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A continuous-time stochastic model to study the abandonment strategy of carbon capture and storage project

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We build a continuous-time stochastic real options model to study the abandonment strategy of carbon capture and storage (CCS) project. Based on the stochastic optimal control theory, we solve the problem with the Hamilton-Jacobi-Bellman variational inequality (HJBVI) to derive the evolution of the optimal CCS investment over time. Using optimal stopping time, we establish a free boundary for each time node over the entire CCS construction stage as a function of the market carbon price and the individual project's remaining total deployment investment. The boundary is to help the investors decide whether to keep investing or abandon the project. Numerical simulations based on Chinese data are conducted by applying the finite element method with the power penalty. Concerning a hypothetical CCS project with a remaining total deployment investment of 10 billion RMB, our projected critical carbon prices relevant to its decisions on CCS project in 2020 are, respectively, 137.27 RMB/ton CO₂ (0.123 RMB/kW·h) and 104.14 RMB/ton CO₂ (0.093 RMB/kW·h). Being well below either threshold, if the current price prevails in 2020, the private investors will have no incentive to keep investing in or operate the above CCS project. It seems to us that this should indicate the exact right moment for the government to consider subsidizing them with at least the amount of money to prevent their abandonment of CCS from happening.

Key words: CCS; HJBVI; optimal stopping time; real options; finite element method.

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1. Introduction

Carbon capture and storage (CCS) technology, considered an important option to reduce greenhouse gas (GHG) emissions, has gained much attention in recent years (Macdowell et al. 2010, Boot-Handford et al. 2014, Bui et al. 2018). International Energy Agency (2009) projects that, without CCS, the overall cost of reducing GHG emissions to the 2005 level by 2050 would increase by 70%. CCS has been applied mainly in the power sector. For example, large fixed emitters, such as power stations and large industrial facilities, are well suited for CCS (Labatt and White 2007).

A carbon price is a governmental measure to fix the market failure associated with greenhouse gas emissions, being an externality. If the current and historical carbon prices can help guide the future prices, most CCS projects will continue to remain economically unfeasible (Rammerstorfer and Eisl 2011, Viebahn et al. 2012). In other words, when a carbon price would be insufficient, additional subsidies are required.

In one strand of the literature, the dynamic general equilibrium models of pollution are developed, typically in a deterministic setting. And the papers investigate the steady-state equilibrium of the full-fledged model (Li and Pan 2014, Li 2016). In the other strand, the stochastic models are built while construction/investment of the CCS project, once set the wheels in motion, is assumed to take one period to complete. Under the static setting in this aspect, the papers typically focus on the timing of the launch of CCS investment (Wang and Du 2015). While timing is no doubt an important issue, we argue that, it would be equally, if not more, important to study, in a stochastic environment, the evolution of the optimal decisions on CCS investment and operation over the project's entire lifespan, considering the nature of any CCS project that requires a huge upfront investment cost and a relatively long construction period, along with numerous uncertainties exposed to, including the carbon price, the interest rate, technological upgrades, and policy changes. These decisions on CCS investment and operation, like most other economic decisions, should be made by the forward-looking decision makers after taking into account their expectations of the future. To address exactly these intertemporal issues, we build a continuous-time stochastic optimal control model in which the length of CCS construction – how many periods of time it takes to complete the CCS construction – and the associated construction cost are both treated to be random.

From a technical viewpoint, we apply the stochastic dynamic programming and solve the problem with the Hamilton-Jacobi-Bellman variational inequality (HJBVI). Our result shows that the firm's optimal CCS investment conforms to a bang-bang control strategy at each time node. Moreover, after applying optimal stopping time, we are able to establish a boundary for each time node that is to help the private investors make decisions on CCS investment on one hand and the government make the related subsidy decisions on the other hand. Specifically, the free boundary is a function of two state variables, the current carbon price and the CCS project's remaining total deployment investment. The latter refers to how much more investment needs to be made before the completion of CCS construction.

Based on Chinese data, numerical simulations are conducted by applying the finite element method with the power penalty. Concerning a hypothetical CCS project with a remaining total deployment investment of 10 billion RMB, our projected critical carbon prices relevant to its decisions on CCS project in 2020 are, respectively, 137.27 RMB/ton CO₂ (0.123 RMB/kW·h) and 104.14 RMB/ton CO₂ (0.093 RMB/kW·h). Being well below either threshold, if the current price prevails in 2020, the private investors will have no incentive to keep investing in or operate the above CCS project. And we suggest that this should indicate the exact right moment for the government to consider subsidizing them with at least the amount of money to prevent their abandonment of CCS from happening.

The papers we are going to review have made important contributions to the CCS literature. First, in a typical stochastic model studying CCS, the construction lasts for one period, i.e., it takes one period from the launch to the completion of CCS construction. Moreover, the construction cost is often treated to be deterministic. In contrast, in our model, CCS construction is assumed to take multiple periods. Moreover, the exact length of CCS construction and the associated construction cost are both random. This dynamic stochastic setting enables our model to move one step closer to the real world. It allows us to tackle the intertemporal issues related to CCS investment in a stochastic environment. This renders our results to be particularly useful in practice. Second, the government subsidy is often treated to be exogenous in the literature. In comparison, in our model, the subsidy sufficient to prevent CCS abandonment from happening is determined endogenously by the free boundary that is established by applying optimal stopping time. Third,

the numerical simulations are conducted in this paper by applying the finite element method with the power penalty. Instead, the typical numerical simulations in the CCS literature are based on Monte Carlo method. However, applying the latter in our model would be inappropriate because the future remaining total deployment investment is unknown at present and will depend on the future annual investment expenditure – a series of the control variable that unfolds gradually over time in response to the realized exogenous shocks. Rather, we carry out numerical simulation as follows. We first introduce the stopping time to mark the timing of the completion of CCS construction. This enables us to divide the whole CCS lifespan into the construction stage and the operational stage. We then apply the finite element method with the power penalty to numerically solve the HJBVI problem.

Given a large body of literature studying CCS, below we focus on only the papers that are related to our work closely. Yang and Blyth (2007) build a comprehensive modeling framework based on the uncertainty of climate policy and investment. Fuss et al. (2008) develop a real options model to assess the value of coal-fired power plant retrofitted with CCS technology for the EU electricity system. Abadie and Chamorro (2008) apply the binomial lattice method to find the optimal investment strategy for coal-fired power plant retrofitted with CCS technology in Spain. Rammerstorfer and Eisl (2011) bring together the market modeling of CO₂ certificates and the real options approach to storage, in order to show how the rational investors would make decision under various environmental scenarios. To investigate the development of full-scale commercial CCS plants in EU, Eckhause and Herold (2014) present a real options framework to address the optimal project selection and funding strategy.

Focusing on China, there has been a great interest in CCS technology. The related research, applying mainly real options method, has been undertaken to investigate CCS from various aspects. Zhou et al. (2010), for example, propose a real options model to study the optimal decisions on CCS investment made by the investors who face possible technological change and policy uncertainty. Zhu and Fan (2011) present a real options model for CCS investment evaluation to assess the cost saving effect and the amount of CO₂ emission reduction by the least squares Monte Carlo method. Dahowski et al. (2012) are considered the first to build a detailed, nation-wide assessment of CCS potential across geographically, geologically, and industrially diverse landscape

of China, through the lens of an integrated CCS cost curve. Mo et al. (2015) construct a three-stage model in which three related but distinct decisions are made sequentially: decommission, CCS retrofit, and CCS operation. Compared to our model, their model assumes that CCS retrofit takes one year to complete and the construction cost is constant. In Wang and Du (2015), the carbon price and the coal price follow a quadrinomial process. Their model features the learning effect of construction cost. Like the other papers in the literature, CCS construction in their model is treated to be static by assuming that it will be completed in one year. Last, the government subsidy is often modeled to be exogenous. Chen et al. (2016) study the government subsidy and relate its size by a parameter to the electricity price. Based on Monte Carlo method, they then study investment strategy under various government subsidy schemes.

The remainder of the paper is organized as follows. In the next section, we lay out the model. In section 3, after solving the model with HJBVI, we derive the investor's optimal investment strategies over time and establish the free boundary. In addition, the artificial boundary condition is presented and discussed. In section 4, we conduct numerical simulations based on Chinese data and then shed light on some important policy issues, including whether CCS as a potential GHG emission reduction option can be taken seriously in China. Sensitivity analysis is performed in section 5. Conclusions follow in section 6.

2. Model

We build a continuous-time stochastic model with two stages. The first stage involves CCS construction and investment. The investors would not receive any cash inflow from the CCS project until the construction is done and CCS enters the second stage: the operational stage. The overall value of the project is evaluated by applying real options.

2.1. Carbon Price

The carbon price is assumed to follow a geometric Brownian motion. For $s > t$,

$$dP_C(s) = \gamma P_C(s) ds + \sigma_C P_C(s) dW_C(s), \quad (1)$$

with the initial condition $P_C(t) = p_C$, in RMB/kW·h of electricity. The parameters γ and σ_C denote, respectively, the drift rate and the volatility of the carbon price. $dW_C(s)$ is the independent increment of a Wiener process.

2.2. CCS Investment

In the first stage, the length of CCS construction and the associated construction cost are both treated to be random. This stochastic nature is captured by a state variable K denoting the remaining total deployment investment, i.e., how much more investment needs to be made before the completion of CCS construction. K follows a controlled diffusion process (Pindyck 1993, Schwartz 2004). Specifically, for $s > t$,

$$dK(s) = -I(s) ds + \beta \sqrt{I(s)K(s)} dW(s), \quad (2)$$

with the initial condition $K(t) = k$. The parameter β captures technological uncertainty. The variable I denotes the annual investment expenditure and is bounded within $\Lambda = [I_{\min}, I_{\max}]$. Note that $K(s) = 0$ represents the case in which CCS construction is completed.

2.3. Cash Flows

The cash flows are described below separately for each of the two stages

$$f(s, P_C(s), K(s); I(s)) = \begin{cases} -I(s), & \text{if } K(s) > 0, \\ q \cdot [r_C \cdot P_C(s) - e_C \cdot p_D], & \text{if } K(s) = 0. \end{cases} \quad (3)$$

During the construction stage marked by $K(s) > 0$, there are only the cash outflows, measured by $-I(s)$. In the operational stage featuring $K(s) = 0$, both the cash inflows and the outflows are involved. The former is the revenues the CCS firm receives from selling its carbon emission right and the latter refers to the expenses it spends on the activities related to CO₂ avoidance. Three parameters need to be introduced here. r_C refers to the capture rate of CCS technology and is relevant to the cash inflows. e_C denotes the emission factor of coal-fired generation and is relevant to the cash outflows. q denotes the electricity generation of the CCS project and is used here to capture the size effect.

The termination payoff function is assumed to be zero in either CCS is abandoned (if $s < T$) or completed (if $s = T$)

$$g(s, P_C(s), K(s)) = \begin{cases} 0, & \text{if } K(s) > 0 \text{ and } s < T, \\ 0, & \text{if } s = T. \end{cases} \quad (4)$$

Zero termination payoff in each case is a direct result of our assumption that there is neither abandonment benefit nor recycling benefit.

2.4. Timing Notations

The timing of the completion of CCS construction is

$$\tau_G = \inf \left\{ s : (s, P_C(s), K(s)) \notin G := [t, T) \times (0, +\infty) \times (0, +\infty) \right\}.$$

The periods of time covering the whole construction stage are described as

$$\mathcal{T}_G := [t, \tau_G).$$

The timing of the actual ending of the project, $\tau \in \mathcal{T}$, varies and depends on whether the CCS project is abandoned during the construction stage or kept alive to the end. Thus, we have

$$\mathcal{T} := \mathcal{T}_G \cup \{T\}. \quad (5)$$

2.5. Value of the CCS Project

The net present value of the CCS project is described as follows

$$\begin{aligned} V_\tau^I(t, p_C, k) &= \int_t^\tau e^{-r(s-t)} \cdot f(s, P_C(s), K(s); I(s)) \, ds \\ &\quad + e^{-r(\tau-t)} \cdot g(\tau, P_C(\tau), K(\tau)), \end{aligned} \quad (6)$$

where V_τ^I denotes the CCS project's net present value over an interval that starts at t with the state variables (p_C, k) and ends at $\tau \in \mathcal{T}$ and during which the investment strategy I is applied. The parameter r denotes the discount rate. The first term on the right-hand side of (6) is the discounted cash flows during the time interval $[t, \tau]$. The second term is the present value of the termination payoff that the investors will get when the project ends. According to (4), this payoff is always zero.

In our model, the investor's goal is to find the optimal stopping time and the optimal investment strategy that maximize the expected net present value in (6). Mathematically speaking, the problem can be defined as a stochastic optimal control problem: for all $(t, p_C, k) \in [0, T) \times (0, +\infty) \times [0, +\infty)$, we are to find the stopping time $\tau^* \in \mathcal{T}$ and the control $I^* \in \mathcal{A}$, such that

$$\begin{aligned} \Phi(t, p_C, k) &= \sup_{\tau \in \mathcal{T}, I \in \mathcal{A}} E^{t, p_C, k} [V_\tau^I(t, p_C, k)] \\ &= E^{t, p_C, k} [V_{\tau^*}^{I^*}(t, p_C, k)], \end{aligned} \quad (7)$$

subject to (1) and (2). In (7), \mathcal{A} is a given family of admissible controls, contained in the set of all adapted processes $\{I(s) : s \in [t, T]\}$ with the values in Λ . In particular, if $K(s) = 0$, \mathcal{A} would contain only the zero function. This simply means that no additional investment is needed once construction is done. If the stopping time τ^* , the control I^* and Φ exist, they will be called the optimal stopping time, the optimal control, and the value function, respectively.

3. Theoretical Results

In this section, the value function Φ defined in (7) is solved by the Hamilton-Jacobi-Bellman variational inequality (HJBVI). Then, the optimal stopping time τ^* and the optimal control I^* are derived.

3.1. HJBVI

In this subsection, we elaborate on the HJBVI that solves the value function. Having divided the CCS project into two stages, we employ separately an HJBVI for each stage: construction ($k > 0$) and operation ($k = 0$). By extending the domain of the termination payoff function g , we merge the two models into one.

3.1.1. HJBVI in the Stage of Operation Let's consider first the case of $k = 0$. Once the construction is completed ($\mathcal{T}_G = \emptyset$), the CCS project will enter the stage of operation and any subsequent annual investment expenditure I will remain zero ($I^* = 0$). By our assumption, the investor cannot abandon the project in the operational stage ($\tau^* = T$). In short, if $k = 0$, we have $\mathcal{T}_G = \emptyset$, $\tau^* = T$, and $I^* = 0$, and the value of the project becomes

$$\Phi(t, p_C, 0) = E^{t, p_C, k} [V_T^0(t, p_C, 0)].$$

According to the stochastic control theory (Øksendal 2003), Φ satisfies the following HJB equation

$$\frac{\partial \Phi}{\partial t}(t, p_C, 0) - (\mathcal{L}^0 \Phi)(t, p_C, 0) - r\Phi(t, p_C, 0) + f(t, p_C, 0; 0) = 0, \quad (8)$$

where $(\mathcal{L}^0 \Phi)(t, p_C, 0)$ is a specific case of $(\mathcal{L}^I \phi)(t, p_C, k)$ if $k = 0$, $I = 0$, and $\phi = \Phi$. In general, for each control $I \in \Lambda$ and $\phi \in C_0^2(\mathbb{R} \times \mathbb{R} \times \mathbb{R})$, we have

$$\begin{aligned} (\mathcal{L}^I \phi)(t, p_C, k) := & - \left[-I \frac{\partial \phi}{\partial k}(t, p_C, k) + \gamma p_C \frac{\partial \phi}{\partial p_C}(t, p_C, k) \right] \\ & - \frac{1}{2} \left[\beta^2 I k \frac{\partial^2 \phi}{\partial k^2}(t, p_C, k) + \sigma_C^2 p_C^2 \frac{\partial^2 \phi}{\partial p_C^2}(t, p_C, k) \right]. \end{aligned} \quad (9)$$

3.1.2. HJBVI in the Stage of Construction Let's move along and consider the second case of $k > 0$ in which the construction is not done yet. According to the definition of \mathcal{T} in (5), the value function Φ can be broken down into the following two parts

$$\Phi(t, p_C, k) = \max \left\{ \sup_{\tau \in \mathcal{T}_G, I \in \mathcal{A}} E^{t, p_C, k} [V_\tau^I(t, p_C, k)], \sup_{I \in \mathcal{A}} E^{t, p_C, k} [V_T^I(t, p_C, k)] \right\}. \quad (10)$$

The first part captures the case of CCS being abandoned during the construction stage. The second one represents the case that construction is done and the project is kept alive until the end of its economic life. Overall, the value of the CCS project depends on which strategy dominates and prevails.

Since the second part in the right-hand side of (10) is similar to the case of $k = 0$ that we discuss above, below we focus on the first part and take a closer look at it. During the entire construction stage, $k > 0$, there are only expenses paid out but no revenue received. As a result, the negative cash flows accumulate as time passes. In this case, the optimal stopping time would be $\tau^* = t$. To put it another way, if the project is deemed unprofitable and must be abandoned during the construction stage ($\tau \in \mathcal{T}_G$), the best thing the investors can do is to abandon it immediately ($\tau^* = t$). If this actually happens, the first part will be reduced to the termination payoff function. As a result, (10) becomes

$$\Phi(t, p_C, k) = \max \left\{ g(t, p_C, k), \sup_{I \in \mathcal{A}} E^{t, p_C, k} [V_T^I(t, p_C, k)] \right\}.$$

Recall that the termination payoff function $g(t, p_C, k)$ is zero in (4). Obviously, Φ should be no less than g . That is

$$\Phi(t, p_C, k) \geq g(t, p_C, k), \quad \text{in } [0, T] \times (0, +\infty) \times (0, +\infty).$$

We then apply dynamic programming (Øksendal and Sulem 2005). For $k > 0$, we have

$$\begin{aligned} \max \left\{ g(t, p_C, k) - \Phi(t, p_C, k), \frac{\partial \Phi}{\partial t}(t, p_C, k) - r\Phi(t, p_C, k) \right. \\ \left. + \sup_{I \in \mathcal{A}} [-(\mathcal{L}^I \Phi)(t, p_C, k) + f(t, p_C, k; I)] \right\} = 0. \end{aligned} \quad (11)$$

3.1.3. Putting Two Stages Together Recall two HJB equations shown in (8) and (11), derived for $k = 0$ and $k > 0$, respectively. To put two stages together, we need to first make the two equations consistent. This is done by extending the domain of g and set

$$g(t, p_C, 0) = -\infty.$$

This assumption imposes the harshest penalty on those investors who intend to abandon the CCS project in the stage of operation ($k = 0$). Essentially, it leads to a result that nobody finds it worthwhile to abandon CCS during the operational stage $[\tau_G, T]$.

Then for all $(t, p_C, k) \in [0, T) \times (0, +\infty) \times [0, +\infty)$, we have

$$\max \left\{ g(t, p_C, k) - \Phi(t, p_C, k), \frac{\partial \Phi}{\partial t}(t, p_C, k) - r\Phi(t, p_C, k) \right. \\ \left. + \sup_{I \in \mathcal{A}} [-(\mathcal{L}^I \Phi)(t, p_C, k) + f(t, p_C, k; I)] \right\} = 0, \quad (12)$$

with the terminal condition

$$\lim_{t \rightarrow T^-} \Phi(t, p_C, k) = g(T, p_C, k), \quad \text{in } (0, +\infty) \times [0, +\infty). \quad (13)$$

Equation (12), a combination of the Hamilton-Jacobi-Bellman (HJB) equation of the stochastic control and the variational inequality (VI) of the optimal stopping time, is termed HJBVI in Øksendal and Sulem (2005).

Based on the first order condition of (12), we derive the control variable as follows

$$I^*(t, p_C, k) = \begin{cases} I_{\max}, & \text{if } k > 0 \text{ and } -\frac{\partial \Phi}{\partial k}(t, p_C, k) + \frac{1}{2}\beta^2 k \frac{\partial^2 \Phi}{\partial k^2}(t, p_C, k) > 1, \\ I_{\min}, & \text{if } k > 0 \text{ and } -\frac{\partial \Phi}{\partial k}(t, p_C, k) + \frac{1}{2}\beta^2 k \frac{\partial^2 \Phi}{\partial k^2}(t, p_C, k) \leq 1, \\ 0, & \text{if } k = 0. \end{cases}$$

The equation above indicates that the optimal annual investment expenditure I^* is governed by a bang-bang control.

3.1.4. Free Boundary We are now able to spell out the free boundary, an important result from which the main idea of this paper emerges. First, we define the free boundary as follows. At any given time t , the free boundary is a set defined by

$$\left\{ (p_c, k) : \Phi(t, p_c, k) = \varepsilon > 0, \text{ where } \varepsilon \text{ is sufficiently small} \right\}.$$

The definition implies that the free boundary represents a borderline between the investors' two options in the construction stage ($k > 0$): abandoning the CCS project or keeping it alive, as both lead to $\Phi = 0$. Specifically, the free boundary at each time node is a function of two state variables: the market carbon price p_c and the individual project's remaining total deployment investment k . In general, any value of Φ will fall into one of the three categories: on the free boundary, to the right of the free boundary, or to the left of the free boundary.

First, if Φ happens to be on the free boundary, it makes no difference for the investors to choose between abandoning and keeping investing in the CCS project. Second, Φ may end up to the right of the free boundary. This could happen if the carbon price is fairly high and/or the project is pretty close to completion. We consider it a good state for combating global warming – the private investors find it optimal to keep investing in CCS and thus, on the flip side, the government finds no subsidy is necessary as the market forces work properly in this case. Last, Φ may be to the left of the boundary, if the carbon price is too low (which lowers the revenue brought into the CCS project) and/or CCS construction is far from complete (which requires a huge chunk of money to be chipped in). This is the case in which the private investors would abandon the project, were there no government for them to lean on.

We consider the last a bad state in a sense that an important (indeed vital) public good – combating global warming – suffers from market failure. To turn the table, apparently the government must jump in and play a role. Besides the call for it to do something to boost the market carbon price, we suggest that this bad state should indicate the exact right moment for the government to consider subsidizing the private investors of the CCS project. Moreover, the amount of the subsidy should be enough to effectively prevent CCS abandonment from happening. That is, after taking into account the government subsidy they will receive, the investors find it no longer optimal for them to abandon the CCS project in the first place.

3.2. Artificial Boundary Condition

The HJBVI problem stated in (12) with the terminal condition in (13) was initially defined in an unbounded domain. To solve it by the method of artificial boundary condition, we first need to set a finite domain as $[0, \bar{p}_C] \times [0, \bar{k}]$. Below we discuss the artificial boundary conditions and

their economic implications in three extreme cases: the initial carbon price p_C hits its lower bound and the upper bound, respectively; and the remaining total deployment investment k reaches its upper bound.

First, consider the initial carbon price p_C at its lower bound of zero. According to the definition of P_C in (1), if $p_C = 0$, P_C will remain permanently in the state of 0. As a result, the carbon emission right becomes worthless. Since there is no benefit to reap, the investor would not launch CCS investment in the first place. Apparently, in this case, the value of the project is zero

$$\Phi(t, 0, k) = 0.$$

Second, consider the case that k reaches its upper bound. This represents the situation that construction of a gigantic CCS project is far from completion and a hefty investment still needs to put in. In this case, we have

$$\Phi(t, p_C, \bar{k}) = 0.$$

that holds for all $0 \leq p_C \leq \bar{p}_C$. Below is how we attempt to make sense of this result. Since the remaining total deployment investment k is so large that, even at \bar{p}_C , the highest possible carbon price, this gigantic CCS project would still not be able to yield a positive net present value. This prompts the investors to become unwilling to invest in it. Like the previous case, the value of the project becomes zero too.

Third, consider the case of p_C hitting its upper bound. The CCS investors experience a windfall as a very high carbon price enables them to reap a great benefit from selling the carbon emission rights. To take advantage, the investors would speed up construction process by deploying the largest possible annual investment expenditure I_{\max} , with the hope to complete construction as quickly as possible because the opportunity cost of delay is simply too big. In this case, Φ is determined in (6) where $I^* = I_{\max}$ and $\tau^* = T$. Actually, an analytical solution exists

$$\Phi(t, \bar{p}_C, k) = \max \left\{ g(t, \bar{p}_C, k), \bar{V}(t, \bar{p}_C, k) \right\},$$

where

$$\begin{aligned}
\bar{V}(t, \bar{p}_C, k) &= \int_t^{\bar{\tau}} e^{-r(s-t)} \cdot (-I_{\max}) ds \\
&+ \int_{\bar{\tau}}^T e^{-r(s-t)} \cdot q(r_C \cdot \bar{p}_C - e_C \cdot p_D) ds + e^{-r(T-t)} \cdot g(T, \bar{p}_C, 0) \\
&= (-I_{\max}) \frac{1 - e^{-r\bar{\tau}}}{r} + q[r_C \cdot \bar{p}_C - e_C \cdot p_D] \frac{e^{-r\bar{\tau}} - e^{-r(T-t)}}{r} \\
&= -\frac{I_{\max}}{r} + \frac{e^{-r\bar{\tau}}}{r} \cdot \left(I_{\max} + q(r_C \cdot \bar{p}_C - e_C \cdot p_D) \right) \\
&\quad - \frac{e^{-r(T-t)}}{r} \cdot q(r_C \cdot \bar{p}_C - e_C \cdot p_D),
\end{aligned}$$

and $\bar{\tau} = t + k/I_{\max}$. Note that, if k is sufficiently large, $\bar{V}(t, \bar{p}_C, k)$ could become negative to ensure that the artificial boundary conditions remain continuous. The reason is similar to the one discussed above in the second case: the positive effect on the net present value $V(t, p_C, k)$ of \bar{p}_C could be outweighed by the negative effect of a very large k . Since the investors have no incentive to invest in a CCS project with a negative net present value, the value Φ remains zero. This actually happens in our numerical simulations described in which we set \bar{p}_C to be 3 RMB/kW·h and \bar{k} to be 3×10^{10} RMB. In comparison with these extreme values, their initial values are set to be $p_C = 0.12$ RMB/kW·h and $k = 1 \times 10^{10}$ RMB in the baseline simulation. Please see Appendix B for more details.

4. Numerical Simulations Based on Chinese Data

We conduct numerical simulations by applying the finite element method with the power penalty (Wang et al. 2000, Zhang et al. 2009). The details are provided in Appendix B. To be specific, we consider a hypothetical power enterprise in China that makes its dynamic decisions on CCS project in a stochastic environment. The simulation period is 2015 – 2035, which we consider to be an important time span in the post-Kyoto protocol era.

4.1. Parameter Values

The parameter values used in the baseline simulation are summarized in Table 1. These values are based on the Chinese data and are taken mainly from the National Bureau of Statistics of China (2016) and Zhu and Fan (2011), as well as from the other CCS studies.

Parameter	Value	Parameter	Value
γ	2.00 percent	q	2.5×10^{10} kW·h per year
σ_C	11.50 percent	T	20 years
p_D	217.1 RMB per ton CO ₂	r_C	90 percent
β	0.5	r	5.00 percent per year
I_{\min}	200 million RMB	I_{\max}	2 billion RMB
e_C	893 g CO ₂ per kW·h	/	/

Table 1 Parameter values in the baseline simulation

4.2. Discussions of Simulation Results

We focus our discussions on the following two results: the value of the CCS project Φ and the free boundary established for each time node.

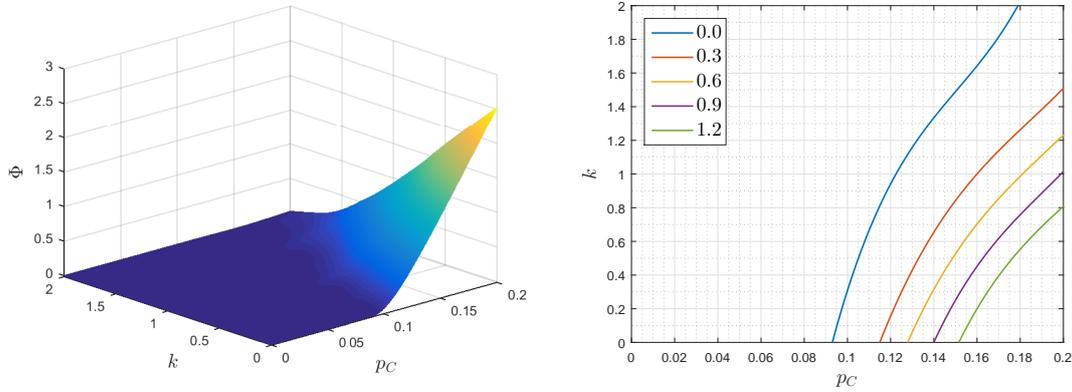


Figure 1 The value of the CCS project and its contour lines for 2020.

The scales are 1×10^{10} RMB for both Φ and k and 1 RMB/kW·h for p_C .

Figure 1 plots the numerical result for Φ with respect to p_C and k for the year 2020. In the three-dimensional graph on the left, it shows that a higher carbon price p_C and/or a lower remaining total deployment investment k lead to a greater Φ . The same idea is depicted, on the right, in a two-dimensional graph by the stacked contour lines. Each line represents a specific value of Φ . In case the contour lines do not sound familiar, recall the indifference curves commonly used in economics – they are the contours of the utility surface, joining points of equal utility. In the right

graph, the higher contour line, representing a greater Φ , is positioned closer to the bottom right corner.

The contour line in blue represents $\Phi = 0$ and depicts the free boundary. The region to its right is called the continuation region, in which $\Phi > 0$ and the private investors have no problem keeping the CCS project alive. On the flip side, the government finds no subsidy is necessary in this situation. Conversely, the region to the left of the free boundary is called the stopping region, in which the investors feel compelled to abandon CCS investment and thus lead to $\Phi = 0$. To prevent abandonment from happening, the government should consider subsidizing the private investors, if, for example, the call for it to boost the market carbon price is deemed too costly on the overall economic growth and job creation.

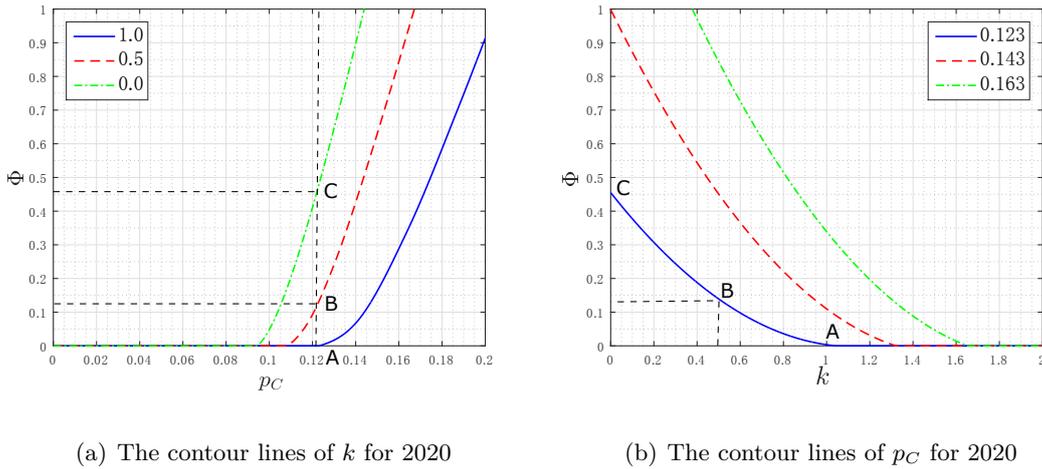


Figure 2 The contour lines of k and p_C for 2020

Figure 2 plots the same relationship among Φ and two state variables, p_C and k , in an alternative way by drawing the contour lines of p_C and k , respectively. The left graph shows that, for a given value of k , a higher carbon price will boost the value of the CCS project. This makes sense as a higher carbon price allows the firm to bring in more revenue from the sale of the carbon emission right. In the right graph, for a given value of p_C , a negative relation between Φ and k emerges. As CCS construction gets closer to completion and k approaches zero, the risk of CCS abandonment is lessened and this helps lift Φ . Conversely, the abandonment risk will be aggravated if k is large.

Overall, our numerical simulations reveal that, for k at 1.0×10^{10} RMB, the critical carbon price associated with $\Phi = 0$ that is relevant to the hypothetical power station's decision on CCS

investment turns out to be 0.123 RMB/kW·h (equivalent to 137.27 RMB/ton CO₂). This is shown, in the left graph, at the intersection point of the blue curve and the horizontal axis. If k decreases all the way down close to zero, we will get the critical carbon price of 0.093 RMB/kW·h (104.14 RMB/ton CO₂) relevant to its decision on CCS operation. The current carbon price in China is well below either threshold. Suppose it prevails in 2020, the private investors, if left to their own devices without any government subsidy to lean on, will have no incentive to either keep investing in or operate CCS in that hypothetical power station.

In short, two graphs in Figure 2 conveys the same information in the alternative ways. For example, the points A , B , and C in the left graph are transformed to the points A , B , and C , respectively, in the right graph. Portrayed in both graphs, the message is consistent and clear: a large CCS project far from completion can survive only in some favorable conditions when the market carbon price is relatively high, while a smaller one or the one closer to completion is able to go through more adverse conditions when the market carbon price is lower.

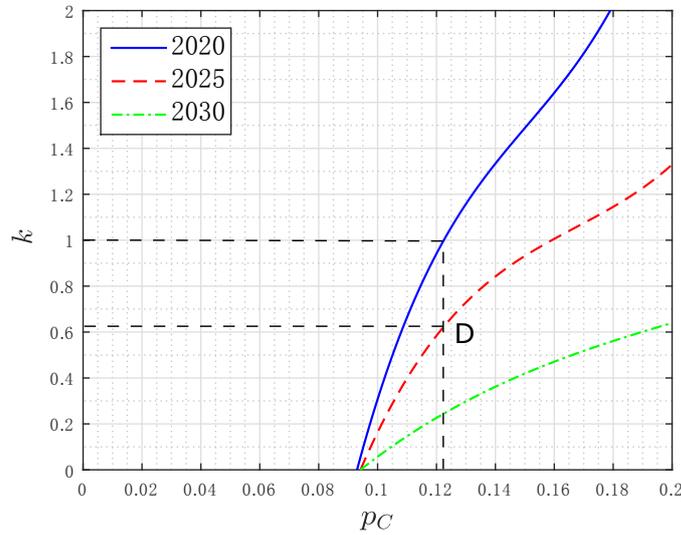


Figure 3 The free boundaries for the years 2020, 2025, 2030, respectively

In Figure 3, three free boundaries ($\Phi=0$) are depicted for the years 2020, 2025, 2030, respectively. Recall that the value of the CCS project Φ is determined by t , p_C , and k jointly. To separate the individual effect of t , we fix p_C and k while vary only t and see what happens. Consider, for example, p_C at 0.123 RMB/kW·h and k at 0.625×10^{10} RMB. For the year 2025, Φ is zero and shown at point D on the free boundary of 2025 (the red curve). If the same state occurs in 2020,

Φ is greater than zero and positioned to the right of the free boundary of 2020 (the blue curve). In 2030, Φ is to the left of the free boundary of 2030 (the green curve).

We believe part of the result is driven by setting the year 2035 to be the termination date, meaning that beyond it all CCS projects will become worthless. To elaborate, let's compare two cases: the project reaches $k = 0.625 \times 10^{10}$ RMB in 2020 vs. in 2030. It is plausible to claim that, in the former case, CCS construction would be done earlier and thus the project would have a longer operational period to reap the benefit from selling carbon emission rights. As a result, Φ would be greater for the former case of 2020 than for the latter of 2030. This is the economic intuition we come up with behind the simulation result depicted in Figure 3.

From a mathematical viewpoint, to make sense of the above simulation results, recall (3) and (6). In order to have $\Phi = 0$ associated with the free boundary, the cash outflows attributed to $-I(s)$ to run down K in the construction stage must be completely offset by the following discounted net cash flows in the operational stage

$$V_{\tau}^I(t, p_C, k) = \int_{\tau_G}^T e^{-r(s-t)} \cdot q \cdot [r_C \cdot P_C(s) - e_C \cdot p_D] ds. \quad (14)$$

If CCS construction could be done earlier, the interval of the above integral, representing the length of the operational stage, would be wider. To yield the same value of the integral, the integrand could be smaller, which implies a lower critical carbon price relevant to the decision on CCS investment. In short, considering the same value of k , the critical carbon price for CCS investment would be lower for 2020 than for 2030. As time passes, the free boundary moves towards the bottom right corner and the continuation region shrinks.

It is worth noting that three free boundaries all intersect with the horizontal line at about the same point $(p_C, k) = (0.093, 0)$. This indicates that the critical carbon prices relevant to the decision on CCS operation are largely the same at 0.093 RMB/kW·h, no matter the construction is done in 2020, 2025, or 2030. The intersection of the free boundary with the horizontal line represents $\Phi = 0$ and k . The latter implies that construction is completed and there is no additional cash outflows $-I(s)$ to be offset. As a result, to yield $\Phi = 0$, the discounted cash flows in the operational stage captured in (14) needs simply to be zero, which in turn requires the integrand to be zero. In this case, the interval of the integral, capturing the length of the operational stage, becomes irrelevant.

REMARK 1. To shed light on the policy issues, in particular on whether CCS as a potential GHG emission reduction option could be taken seriously in China, we need first to let the simulation results based on the theoretical model meet with the data in reality. In China, the carbon price fluctuated, over the past five years, within 40 and 70 RMB/ton CO₂, equivalent to 0.036 and 0.063 RMB/kW·h, respectively. It has always been well below 104.14 RMB/ton CO₂ (0.093 RMB/kW·h), our projected critical carbon price relevant to CCS operation in that hypothetical power station. Actually, to reach the above threshold so the CCS operation could become economically feasible, the market carbon price needs to be almost doubled in China. Moreover, to make CCS investment economically feasible, the carbon price has to be even higher, being at least 137.27 RMB/ton CO₂ (0.123 RMB/kW·h). Can these be achieved at all, let alone without the dire consequences to economic growth and job creation? If such drastic changes in the market carbon price are off the table, to make CCS investment and operation viable in that hypothetical Chinese power station, it seems to us the government subsidy is inevitable.

5. Sensitivity Analysis

The sensitivity analysis is carried out of four parameters: γ , σ_C , I_{\max} , and q . The results are displayed below in Figures 4 - Figure 7, respectively.

5.1. Sensitivity Analysis of the parameters γ and σ_C

An increase in γ , i.e., a higher growth rate of the carbon prices, and an increase in σ_C , i.e., the more volatile carbon prices, turn out to be both good news to the CCS investors. In the first case, if the prospect of the future carbon price is brighter, the investors will be more tolerant to a depressed carbon price at present. In the second case, like holding the option, the CCS investors have the right to abandon the project if things go wrong. As a result, more volatile carbon prices enable the investors to reap the great benefit in some situations but not to be exposed to the great loss in the others. Shown in Figure 4 and Figure 5, both cases move the free boundaries towards the upper left corner, leading to a shrinking stopping region and the lower critical carbon prices for CCS project. The last result is captured in Figure 4 by $G_1 < R_1 < B_1$ and $G_2 < R_2 < B_2$, respectively.

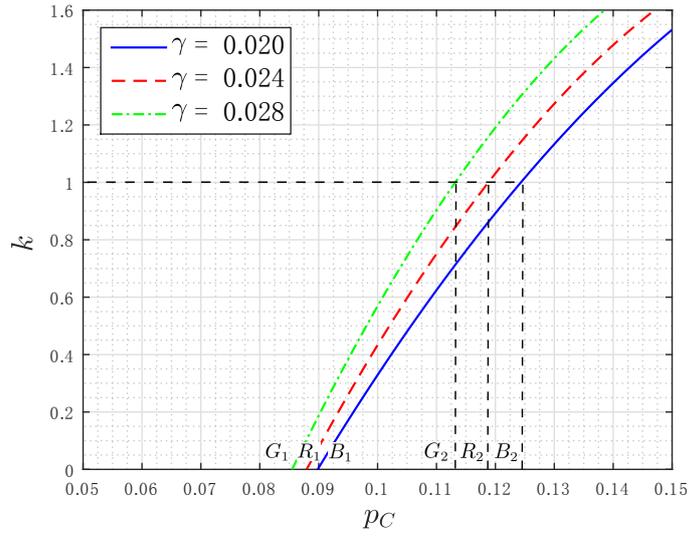


Figure 4 The effect of γ on the free boundary

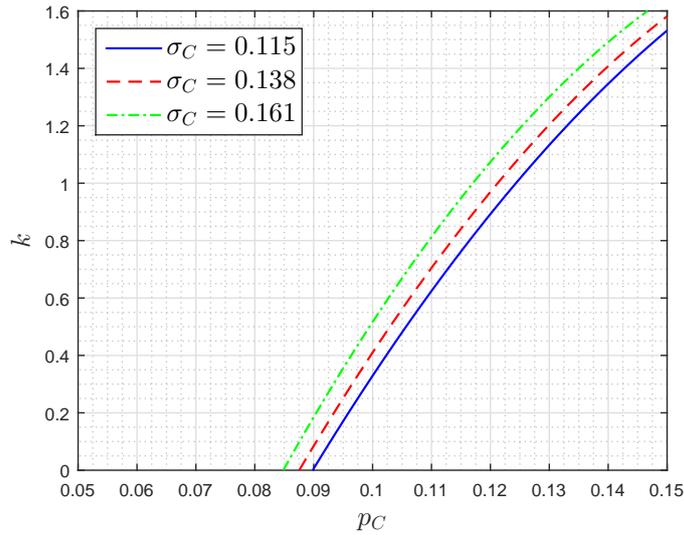


Figure 5 The effect of σ_C on the free boundary

5.2. Sensitivity Analysis of the parameter I_{\max}

Figure 6 presents the effect on the free boundary of an increase in the upper bound of the annual investment expenditure, I_{\max} . Here we consider two offsetting effects. On one hand, a greater I_{\max} make it possible to speed up CCS construction and thus to leave the CCS project with a longer operational stage. This effect leads to a greater Φ . On the other hand, a greater I_{\max} could make $K(s)$ more volatile and thus leads to a smaller Φ . Overall, according to our simulation results, the former positive effect outweighs the latter negative effect. As a result, a greater I_{\max} is associated with a greater Φ , a shrinking stopping region, and a lower critical carbon price

for CCS investment. The related policy implication is, to encourage the investors to speed up CCS construction, the government could consider offering a regressive tax credit scheme on CCS investment expenditures, in which the marginal tax credit would be more generous towards the larger annual investment expenditure I . Last, we notice that any change in I_{\max} barely has any effect on the critical carbon price for CCS operation ($k = 0$). This makes sense. In the operational stage, $K(s)$ and $I(s)$ are both zero and the critical carbon price for CCS operation is determined by $r_C \cdot P_C(s) - e_C \cdot p_D = 0$, in which I_{\max} plays no role.

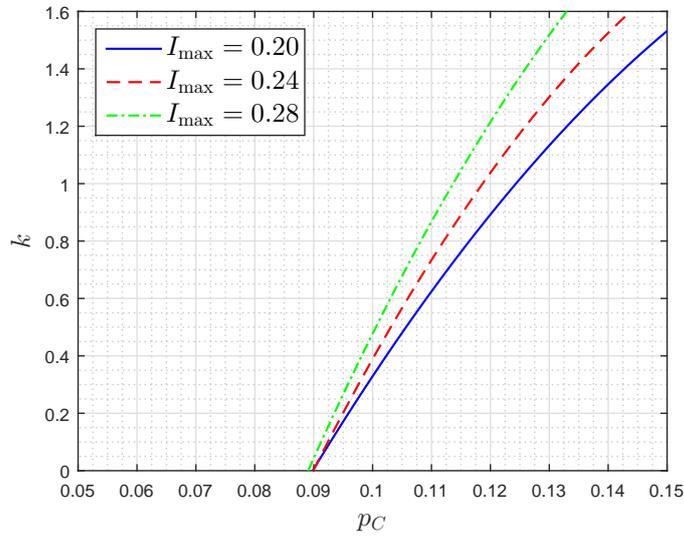


Figure 6 The effects of I_{\max} on the free boundary

5.3. Sensitivity Analysis of the parameter q

Figure 7 depicts the size effect on the free boundary, captured by varying the electricity generation capacity of the CCS-retrofitted power plant. Above the horizontal axis, $K(s)$ and $I(s)$ are both positive. On the free boundary ($\Phi = 0$), to fully offset the negative cash outflows of $-I(s)$, the CCS project must generate the same amount of the positive cash flows from $q \cdot [r_C \cdot P_C(s) - e_C \cdot p_D]$. Given the unit cash inflow represented in the square bracket, a greater q would make the positive cash flows larger. This is the size effect. As a result, as q increases, the free boundary moves towards the upper left corner. In addition, Figure 7 shows that varying q barely has any effect on the critical carbon price for CCS operation ($k = 0$). The reason is similar to the one discussed above in the case of I_{\max} .

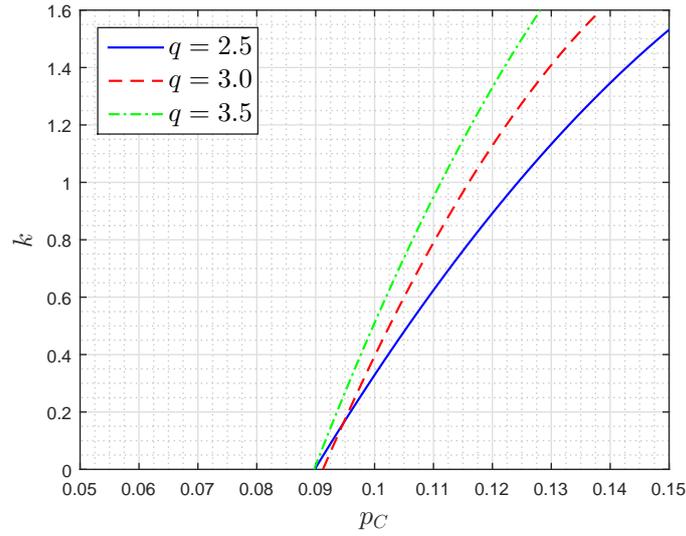


Figure 7 The effects of q on the free boundary

Table 5 presents a summary of the results obtained in the sensitivity analysis. It exhibits the effects of those four parameters on the critical carbon price relevant to the decision on CCS investment. For example, it displays in the table that, from its baseline value of 0.020, if the value of γ is increased by 20% and 40%, the critical carbon price would fall by 4.03% and 8.87%, respectively, from 0.123 RMB/kW·h obtained in the baseline simulation concerning the hypothetical CCS project with $k = 1 \times 10^{10}$ in 2020.

	γ	σ_C	I_{\max}	q
	0.020	0.115	0.20	2.5
+20%	-4.03%	-2.42%	-4.03%	-6.45%
+40%	-8.87%	-5.65%	-8.06%	-9.68%

Table 2 The effects of four parameters on the critical carbon price relevant to CCS investment in a hypothetical project with $k = 1 \times 10^{10}$ in 2020

6. Conclusions

We build a continuous-time stochastic model to study the abandonment strategy of CCS project. Applying HJBVI, we derive the optimal investment expenditure as a bang-bang control at each

time node. Using the optimal stopping time, we establish a free boundary for each time node over the construction stage. The numerical simulations are carried out by employing the finite element method with the power penalty. To extend the model, one may consider setting up a government sector besides the private sector, with the former taking account of the societal benefit from CCS. The interaction between these two sectors could be explored in a differential game model (Krawczyk and Zaccour 1999, Jørgensen and Zaccour 1999).

Acknowledgments

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Appendix A: The Dimensionless HJBVI

Set characteristic length

$$\begin{aligned} P_{C,\infty} = P_{D,\infty} &= 1 \text{ RMB/kW}\cdot\text{h}, \quad I_\infty = f_\infty = 10^{10} \text{ RMB/year}, \\ q_\infty &= 10^{10} \text{ kW}\cdot\text{h/year}, \quad T_\infty = 1 \text{ year}, \quad \Phi_\infty = K_\infty = g_\infty = 10^{10} \text{ RMB}. \end{aligned}$$

It is straightforward to nondimensionalize the HJBVI in (12) with the following substitutions, i.e.,

$$\begin{aligned} \max \left\{ \frac{g_\infty}{\Phi_\infty} \frac{g}{g_\infty} - \frac{\Phi}{\Phi_\infty}, \frac{\partial(\Phi/\Phi_\infty)}{\partial t} - r \frac{\Phi}{\Phi_\infty} + \sup_{(I/I_\infty) \in \mathcal{A}} \left[- \frac{I_\infty}{K_\infty} \frac{I}{I_\infty} \frac{\partial(\Phi/\Phi_\infty)}{\partial(k/K_\infty)} \right. \right. \\ \left. \left. + \gamma \frac{p_C}{P_{C,\infty}} \frac{\partial(\Phi/\Phi_\infty)}{\partial(p_C/P_{C,\infty})} + \frac{1}{2} \sigma_C^2 \left(\frac{p_C}{P_{C,\infty}} \right)^2 \frac{\partial^2(\Phi/\Phi_\infty)}{\partial(p_C/P_{C,\infty})^2} \right. \right. \\ \left. \left. + \frac{1}{2} \beta^2 \frac{I_\infty}{K_\infty} \frac{I}{I_\infty} \frac{k}{K_\infty} \frac{\partial^2(\Phi/\Phi_\infty)}{\partial(k/K_\infty)^2} + \frac{f}{f_\infty} \frac{f_\infty}{\Phi_\infty} \right] \right\} = 0. \end{aligned} \quad (15)$$

For simplicity, we also denote the dimensionless HJBVI as

$$\begin{aligned} \max \left\{ g - \Phi, \frac{\partial \Phi}{\partial t} - r \Phi + \sup_{I \in \mathcal{A}} \left[- I \frac{\partial \Phi}{\partial k} + \gamma p_C \frac{\partial \Phi}{\partial p_C} + \frac{1}{2} \sigma_C^2 p_C^2 \frac{\partial^2 \Phi}{\partial p_C^2} \right. \right. \\ \left. \left. + \frac{1}{2} \beta^2 I k \frac{\partial^2 \Phi}{\partial k^2} + f \right] \right\} = 0, \end{aligned} \quad (16)$$

with the terminal condition $\Phi(T, \cdot) = g(T, \cdot)$, where

$$f(t, p_C, k; I) = \begin{cases} -I, & \text{when } k > 0, \\ q \cdot [r_C \cdot p_C - e \cdot p_D], & \text{when } k = 0, \end{cases}$$

and

$$g(t, p_C, k) = \begin{cases} 0, & \text{when } k > 0 \text{ and } t < T, \\ 0, & \text{when } t = T, \\ -\infty, & \text{when } k = 0 \text{ and } t < T. \end{cases}$$

Appendix B: Numerical Scheme

HJBVI (16) will be solved by the power penalty method, developed by Zhang et al. (2009), Zhang and Wang (2011).

It is fairly straightforward to check that

$$\lim_{\varepsilon \rightarrow 0^+} \frac{\exp(-y/\varepsilon)}{\varepsilon} = \begin{cases} 0, & \text{when } y > 0, \\ -\infty, & \text{when } y \leq 0, \end{cases} \quad \lim_{\varepsilon \rightarrow 0^+} \varepsilon \exp(y/\varepsilon) = \begin{cases} +\infty, & \text{when } y > 0, \\ 0, & \text{when } y \leq 0, \end{cases}$$

$$\lim_{\varepsilon \rightarrow 0^+} \exp(-y/\varepsilon) = \begin{cases} 0, & \text{when } y > 0, \\ 1, & \text{when } y = 0. \end{cases}$$

So the problem can be reduced to an equivalent non-linear equation

$$\frac{\partial \Phi^\varepsilon}{\partial t}(t, p_C, k) - r\Phi^\varepsilon(t, p_C, k) + \sup_{I \in \mathcal{A}} \left[-(\mathcal{L}^I \Phi^\varepsilon)(t, p_C, k) + f^\varepsilon(t, p_C, k) \right] + \pi_\varepsilon(g^\varepsilon(t, p_C, k) - \Phi^\varepsilon(t, p_C, k)) = 0, \quad (17)$$

with the terminal condition

$$\lim_{t \rightarrow T^-} \Phi^\varepsilon(t, p_C, k) = g^\varepsilon(T, p_C, k),$$

where $\pi_\varepsilon(y) = \varepsilon \exp(y/\varepsilon)$, and

$$f^\varepsilon(t, p_C, k; I) = q(r_C \cdot p_C - e \cdot p_D) \cdot \exp(-k/\varepsilon) - I \cdot (1 - \exp(-k/\varepsilon)),$$

$$g^\varepsilon(t, p_C, k) = -\frac{T-t}{\varepsilon} \cdot \exp(-k/\varepsilon),$$

which are the smoothing version of the functions f and g , respectively.

Let the set $\{t_n = n\Delta t\}_{n=0}^N$ be an equipartition of $[0, T]$ with the interval length denoted by Δt . The Crank-Nichoson scheme will be used to discrete the time as follow: for $n = 0, \dots, N-1$,

$$\frac{\Phi^\varepsilon(t_{n+1}, p_C, k) - \Phi^\varepsilon(t_n, p_C, k)}{\Delta t} - \frac{1}{2} \left(r\Phi^\varepsilon(t_n, p_C, k) + r\Phi^\varepsilon(t_{n+1}, p_C, k) \right) + \frac{1}{2} \sup_{I \in \mathcal{A}} \left[-\mathcal{L}^I u(t_n, p_C, k) + f^\varepsilon(t_n, p_C, k; I) - \mathcal{L}^I \Phi^\varepsilon(t_{n+1}, p_C, k) + f^\varepsilon(t_{n+1}, p_C, k; I) \right] + \frac{1}{2} \left(\pi_\varepsilon(g^\varepsilon(t_n, p_C, k) - \Phi^\varepsilon(t_n, p_C, k)) + \pi_\varepsilon(g^\varepsilon(t_{n+1}, p_C, k) - \Phi^\varepsilon(t_{n+1}, p_C, k)) \right) = 0; \quad (18)$$

for $n = N$,

$$\Phi^\varepsilon(t_N, p_C, k) = g^\varepsilon(t_N, p_C, k).$$

After we set I^* to be the maximal value point of (17), (18) becomes

$$\Phi^\varepsilon(t_n, p_C, k) + \frac{\Delta t}{2} \left[(\mathcal{L}^{I^*} \Phi^\varepsilon)(t_n, p_C, k) + r\Phi^\varepsilon(t_n, p_C, k) - \pi_\varepsilon(g(t_n, p_C, k) - \Phi^\varepsilon(t_n, p_C, k)) \right] = \Phi^\varepsilon(t_{n+1}, p_C, k) - \frac{\Delta t}{2} \left[(\mathcal{L}^{I^*} \Phi^\varepsilon)(t_{n+1}, p_C, k) + r\Phi^\varepsilon(t_{n+1}, p_C, k) - \pi_\varepsilon(g(t_{n+1}, p_C, k) - \Phi^\varepsilon(t_{n+1}, p_C, k)) - f^\varepsilon(t_n, p_C, k; I^*) - f^\varepsilon(t_{n+1}, p_C, k; I^*) \right].$$

Unbounded region will be replaced by the bounded region $\Omega = (0, \bar{p}_C) \times (0, \bar{k})$, where $\bar{p}_C = 3$ and $\bar{k} = 3$. In the baseline simulation, we focus on the model's results if the state variables are taken the values around $(p_C, k) = (0.12, 1)$.

In the rest of the section, we apply the finite element method to solve the partial differential equation (PDE) numerically. For simplicity, p_C and k are denoted below by x_1 and x_2 , respectively. We then define the bilinear form as follows. For all $u, v \in C_0^\infty(U)$,

$$a^I(u, v) = \int_U uv + \frac{\Delta t}{2} \left[- \sum_{i=1}^2 b_i^I u_{x_i} v + \sum_{i,j=1}^2 \frac{\partial(a_{ij}^I v)}{\partial x_i} u_{x_j} + ruv \right] dx,$$

and

$$b(u, v) = \int_U \pi_\varepsilon(g - u)v dx,$$

$$n(u, v) = \int_U \sum_{i=1}^2 u_{x_i} v_{x_i} dx,$$

$$F^I(u, v) = \int_U uv - \frac{\Delta t}{2} \left[- \sum_{i=1}^2 b_i^I u_{x_i} v + \sum_{i,j=1}^2 \frac{\partial(a_{ij}^I v)}{\partial x_i} u_{x_j} + ruv - \pi_\varepsilon(g - u)v \right] + \frac{\Delta t}{2} (f^I(t - \Delta t) + f^I(t))v dx,$$

where

$$(b_i^I)_{2 \times 1} = \begin{pmatrix} \gamma x_1 \\ -I \end{pmatrix}, \quad (a_{ij}^I)_{2 \times 2} = \begin{pmatrix} \frac{1}{2} \sigma_C^2 x_1^2 & 0 \\ 0 & \frac{1}{2} \beta^2 I x_2 \end{pmatrix}$$

and

$$\frac{\partial(a_{11}^I v)}{\partial x_1} = \frac{1}{2} \sigma_C^2 (2x_1 v + x_1^2 v_{x_1}), \quad \frac{\partial(a_{22}^I v)}{\partial x_2} = \frac{1}{2} \beta^2 (Iv + Ix_2 v_{x_2}).$$

Next, we use the finite element method to solve the following variational problem. For $n = N - 1, N - 2, \dots, 0$, and $\forall v \in H_0^1(U)$, we have

$$\sup_{I \in \Lambda} \left[a^I(u_n, v) - \frac{\Delta t}{2} b(u_n, v) + \varepsilon n(u_n, v) - F^I(u_{n+1}, v) \right] = 0, \quad (19)$$

and $u_N(\cdot) = g(T, \cdot)$. Since a stable term $\varepsilon n(u_n, v)$ is added, the problem is strictly elliptical.

Because π_ε is a non-linear function, we should make linearization of it by Newton's method. Based on the non-linear functions

$$a^{I^*}(u_n, v) - \frac{\Delta t}{2} b(u_n, v) + \varepsilon n(u_n, v) = F^{I^*}(u_{n+1}, v),$$

with the initial value u_n , we obtain u'_n by one Newton's step that satisfies

$$\begin{aligned} a^{I^*}(u'_n, v) + \frac{\Delta t}{2} \int_U \exp((g - u_n)/\varepsilon) u'_n v dx + \varepsilon n(u'_n, v) \\ = F^{I^*}(u_{n+1}, v) + \frac{\Delta t}{2} \int_U \exp((g - u_n)/\varepsilon) u_n v dx + \frac{\Delta t}{2} b(u_n, v). \end{aligned}$$

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Figures

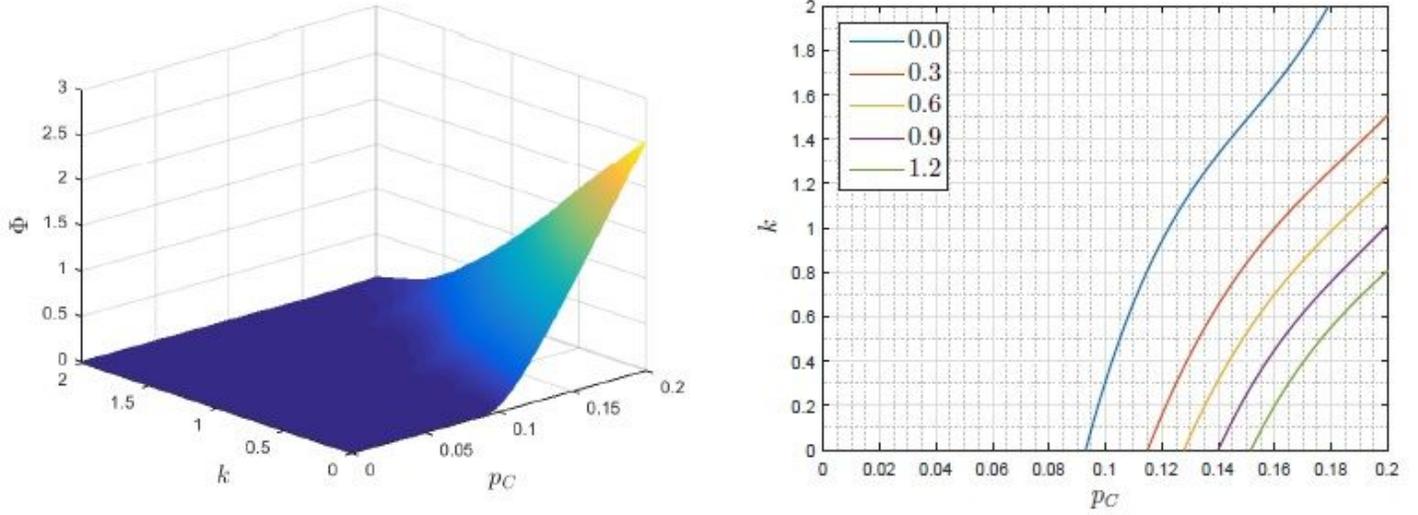
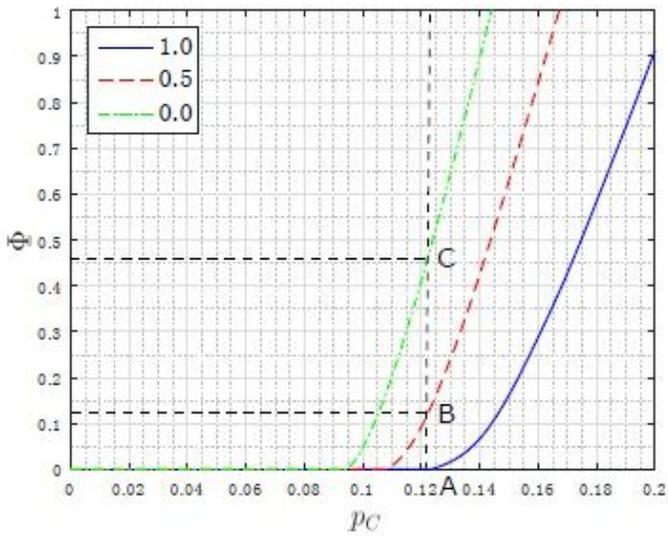
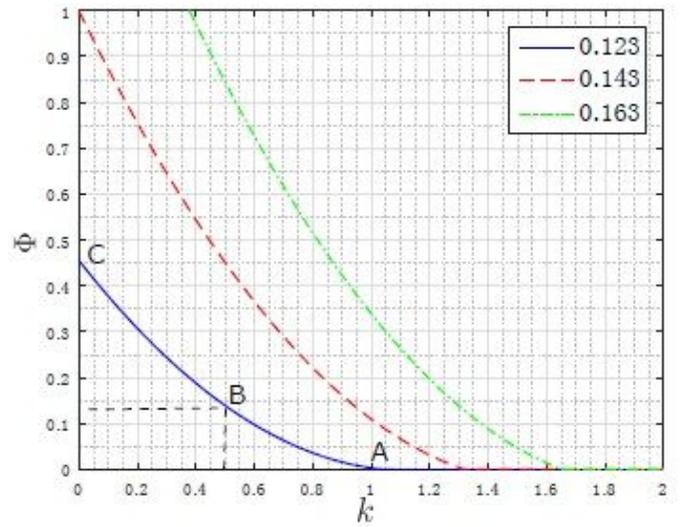


Figure 1

The value of the CCS project and its contour lines for 2020. The scales are 1×10^{10} RMB for both Φ and k and 1 RMB/kW·h for p_C .



(a) The contour lines of k for 2020



(b) The contour lines of p_C for 2020

Figure 2

The contour lines of k and p_C for 2020

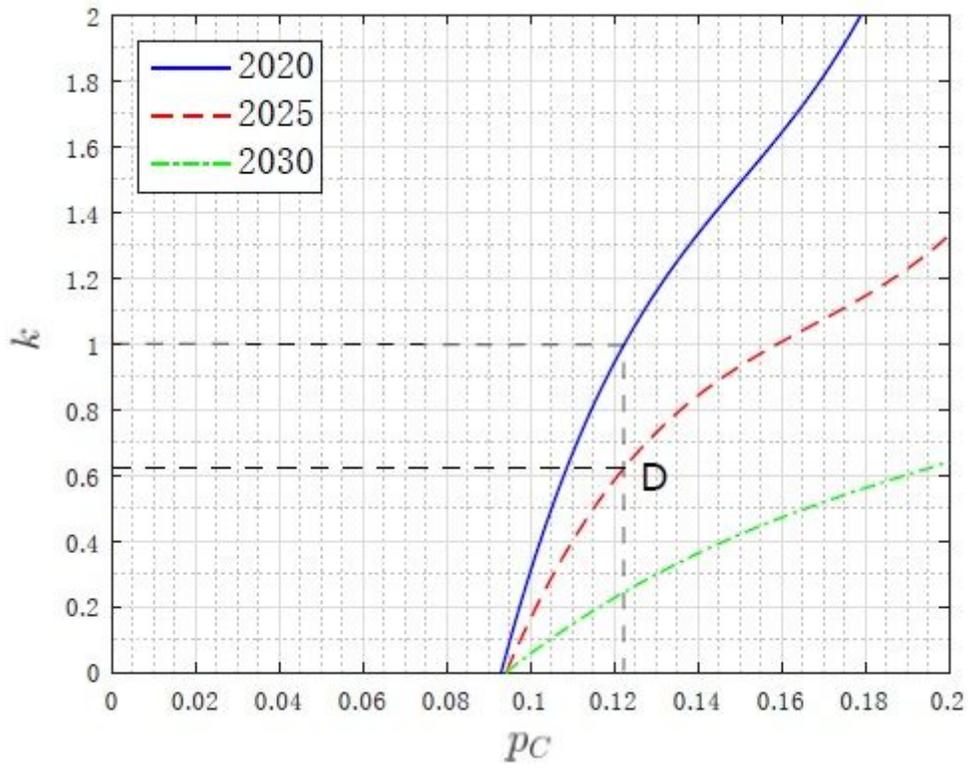


Figure 3

The free boundaries for the years 2020, 2025, 2030, respectively

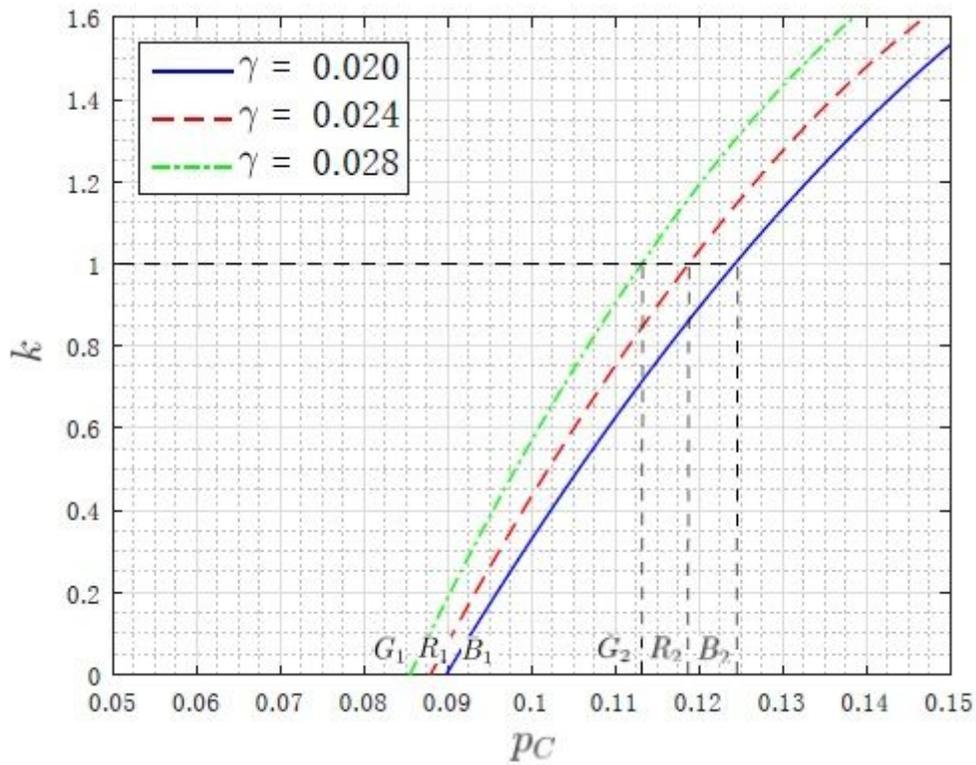


Figure 4

The effect of γ on the free boundary

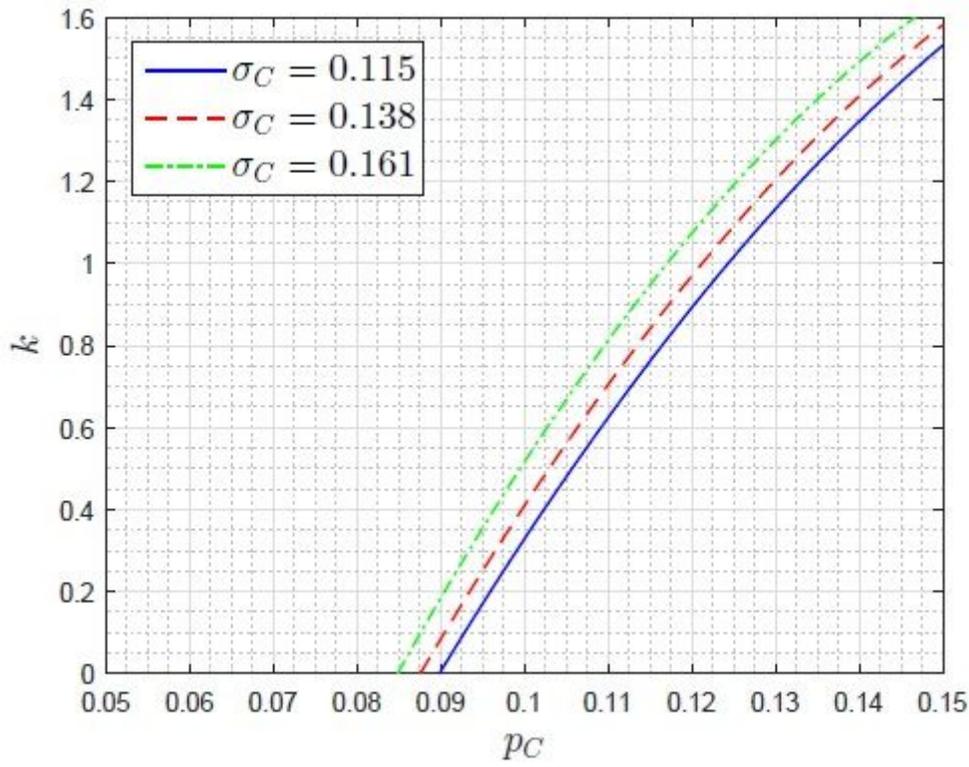


Figure 5

The effect of σ_C on the free boundary

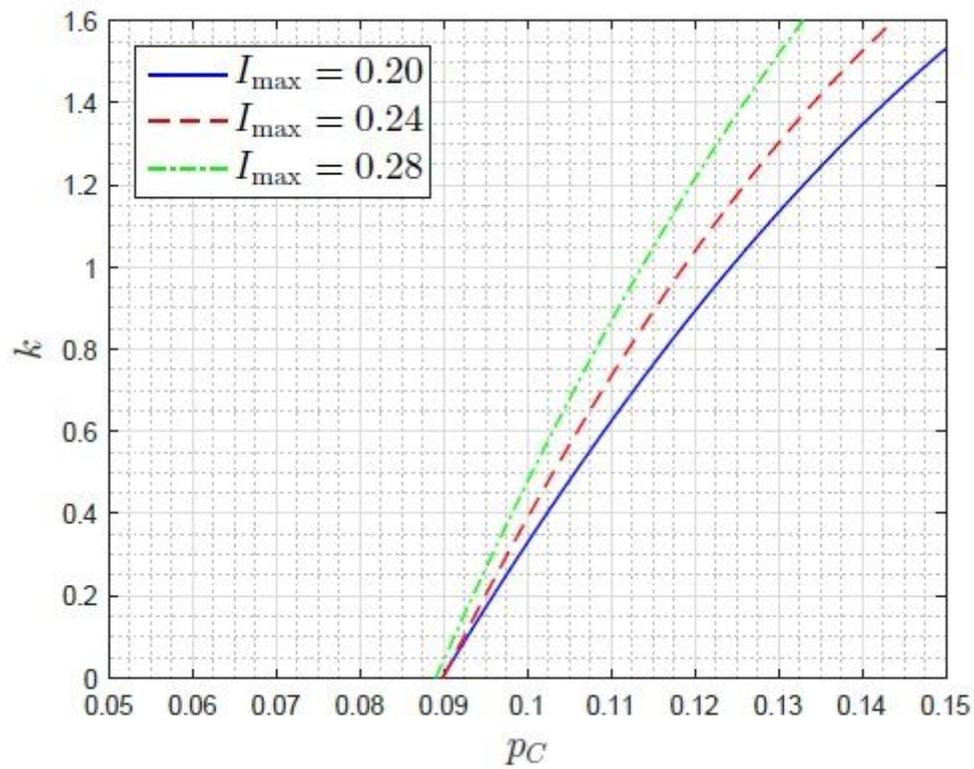


Figure 6

The effects of l_{max} on the free boundary

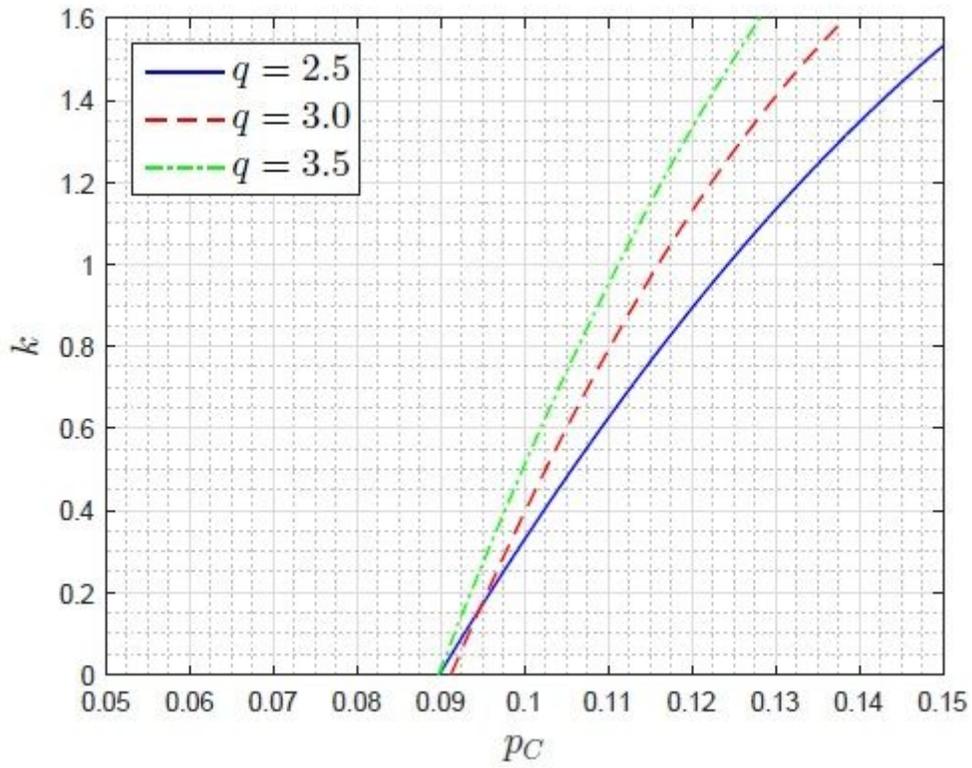


Figure 7

The effects of q on the free boundary