and an adjustable (“wait and see”) variables, that is $x = (u^T, v^T)^T$, where the sub-vector u indicates the non-adjustable variable and v indicates the adjustable variable.

Adjustable Robust Counterpart (ARC). The ARC equation distinguishes the non-adjustable and adjustable variables and then normalizes the objective function as follows, _

$$\left\{ \min_{u,v} c^\top u : Uu + Vv \leq b, \right\}_{\xi=[U,V,b] \in Z} \quad (2)$$

When V is a certain constant matrix, it is recognized as a fixed recourse that corresponds to a “two-stage stochastic programming problem”, in which u is the first-stage (“here and now”) decision and v is the second-stage (“wait and see”) decision. This uncertain linear optimization problem which will be addressed next, can be rewritten as _

$$\left\{ \min_{u,v} c^\top u : Uu + Vv \leq b \right\}_{\xi=[U,b] \in Z} \quad (3)$$

Definition. The Adjustable Robust Counterpart (ARC) for the uncertain linear programming problems can be defined as

$$(\mathbf{ARO}): \min_u \left\{ c^\top u : \forall (\xi = [U, V, b] \in Z), \exists v : Uu + Vv \leq b \right\} \quad (4)$$

and the classical Robust Optimization (RO) is :

$$(\mathbf{RO}): \min_u \left\{ c^\top u : \exists v, \forall (\xi = [U, V, b] \in Z) : Uu + Vv \leq b \right\} \quad (5)$$

Equations (14-15) demonstrate that the ARC is more flexible than the RO and is able to obtain better optimal values.

Affinely Adjustable Robust Counterpart (AARC). The AARC (Ben-Tal et al. 2004) is then introduced to deal with the computational intractability of the ARC, which is an NP-hard problem even in fixed resource cases, as the linear inequality constraints between the adjustable decisions and random variables can be limited appropriately using affine functions. As in the fixed resource case, the non-variable u can be assumed as given, and U, v, b adopt common assumptions by affinely depending on uncertainty set Z through parameterization mapping $[U, v, b] = [U^0, v^0, b^0] + \sum_{\kappa} \xi_{\kappa} [U^{\kappa}, v^{\kappa}, u^{\kappa}]$. The AARC formulation is:

$$\min_{u, v^0, v^1, \dots, v^{\kappa}} \left\{ c^\top u : [U^0 + \sum \xi_{\kappa} U^{\kappa}]u + V[v^0 + \sum \xi_{\kappa} v^{\kappa}] \leq [b^0 + \sum \xi_{\kappa} b^{\kappa}], \forall \xi \in Z \right\} \quad (6)$$

3 Model Formulation

As shown in Fig. 1, a specific water allocation system for sustainable development in drought-prone regions under uncertainty is considered in this paper. This section details the mathematical formulations for the bi-level multi-stage framework with uncertainty.

The parameters and decision variables are displayed in Table 1.

Table 1 Notations

Index	
i	water subarea $i, i = 1, 2, \dots, I$
j	water department $j, j = 1, 2, \dots, J$
t	stage for the water resources planning and management $t, t = 1, 2, \dots, T$
Certain parameters	
$e_{it}^{\min}$	minimum ecological water demand in subarea i at period t
b_{ijt}	benefit parameter per unit for water department j in subarea i at period i
$L_{ijt}^{\min}$	minimum water demand of water department j in subarea i at period i
$L_{ijt}^{\max}$	maximum water demand of water department j in subarea i at period i
$V_{\min}$	minimum water capacity of stored water
$V_{\max}$	maximum water capacity of stored water
Uncertain Parameters	
WA_t	regional available water at period t
WD_{ijt}	water demand for water department j in subarea i at period i
State Variables	
$V(0)$	regional initial stored water at the beginning
$V(t)$	regional stored water at period t at the beginning
Decision variables	
x_{it}	water allocated to subarea i at period t
y_{ijt}	water allocated to water department j in subarea i at period i
e_{it}	water allocated for ecological use in subarea i at period t

3.1 Upper-level regional water allocation decision process

3.1.1 Upper-level objective

In areas where there is a sharp contradiction between water supply and demand, the difference and irrationality of resource allocations are more likely to trigger conflicts.

To reduce regional imbalances and promote coordinated development in the whole basin, it is necessary to guarantee water resource allocation equity. The Gini coefficient, which is an effective tool for measuring impartiality, has been widely used to weight water management inequalities (Xu et al. 2019); the larger the Gini coefficient, the more

unfair the resource allocation, and if the Gini coefficient is 0, absolute fairness has been achieved.

When determining regional water allocations in drought-prone areas, water authority need to ensure greater socio-economic benefits and adequate water resources to maintain production and residential water security. Therefore, to measure the resource allocation rationality and fairness, a comprehensive Gini coefficient system is proposed that considers living standards and economic development level, population and Gross Domestic Product (GDP) in each region as the criteria. These criteria were chosen for the following reasons: (1) as water resource consumption is positively proportional to the population, population growth inevitably leads to increased water use, and as production and daily life water must be guaranteed, population levels are an appropriate index for balanced resource allocation; (2) socio-economic development cannot be separated from the water resource support that is, no industry can survive without water resources, and the better the development, the greater the demand for water. Therefore, sub-areas' population and GDP were selected as indicators for the fairness assessment.

Moreover, for sustainability, equity must consider both spatial and temporal dimensions and focus on both individual and intergenerational issues (Kverndokk et al. 2014). The spatial equity considerations need to balance the solutions in the different regions for the current generation and the temporal equity considerations need to maintain equilibrium between the current and future generations (different periods).

The equity functions are:

Intra-generational equity:

$$\min F_{\text{intra-equity}} = 0.5 \frac{1}{T} \sum_t \sum_{k=1}^K w_k \frac{1}{2n \sum_{i=1}^n \frac{x_{it}}{h_{kii}}} \sum_{u=1}^I \sum_{v=1}^I \left| \frac{x_{ut}}{h_{kut}} - \frac{x_{vt}}{h_{kvt}} \right| \quad (7)$$

Inter-generational equity:

$$\min F_{\text{inter-equity}} = \frac{1}{I} \sum_i \sum_{k=1}^K w_k \frac{1}{2T \sum_{t=1}^T \frac{x_{it}}{h_{kit}}} \sum_{l=1}^T \sum_{q=1}^T \left| \frac{x_{il}}{h_{kil}} - \frac{x_{iq}}{h_{kiq}} \right| \quad (8)$$

Thus, the Gini coefficient minimization function F_{equity} is described as follows:

$$\min F = \omega \frac{1}{T} \frac{1}{K} \sum_t \sum_{k=1}^K \frac{1}{2n \sum_{i=1}^n \frac{x_{it}}{h_{kii}}} \sum_{u=1}^I \sum_{v=1}^I \left| \frac{x_{ut}}{h_{kut}} - \frac{x_{vt}}{h_{kvt}} \right| + \theta \frac{1}{I} \frac{1}{K} \sum_i \sum_{k=1}^K \frac{1}{2T \sum_{t=1}^T \frac{x_{it}}{h_{kit}}} \sum_{l=1}^T \sum_{q=1}^T \left| \frac{x_{il}}{h_{kil}} - \frac{x_{iq}}{h_{kiq}} \right| \quad (9)$$

where u and v are watershed areas $u, v \in i (i=1, 2, \dots, I)$, l and q are the periods $l, q \in t (t=1, 2, \dots, T)$, and h_{ki} is the Gini coefficient assessment.

3.1.2 Upper-level constraints

State transit of stored water: In arid and semiarid areas, the reservoir water storage in flood periods plays a significant role in providing both spatial and temporal dry season water resources. Therefore, there is a certain functional relationship between the state of the storage water variables and decision-making, that is, the stored water $V(t)$ at the end of period t can be determined from the water storage in the $t-1$ period $V(t-1)$, the water availability WA_t and the allocated water $\sum_i x_{it}$ in period t , as follows:

$$V(t) = V(t-1) + WA_t - \sum_i x_{it}, \forall t \quad (10)$$

The stored water reservoir capacity is limited within a certain range $[V_{\min}, V_{\max}]$,

with its function being to meet several needs, such as power generation, flood control, and the fishing industry. The water storage constraint is:

$$V_{\min} \leq V(t) \leq V_{\max}, \forall t \quad (11)$$

which can be rewritten with Equation 10 as follows:

$$V_{\min} \leq V(t) = V(t-1) + \tilde{W}A_t - \sum_i x_{it} \leq V_{\max}, \forall t \quad (12)$$

where the initial parameter $V(0)$ is determined from the relevant report.

Ecological water requirement constraint: Arid and semi-arid areas have fragile ecological environments and are vulnerable to eco-environmental damage as they have poor resilience and are sensitive to changes in external conditions. Therefore, the ecological water supply is crucial to the healthy maintenance of aquatic organisms and plants. To meet this sustainable development requirement, the water allocated to ecosystem e_{it} care must be guaranteed that meets the minimum ecological water demand $e_{it}^{\min}$, as shown in the following constraint:

$$e_{it} \geq e_{it}^{\min}, \forall i, t \quad (13)$$

Water availability constraint for sub-area allocation: The volume of water distributed from region to sub-areas, including ecological water e_{it} , and domestic, industrial, agricultural water $\sum_j y_{ijt}$, cannot exceed the regional water supply:

$$\sum_j y_{ijt} + e_{it} \leq x_{it}, \forall i, t \quad (14)$$

Non-negative constraint: The water withdrawal cannot be negative; therefore, there is the following constraint on the decision variables:

$$x_{it} > 0 \quad (15)$$

3.2 Lower-level sub-area water allocation decision process

3.2.1 Lower-level objective

In the lower level, subarea managers focus more on their own respective benefits and development, instead of taking a panoramic view of the system with the leader with economic benefit maximization $\sum_j \sum_t b_{ijt} y_{ijt}$ being the primary goal. Each subarea provides water to three water users (domestic, industrial, and agricultural) where b_{ijt} is the economic profit parameter per unit of used water (CNY / m^3). The benefit maximization is therefore expressed as:

The objective of maximize economic benefit:

$$\max f_i^{be} = \sum_j \sum_t b_{ijt} y_{ijt} \quad (16)$$

In arid and semi-arid regions and especially when there is extreme water insufficiency, each sub-area manager seeks to reduce their water deficiencies. Thus, water shortage minimization between water demand $\sum_i \sum_j \sum_t \bar{d}_{ijt}$ and water supply $\sum_i \sum_j \sum_t y_{ijt}$ is the second lower-level objective, which is calculated by the following equation:

The objective of minimize water deficiency:

$$\min f_i^{sh} = \sum_i \sum_j \sum_t (\bar{w}D_{ijt} - y_{ijt}) \quad (17)$$

3.2.2 Lower-level constraints

Water allocation constraint for each sub-area: To ensure no waste or inefficiencies, the allocated water y_{ijt} for each water user is constrained within a certain range, that is, between the minimum water requirements $L_{ijt}^{\min}$ to be satisfied and the maximum withdrawal targets $L_{ijt}^{\max}$, as follows:

$$L_{ijt}^{\min} \leq y_{ijt} \leq L_{ijt}^{\max}, \forall i, j, t \quad (18)$$

Non-negative constraint: As the distributed water cannot be negative, there is the following constraint on the decision variables:

$$y_{ijt} > 0, \forall i, j, t \quad (19)$$

3.3 Allocation principles

When there is extreme water insufficiency, water distribution among the departments has become the key and difficult problem. The managers of sub-areas are responsible for making an optimal strategy for maintaining social life and development. As the sub-area managers are responsible for developing optimal strategies to ensure the maintenance of socio-economic development, to deal with the conflicts quickly and efficiently and obtain appropriate solutions, certain water resource allocation priority principles were established as lower-level management tools for drought-prone regions based on China's specific administrative policies. First, to ensure production and residential water in water shortage situations, a certain reservoir inventory is needed, as shown in Eq. 12. Second, when the ecological environmental water demand is satisfied ((Eq. 13) of the upper constraints), the domestic water demand should be satisfied first, followed by the industrial water demand, with the residual amount being distributed to the agricultural sector. As explain in Hatem et al. (2020), water scarcity can be mitigated using rational spatial cropping patterns, that is, less allocated water should motivate farmers to adopt more effective irrigation techniques and strategies (Chukalla et al. 2015). A goal preference programming method is therefore utilized to realize the

lower-level distribution principles, with the lower-level objectives and constraints (Eq. 16-19) being represented using goal programming:

$$\begin{aligned} \min_{y_{ijt}} \quad & f_i = P_0 df_i^- + \sum_{jt} P_1 d_{1jt}^- + P_2 (d_{2jt}^- + d_{2jt}^+) + P_3 d_{3jt}^+ \\ \text{s.t.} \quad & \begin{cases} f_i^{be} - \sum_j \sum_t b_{ijt} y_{ijt} + df_i^- - df_i^+ = f_{i\max}^{be}, \forall i \\ y_{ijt} + d_{ijt}^- - d_{ijt}^+ = \tilde{W}D_{ijt}, \forall i, j, t \\ L_{ijt}^{\min} \leq y_{ijt} \leq L_{ijt}^{\max}, \forall i, j, t \\ y_{ijt} > 0, \forall i, j, t \end{cases} \end{aligned} \quad (20)$$

where P_0 represents the preferences for the first objective f_i^{be} , P_1, P_2, P_3 represent the water user preferences in the second objective f_i^{sh} , $f_{i\max}^{be}$ refers to the ideal values, $df_i^-, df_i^+, d_{ijt}^-, d_{ijt}^+$ refers to the slack variables. $P_1 > P_2 > P_3$ is representative for the priority, $\min P_1 d_{1jt}^-$ indicates the needs are met and the shortage of domestic water is minimized, $P_2 (d_{2jt}^- + d_{2jt}^+)$ indicates that the industrial water requirement is not exceed, and $P_3 d_{3jt}^+$ means that the agricultural one does not exceed the demand.

3.4 Global optimization model

In the bi-level multi-period framework, the regional authority as the leader considers the overall socio-economic development when allocating the water to the subareas, with the regional objective function (Eq. 9) being to maximize the intra- and intergenerational equity under the respective constraints (Eq. 11-15). Water deficiencies increase environmental protection tension and ecological environmental issues, which means the upper level must guarantee the minimum ecological water before the lower-level allocations. Each sub-area follower then makes water withdrawal decisions based on the economic benefit maximization and the water deficiency

minimization objective functions and constraints, with the priority allocations being given to the domestic sector first, followed by the industrial and agricultural sectors (Eq. 20). To deal with the contradictions between the leader and the followers, a bi-level method with interactive relationships is proposed for the sustainable water resources allocation system in water deficient regions with uncertainty, as follows:

$$\begin{aligned}
 \min F = & \omega \frac{1}{T} \frac{1}{K} \sum_t \sum_{k=1}^K \frac{1}{2n \sum_{i=1}^n \frac{x_{it}}{h_{kii}}} \sum_{u=1}^I \sum_{v=1}^I \left| \frac{x_{ut}}{h_{kut}} - \frac{x_{vt}}{h_{kvt}} \right| + \theta \frac{1}{I} \frac{1}{K} \sum_i \sum_{k=1}^K \frac{1}{2T \sum_{t=1}^T \frac{x_{it}}{h_{kit}}} \sum_{l=1}^T \sum_{q=1}^T \left| \frac{x_{il}}{h_{kil}} - \frac{x_{iq}}{h_{kiiq}} \right| \\
 s.t. \left\{ \begin{aligned}
 & V(t) = V(t-1) + \bar{W}A_t - \sum_i x_{it}, \forall t \\
 & V_{\min} \leq V(t) \leq V_{\max}, \forall t \\
 & \sum_j y_{ijt} + e_{it} = x_{it}, \forall i, t \\
 & e_{it} \geq e_{it}^{\min}, \forall i, t \\
 & \min_{y_{ijt}} f_i = P_0 df_i^- + \sum_{jt} P_1 d_{1jt}^- + P_2 (d_{2jt}^- + d_{2jt}^+) + P_3 d_{3jt}^+ \\
 & \left\{ \begin{aligned}
 & f_i^{be} - \sum_j \sum_t b_{ijt} y_{ijt} + df_i^- - df_i^+ = f_{i\max}^{be}, \forall i \\
 & y_{ijt} + d_{ijt}^- - d_{ijt}^+ = \bar{W}D_{ijt}, \forall i, j, t \\
 & L_{ijt}^{\min} \leq y_{ijt} \leq L_{ijt}^{\max}, \forall i, j, t \\
 & y_{ijt} > 0, \forall i, j, t
 \end{aligned} \right. \\
 & x_{it} > 0, e_{it}^{\min}, y_{ijt} > 0, \forall i, j, t
 \end{aligned} \right.
 \end{aligned}
 \tag{21}$$

4 Problem solutions

Bilevel optimization model is known to be a typical non-deterministic polynomial complete problem that is surprisingly difficult to implement directly. Extensive

methodology, including game theory, descent method, penalty function (Aiyoshi et al. 1984), evolutionary algorithm, single-level reduction, Karush-Kuhn-Tucker (KKT) conditions, are turned out to be effective solution for the complex bilevel programming problem. KKT conditions is a sophisticated way to deal with hierarchical structures with multiple independent followers through replacing the lower level optimization problem with equivalent constraints. Hence, the equivalent single-level programming model of Eq.(13) through KKT method, as demonstrated in Eq.(14).

At the beginning of each year, the water authority needs to allocate the water to the different areas and sectors to meet the random demand $\tilde{W}D_{ijt}$, the value for which is determined from the uncertain set Z . Although there is no information regarding the demand $\tilde{W}D_{ijt}$ in period t , the actual values from years $\{1, 2, \dots, t-1\}$ can be easily acquired, with y_{ijt1} being the “here and now” decisions, and y_{ijt} being the “wait and see” decisions based on the information from the previous periods $l \in I_t = \{1, 2, \dots, t-1\}$. The AARC methodology assumes that the adjustable variable y_{ijt} is an affine function of $\tilde{W}D_{ij}^r : y_{ijt} = \pi_{ijt}^0 + \sum_{r \in I_t} \pi_{ijt}^r \tilde{W}D_{ij}^r$, with the i_{th} water quantity allocated x_{it} being a non-exact value that depends on data water availability $\tilde{W}A^r : r \in I_t$. x_{it} is then mapped from the vector $\tilde{W}A^r$ with the affine decision rules as follows: $x_{it} = \eta_{it}^0 + \sum_{r \in I_t} \eta_{it}^r \tilde{W}A^r$, with the coefficients π_{ijt}^r and η_{it}^r being the new nonadjustable variables.

The uncertainties can then be converted to the following uncertain programming problem using the AARC approach:

$\min \chi$

$$\begin{aligned}
& 0.5F_{\text{intra-equity}} + 0.5F_{\text{inter-equity}} \leq \chi \\
& F_{\text{intra-equity}} = \frac{1}{T} \frac{1}{K} \sum_t \sum_{k=1}^K \left| \frac{\sum_{u=1}^n \sum_{v=1}^n \left(\frac{\eta_{it}^0 + \sum_{r=1}^t \eta_{ut}^r \mathbf{wA}_r}{h_{kut}} - \frac{\eta_{it}^0 + \sum_{r=1}^t \eta_{vt}^r \mathbf{wA}_r}{h_{kvt}} \right)}{2n \sum_{i=1}^n \frac{\eta_{it}^0 + \sum_{r=1}^t \eta_{it}^r \mathbf{wA}_r}{h_{kit}}} \right| \\
& \min F_{\text{inter-equity}} = \frac{1}{I} \frac{1}{K} \sum_t \sum_{k=1}^K \left| \frac{\sum_{l=1}^T \sum_{q=1}^T \left(\frac{\eta_{il}^0 + \sum_{r=1}^l \eta_{il}^r \mathbf{wA}_r}{h_{kil}} - \frac{\eta_{iq}^0 + \sum_{r=1}^q \eta_{iq}^r \mathbf{wA}_r}{h_{k iq}} \right)}{2T \sum_{t=1}^T \frac{\eta_{it}^0 + \sum_{r=1}^t \eta_{it}^r \mathbf{wA}_r}{h_{kit}}} \right| \\
& V_{\min} \leq V(0) - \sum_{s=1}^t \sum_i \eta_{is}^0 + \sum_{r=1}^t v_t^r \mathbf{wA}_r \leq V_{\max}, \forall t \\
& \left\{ \begin{aligned} & \sum_j (\pi_{ijt}^0 + \sum_{r \in I_t} \pi_{ijt}^r \mathbf{wD}_{ijr}) + \mathbf{e}_{it} \leq \eta_{it}^0 + \sum_{r=1}^t \eta_{it}^r \mathbf{wA}_r, \forall i, t \\ & \mathbf{e}_{it} \geq \mathbf{e}_{it}^{\min}, \forall i, t \end{aligned} \right. \quad (22) \\
& \min \gamma \\
& \left\{ \begin{aligned} & P_0 df_i^- + \sum_{jt} P_1 d_{1jt}^- + P_2 (d_{2jt}^- + d_{2jt}^+) + P_3 d_{3jt}^+ \leq \gamma, \forall i \\ & \sum_j \sum_t b_{ijt} (\pi_{ijt}^0 + \sum_{r \in I_t} \pi_{ijt}^r \mathbf{wD}_{ijr}) + df_i^- - df_i^+ = f_{i\max}^{be}, \forall i \\ & \pi_{ijt}^0 + \sum_{r \in I_t} \pi_{ijt}^r \mathbf{wD}_{ijr} + d_{ijt}^+ - d_{ijt}^- = \mathbf{wD}_{ijr}, \forall i, j, t \\ & L_{ijt}^{\min} \leq \pi_{ijt}^0 + \sum_{r \in I_t} \pi_{ijt}^r \mathbf{wD}_{ijr} \leq L_{ijt}^{\max}, \forall i, j, t \\ & P_0 = P_1 > P_2 > P_3 \end{aligned} \right. \\
& \eta_{it}^0 + \sum_{r=1}^t \eta_{it}^r \mathbf{wA}_r > 0, \pi_{ijt}^0 + \sum_{r \in I_t} \pi_{ijt}^r \mathbf{wD}_{ijr} > 0
\end{aligned}$$

Water availability $\bar{WA}_t$ in the uncertainty set is based on the given parameter perturbations α and the nominal supply $WA_t^* : \bar{WA}_t \in [WA_t^* - \sigma WA_t^*, WA_t^* + \sigma WA_t^*]$, and the future demand $\bar{WD}_{ijt}$ in the uncertainty set is based on the given parameter perturbations θ and the nominal demand $WD_{ijt}^* : \bar{WD}_{ijt} \in [WD_{ijt}^* - \theta WD_{ijt}^*, WD_{ijt}^* + \theta WD_{ijt}^*]$. Therefore, from the uncertainty set and the equivalence proposed by Ben-Tal et al. (2005), the uncertain AARC problem can be straightforwardly converted to a computationally tractable equivalent linear programming equation, as follows:

$\min \chi$

$$\begin{aligned}
& \omega F_{\text{intra-equity}} + \theta F_{\text{inter-equity}} \leq \chi \\
& F_{\text{intra-equity}} = \frac{1}{T} \frac{1}{K} \sum_t \sum_{k=1}^K \frac{\sum_{u=1}^n \sum_{v=1}^n \left| \frac{\eta_{it}^0 + \sum_{r=1}^t \eta_{it}^r \text{WA}_r^* - \alpha \sum_{r=1}^t \zeta_{it}^r \text{WA}_r^*}{h_{kut}} - \frac{\eta_{it}^0 + \sum_{r=1}^t \eta_{it}^r \text{WA}_r^* - \alpha \sum_{r=1}^t \zeta_{it}^r \text{WA}_r^*}{h_{kvt}} \right|}{2n \sum_{i=1}^n \frac{\eta_{it}^0 + \sum_{r=1}^t \eta_{it}^r \text{WA}_r^* - \alpha \sum_{r=1}^t \zeta_{it}^r \text{WA}_r^*}{h_{kit}}} \\
& \min F_{\text{inter-equity}} = \frac{1}{I} \frac{1}{K} \sum_t \sum_{k=1}^K \frac{\sum_{l=1}^T \sum_{q=1}^T \left| \frac{\eta_{il}^0 + \sum_{r=1}^l \eta_{il}^r \text{WA}_r^* - \alpha \sum_{r=1}^l \zeta_{il}^r \text{WA}_r^*}{h_{kil}} - \frac{\eta_{iq}^0 + \sum_{r=1}^q \eta_{iq}^r \text{WA}_r^* - \alpha \sum_{r=1}^q \zeta_{iq}^r \text{WA}_r^*}{h_{kiq}} \right|}{2T \sum_{t=1}^T \frac{\eta_{it}^0 + \sum_{r=1}^t \eta_{it}^r \text{WA}_r^* - \alpha \sum_{r=1}^t \zeta_{it}^r \text{WA}_r^*}{h_{kit}}} \\
& -\zeta_{it}^r \leq \eta_{it}^r \leq \zeta_{it}^r \\
& V(0) - \sum_{s=1}^t \sum_i \eta_{is}^0 + \sum_{r=1}^t v_t^r \text{WA}_r^* - \alpha \sum_{r=1}^t \mu_t^r \text{WA}_r^* \leq V_{\max}, \forall t \\
& V(0) - \sum_{s=1}^t \sum_i \eta_{is}^0 + \sum_{r=1}^{t-1} v_t^r \text{WA}_r^* + \alpha \sum_{r=1}^t \mu_t^r \text{WA}_r^* \geq V_{\min}, \forall t \\
& 1 - \sum_i \sum_{s \leq t, r \in I_s} \eta_{is}^r = v_t^r, -\mu_t^r \leq v_t^r \leq \mu_t^r \\
& \sum_j (\pi_{ijt}^0 + \sum_{r \in I_t} \pi_{ijt}^r \text{WD}_{ijr}^* + \beta \sum_{r \in I_t} g_{ijt}^r \text{WD}_{ijr}^*) + e_{it} \leq \eta_{it}^0 + \sum_{r=1}^t \eta_{it}^r \text{WA}_r^* - \alpha \sum_{r=1}^t \zeta_{it}^r \text{WA}_r^*, \forall i, t \\
& e_{it} \geq e_{it}^{\min}, \forall i, t \\
& \min \gamma \\
& \left\{ \begin{aligned} & P_0 df_i^- + \sum_{jt} P_1 d_{1jt}^- + P_2 (d_{2jt}^- + d_{2jt}^+) + P_3 d_{3jt}^+ \leq \gamma, \forall i \\ & \sum_j \sum_t b_{ijt} (\pi_{ijt}^0 + \sum_{r \in I_t} \pi_{ijt}^r \text{WD}_{ijr}^* - \beta \sum_{r \in I_t} g_{ijt}^r \text{WD}_{ijr}^*) + df_i^- - df_i^+ = f_{i\max}^{be}, -g_{ijt}^r \leq \pi_{ijt}^r \leq g_{ijt}^r, \forall i \\ & \pi_{ijt}^0 + \sum_{r \in I_t} \pi_{ijt}^r \text{WD}_{ijr}^* - \beta \sum_{r \in I_t} g_{ijt}^r \text{WD}_{ijr}^* + d_{ijt}^+ - d_{ijt}^- = (1 + \beta) \text{WD}_{ijt}^*, \forall i, j, t \\ & \pi_{ijt}^0 + \sum_{r \in I_t} \pi_{ijt}^r \text{WD}_{ijr}^* - \beta \sum_{r \in I_t} g_{ijt}^r \text{WD}_{ijr}^* \geq L_{ijt}^{\min}, \forall i, j, t \\ & \pi_{ijt}^0 + \sum_{r \in I_t} \pi_{ijt}^r \text{WD}_{ijr}^* + \beta \sum_{r \in I_t} g_{ijt}^r \text{WD}_{ijr}^* \leq L_{ijt}^{\max}, \forall i, j, t \\ & P_0 = P_1 > P_2 > P_3 \end{aligned} \right. \\
& \eta_{it}^0 + \sum_{r=1}^t \eta_{it}^r \text{WA}_r^* - \alpha \sum_{r=1}^t \zeta_{it}^r \text{WA}_r^* > 0, \pi_{ijt}^0 + \sum_{r \in I_t} \pi_{ijt}^r \text{WD}_{ijr}^* - \beta \sum_{r \in I_t} g_{ijt}^r \text{WD}_{ijr}^* > 0,
\end{aligned}$$

(23)

5 Case study

To demonstrate the effectiveness of the proposed robust optimization methodology under uncertainty, this section gives a practical application from Neijiang, China.

5.1 Case study system overview

Neijiang, which is a prefecture-level city located on the Tuo River in southeast Sichuan province, China, has a per capita water resource that is usually below $400 \text{ m}^3 / \text{per}$ and a water resource utilization above 50%. The location of Neijiang area is shown in Fig. 2. Although it is surrounded by several river basins, most of the transit flood water is excluded from the territory due to the poor water conservancy storage capacity. Therefore, it has been found to have typical seasonal and year water shortages and is prone to droughts and floods; however, it is more vulnerable to droughts. Neijiang has five subareas: Shizhong district, Dongxing district, Zizhong county, Longchang county, and Weiyuan county, with $i=1, 2, 3, 4,$ and 5 respectively representing each subarea. After meeting the minimum ecological water demand, the water resources are supplied to the three sectors: $j=1$ is the domestic sector, $j=2$ is the industrial sector, and $j=3$ is the agricultural sector.

Fig. 2 Research region-Neijiang location

The data and parameters for this problem were extracted from the 2011 to 2019 Statistical Yearbooks and Water Resources Bulletins from the Regional Bureau of Statistics in Neijiang and expert opinion. Essential information for the determined parameters and expert decisions are shown in Tables 2 and 3. The parameters assumed to be constant $(b_{ijt}, L_{ijt}^{\min}, L_{ijt}^{\max}, WD_{ijt}^*)$ are shown in Table 2, and those assumed to be variable $(e_{it}^{\min}, G_{it}, PO_{it}, WA_t^*)$ are shown in Table 3, in which the nominal water supply gradually reduces to indicate the increasing water stress. To meet the allocation principle requirements, the preference weights (P_j) were diverse, where $P_1 > P_2 > P_3$ illustrated that the water needs of the domestic sector were satisfied first, followed by the industry and agricultural sectors. To prevent any irreversible ecological water shortage damage to the environment, the planning period was set to three years, with the time horizon being $T = 3$ periods and with the predicted values for the next three

years being taken as the model reference. In this case, equal importance was given to the intra- and inter-generational equity.

Table 2 Parameters I of Neijiang City

Parameters	Water user j	Sub-area i				
		$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
$b_{ijt} (CNY / 10^4 m^3)$	$j = 1$	15.83	18.21	22.63	19.28	27.33
	$j = 2$	18.02	23.88	35.09	25.16	33.33
	$j = 3$	11.38	15.85	9.68	7.76	9.97
$L_{ijt}^{\min} (10^4 m^3)$	$j = 1$	2298.89	3464.98	3300.00	2979.62	2692.00
	$j = 2$	5398.00	3228.00	2879.00	5065.87	5363.00
	$j = 3$	3010.00	5643.70	8041.30	6316.12	7957.00
$L_{ijt}^{\max} (10^4 m^3)$	$j = 1$	3796.88	5079.21	6150.83	5827.76	3863.63
	$j = 2$	12619.18	6077.82	6420.41	8899.44	10599.81
	$j = 3$	4863.00	10163.30	20451.00	16200.00	19930.77
$WD_{ijt}^* (10^4 m^3)$	$j = 1$	2975.41	3795.60	5028.46	3191.66	3118.55
	$j = 2$	7492.54	4933.77	4066.89	6712.93	7261.78
	$j = 3$	3775.66	7193.53	14390.27	8941.70	10827.26

Table 3 Parameters II of Neijiang City

Parameters	Water user j	Sub-area i				
		$i = 1$	$i = 2$	$i = 3$	$i = 4$	$i = 5$
$e_{it}^{\min} (10^4 m^3)$	$t = 1$	86.72	70.71	92.96	86.58	112.74
	$t = 2$	92.91	75.47	99.61	94.65	127.14
	$t = 3$	99.10	80.22	106.25	102.71	141.54
G_{it}	$t = 1$	3.020	2.486	2.961	3.070	3.796
	$t = 2$	3.208	2.593	3.114	3.264	4.001

	$t = 3$	3.396	2.700	3.267	3.457	4.206
PO_{it}	$t = 1$	53.0	79.5	114.6	63.0	59.2
	$t = 2$	53.7	78.6	114.1	63.4	58.7
	$t = 3$	53.2	77.0	113.9	64.3	58.5
WA_t^*	$t = 1$			93691.23		
	$t = 2$			83691.23		
	$t = 3$			73691.23		

5.2 Results and discussions

This section comprehensively discusses the calculation results; water allocation strategies, comparative studies, and managerial insights; to provide a valuable reference for arid and semi-arid regional decision makers under uncertain environments.

5.2.1 Regional strategies analysis under several uncertainty levels

Several experiments were conducted under different uncertainty levels (from 0.025 to 0.2) and different uncertainty conditions (only WA or WD were uncertain or both were uncertain), with the optimal objective value for each respectively shown in Fig. 3, 4 and 5, and the water withdrawal solutions illustrated in Fig. 6. Figures 3, 4 and 5 and Table 3 reveal the regional system economic benefits and water shortage equity in the lower-level subareas for the three uncertain scenarios: Scenario 1, in which the water demand is deterministic and the water availability is indeterministic; Scenario 2, in which the water availability is uncertain while the water demand is certain; and Scenario 3, in which both the water availability and demand are uncertain.

Fig.3 Values of system objective functions in Scenario 1

Fig.4 Values of system objective functions in Scenario 2

Fig.5 Values of system objective functions in Scenario 3

Fig.6 Regional water allocation strategies under uncertainties of WA and WD

(1) Equity, benefit, water shortage analysis

The Gini coefficient ranges from 0 to 1 with the smaller the value the greater the equity, with 0 being absolutely fairness. From the equity results shown in Figure 3 under the various uncertainty levels, the minimum was 0.0510, the maximum was 0.0650, an increase of 21%, and the average was 0.0554, that is, all results were between 0 and 0.1, which indicated that the water resources allocations had high equity even though there were fluctuating uncertainty levels, which demonstrated that the AARC

water allocation framework is able to make sound decisions even when there is significant uncertainty. From the regional total economic benefits for the various uncertainty sets shown in Fig. 4, it can be seen that the results were sensitive to fluctuations in WA and WD, with the maximum being 4,733,704 CNY, 653,504 CNY higher than the minimum with a 16% fluctuation. The water shortages were shown to be more vulnerable to indefinite circumstances as they moved from 24,454.12 104m³ to 110,964.6 104m³, which clearly indicated that the uncertainties have a significant impact on water resource management systems and cannot be ignored.

(2) Strategy analysis comparison under uncertainty

The results from the three scenarios indicated that there were large differences in the economic benefit and water shortage targets; however, the equity was similar regardless of the uncertainty. Scenario 2 had the largest economic benefit on average at 4,612,948 CNY, followed by Scenario 1 at 4,225,067 CNY and Scenario 3 at 4,208,298 CNY, which indicated that any variations in water demand has a greater impact on the system than the water supply. The better result in Scenario 2 was mainly because dynamic water resource allocations increase the possibility of greater water provision in the region. The benefit in the Scenario 1 was more likely affected by water demand fluctuations as indicated by the higher standard deviation (105139) than Scenarios 2 (90294) and 3 (76578). Therefore, it was concluded that water demand management is imperative in improving regional development stability. The lowest regional water shortages were found when the water availability was uncertain and the highest regional water shortages were when there were uncertainties in both the water supply and water

demand. When only the water supply was uncertain, the result was significantly lower, indicating that water shortages were more vulnerable to fluctuations in the water demand, as shown by the maximum standard deviation (27579) in Scenario 1. Interestingly, the result was not as volatile in Scenario 3 when there were double uncertainties, indicating that it is possible that the water resources supply interval uncertainty alleviated the impact of the dynamic demand.

5.2.2 Subarea water allocation strategy analysis

As displayed in Table 2, to illustrate a serious water stress situation, the nominal demand WD_{ijt}^* took the average of the previous years and the WA_t^* reduced over time, with an average of $93691.23 \times 10^4 m^3$ in the first period (WA_1^*) and a decrease of $10^4 m^3$ in the following year. In an actual situation, water resource supply and demand are uncertain and decision makers do not have complete information in advance when formulating their plans. Therefore, multi-period sub-area and departmental water allocation strategies under several vital uncertainty levels were examined, as shown in Fig. 6, for which the water shortages were based on the maximum possible water loss.

The total water distribution indicated that Weiyuan was the largest area and had different uncertainty levels in different time periods, while Dongxing has the least amount of water allocated and a difference of about 12000 m³ in each uncertainty level, which was mainly due to its higher GDP and larger water demand. Although Zizhong has the largest population, its GDP and industrial water demand were significantly lower than in Weiyuan County. The comparison of the two sub-areas indicated that the

bi-level model trade-off conflicts between the upper level and the lower level were taking the followers' interests into account. From a water distribution perspective, the system strictly abides by the water shortage policy constraint priority principle from the domestic sector, to the industry sector to the agricultural sector. For example, in the Shizhong sub-area when there is a 10% WA and WD uncertainty level, from the first period when there are sufficient water resources to the second water reduction period, the agricultural water sector allocation decreased from $4153.23 m^3$ to the minimum of $3010.00 m^3$ (by $1143.226 m^3$), while the industrial water consumption reduced by $704 m^3$ and there was little decrease in the domestic sector; however, in the third higher water shortage period, each sector only obtained the minimum water requirements.

The water allocations in the various periods t were somewhat similar, especially in the latter two periods. As shown in Fig. 6, to obtain high intra-generational equity and especially high inter-generational equity, the Gini coefficients were respectively 0.0564, 0.0578, 0.0511, 0.0511 and 0.0535 (all less than 0.1) as the uncertainty grew, which indicated that the water resource distribution has small fluctuations in spatial and temporal heterogeneity when there is increasingly limited water resource availability, which is mainly reservoir storage and the emergency water provision. Therefore, to reduce annual water imbalances and seasonal water, water conservancy projects (e.g., reservoir) are needed to regulate water storage and water deficiencies.

5.2.3 Comparison with deterministic programming

To further demonstrate the superiority and effectiveness of the AARC model's response

to regional water allocations under dynamic water supply and demand uncertainties, it was compared with conventional deterministic programming. The deterministic optimization model was formulated with no uncertainty, that is, $\alpha = 0$ and $\beta = 0$, and with the water availability and water demand taking average values for an ideal situation with complete information. As shown in Table 4, the linear programming determined the allocation strategies to be a 0.5682 intra- and inter-generational equity, 4472632 CNY economic benefit, and a 29678 m³ water shortage.

Compared with the indeterministic model, the proposed model had higher spatial and temporal equity in all scenarios with little “price of robustness”. However, the economic benefit maximization and water shortage minimization goal results were similar to the certainty case and significantly better than the other two scenarios in which there was uncertain demand. Consequently, it was concluded that accurate water demand descriptions are vital for regional water resources planning. Although the water shortages in uncertain environments are greater than in certain environments, the proposed allocation scheme obtained a higher equity and a regional economic benefit close to the certain environment. Therefore, the AARC model, which incorporates uncertainty and robustness, provides more flexible, less conservative solutions, and gives deeper insights in the sub-areas and water user water distribution even in constantly changing uncertain environments, which allows decision makers to adjust their uncertainty coefficients to express varying attitudes and diverse future situations.

5.2.4 Managerial suggestions

In response to water insufficiency and possibly more serious states, it is necessary to develop water resource allocation frameworks that encompass sustainable development, balance supply and demand, and provide efficient utilization, water conservation, and water security for both the short and long term. Water resources management is a sophisticated complex system, with much of the information not accurately known before the decision-making and optimal solutions needing to be made that embrace the uncertainties. Based on these results, several policy implications for water resources management in regional authorities are proposed. First, regional authority managers need to consider the impact of uncertain parameters on their decision-making to account for the lack of advance information, which means that to cope with water deficiencies, they need to establish water resources planning frameworks that enable information fluctuations. The results analysis indicated that the more unstable the water supply security and the greater the contradiction between supply and demand, the greater the system losses. More specifically, uncertain water demand results in greater fluctuations in overall system development, an area that has received little research attention. Second, when there are serious water shortages, the administration can set simple constraints to prioritize the water resources allocation to first ensure sustainable ecosystem development, then respectively meet the domestic, industrial production, and agricultural production requirements. Administrative measures need to be able to directly and quickly respond to emergencies by focusing on contingency plans for all sectors. Finally, reservoirs play an important role in regulating the annual and long-term distribution of water resources, especially when there are long-term water

shortages and seasonal droughts. Governments need to establish water conservancy facilities (reservoirs) to ensure emergency water demand, improve water resources storage efficiencies, and reduce water shortages. However, as conservancy project construction inevitably changes the surrounding environment and ecology, corresponding policies and regulations need further study to identify alternative schemes.

6 conclusions

In this paper, a bi-level robust optimization methodology for regional sustainable water resources management was proposed to balance the trade-off among sub-areas and water-use departments and to provide government and regional authority with a practicable and rational water allocation planning framework under deep uncertainty. In this hierarchal model framework, the upper-level regional authority water allocations to the sub-areas sought to maximize the equity between the current and future generations, whereas the lower-level sub-areas distributed water to their various water sectors to maximize their own benefits and minimize any water shortages. The study closely examined the supply and demand uncertainties and constructed an adjustable robust optimization programming function to provide flexible, less conservative solutions for governments and managers. The affine functions between the water allocated and water availability or demand were defined using linear rules that transformed the bi-level model into a linear programming problem that was computationally tractable. Priority allocation principles were also established on the

lower level using goal programming with preferences to ensure ecological sustainability and to adhere to administrative policies. The model was then applied to Neijiang, a drought prone area in Sichuan Province, China, to demonstrate the applicability and rationality of the optimization method and elucidate sustainable water allocation solutions. An in-depth water deficiency analysis was conducted by varying the uncertain coefficients, which provides regional authorities and sub-area managers with a viable methodology to deal with real-world water supply and demand uncertainty problems.

The major contributions of this work are therefore as follows. (i) A bi-level affinely adjustable robust model was developed that considered the conflicts between the participants. (ii) It was demonstrated that to meet the requirements, future decisions must adjust to the revealed uncertainties from earlier periods. (iii) Strong administrative allocation frameworks and established priority rules are needed in drought-prone areas. Regardless of the positive results, however, there were some research gaps that need to be addressed in future research. For example, the effect of groundwater pumping on surface water flows in future years (Elbakidze et al. 2012; Kuwayama et al. 2013) and the complex and interrelated relationship between water supply uncertainty and water demand uncertainty were not considered.

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Competing interests

The authors declare that they have no conflict of interest.

Availability of data and material

Data and material used in this paper are listed in Tables 2 and 3.

Code availability

There is no code in this paper.

Authors contributions

Yuhong S. led the methodology, formulation solutions and writing of the draft. Liming Y. supported the review and editing of the paper and the development of the methodology.

Ethics approval

Not applicable

Consent to participate

Not applicable

Consent for publication

All authors have duly consented to publish this manuscript.

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Figures

Figure 1

Water allocation decision making framework for drought-prone regions

Figure 2

Research region-Neijiang location Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.

Figure 3

Values of system objective functions in Scenario 1

Figure 4

Values of system objective functions in Scenario 2

Values of objectives in Scenario 3

Figure 5

Values of system objective functions in Scenario 3

Figure 6

Regional water allocation strategies under uncertainties of WA and WD