Tunable Nonlinear Band Gaps in a Sandwich-Like Meta-Plate

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Tunable nonlinear band gaps in a sandwich-like meta-plate

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Abstract

This paper aims to explore the actual working mechanism of sandwich-like meta-plates by periodically attaching nonlinear mass-beam-spring (MBS) resonators for low-frequency wave absorption. The nonlinear MBS resonator consists of a mass, a cantilever beam and a spring that can provide negative stiffness in the transverse vibration of the resonator, and its stiffness is tunable by changing the parameters of the spring. Considering the nonlinear stiffness of the resonator, the energy method is applied to obtain the dispersion relation of the sandwich-like meta-plate and the band-gap bounds related to the amplitude of resonator is derived by dispersion analysis. For the finite sized sandwich-like meta-plate with the fully free boundary condition subjected to external excitations, its dynamic equation is also established by the Galerkin method. The frequency response analysis of the meta-plate is carried out by the numerical simulation, whose band-gap range demonstrates good agreement with the theoretical one. Results show that the band-gap range of the present meta-plate is tunable by the design of the structural parameters of the MBS resonator. Furthermore, by analyzing the vibration suppression of the finite sized meta-plate, it can be observed that the nonlinearity of resonators can widen the wave attenuation range of meta-plate.

Keywords: Meta-plate; Nonlinear band gaps; Wave transmittance; MBS resonator

1. Introduction

Over the past decades, the investigations on phononic crystals have attracted considerable attention due to their scattering mechanism (Bragg scattering) [1-4], which means that the wave scattering at the periodic lattice interface creates frequency bands that stop the propagation of phonons/photons. On this basis, acoustic metamaterials, which are the artificially periodic structures with peculiar properties, e.g. noise reduction [5-6], elastic wave cloaking [7-8] and band-gaps [9-10], were proposed and became a research hotspot because of a number of potential applications. Generally, acoustic metamaterials are considered as a smart design of composite structures to realize manipulation and actuation for mechanical waves [11-13].

Comparing to phononic crystals devised to achieve wave manipulation by Bragg scattering, locally resonant metamaterials comprised of unit cells with subwavelength dimensions can guide and control wave propagation by the locally resonant properties [12, 14-16], which are not rely on design periodicity. In locally resonant metamaterials, the band-gaps yielding low-frequency wave attenuation are generally located within the resonant frequency of resonators, which is influenced by the inner attached mass and restoring force [17]. Note that the wave transmission properties of metamaterials are commonly independent of the constituent materials, but on the coupling characteristics between artificial microstructure units and elastic waves [18-20]. Therefore, the generation and adjustability of locally resonant band-gaps are attracting increasing interest. Furthermore, the various methods, including the PWE method [21-22], the harmonic balance method [23-24], the finite element method [21, 25-26] and the transfer matrix method [27-28], were

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1 applied to calculate the band gap and wave attenuation of locally resonant metamaterials. In addition,
2 the numerical simulation [14,15,21,29] and experiments [18,20,30] were also utilized to reveal the
3 wave propagation characteristics in locally resonant metamaterials.

4 In order to clarify the problem easily, the linear local resonators [21,23,31-34] are always
5 applied to analyze the band-gap properties of structures. Nevertheless, in such linear design, the
6 resulting band-gap range is limited by the requirement of lightweight and compact designs and
7 application to large-amplitude excitation. Especially, due to the added mass constraints, it is difficult
8 for linear meta-plates to achieve both low frequency and wide band-gap for vibration suppression,
9 and the linear resonators increase the number of resonance peaks in the passband of the finite
10 structure. These factors limit the application of locally resonant meta-plates in the new devices and
11 new structures.

12 On the other hand, to realize the further application of metamaterials, it is inevitable to analyze
13 structural nonlinearity. The nonlinear metamaterials are the artificially periodic structures with
14 nonlinear dynamic properties [35-36], such as nonlinear resonance, internal resonance, modal
15 coupling, and chaos, which are expected to provide more control mechanisms for the nonlinear
16 elastic wave. Currently, metamaterial nonlinearity is attracting increasing attention [28,37-41].
17 Wang et al [42] designed a metamaterial plate with high-static-low-dynamic-stiffness resonators and
18 explored the nonlinear frequency response of meta-plate under the excitation with different
19 amplitudes. Fang et al [43] presented the chaotic band-gaps of nonlinear acoustic metamaterials and
20 achieved the ultralow- and ultrabroadband wave suppressions. Emerson and Manimala [44] utilized
21 the tunability of amplitude-dependent response of nonlinear local resonator to manipulate the
22 direction and bandwidth of wave propagation in the metamaterials. Based on the chaotic vibrations
23 of the bistable resonators under large-amplitude excitation, Xia et al [45] investigated the nonlinear
24 wideband attenuation in a metamaterial beam, whose validity was also demonstrated by experiment.
25 These investigations shed light on the design of nonlinear metamaterials to adjust the band gap
26 range. However, to our best knowledge, the mathematical formulations and numerical simulation
27 on the wave propagation characteristics in the sandwich-like meta-plate with periodically attached
28 nonlinear resonators are rare.

29 Our previous work [46] investigated the wave propagation properties of meta-plate with linear
30 resonators (mass-beam type). The present work aims to study the influence of the nonlinear
31 resonator (mass-beam-spring type) on the band-gap characteristics of the sandwich-like meta-plate.
32 The stiffness of MBS resonator is tunable by changing the compressibility of the spring at the
33 equilibrium position. Considering the nonlinear stiffness of the resonator and using the energy
34 method, the band-gap bounds related to the amplitude of resonator is derived by dispersion analysis.
35 The dynamic equation of the sandwich-like meta-plate under external excitations is also established
36 by the Galerkin method. The wave propagation properties of the finite sized meta-plate are
37 investigated by the finite element simulation and Galerkin method, whose band-gap range is
38 compared with the theoretical one. The present research is helpful to design meta-plate for very low-
39 frequency vibration control.

2. Mathematical formulations

2.1 Design concept

Fig. 1 (a) Schematic diagram of the sandwich-like meta-plate with periodically attached MBS resonators. (b) The unit cell of the meta-plate. (c) Physical prototype of MBS resonator in a static equilibrium state. (d) The deformation of MBS resonator under restoring force.

Fig. 1(a) shows the schematic diagrams of the sandwich-like meta-plate with nonlinear resonators, where the two faceplates are connected by several vertical rods and the nonlinear MBS resonators are attached periodically on the vertical rods. The unit cell of the metamaterial plate in Fig. 1(b) shows that the nonlinear resonator is made of a cantilever beam, a mass and a spring, which can provide negative stiffness in the transverse vibration of the resonators. When the resonator is in a static equilibrium state, corresponding to zero external force, the spring and cantilever beam sit in the same horizontal plane, as shown in Fig. 1(c). It is assumed that the mass can only oscillates in the vertical direction, Fig. 1(d) shows the deformation of the MBS resonator under restoring force, which is given as follows:

$$f_r = \frac{3E_b I_b}{l^3} w_r - k \left(\frac{a_0}{1-\eta} - \sqrt{a_0^2 + w_r^2} \right) \frac{w_r}{\sqrt{a_0^2 + w_r^2}} \quad (1)$$

where l , E_b and I_b are the length, elastic modulus and moment of inertia of the cantilever beam, respectively. w_r is the transverse displacement of the mass. a_0 is the length of the spring at $w_r = 0$. k is the stiffness of the spring. η ($0 \leq \eta < 0.6$) is the pre-compression ratio of the spring, which is defined as the proportion of the compressed length ($L-a$) of the spring at $w_r = 0$ to the original length (L) of the spring. It should be pointed out that when $\eta < 0$, the spring will always be in tension, this case is not considered in this article. The stiffness of the nonlinear resonator is derived by differentiating Eq. (1) with respect to w_r as below:

$$k_r = P \left(1 - \mu \left(\frac{1}{(1-\eta)(1+(w_r/a_0)^2)^{3/2}} - 1 \right) \right) \quad (2)$$

in which $P = 3E_b I_b / l^3$, called the cantilever stiffness, and $\mu = k/P$ is the ratio of spring stiffness to

cantilever stiffness. Using Taylor series at zero equilibrium, Eq. (2) can be presented as:

$$K_r = \beta_0 + \beta_1 w_r^2 + o \quad (3)$$

where

$$\beta_0 = P \left(1 - \mu \left(\frac{1}{(1-\eta)} - 1 \right) \right) \quad (4)$$

$$\beta_1 = \frac{3\mu P}{2(1-\eta)a_0^2} \quad (5)$$

2.2 Dispersion relation of infinite sized meta-plate with MBS resonators

Fig. 2 A unit cell of the meta-plate with MBS resonators: (a) front view and (b) side view.

In this section, a unit cell shown in Fig. 2 is applied to analyze dispersion relation of the infinite sized sandwich-like meta-plate with nonlinear resonator. As for the MBS resonator, the deformation of cantilever beam is assumed to be linear elastic and the weights of the spring and the cantilever beam are also neglected. According to Newton's second law, the governing equation of the nonlinear resonator is expressed as:

$$m\ddot{w}_r = K_r (w_r - w_0) = (\beta_0 + \beta_1 w_r^2) (w_r - w_0) \quad (6)$$

in which w_0 is the transverse displacement of the faceplate at the vertical rods position, corresponding to the coordinate origin, w_r is the resonator's displacement.

For the free vibration of the nonlinear resonator without counting the effect of the resonator on the faceplates, Eq. (6) can be rewritten as following form:

$$\ddot{w}_r + \omega_0^2 (w_r + \varepsilon w_r^3) = 0 \quad (7)$$

where $\omega_0^2 = \beta_0/m$, $\varepsilon = \beta_1/\beta_0$. Based on the Harmonic Balance Method (HBM) [35], the amplitude-related natural frequency ω_r for a single-degree-of-freedom nonlinear resonator is as follows:

$$\omega_r^2 = \omega_0^2 \left(1 + \frac{3}{4} \varepsilon r^2 \right) \quad (8)$$

where r is the amplitude of the resonator.

As for two thin faceplates, the strain-displacement relations based on the classical plate theory are expressed as follows:

$$\{\varepsilon_x, \varepsilon_y, \gamma_{xy}\}^T = \left\{ -z \frac{\partial^2 w}{\partial x^2}, -z \frac{\partial^2 w}{\partial y^2}, -2z \frac{\partial^2 w}{\partial x \partial y} \right\}^T \quad (9)$$

where w is the transverse displacement of the faceplates.

The constitutive relations of faceplates are presented as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \quad (10)$$

where

$$Q_{11} = Q_{22} = \frac{E}{1-\nu^2}, Q_{12} = Q_{21} = \frac{\nu E}{1-\nu^2}, Q_{66} = \frac{E}{2(1+\nu)} \quad (11)$$

in which E and ν are Young's modulus and Poisson's ratio of the faceplate, respectively. Then the elastic energy U and kinetic energy T of the faceplates can be defined as:

$$U = \frac{1}{2} \int_V (\varepsilon_x \sigma_x + \varepsilon_y \sigma_y + \tau_{xy} \gamma_{xy}) dV \quad (12)$$

$$T = \frac{1}{2} \int_V \rho \dot{w}^2 dV \quad (13)$$

where ρ and V are the density and volume of the faceplates. In the vertical rod position, the non-conservative work W are given by:

$$W = K_r (w_r - w_0) w_0 = (\beta_0 + \beta_1 w_r^2) (w_r - w_0) w_0 \quad (14)$$

When the elastic waves propagate in the infinite sized meta-plate composed of the above periodic unit cell, w , w_0 and w_r are defined in the form of the exponential function as:

$$w = q e^{j(\alpha x + \beta y - \omega t)}, w_0 = q e^{-j\omega t}, w_r = r e^{-j\omega t} \quad (15)$$

in which q is the displacement amplitude of meta-plate, ω is the wave frequency. The wavenumbers along the x and y axis are the $\alpha = 2\pi/\lambda_1$ and $\beta = 2\pi/\lambda_2$ where λ_1 and λ_2 are the corresponding wavelengths.

Based on the following Hamilton's principle:

$$\int_{t_1}^{t_2} \delta(T - U) dt + \int_{t_1}^{t_2} \delta W dt = 0 \quad (16)$$

Substituting Eqs. (12)-(14) into Eq. (16), then substituting Eq. (15) into Eq. (6) and (16), the following results are presented:

$$\begin{bmatrix} \frac{8 \sin(\alpha a / 2) \sin(\beta b / 2)}{\alpha \beta} \left[\rho h \omega^2 - \frac{h^3}{12} Q_{11} (\alpha^2 + \beta^2)^2 \right] - K_r & K_r \\ K_r & m \omega^2 - K_r \end{bmatrix} \begin{Bmatrix} q \\ r \end{Bmatrix} = 0 \quad (17)$$

Eq. (17) shows that the non-zero solutions of q and r exist only if the determinant of the matrix is zero:

$$\frac{8 \sin(\alpha a / 2) \sin(\beta b / 2)}{\alpha \beta} \left(\rho h \omega^2 - \frac{h^3}{12} Q_{11} (\alpha^2 + \beta^2)^2 \right) \left(\omega^2 - \frac{K_r}{m} \right) - \frac{K_r}{m} m \omega^2 = 0 \quad (18)$$

where K_r/m is taken as ω_r^2 . Substituting Eq. (8) into Eq. (18) and the dispersion equation of the

infinite sized meta-plate with nonlinear resonators is expressed as:

$$\frac{8 \sin(\alpha a / 2) \sin(\beta b / 2)}{\alpha \beta} \left(\rho h \omega^2 - \frac{h^3}{12} Q_{11} (\alpha^2 + \beta^2)^2 \right) \left(\omega^2 - \omega_0^2 \left(1 + \frac{3}{4} \varepsilon r^2 \right) \right) - \left(1 + \frac{3}{4} \varepsilon r^2 \right) \omega_0^2 m \omega^2 = 0 \quad (19)$$

The wave frequency ω related to α and β is obtained by solving Eq. (19). The lower bound of the band-gap corresponds to wave frequency ω with α and $\beta \rightarrow \infty$ and the upper bound of the band-gap corresponds to wave frequency ω with α and $\beta \rightarrow 0$. Divide the wave frequency of the band-gap bounds by 2π . The band-gap is presented as:

$$\left(\frac{\omega_0}{2\pi} \sqrt{1 + \frac{3}{4} \varepsilon r^2}, \quad \frac{\omega_0}{2\pi} \sqrt{\left(1 + \frac{3}{4} \varepsilon r^2 \right) (1 + \gamma)} \right) \quad (20)$$

where ω_0 is the natural frequency corresponding to linear stiffness, $\gamma = m/2abhp$ is the mass ratio of the resonator and faceplates. Eqs. (3)-(5) and Eq. (20) show that the lower bound and width of band-gap can be modified by adjusting parameters η , μ and γ . Moreover, it is noted that the band-gap bound of the infinite sized meta-plate with nonlinear resonators is related to the resonator amplitude and as the amplitude increases, the lower bound of band-gap moves to higher frequency.

2.3 Dynamic model of finite sized meta-plate with MBS resonators

According to the Galerkin method, the transverse displacement of the faceplates at this point (x, y) shows as

$$w(x, y, t) = \sum_{i=1}^m \sum_{j=1}^n \xi_{ij}(x, y) p_{ij}(t) = \xi^T(x, y) \mathbf{p}(t) \quad (21)$$

where m and n are the order of the eigenfunction, $\mathbf{p}(t)$ is generalized coordinate vector, $\xi(x, y)$ is the displacement shape function which is assumed to satisfy the fully free boundary condition [42] and it is given by

$$\xi_{ij}(x, y) = \Phi_i(x) \Psi_j(y) \quad (22)$$

where

$$\Phi_i(x) = \sin \gamma_{xi} x + \sinh \gamma_{xi} x + \frac{\sin \gamma_{xi} L_x - \sinh \gamma_{xi} L_x}{\cosh \gamma_{xi} L_x - \cos \gamma_{xi} L_x} (\cos \gamma_{xi} x + \cosh \gamma_{xi} x) \quad (23)$$

$$\Psi_j(y) = \sin \gamma_{yj} y + \sinh \gamma_{yj} y + \frac{\sin \gamma_{yj} L_y - \sinh \gamma_{yj} L_y}{\cosh \gamma_{yj} L_y - \cos \gamma_{yj} L_y} (\cos \gamma_{yj} y + \cosh \gamma_{yj} y) \quad (24)$$

in which the eigenvalue γ_{xi} and γ_{yj} can be obtained by solving the following eigenequations:

$$\begin{cases} \cos \gamma_{xi} L_x + \cosh \gamma_{xi} L_x = 1, & i = 1, 2, 3, \dots \\ \cos \gamma_{yj} L_y + \cosh \gamma_{yj} L_y = 1, & j = 1, 2, 3, \dots \end{cases} \quad (25)$$

where L_x and L_y denote the length and width of the meta-plate. In order to obtain the governing equations of the meta-plate subjected to external excitation $f_q(t)$, substituting Eqs. (21)-(24) into Eq. (16) and combining Eq. (6), the detail expressions can be given by:

$$\begin{cases} \mathbf{M}\ddot{\mathbf{p}}(t) + \mathbf{K}\mathbf{p}(t) = \boldsymbol{\xi}^T(x_0, y_0)f_q(t) + \sum_{r=1}^R \boldsymbol{\xi}^T(x_r, y_r)f_r(x_r, y_r, t) \\ m\ddot{w}_r(t) + f_r(x_r, y_r, t) = 0 \end{cases} \quad (26)$$

where $\mathbf{M}$ and $\mathbf{K}$ are the mass matrix and stiffness matrix, respectively.

The restoring force $f_r(x_r, y_r, t)$ caused by the resonator at point (x_r, y_r) is presented as

$$f_r(x_r, y_r, t) = \frac{3E_b I_b}{l^3} [w_r(t) - \boldsymbol{\xi}^T(x_r, y_r)\mathbf{p}(t)] - k \left(\frac{1}{(1-\eta) \left(1 + \left((w_r(t) - \boldsymbol{\xi}^T(x_r, y_r)\mathbf{p}(t)) / a_0 \right)^2 \right)^{3/2}} - 1 \right) [w_r(t) - \boldsymbol{\xi}^T(x_r, y_r)\mathbf{p}(t)] \quad (27)$$

Considering the damping of the resonator and the modal damping of the faceplates, Eq. (26) can be rewritten as

$$\begin{cases} \mathbf{M}\ddot{\mathbf{p}}(t) + \mathbf{C}\dot{\mathbf{p}} + \mathbf{K}\mathbf{p}(t) = \boldsymbol{\xi}^T(x_0, y_0)f_q(t) + \sum_{r=1}^R \boldsymbol{\xi}^T(x_r, y_r)f_d(x_r, y_r, t) \\ \quad + \sum_{r=1}^R \boldsymbol{\xi}^T(x_r, y_r)f_r(x_r, y_r, t) \\ m\ddot{w}_r(t) + f_d(x_r, y_r, t) + f_r(x_r, y_r, t) = 0 \end{cases} \quad (28)$$

By introducing structural damping, the transient response can quickly converge in the numerical simulation. $\mathbf{C}$ and $f_d(x_r, y_r, t)$ are given by

$$\mathbf{C} = 2\zeta\sqrt{\mathbf{M}\mathbf{K}}, \quad f_d(x_r, y_r, t) = 2\zeta_0\sqrt{m\beta_0} [\dot{w}_r(t) - \boldsymbol{\xi}^T(x_r, y_r)\dot{\mathbf{p}}(t)] \quad (29)$$

in which ζ is the modal damping ratio, and ζ_0 is damping ratio of the resonator. Note that the generalized coordinate $\mathbf{p}(t)$ can be obtained by solving Eq. (28), then using Eq. (21), the steady-state response of the meta-plate under external force f_q is presented.

3. Results and discussions

3.1 Verification of the band-gap properties

Table 1 Material properties of the sandwich-like meta-plate.

Components	Material	E (Gpa)	ρ (kg/m ³)	ν
Faceplate/Cantilever beam	Aluminium	70	2700	0.3
Mass	Copper	110	8960	0.3
Screw	304 Steel	210	7780	0.3

Table 2 Geometric parameters of the sandwich-like meta-plate.

Geometric parameters	Notation	Value
l	Effective length of the cantilever beam	0.035m
h_b	Thickness of the cantilever beam	0.001m
l_b	Width of the cantilever beam	0.005m
a	Length of the unit-cell	0.06m
b	Width of the unit-cell	0.03m
h_c	Length of the screw	0.025m
h	Thickness of the faceplate	0.0003m
$n_x(n_y)$	Unit-cell number along $x(y)$ axis	8
m	Mass of the resonator	0.0086kg
k	Spring stiffness	2700N/m
a_0	Length of the deformed horizontal spring at $z = 0$	0.02m
ζ	Model damping ratios	0.02
ζ_0	Damping ratio of resonator	0.01
q_0	Amplitude of harmonic excitation	1N

In this section, the numerical simulations based on Finite Element Method (FEM) and Galerkin Method are carried out to demonstrate the validity of theoretical band-gap. The material properties and geometrical parameters of sandwich-like meta-plate are listed in Table 1 and Table 2. The numerical simulation model of the sandwich-like meta-plate with 8×8 unit cells is presented in Fig. 3. It is assumed that a harmonic excitation $f_q = q_0 \sin \omega t$ with a bandwidth from 1 to 300 Hz acts on the origin of the meta-plate (P_s in Fig. 3), whose response is adopted as the input signal, and the time history of displacement for the test point P_r in Fig. 3 is chosen as the output signal. The wave attenuation property is estimated by wave transmittance in decibel (dB), which is defined as

$$T_r = 20 \log \frac{|q_{out}|}{|q_{in}|} \quad (30)$$

In the numerical simulation based on FEM, the COMSOL Multiphysics® 5.5 is applied to perform the wave transmittance analysis of the finite sized meta-plate with opposite boundary completely clamped support and others free. Note that the q_{in} and q_{out} are the input acceleration amplitude and the output acceleration amplitude, respectively. The wave transmission curves are shown by the solid red lines in Fig. 4. As for the numerical simulation based on Galerkin Method, the q_{in} and q_{out} shown in Eq. (30) are the input and output root-mean-squared (RMS) displacement of the response obtained by solving Eq. (28) using the Runge–Kutta method at a certain frequency. Take into account the damping and nonlinear stiffness of the resonator, the solid blue lines shown

in Fig. 4 present the transmission spectrums, where the harmonic excitation amplitude q_0 is take as 1N.

It can be seen from Fig. 4 that the sharp drop and increase appear in the frequency region with the minus transmittance in dB, and the corresponding frequency range represents a band-gap. To demonstrate the validity of present theoretical band-gaps, the band-gap ranges (gray shaded area) obtained by dispersion analysis (Eq. (20)) are given as the comparison in Fig. 4. Note that the amplitude r is taken as 0.001m in Eq. (20). Clearly, the attenuation ranges of the measured transmission spectra are in excellent agreement with the theoretical band-gaps predicted by the Eq. (20). Consequently, the theoretical band-gaps of meta-plate can be demonstrated by the wave transmittance using the FEM and Galerkin Method.

Fig. 3 Numerical simulation model of the sandwich-like meta-plate with 8×8 unit cells.

Fig. 4 Comparisons of transmission spectrums obtained by finite element method and Galerkin method and band-gap obtained by theoretical analysis.

3.2 Dispersion properties

Fig. 5 Comparisons of dispersion surfaces and band-gaps: (a) and (b) denote results of the meta-plate, (c) and (d) denote results of the plain sandwich plate.

Fig. 6 Comparisons of linear ($\beta_1 = 0$) and nonlinear ($\beta_1 \neq 0$) band-gaps.

In the proposed infinite sized meta-plate with nonlinear resonator, a unit cell shown in Fig. 2 is used to analyze dispersion relation. It is noted from Eq. (19) that the wave frequency ω is related to each pair of α and β . The material properties and geometrical parameters of the unit cell for dispersion analysis are tabulated in Table 1 and Table 2.

Fig. 5(a) and Fig. 5(c) shows the 3D plots of the dispersion surfaces in terms of the wave frequency ω as wave number α and β . According to Eq. (19), two real and positive values of ω can be solved for each pair of α and β , corresponding to two dispersion surfaces in Fig. 5(a). Meanwhile, Fig. 5(b) and Fig. 5(d) present the frequency value corresponding to four edges (**M-Γ-II-X-M**) of dispersion surfaces. It is clearly observed from Fig. 5(b) that the band-gap among the two dispersion curves is 63.6-126.5Hz, which means that the structural vibration is suppressed in this frequency range, where the pre-compression ratio η is 0.2 and the resonator amplitude r is 0.001m. However, for the sandwich plate without resonators, there is only one dispersion curve, as shown in Fig. 5(d). The reason is that when the harmonic wave propagates in the plain sandwich plate, the faceplate is self-dispersive as the wave speed changes. It is noted from Eq. (3) that the stiffness of resonator is composed of linear term and nonlinear term. Eq. (20) shows that the dispersion solutions of nonlinear meta-plate depend on the resonator amplitude r . Therefore, the linear ($\beta_1=0$) and nonlinear ($\beta_1 \neq 0$) dispersion curves are shown in Fig. 6. For the nonlinear meta-plate with MBS resonators, as resonator amplitude increases, the band-gap range shifts to high frequency, which means that the nonlinearity of the resonator can realize the band-gap adjustment of the meta-plate.

3.3 Band-gap adjustment by structural parameters

In Section 3.2, a unit cell of infinite sized meta-plate is analyzed to predict the dispersion relation. In this section, for the finite sized meta-plate with periodically attached MBS resonators, the effect of structural parameters on the wave transmittance is investigated. Considering the nonlinear stiffness of the MBS resonator, the governing equations of the sandwich-like meta-plate subjected to external excitation are derived by the Galerkin method in Section 2.3. The material properties and geometrical parameters of Section 3.2 are used to be consistent with parameters applied in the following finite sized meta-plate. The four edges of finite sized meta-plate are the fully free according to the shape function expressions in Eqs. (23)-(24). The numerical simulation model of the meta-plate with 8×8 unit cells is the same as the model in Section 3.1, as shown in Fig. 3.

Firstly, the effect of structural parameters on the nonlinear stiffness of MBS resonator is analyzed. Based on Eq. (2) and using k_r/P , the dimensionless expression of resonator stiffness is presented as follows:

$$\bar{k}_r = 1 - \mu \left(\frac{1}{(1-\eta) \left(1 + \frac{1}{(1-\eta)^2} \bar{w}_r^2 \right)^{3/2}} - 1 \right) \quad (31)$$

where $\bar{k}_r = k_r/P$, $\bar{w}_r = w_r/L$.

The relationship between $\bar{k}_r$ and μ, η is present in Fig. 7 at the equilibrium position $\bar{w}_r = 0$. It is noted from Fig. 7(b) that the curve L_1 , corresponding to $\bar{k}_r = 0$, divides the projection of $\bar{k}_r$ on xy -plane into two areas S_1 ($\bar{k}_r \geq 0$) and S_2 ($\bar{k}_r \leq 0$). In order to ensure the effectiveness of nonlinear resonator stiffness, the values of μ and η should be taken in the S_1 area. Moreover, the effects of the pre-compression ratio of the spring and non-dimensional transverse displacement of the mass on the non-dimensional stiffness of nonlinear resonator are shown in Fig. 8. From the 3D plot, one can

find that the stiffness of resonator decreases as the pre-compression ratio η increases at $\bar{w}_r = 0$ and the minimum stiffness of resonator is obtained at the equilibrium position. Then, figure 9 shows the non-dimensional stiffness of nonlinear resonator with different ratio μ . It can be observed that when the value of ratio μ is taken as 0, corresponding to mass-beam linear resonator [46], the stiffness of nonlinear resonator remains constant with the change of pre-compression ratio η and transverse displacement $\bar{w}_r$. However, in Fig. 9 (a), as μ increases to 1.5, the stiffness of resonator can be conveniently arranged to any values from zero to the stiffness of the cantilever by changing the pre-compression ratio η at $\bar{w}_r = 0$, which means that the novel sandwich-like meta-plate with the MBS resonators can obtain a lower band-gap than that with mass-beam linear resonator. It is also noted from Fig. 9(b) that the smaller μ , the weaker the nonlinear property of the MBS resonator. These results are helpful for the design of the proposed nonlinear resonator in the numerical simulations.

Based on the nonlinear stiffness analysis of MBS resonator, the effects of η and μ on the wave transmission obtained by Galerkin method ($q_0=1\text{N}$) and band-gap obtained by dispersion analysis ($\beta_1 \neq 0$, $r=0.001\text{m}$) are investigated, the results are shown in Fig. 10 and Fig. 11. Note that the ratio μ is taken as 1.323 in Fig. 10 and the pre-compression ratio η is taken as 0.3 in Fig. 11. It is observed from Fig. 10 that as the pre-compression ratio η increases, the width of the band-gap becomes narrow and the lower bound of band-gap move from high-frequency region to a low one. Moreover, the wave attenuation range shifts to low frequency when the ratio μ increases, as shown in Fig. 11. The above band-gap change is the same as the change of the resonator stiffness with μ and η , as shown in Fig. 7(b). As is well known, the band gap of the locally resonant metamaterials is determined by the stiffness of the resonator. Therefore, for the nonlinear sandwich-like meta-plate with MBS resonators, the tunable feature of band-gaps can be achieved by the design of the structural parameters of the MBS resonator.

Fig. 7 The stiffness adjustment diagram of nonlinear resonator at the equilibrium position $\bar{w}_r = 0$. (a) 3D plot; The projections of $\bar{k}_r$ on (b) xy -plane

Fig. 8 3D plot of the dimensionless stiffness of the nonlinear resonator with respect to dimensionless displacement and pre-compression ratio of the spring. ($\mu = 2/3$)

Fig. 9 (a) The dimensionless stiffness $\bar{k}_r$ vs. dimensionless displacement $\bar{w}_r$; (b) The dimensionless stiffness $\bar{k}_r$ vs. pre-compression ratio η of the spring.

Fig. 10 Effect of pre-compression ratio η on transmission spectrums obtained by Galerkin method ($q_0=1\text{N}$) and band-gap obtained by theoretical analysis (gray shaded area, $\beta_1 \neq 0$, $r=0.001\text{m}$). (a) $\eta = 0$, (b) $\eta = 0.3$, (c) $\eta = 0.4$.

Fig. 11 Effect of ratio μ of spring stiffness to cantilever stiffness on transmission spectrums obtained by Galerkin method ($q_0=1\text{N}$) and band-gap obtained by theoretical analysis (gray shaded area, $\beta_1 \neq 0$, $r=0.001\text{m}$). (a) $\mu = 0$, (b) $\mu = 0.6$, (c) $\mu = 1.2$.

3.4 Effect external excitation on the wave transmittance

As mentioned in Section 3.3, the wave transmittances of the meta-plate with MBS resonators are investigated in Fig. 10 and Fig. 11, the corresponding low excitation amplitude q_0 is 1N. It is also noted from Fig. 9(a) that the nonlinearity of the resonator stiffness is related to displacement response of the resonator. Therefore, the effect of external excitation on the wave transmittance will be analyzed in the following research. The material properties and geometrical parameters are the same as the model in Section 3.1.

Fig. 12 shows the wave transmittance curves of meta-plate with nonlinear resonators under the excitation with different amplitudes, where the pre-compression ratio η is 0.3. The band-gap ranges (gray shaded area) obtained by Eq. (20) are given as the comparison in Fig. 12(b) and Fig. 12(c). Note that the theoretical band gap is linear ($\beta_1 = 0$) in Fig. 12(b) and is nonlinear ($\beta_1 \neq 0$, $r = 0.0049\text{m}$) in Fig. 12(c), where the amplitude r of Eq. (20) is taken as the maximum value of the RMS displacement of the resonator in the band-gap range by solving Eq. (28). It can be found from Fig. 12 that with the increase of the excitation amplitude, the wave attenuation range is evidently shifted from a low frequency region into a high one. This trend is consistent with the change of band-gaps obtained by Eq. (20) as the resonator amplitude increases. Fig. 13 shows the RMS displacements of the faceplate and resonators under large-amplitude excitation ($q_0 = 10\text{N}$), where $\omega = 40\text{Hz}$ and $\omega = 140\text{Hz}$ are in the passband range, $\omega = 80\text{Hz}$ is in the band-gap range. It is observed from Fig. 13(c) and Fig. 13(d) that the vibration of the faceplate and resonators mainly occurs near the excitation point, while in other areas, the faceplate vibration is negligible. It can be ascribed to a fact that the resonator vibrates in resonance with the incoming excitation within the band-gap. Meanwhile, it is an interesting phenomenon that even though the RMS displacements of the resonators and faceplate in the low frequency region are at the same order of magnitude, the resonators have little effect on the vibration suppression of the faceplate, as shown in Fig. 13(a) and Fig. 13(b). The reason is that when the excitation frequency is in the low-frequency passband region, the resonators cannot provide reaction force to suppress faceplate vibration due to the zero-phase-difference between the displacement response of the resonator and the faceplate. Moreover, it is observed that there are many peaks in the band-gap range for the case of large-amplitude excitation, as shown by the purple dotted line in Fig. 12(a). The reason is that the large-amplitude excitation results in complicated dynamic behaviors, such as coupled vibration and chaos, which can weaken the wave attenuation at certain frequencies. Therefore, when the excitation frequency is taken as a constant value within the wave attenuation range, such as $\omega = 75\text{Hz}$, the transient responses of the meta-plate under different excitation amplitudes are shown in Fig. 14. Compared with transient responses under small-excitation amplitude, it is interesting to note that the large-amplitude excitation significantly diminishes the wave attenuation efficiency.

Fig. 12 (a) Effects of the excitation amplitude on the transmission spectrum of the meta-plate with nonlinear local resonators. Comparison of wave attenuation range and theoretical band-gap corresponding to (b) linear ($\beta_1 = 0$) and (c) nonlinearity ($\beta_1 \neq 0$) of MBS resonators.

Fig. 13 RMS displacements of faceplate (a) and resonators (b) under the different excitation frequency when $q_0 = 10\text{N}$ and $\eta = 0.3$.

Fig. 14 Transient responses of the sandwich-like meta-plate under different excitation amplitude. (a) $q_0 = 1\text{N}$, (b) $q_0 = 20\text{N}$.

To further demonstrate the effect of nonlinearity on dynamic behaviors of meta-plate, the bifurcation diagrams of the displacement amplitude versus excitation amplitude at points P_s and P_r is shown in Fig. 15. Note that here, the harmonic excitation frequency is taken as 60 Hz, which is within the band-gap ranges. It is observed from Fig. 15 that as the excitation amplitude increases from 0 N to 20 N, a route to chaos, corresponding to the change in the displacement amplitude of meta-plate from the gradient growth to interwell oscillations, is presented. As for the meta-plate in this article, when the excitation amplitude q_0 is greater than 6.9 N, the dynamic behaviors change into chaos. Comparing the displacement amplitude of the ‘excitation source’ (Fig. 15(a)) and ‘test point’ (Fig. 15(b)), it is obviously seen that the bifurcation phenomenon becomes weak at point P_r . The reason is that the sandwich-like meta-plate can control wave propagation by the locally resonant properties.

Moreover, the phase portraits and the corresponding Poincare sections under different excitation amplitude are shown in Fig. 16. For the small-amplitude excitation, the phase portrait is an elliptic orbit (Fig. 16(a), Fig. 16(d) and Fig. 16(e)) or a line-like flat curve (Fig. 16(b)) and the corresponding Poincare sections are clustered near a point due to the periodicity of dynamic behavior. However, as the excitation amplitude q_0 increases to 20N, the phase portraits of meta-plate exhibit a cluttered state and the Poincare sections become dispersed, as shown in Fig. 16(c) and Fig. 16(f), which mean that the dynamic behavior of the meta-plate is chaotic. In addition, the transverse displacement responses at points P_s (Fig. 16(a), Fig. 16(b) and Fig. 16(c)) and P_r (Fig. 16(d), Fig. 16(e) and Fig. 16(f)) are also compared to reveal the wave attenuation properties of meta-plate by the MBS resonators. Especially for small-amplitude excitation ($q_0 = 1\text{N}$ and $q_0 = 5\text{N}$), there are two orders of magnitude difference in the transverse displacements of point P_s and P_r . However, when the large-amplitude excitation ($q_0 = 20\text{N}$) is applied to point P_s , the corresponding responses are chaotic (Fig. 16(c) and Fig. 16(f)), the displacement response is at the same order of magnitude and its attenuation is unobvious. This interesting phenomenon is helpful for wave manipulation.

Fig. 15 Bifurcation diagram of the displacement responses with respect to the excitation amplitude at points (a) $P_s(0,0)$ and (b) $P_r(L_x, L_y)$ when $\omega=60\text{Hz}$ and $\eta=0.3$.

Fig. 16 Phase portraits (blue curves) and the corresponding Poincaré sections (red markers) under different excitation amplitude when $\omega=60\text{Hz}$ and $\eta=0.3$.

4. Conclusion

In this paper, the concept of a sandwich-like meta-plate is considered for wave manipulation while accounting for the mass-beam-spring (MBS) resonator's nonlinearity. Based on Hamilton's principle and harmonic balance method, the expression of band-gap bounds related to the amplitude of resonator is derived by dispersion analysis. A coupled mathematical model is established to analyze the nonlinear dynamic behavior of a finite sized meta-plate periodically attached to MBS-damper systems acting as local resonators. Some conclusions can be drawn as follows. The attenuation ranges of transmission spectrums obtained by the FEM and Galerkin method match very well with the band-gaps predicted by dispersion analysis. By investigating the dispersion relation of the infinite sized meta-plate, it is noted that with the increase of resonator amplitude, the nonlinear theoretical band-gap shifts to high frequency. The stiffness of MBS resonator can be adjusted to any low value by increasing the pre-compression of the springs at the equilibrium position, and thus the ultra-low frequency band-gaps can be obtained through the design of the meta-plate with MBS resonators. Nevertheless, for the case of large-amplitude excitation, corresponding to the nonlinearity of resonators, the frequency within the wave attenuation range is evidently higher and the wave attenuation becomes worse in band-gap range compared with that of the linear finite sized meta-plate. Moreover, the nonlinearity of MBS resonators widens the band-gap of sandwich-like meta-plate.

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Compliance with ethical standards

Conflict of interest

The authors declare that they have no conflict of interest concerning the publication of this manuscript.

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Figures

Figure 1

(a) Schematic diagram of the sandwich-like meta-plate with periodically attached MBS resonators. (b) The unit cell of the meta-plate. (c) Physical prototype of MBS resonator in a static equilibrium state. (d) The deformation of MBS resonator under restoring force.

Figure 2

A unit cell of the meta-plate with MBS resonators: (a) front view and (b) side view.

Figure 3

Numerical simulation model of the sandwich-like meta-plate with 8×8 unit cells

Figure 4

Comparisons of transmission spectrums obtained by finite element method and Galerkin method and band-gap obtained by theoretical analysis.

Figure 5

Comparisons of dispersion surfaces and band-gaps: (a) and (b) denote results of the meta-plate, (c) and (d) denote results of the plain sandwich plate.

Figure 6

Comparisons of linear ($\beta_1 = 0$) and nonlinear ($\beta_1 \neq 0$) band-gaps.

Figure 7

The stiffness adjustment diagram of nonlinear resonator at the equilibrium position " $w \approx r$ ". (a) 3D plot; The projections of " $k \approx r$ " on (b) xy-plane

Figure 8

3D plot of the dimensionless stiffness of the nonlinear resonator with respect to dimensionless displacement and pre-compression ratio of the spring. ($\mu = 2/3$)

Figure 9

(a) The dimensionless stiffness $k(-)r$ vs. dimensionless displacement " w " $\bar{w}_r$; (b) The dimensionless stiffness $k(-)r$ vs. pre-compression ratio η of the spring.

Figure 10

Effect of pre-compression ratio η on transmission spectrums obtained by Galerkin method ($q_0=1N$) and band-gap obtained by theoretical analysis (gray shaded area, $\beta_1 \neq 0$, $r=0.001m$). (a) $\eta = 0$, (b) $\eta = 0.3$, (c) $\eta = 0.4$.

Figure 11

Effect of ratio μ of spring stiffness to cantilever stiffness on transmission spectrums obtained by Galerkin method ($q_0=1\text{N}$) and band-gap obtained by theoretical analysis (gray shaded area, $\beta_1 \neq 0$, $r=0.001\text{m}$). (a) $\mu = 0$, (b) $\mu = 0.6$, (c) $\mu = 1.2$.

Figure 12

(a) Effects of the excitation amplitude on the transmission spectrum of the meta-plate with nonlinear local resonators. Comparison of wave attenuation range and theoretical band-gap corresponding to (b) linear ($\beta_1 = 0$) and (c) nonlinearity ($\beta_1 \neq 0$) of MBS resonators.

Figure 13

RMS displacements of faceplate (a) and resonators (b) under the different excitation frequency when $q_0=10\text{N}$ and $\eta=0.3$.

Figure 14

Transient responses of the sandwich-like meta-plate under different excitation amplitude. (a) $q_0 = 1$ N, (b) $q_0 = 20$ N.

Figure 15

Bifurcation diagram of the displacement responses with respect to the excitation amplitude at points (a) $P_s(0,0)$ and (b) $P_r(L_x, L_y)$ when $\omega = 60$ Hz and $\eta = 0.3$.

Figure 16

Phase portraits (blue curves) and the corresponding Poincaré sections (red markers) under different excitation amplitude when $\omega=60\text{Hz}$ and $\eta=0.3$.