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Effect of Proximity to Bed on 30° and 45° Inclined Dense Jets: A Numerical Study

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Abstract

Employing inclined dense jets is a common way for the disposal of brine effluent from coastal desalination plants. This paper numerically analyzes the mixing and geometrical properties of 30° and 45° inclined dense jets when they discharge close to the bed. For this purpose, two series of numerical simulations were developed. First, the nozzle acts as a free jet when it is placed far enough from the lower boundary. Meanwhile, in the second series, the distance between the nozzle tip and seabed is substantially reduced. Consequently, by comparing these two series, the effect of proximity to bed on the behavior of dense jets is investigated. The governing equations are solved by modifying a solver within the CFD package of OpenFOAM. The numerical results are presented in comparative figures and compared to the previous works. Comparisons indicated that the numerical model predicts the geometrical characteristics of dense jets in good agreement with the past experimental studies. However, the dilution predictions are conservative. It has been observed that proximity to the bed has almost no appreciable effects on the behavior of 45° jets. However, for 30° jets, when the bed proximity parameter (y_0/L_M) falls below 0.14, normalized values of horizontal and vertical locations of centerline peak and return point dilution are slightly reduced while the terminal rise height remains untouched.

Keywords Dense jet . Negatively buoyant jet . Mixing . Coanda effect . CFD . OpenFOAM

Article Highlights

- The mixing and geometrical properties of 30° and 45° inclined dense jets were numerically analyzed.
- Some characteristics of 30° jets were influenced by the lower boundary.
- The behavior of 45° inclined dense jets was almost insensitive to variations of the bed proximity parameter.
- The RANS approach was able to predict geometrical characteristics while dilution predictions were conservative.

1 Introduction

In recent decades, seawater desalination has attracted a lot of attention as a new and unconventional resource of freshwater. The main by-product of the desalination process is a concentrate effluent or brine, which is commonly disposed of into the sea. The disposal of this effluent has raised serious concerns due to its potential impacts on marine biota, especially benthic communities. Submerged discharge in the form of a inclined dense jet is a common way to mitigate these impacts. By using inclined nozzles, brine efficiently mixes with the surrounding water, and consequently, the concentration reduces to the accepted levels for the marine ecosystems (Abessi and Roberts 2015).

Anticipating the behavior of flow is possible through the experimental, analytical, and numerical models. Zeitoun et al. (1970) conducted several experiments on the effect of nozzle orientation on dense discharges. They reported 60° angle as the most appropriate inclination since it produces the longest trajectory and the highest dilution compared to 30°, 45°, and 90° jets. Afterward, the 60° accepted as the de facto standard for brine discharge, although this has been questioned in later studies. Based on the latter suggestion, Roberts et al. (1997) carried out detailed measurements on inclined dense jets at 60° using Laser-Induced Fluorescence (LIF) and a micro-conductivity probe. They derived flow properties along the near-field and presented experimental coefficients for various flow parameters such as impact point and the end of the initial mixing zone. Kikkert et al. (2007) performed experiments using Light Attenuation (LA) and LIF techniques to validate their analytical solutions. Their comparisons indicated that the analytical solutions reasonably predict the flow's path and terminal rise height, but the minimum dilution at the location of the return point was underestimated. Lai and Lee (2012) reported a comprehensive series of experimental work on dense discharge in stationary water. They employed LIF and Particle Image Velocimetry (PIV) techniques to measure tracer concentration and velocity fields, respectively. According to their observations, the dimensionless maximum rise height $y_t/(d.Fr_d)$ is independent of source conditions for $Fr_d \geq 25$, and the impact point dilution is not sensitive to nozzle angle within the range of 38°–60°. Abessi and Roberts (2015) conducted experimental studies on single-port dense jets oriented at angles from 15° to 85° to the horizontal using the Three-Dimensional LIF (3D-LIF) technique. The experimental results showed that the dilution at the impact point is almost insensitive to nozzle angle over the range of about 45°–65°, while the near-field dilution is more sensitive to nozzle inclination and is highest for 60°. Furthermore, they inquired about the bottom boundary effects on the impact point dilution. They observed that the time-average dilution along the jet centerline first increased and then decreased in a thin layer close to the wall.

Although the 60° inclined dense jets are suggested for brine discharge from desalination plants, the associated terminal rise height is relatively high for shallow coastlines; therefore, smaller nozzle angles seem more practical in this condition. Shao and Law (2010) carried out comprehensive experiments on the mixing behavior of dense jets at smaller angles of 30° and 45° using combined PIV and Planar LIF (PLIF) techniques. The effect of proximity to the bed on the flow behavior was also examined. They reported that in 30° dense jets, the flow is significantly affected by the bottom boundary for the jet proximity parameter (y_0/L_M) less than 0.15. Abessi and Roberts (2016) performed experiments on inclined dense jets with nozzles oriented at 30°, 45°, and 60° in shallow water, employing a 3D-LIF system. They proposed the 30° jet for shallower water since it has fewer surface interactions and visual impacts and gives slightly better dilution at the impact point compared to other angles in such a shallow condition.

Besides experimental and analytical studies, various numerical studies have also been reported in the context of dense jets. Oliver et al. (2008) investigated the initial mixing of negatively buoyant jets using the ANSYS CFX model. They tried to improve the predictions by adjusting the turbulent Schmidt number in the tracer transport equation for positively buoyant vertical jets and then applying it to inclined negatively buoyant jets. However, the overall effect of this approach on the quality of the results was small, and the dilution predictions were underestimated. Kheirkhah Gildeh et al. (2015) studied the geometrical and mixing characteristics of 30° and 45° inclined dense jets in stationary ambient using the OpenFOAM finite volume model. They evaluated the effect of several turbulence models on the accuracy of CFD predictions. Comparing the numerical results to previous experimental data indicated that the LRR and realizable $k - \varepsilon$ turbulence models are able to predict flow behavior more accurately. Zhang et al. (2017) performed a numerical study on 45° and 60° inclined dense jets in the stagnant water with the Large Eddy Simulation (LES) approach. Their simulations could reproduce concentration build-up at the impact point reported by Abessi and Roberts (2015); however, dilutions at the impact point and within the spreading layer were underpredicted. In a recent study, Tahmooreesi and Ahmadyar (2021) examined the effects of the turbulent Schmidt number on CFD predictions of 45°

inclined dense jets. They reported reducing the turbulent Schmidt number from 1.0 to 0.4 improves the dilution predictions, whereas this change adversely affects the geometrical parameters and cross-sectional distribution of concentration.

This paper numerically investigates the effects of bed proximity on mixing and geometrical characteristics of 30° and 45° inclined dense jets with the Reynolds-Averaged Navier-Stokes (RANS) approach. For this purpose, two series of numerical models were considered. In the first series, the nozzles are placed far enough from the floor to act as a free jet. Meanwhile, in the second series, the distance of nozzles from the bed is substantially reduced. The governing equations are implemented and solved within the OpenFOAM finite volume model, and the realizable $k - \varepsilon$ turbulence model is employed for turbulent closure. The results of these two numerical series are compared to each other and also previous experimental studies. Through this comparison, assessing the probable effects of proximity to bed on the flow behavior would be possible.

2 Dimensional analysis

The schematic side view of an inclined dense jet is shown in Fig. 1. The jet discharges upwardly from a round nozzle of d , with jet velocity U_0 , jet density ρ_0 , and at the initial angle of θ to the horizontal. The jet rises and mixes with ambient water ($\rho_a < \rho_0$) due to its initial momentum until it reaches a maximum height (y_t) and then falls back to the floor because of its negative buoyancy. Finally, it impacts the bed at a horizontal distance of x_i from the source and continues spreading as a density current.

The dimensional analysis of free jets in deep water is well known and is described in Fischer et al. (1979), Roberts and Toms (1987), and Roberts et al. (1997), but for jets which placed at a close distance to the bed, the height of the nozzle tip from the bed y_0 should be added. Based on the mentioned references, for fully turbulent jets in which the flow is independent of viscosity, all the dependent variables of the flow φ , could be characterized by the discharge angle θ , the jet kinematic fluxes of volume Q_0 , momentum M_0 , buoyancy B_0 , and the height of the nozzle tip from the bed y_0 :

$$\varphi = f(Q_0, M_0, B_0, \theta, y_0) \quad (1)$$

and

$$Q_0 = \frac{\pi}{4} d^2 U_0; \quad M_0 = U_0 Q_0; \quad B_0 = g'_0 Q_0 \quad (2)$$

where $g'_0 = g(\rho_0 - \rho_a)/\rho_a$ is the modified acceleration due to gravity. These fluxes can form the jet-to-plume transition length scale, $L_M = M_0^{3/4}/B_0^{1/2}$; that is a distance from the source of discharge in which the flow is dominated by initial momentum flux (jet-like). This characteristic length scale is also equal to $(\pi/4)^{1/4} d Fr_d$ where Fr_d is the jet densimetric Froude number:

$$Fr_d = \frac{U_0}{\sqrt{g'_0 d}} \quad (3)$$

For high Froude numbers ($Fr_d > 20$), the initial volume flux Q_0 is not dynamically important, and it can be neglected (Roberts et al. 1997). Therefore, for boundary-affected jets at a specific angle θ , the dependent variables are a function of M_0 , B_0 , and y_0 :

$$\varphi = f(M_0, B_0, y_0) \quad (4)$$

Following a dimensional analysis, any characteristic length χ , can be expressed as:

$$\frac{\chi}{L_M} = f\left(\frac{y_0}{L_M}\right) \quad (5)$$

and dilution as:

$$\frac{s}{Fr_d} = f\left(\frac{y_0}{L_M}\right) \quad (6)$$

where dilution is defined as $s = (C_0 - C_a)/(C - C_a)$ where C_0 , C_a , and C are jet discharge concentration, ambient concentration, and local time-averaged concentration, respectively.

Fig. 1 A schematic side view of an inclined dense jet

3 Numerical model

3.1 Governing equations

The governing equations consist of the continuity equation, Navier-Stokes equations with the Boussinesq approximation for buoyancy effects, and tracer advection-diffusion equation for incompressible three-dimensional flows. Based on the Boussinesq approximation, when the density variation is not large, the density can be treated as a constant in the unsteady and convection terms and as a variable only in the gravitational term (Ferziger and Perić 2002). After applying the Reynolds-averaging, the governing equations take the following form:

- Continuity equation

$$\frac{\partial U_i}{\partial x_i} = 0 \quad (7)$$

- Navier-Stokes equations with the Boussinesq approximation for buoyancy

$$\frac{\partial U_i}{\partial t} + \frac{\partial (U_i U_j)}{\partial x_j} = -\frac{1}{\rho_r} \frac{\partial P}{\partial x_i} + \nu \frac{\partial^2 U_i}{\partial x_j \partial x_j} - \frac{\partial \overline{u'_i u'_j}}{\partial x_j} + g_i \frac{\rho - \rho_a}{\rho_a} \quad (8)$$

where U_i and u'_i are mean and fluctuating part of the fluid velocity, P is the mean flow hydrodynamic pressure, ρ is the fluid density, and ν is the kinematic viscosity.

- Tracer advection-diffusion equation

$$\frac{\partial C}{\partial t} + \frac{\partial (C U_j)}{\partial x_j} = \frac{\partial}{\partial x_j} \left(D \frac{\partial C}{\partial x_j} \right) - \frac{\partial \overline{u'_j c'}}{\partial x_j} \quad (9)$$

where C and c' are the mean and fluctuating part of the tracer concentration, $D = \nu/Sc$ is the molecular diffusion, and Sc is the Schmidt number. The density of both the jet and the ambient water is calculated using the equation of state for seawater proposed by El-Dessouky and Ettouney (2002).

3.2 Turbulence modeling

The above RANS equations have closure problem due to the Reynolds stress tensor $\tau_{ij} = -\overline{u'_i u'_j}$ and turbulent scalar fluxes $\overline{u'_j c'}$. The Reynolds stress tensor is commonly modeled using the Boussinesq hypothesis, which assumes the Reynolds stress is a linear function of the mean velocity gradients (Moukalled et al. 2016):

$$-\overline{u'_i u'_j} = 2\nu_t S_{ij} - \frac{2}{3}k\delta_{ij} \quad (10)$$

where ν_t is the turbulent viscosity, $k = 1/2 \overline{u'_i u'_i}$ is the turbulence kinetic energy, and S_{ij} is the mean strain-rate tensor:

$$S_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \quad (11)$$

With this assumption, it is required to calculate the turbulent viscosity and turbulence kinetic energy rather than the Reynolds stresses. Several turbulence models have been developed to compute these two terms. The present paper employs the realizable $k - \varepsilon$ model of Shih et al. (1995), which has been used and validated in several studies in the context of dense jets (Kheirkhah Gildeh et al. 2015; Ardalan and Vafaei 2019; Tahmooreesi and Ahmadyar 2021). The formulation of the realizable $k - \varepsilon$ turbulence model is given by:

$$\frac{\partial k}{\partial t} + \frac{\partial(kU_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left(D_k \frac{\partial k}{\partial x_i} \right) + G_k - \varepsilon \quad (12)$$

$$\frac{\partial \varepsilon}{\partial t} + \frac{\partial(\varepsilon U_i)}{\partial x_i} = \frac{\partial}{\partial x_i} \left(D_\varepsilon \frac{\partial \varepsilon}{\partial x_i} \right) + \sqrt{2}C_{1\varepsilon}S_{ij}\varepsilon - C_{2\varepsilon}\frac{\varepsilon^2}{k + \sqrt{\nu\varepsilon}} \quad (13)$$

where $G_k = 2\nu_t S_{ij}^2$ represents the generation of turbulence kinetic energy due to the mean velocity gradients, $D_k = \nu + (\nu_t/\sigma_k)$ is the effective diffusion coefficient for k , and $D_\varepsilon = \nu + (\nu_t/\sigma_\varepsilon)$ is the effective diffusion coefficient for the rate of dissipation of turbulence energy ε . In the model, the turbulent viscosity is obtained using:

$$\nu_t = C_\mu \frac{k^2}{\varepsilon} \quad (14)$$

and C_μ is computed as follows:

$$C_\mu = \frac{1}{A_0 + A_s \frac{kU^*}{\varepsilon}} \quad (15)$$

$$U^* = \sqrt{S_{ij}S_{ij} + \tilde{\Omega}_{ij}\tilde{\Omega}_{ij}} \quad (16)$$

$$\tilde{\Omega}_{ij} = \Omega_{ij} - 2\varepsilon_{ijk}\omega_k \quad (17)$$

$$\Omega_{ij} = \bar{\Omega}_{ij} - \varepsilon_{ijk}\omega_k \quad (18)$$

where $\bar{\Omega}_{ij}$ is the mean rate of rotation tensor and ω_k is the angular velocity. The parameters A_0 and A_s are determined as follows:

$$A_0 = 4, A_s = \sqrt{6} \cos \phi \quad (19)$$

$$\phi = \frac{1}{3} \cos^{-1}(\sqrt{6}W), W = \frac{S_{ij}S_{jk}S_{ki}}{\tilde{S}^3}, \tilde{S} = \sqrt{S_{ij}S_{ij}} \quad (20)$$

$C_{1\varepsilon}$ is defined as follows:

$$C_{1\varepsilon} = \max \left[0.43, \frac{\eta}{\eta + 5} \right], \eta = S \left(\frac{k}{\varepsilon} \right), S = \sqrt{2S_{ij}S_{ij}} \quad (21)$$

The model constants assigned the following values: $C_{2\varepsilon} = 1.9$, $\sigma_k = 1.0$, and $\sigma_\varepsilon = 1.2$.

The turbulent scalar fluxes $\overline{u'_j c'}$ are mostly estimated employing the Standard Gradient Diffusion Hypothesis (SGDH) (Gualtieri et al. 2017):

$$-\overline{u'_j c'} = \frac{\nu_t}{Sc_t} \frac{\partial C}{\partial t} \quad (22)$$

where Sc_t is the turbulent Schmidt number.

3.3 Flow configuration and computational setup

Based on the available experimental setups, a computational domain with dimensions of 1.25 m length, 0.4 m width, and 0.4 m depth was chosen, but only half of the domain ($1.25 \text{ m} \times 0.2 \text{ m} \times 0.4 \text{ m}$) was considered since the problem is symmetrical. The depth and width of the considered domain satisfy the criteria for the deep flow regime ($dFr_d/H < 0.8$) and acting as a single jet ($s/(d.Fr_d) > 2$, where s is port spacing) proposed by Abessi and Roberts (2014, 2016).

Fig. 2 Computational domain: (a) symmetry plane; (b) isometric view

For grid independency analysis, the dilution at the return point was considered as the monitoring point with a 2% tolerance. It was found that a grid of 650,000 cells with increasing grid spacing from the center of the nozzle to the boundaries, as shown in Fig. 2, provides independence of results from the grid cell sizes.

The boundary conditions for the nozzle inlet are as follows:

$$U_x = U_0 \times \cos(\theta), U_y = U_0 \times \sin(\theta), U_z = 0, C = C_0, \text{"fixedFluxPressure"} \quad (23)$$

The initial values for k and ε are calculated using the equation proposed by Huai et al. (2010) as follows:

$$k = 0.06U_0^2; \quad \varepsilon = 0.06 U_0^3/d \quad (24)$$

At the outlet section, a zero-gradient boundary condition perpendicular to the outlet plane is determined for variables U , k , ε , and C , while a Dirichlet boundary condition with constant value zero is applied to P in order to set a reference pressure. For the wall boundaries, the no-slip condition is imposed for velocity, and the improved wall functions presented by Launder and Spalding (1974) are used for k , and ε . Moreover, at these sections, the *fixedFluxPressure* boundary condition is applied to P , which adjusts the pressure gradient so that the boundary flux matches the velocity boundary condition (Greenshields 2015). Finally, for the symmetry plane, the *symmetry* boundary condition is employed. This boundary condition is imposed by setting normal gradient of scalars and velocity vector as well as normal velocity to zero (Moukalled et al. 2016).

The simulations are performed using the open-source software package of OpenFOAM. The modified *buoyantBoussinesqPimpleFoam* solver, which is a transient solver for buoyant, turbulent, incompressible flows, is employed within OpenFOAM. This solver uses the Boussinesq approximation for buoyancy effects and the PIMPLE algorithm to solve the coupled pressure-momentum system. The PIMPLE algorithm is a combination of SIMPLE and PISO algorithms, which allows using of larger Courant numbers and time intervals. In the present study, however, the solver is run in PISO mode and also with the Courant number less than unity to ensure that any important transient information, which can significantly influence the flow, would not be missed. Further details about the PIMPLE algorithm can be found in Holzmann (2018).

The first-order implicit *Euler* scheme is used for temporal discretization. The advection (divergence) and diffusion (laplacian) terms are discretized using the standard Gaussian finite-volume integration in which the interpolation of values on cell faces from cell centers is required. The *limitedLinear* and *linearUpwind* interpolation schemes are employed for the advection terms of scalar variables and velocity, respectively, and also the *linear* scheme for the diffusion of all variables. The discretized equations form linear sets of equations as $Ax=b$, which have to be solved using a linear system solver method. For that purpose, the Preconditioned Conjugate Gradient (PCG) method with the Diagonal Incomplete Cholesky (DIC) preconditioner is used for solving the system of linear equations of the pressure field. Meanwhile, the Preconditioned Biconjugate Gradient (PBiCG) method with the Diagonal Incomplete LU (DILU) preconditioner is employed for the other fields. The convergence criterion of 10^{-7} is set for U , k , ε , and C , while that for P is 10^{-9} . The Sc and Sc_t are considered constant with the values of 600 and 0.7, respectively (Rard and Miller 1979; Nimdeo et al. 2014; Van Reeuwijk et al. 2016).

4 Results and discussions

Totally twelve numerical simulations in two series have been performed to investigate the possible effects of bed proximity on flow behavior. The characteristics of these simulations are presented in

Table 1 Simulations characteristics

Case. No.	Inclined angle θ	Nozzle diameter d (mm)	Distance y_0 (mm)	$\Delta\rho/\rho_a$ (%)	Discharge velocity U_0 (m/s)	Reynolds number Re	Densimetric Froude number Fr_d	Jet to plume characteristic length scale L_M (mm)	Bed proximity parameter y_0/L_M
F1	30	6	27.0	1	0.485	3257	20	112.97	0.24
F2	30	6	27.0	1	0.607	4071	25	141.21	0.19
F3	30	6	27.0	1	0.728	4885	30	169.45	0.16
F4	30	6	27.0	1	0.849	5699	35	197.69	0.14
F5	45	6	40.0	1	0.485	3257	20	112.97	0.35
F6	45	6	40.0	1	0.607	4071	25	141.21	0.28
N1	30	6	11.9	1	0.485	3257	20	112.97	0.11
N2	30	6	11.9	1	0.607	4071	25	141.21	0.08
N3	30	6	11.9	1	0.728	4885	30	169.45	0.07
N4	30	6	11.9	1	0.849	5699	35	197.69	0.06
N5	45	6	12.0	1	0.485	3257	20	112.97	0.11
N6	45	6	12.0	1	0.607	4071	25	141.21	0.08

Table 1. In the table, the far and proximate-to-bed jets have been denoted by F (stands for far) and N (stands for near), respectively.

4.1 General observations

In most cases, the flow reached the steady-state condition about 30-40 s after the beginning of discharge. The simulations, however, were continued until 80 s after the start time. Unlike the Direct Numerical Simulation (DNS) and LES approaches, all the turbulence fluctuations are modeled and represented in terms of the mean-flow characteristics in the RANS approach. Hence, after the flow reaches the steady-state condition, almost no appreciable difference is seen between the instantaneous and time-averaged values of a quantity at a specific point. While in real conditions, flow-dependent quantities at a specific point fluctuate widely; that is, the instantaneous values can be substantially higher or lower than the time-averaged value.

Fig. 3 Non-dimensional central plane tracer concentrations

Fig. 3 shows the non-dimensional time-averaged concentration contour maps and lines of two cases with identical densimetric Froude number in two different series, and consequently, different bed proximity parameter (y_0/L_M) for both inclination angles. The figure does not show a significant difference in the flow regime for these two cases. Like the free jet, the proximate-to-bed jet rises and mixes with the ambient water until a maximum height and then falls back to the floor. It is also observed that due to this process, the concentration of flow is considerably reduced when it impacts the bed.

The centerline trajectory is one of the simplest and most important characteristics of the flow, which is derived through the connection of the maximum velocity or concentration location at each cross-section perpendicular to the flow. It is worth noting that the trajectories that are obtained using the velocity variable almost coincide with the ones obtained through the concentration variable, but the latter descends slightly faster (Shao 2010). The centerline trajectories for various cases in both series that are derived using the concentration variable have been illustrated in Fig. 4. The results are non-dimensionalized with L_M for better comparison. It is seen that the results are in acceptable agreement with previous experimental data, especially before the centreline peak, where the flow behavior is still jet-like. The slope of the ascending and descending parts of the trajectories for both series are almost equal, meaning that the trajectories are relatively symmetrical. The symmetry of centreline trajectories of free jets is in agreement with the observations of Shao and Law (2010). They also reported a slight asymmetry in boundary-affected cases, which is not seen herein. For 30° jets, the major difference between the trajectories of two series F and N is that the trajectories of series N descend sooner in comparison to series F. This difference, which is also reported by Shao and Law (2010), happens due to the effect of Coanda on flow behavior. The Coanda effect is the tendency of a fluid to adhere to a surface because of the reduced pressure due to flow acceleration around the surface (Constantin 2010). However, no notable difference is seen between the two series for 45° jets. This means that nozzle angle, in addition to distance to the bottom boundary, plays an important role in determining the bed influence on flow behavior. So it can be concluded that the steeper the nozzle angle is, the less the effect of the boundary on the flow behavior is.

Fig. 4 Normalized centerline trajectories: (a) 30°; (b) 45°

4.2 Centerline peak

The centerline peak (x_m , y_m) is the location where the centerline trajectory reaches its maximum rise height. The horizontal and vertical locations of the centerline peak are normalized with the jet diameter and plotted against Fr_d in Fig. 5. The experimental results from Cipollina et al. (2005), Kikkert (2006), and Shao and Law (2010), along with the analytical model of Kikkert (2006), are added for comparison. As seen in Fig. 5, the numerical simulations slightly underestimate the centerline peak location in comparison to previous experimental data and the analytical model of Kikkert (2006).

In free jets, the location of this point is controlled by the initial momentum flux and nozzle angle. It means that the centerline peak is situated in a region of flow, where the flow behavior is jet-like, not plume-like. It was also reported by Ghayoor et al. (2019) after analyzing the turbulent concentration fluctuations along with the jet and plume-like region of the dense jet in different locations. To perceive how proximity to the bed affects the location of the centerline peak, the numerical results are normalized with L_M and plotted against the bed proximity parameter in Fig. 6. No appreciable changes is observed in the centerline peak's location with the variation of bed proximity parameter for 45° jets. For 30° jets, however, when $y_0/L_M \leq 0.14$, the normalized values of both the horizontal and vertical components of the centerline peak are reduced compared to the free condition. It is worth mentioning that the reduction in the horizontal component is notably higher than the vertical component. The transition criterion of $y_0/L_M \leq 0.14$ between free and boundary-affected 30° jets is in agreement with Shao and Law (2010), which reported $y_0/L_M \leq 0.15$ for boundary-affected jets. It is notable to mention, as seen in Fig. 6, the last case of series F, which has the least bed proximity parameter in the series, placed on the line extending from the results of series N. It seems that the bed proximity parameter y_0/L_M is the better deciding factor for the determination of the boundary influence in dense jets than y_0/d . It also means that both hydrodynamical and geometrical characteristics of discharge have a role in determining bed influence on the flow behavior.

(a)

(b)

(c)

(d)

Fig. 5 Normalized Centerline peak versus Fr_d : (a) horizontal component, 30° ; (b) horizontal component, 45° ; (c) vertical component, 30° ; (d) vertical component, 45°

(a)

(b)

Fig. 6 Normalized Centerline peak versus bed proximity parameter y_0/L_M : (a) horizontal component, 30°; (b) horizontal component, 45°; (c) vertical component, 30°; (d) vertical component, 45°

4.3 Terminal rise height

As discussed earlier, the dense jet rises due to its initial momentum until it reaches a maximum height. At this point, which is known as terminal rise height y_t , the vertical component of momentum equals zero. This height plays a key role in the design of brine outfalls as it can lead to undesirable visual impacts and distinctive reduction in dilution (Jiang et al. 2014; Abessi and Roberts 2016; Shrivastava and Adams 2019). There is a lack of consensus across past studies about the determination of terminal rise height. Lai and Lee (2012) considered the visual boundary as $0.25C_{max}$ concentration contour, which corresponds to the radial position where turbulent intermittency (the fraction of time that a point occupied by turbulent flow) is 0.5. The famous integral model of CorJet uses two cut-off levels of 3% and 25% for the visual boundary. In the present study, similar to Shao and Law (2010), the cut-off level of 3% is employed for the derivation of terminal rise height. The terminal rise height of both series is normalized with the jet diameter and plotted against Fr_d in Fig. 7. The numerical results are in acceptable agreement with previous experimental data and slightly lower than the analytical solution of Kikkert (2006).

In order to investigate the possible effects of bed proximity on terminal rise height, the numerical results are non-dimensionalized with L_M and plotted against the bed proximity parameter in Fig. 8. As seen in the figure, the terminal rise height is almost insensible to the variations of bed proximity parameter for both angles over the range investigated herein, which is in agreement with the experimental study conducted by Shao and Law (2010).

Fig. 7 Normalized terminal rise height versus Fr_d : (a) 30°; (b) 45°

Fig. 8 Normalized terminal rise height versus bed proximity parameter y_0/L_M : (a) 30°; (b) 45°

4.4 Horizontal distance of return point

After the initial rise of the dense jet, it falls back to the floor due to its negative buoyancy and finally impacts the bed at a location referred to as the impact point. As discussed by Roberts et al. (1997), the minimum dilution along the lower boundary occurs at the impact point. Beyond this point, the dilution increases until it reaches a maximum value, and then entrainment collapses due to the decay of turbulent fluctuations and relaminarization of the flow. They defined the end of the active mixing zone as the location where turbulent intensity falls below 5%. The ultimate dilution at the end of the near field is roughly 60% more than the impact point dilution (Roberts et al. 1997; Abessi and Roberts 2015). Additional mixing and dilution beyond the near field will be mostly because of ambient turbulence, which is significantly lower than the jet's initial mixing.

As mentioned above, the minimum dilution along the bed occurs at the impact point; hence, the dilution at the impact point is commonly used as an indicator of the negative impacts of brine discharge on benthic organisms. However, the impact point's location depends on nozzle elevation and bed slope, so it is site-specific. Therefore, various past studies (Shao and Law 2010; Christodoulou et al. 2015; Crowe et al. 2016; Papakonstantis and Tsatsara 2018) have investigated the return point — which is independent of nozzle height and bed slope — rather than the impact point for more generality. The return point is the location where the flow returns to the nozzle elevation. It is worth noting that in most

practical cases, the nozzle height and bed slope are typically small relative to the entire mixing zone. Thus, in these cases, the difference between the return and impact points would not be remarkable.

The horizontal distance of the return point after normalizing with the jet diameter is plotted against Fr_d in Fig. 9. Similar to the previously investigated geometrical characteristics, the numerical results are in good agreement with experimental data and slightly lower than the analytical model of Kikkert (2006). In Fig. 10, the derived dimensionless horizontal distances are plotted against the bed proximity parameter. It is not seen a meaningful change in horizontal distances with the variations of bed proximity parameter for both angles. While Shao and Law (2010) reported that in boundary-affected 30° cases, the flow returns to the discharge source elevation at a slightly closer distance compared to the free condition.

Fig. 9 Normalized horizontal distance of return point versus Fr_d : (a) 30°; (b) 45°

Fig. 10 Normalized horizontal distance of return point versus bed proximity parameter y_0/L_M : (a) 30°; (b) 45°

4.5 Cross-sectional concentration profiles and jet spread

Mean centerline concentration profiles at three locations along the jet trajectories for a selected case from each series are demonstrated in Fig. 11. The profiles are perpendicular to the flow path and are normalized as C/C_C against r/b_C where C_C , r , and b_C are maximum local concentration at the cross-section, the radial distance, and the radius where $C/C_C = 1/e$, respectively. These three sections are chosen so that the first section is in the jet-like region, the second one is at terminal rise height, and

the last one is in the plume-like region of the flow. As seen in the figures, in the jet-like region where the flow is momentum-dominated, profiles are almost coincident with the Gaussian profile. By distancing from the nozzle along the trajectory, the inner (lower) half of the jet starts to deviate from the Gaussian profile, while the outer (upper) half remains Gaussian. This deviation from the Gaussian profile is caused by detrainment in the lower half, known as buoyant instability. Destroying the jets' axial symmetry due to buoyant instabilities is also reported by Shao and Law (2010), Lai and Lee (2012), and Abessi and Roberts (2015).

Fig. 11 Mean concentration profiles perpendicular to the centerline: (a) 30°; (b) 45°

The $1/e$ width values of the upper half of the jets are extracted based on the normalized concentration profiles along the trajectory and plotted against the non-dimensional centerline length L_C at the corresponding cross-section in Fig. 12 (a). The gradient of jet width to path length is typically used to characterize the spread of dense jets. It can be seen from the figure that the predicted growth rate is slightly higher than the reported value of Lai and Lee (2012) and lower than Jiang et al. (2014). However, as mentioned above, the cross-sectional profiles along the trajectory become asymmetric by distancing away from the source. Consequently, the inner and outer spread of dense jets is not equal to each other throughout the flow. The inner (lower) and outer (upper) widths of discharge and path length

are normalized with $d.Fr_d$ and plotted against each other in Fig. 12 (b). The experimental results of Crowe et al. (2016) are also added for comparison. As can be seen from the figure, the inner and outer widths are almost identical near the source (from nozzle tip until about terminal rise height). Beyond this region, the inner and outer widths start to deviate from each other. The predicted lower widths are in good agreement with experimental data, but the upper width results are slightly bigger than experimental observations.

Fig. 12 Normalized discharge width against normalized path length

4.6 Return point dilution

In order to investigate the possible effects of proximity to bed on return point dilution, the dilution values are normalized with corresponding densimetric Froude number and plotted against bed proximity parameter in Fig. 13. It is seen that the dilution predictions are close to the experimental value of Lai and Lee (2012) and considerably lower than Shao and Law (2010) as well as Abessi and Roberts (2015). The conservative dilution predictions of RANS models are also reported by Zhang et al. (2017) and Robinson et al. (2016). This may be attributed to considering a constant turbulent Schmidt number throughout the fluid flow and the SGD model in which the scalar flux vector is aligned with the mean scalar gradient vector (Pope 2000; Lai and Socolofsky 2019). As a result of the latter, the SGD model is known to inaccurately predict the turbulent effects even in some simple turbulent flows (especially

highly anisotropic flows) (Combest et al. 2011). It is not observed noticeable variation in return point dilution with changes in bed proximity parameter for 45° jets. For 30° jets, however, it can be concluded that boundary-affected cases have a slightly lower dilution compared to free jets, which is in agreement with Shao and Law (2010). It is worth noting that the amount of reduction in their study is more than the present numerical predictions. The decline in return point dilution may be due to reduced entrainment of ambient fluid in the lower boundary.

Fig. 13 Return point dilution versus bed proximity parameter y_0/L_M : (a) 30°; (b) 45°

Table 2 Summary of numerical results with previous experimental data for 30° jets

Quantity	Proportionality coefficients	Present study		Shao and Law (2010)		Lai and Lee (2012)	Kikkert et al. (2007)			Abessi and Roberts (2015)
		0.06 $\leq y_0/L_M \leq 0.14$	$y_0/L_M > 0.14$	0.10 $\leq y_0/L_M \leq 0.15$	$y_0/L_M > 0.15$		LA data	LIF data	Model prediction	
Terminal rise height	$y_t/d.Fr_d$	0.91	0.95	1.05	-	0.95	1.00	1.19	1.02	1.17
Horizontal distance of return point	$x_r/d.Fr_d$	3.01	2.97	2.88	3.00	3.18	3.14	3.44	2.95	-
Vertical location of centerline peak	$y_m/d.Fr_d$	0.55	0.59	0.66	-	0.65	0.56	0.66	0.62	0.79
Horizontal location of centerline peak	$x_m/d.Fr_d$	1.50	1.61	1.70	1.54	1.95	1.75	1.85	1.70	-
Return/imp act point dilution	s/Fr_d	0.65	0.71	1.18	1.45	0.82	-	-	-	1.20

Table 3 Summary of numerical results with previous experimental data for 45° jets

Quantity	Proportionality coefficients	Present study	Shao and Law (2010)	Lai and Lee (2012)	Kikkert et al. (2007)			Abessi and Roberts (2015)
		$y_0/L_M \geq 0.08$	$y_0/L_M \geq 0.05$		LA data	LIF data	Model prediction	
Terminal rise height	$y_t/(d.Fr_d)$	1.47	1.47	1.58	1.60	-	1.61	1.80
Horizontal distance of return point	$x_r/(d.Fr_d)$	3.02	2.83	3.34	3.26	-	3.04	-
Vertical location of centerline peak	$y_m/(d.Fr_d)$	1.03	1.14	1.19	1.06	-	1.13	1.33
Horizontal location of centerline peak	$x_m/(d.Fr_d)$	1.65	1.69	2.09	1.84	-	1.88	-
Return/impact point dilution	s/Fr_d	0.82	1.26	1.09	-	-	-	1.60

5 Summary and conclusions

Using marine outfalls in the form of a dense jet is a common way to mitigate the environmental impacts of brine discharge from seawater desalination plants into the coastal waters. The main objective of the present study was to numerically investigate the mixing and geometrical characteristics of 30° and 45° inclined dense jets when they are placed at a close distance to the lower boundary. This condition may occur when, in addition to reducing the nozzle angle, it is necessary to reduce the nozzle height to prevent surface contact in shallow waters. For this purpose, two numerical series were performed by modifying a solver in the CFD package of OpenFOAM. In the first series, series F, the nozzle was relatively far from the bed to act as a free jet. While in the second series, series N, the distance of the nozzle to the lower boundary was substantially reduced. Consequently, the effects of proximity to the bed became possible by comparing the latter's results with reference cases. The following conclusions can be drawn from the present study:

1. The bed proximity influences on the flow behavior appeared to have dependence not only on the nozzle height but also on its angle, and it is more significant in smaller nozzle angles. While proximity to the bed had almost no appreciable effects on the flow behavior for 45° jets, some changes in both the geometrical and mixing characteristics were observed for 30° jets.
2. The bed proximity parameter, y_0/L_M , was found to be the controlling parameter for bed influence rather y_0/d . For 30° jets, the influences of proximity to the lower boundary became evident when $y_0/L_M \leq 0.14$.
3. Consistent with previous experimental studies, it was found that the geometrical and mixing characteristics of dense jets, including centerline peak, terminal rise height, the horizontal distance of return point, and the return point dilution, have a correlation with the densimetric Froude number. The proportionality coefficients were determined and presented in Tables 2 and 3.

4. In boundary-affected jets, the normalized values of the centerline peak's horizontal and vertical locations were declined compared to their counterparts in free jets. Simultaneously, the terminal rise height and the horizontal distance of the return point were almost untouched. The noted observations are in agreement with Shao and Law (2010) except for the horizontal distance of the return point.
5. The dilution predictions appeared to be conservative for both the free and boundary-affected jets. This also has been reported in various past numerical studies that employed the RANS approach (e.g., Zhang et al. (2017) and Robinson et al. (2016), and Tahmooreesi and Ahmadyar(2021)).
6. A slight reduction in the return point dilution was observed for boundary-affected jets, consistent with Shao and Law (2010). However, they reported more reduction amount in comparison to the present numerical simulations.

Declarations

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Availability of data and material Some or all data are available from the first author by request.

Code availability Code generated or used during the study are available from the first author by request.

Authors' contributions All authors contributed to the study. Material preparation, data collection and analysis were performed by Mohammadmehdi Ramezani, Ozeair Abessi and Ali Rahmani Firoozjae. The first draft of the manuscript was written by Mohammadmehdi Ramezani and all authors commented on previous versions of the manuscript. All authors read and approved the final manuscript.

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Figures

Figure 1

A schematic side view of an inclined dense jet

Figure 2

Computational domain: (a) symmetry plane; (b) isometric view

Figure 3

Non-dimensional central plane tracer concentrations

Figure 4

Normalized centerline trajectories: (a) 30°; (b) 45°

Figure 5

Normalized Centerline peak versus Fr_d : (a) horizontal component, 30°; (b) horizontal component, 45°; (c) vertical component, 30°; (d) vertical component, 45°

Figure 6

Normalized Centerline peak versus bed proximity parameter y_0/L_M : (a) horizontal component, 30°; (b) horizontal component, 45°; (c) vertical component, 30°; (d) vertical component, 45°

Figure 7

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Figure 8

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Figure 9

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Figure 10

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Figure 11

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(a)

(b)

Figure 12

Normalized discharge width against normalized path length

Figure 13

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