On the order of the non-primes of type $6n+5$ and $6n+1$

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Research Article

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The fundamental importance of prime numbers for mathematics is that all other natural numbers can be represented as a unique product of prime numbers¹. For centuries, mathematicians have been trying to find an order in the occurrence of prime numbers. Since Riemann's paper on the number of primes under a given size, the distribution of the primes is assumed to be random². Here, I show that prime numbers are not randomly distributed. Their positions are determined by the order of occurrence of the nonprimes of types $6n+5$ and $6n+1$. By using parametric sine functions, I will show here that all assumptions known today regarding the distribution of prime numbers larger than 3 are wrong. In particular, this refutes Riemann's hypothesis of random distribution of prime numbers. Furthermore, I will show an exact primality test based on these parametric sine functions, which only uses one parameter.

The number sequences $6n+5$ (for all $n \geq 0$) and $6n+1$ (for all $n > 0$) cover all odd natural numbers that are not divisible by 3. For example, $6n+5$ leads to 5, 11, 17, ... and $6n+1$ leads to 7, 13, 19, Because all primes larger than 3 are odd natural numbers and not divisible by 3, they must be of type $6n+5$ or $6n+1$. All potential prime numbers larger than 3 can therefore be shown as zeros of the following two sine functions: $f_1(x) = \sin\left(\left(\frac{\pi}{6}\right)(x - 5)\right)$ for all numbers of type $6n+5$ and $f_2(x) = \sin\left(\left(\frac{\pi}{6}\right)(x - 1)\right)$ for all numbers of type $6n+1$. The allowed range for x reaches for both sine functions from 0 to infinite.

Now the question arises, which natural numbers of type $6n+5$ and $6n+1$ are not prime?

1. Under what conditions are numbers of type $6n+5$ not prime and what regularity do these follow?

$6n+5$ is not a prime number if: $(6n+5) : a = b \leftrightarrow a \cdot b = 6n+5 \leftrightarrow \frac{a \cdot b - 5}{6} = n \quad a, b, n \in \mathbb{N} \text{ and } a, b > 1$

A natural number is divisible by 6 if it is divisible by 2 and by 3. For the divisibility by 2, a and b must be odd natural numbers because an odd natural number multiplied by another odd natural number leads to an odd natural number. And an odd natural number larger than or equal to 7 minus 5 leads to an even natural number. For the divisibility by 3, it is necessary that $a \cdot b$ has the structure $(3x+1)(3y-1)$ for all $x, y \in \mathbb{N}$ because

$$\frac{(3x+1)(3y-1)-5}{3} = \frac{9xy-3x+3y-1-5}{3} = \frac{9xy-3x+3y-6}{3} = 3xy-x+y-2$$

Therefore, for a nonprime number of type $6n+5$, the following must hold for the index n :

$$\frac{(3x+1)(3y-1)-5}{6} = n$$

for all $n \in \mathbb{N}$ and $x, y \in \text{even natural numbers} \geq 2$

For $x = 2$ and $y = 2, 4, 6, 8, 10, \dots$ after inserting n in $6n+5$ this leads to the numbers 35, 77, 119, 161, 203, ... as you can see this is every 42nd number starting at 35.

¹ Gauß, C. F. (1801). *Disquisitiones Arithmeticae*. Chapter 2. Retrieved from: [Disquisitiones.ocr.pdf \(uni-bielefeld.de\)](https://disquisitiones.ocr.pdf.uni-bielefeld.de/).

² Riemann, B. (1859). *Ueber die Anzahl der Primzahlen unter einer gegebenen GröÙe*. *Monatsberichte der Berliner Akademie*, November 1859. Transcribed by D. R. Wilkins, Preliminary Version: December 1998. Retrieved from: [zeta.pdf \(claymath.org\)](https://zeta.pdf.claymath.org/)

For $x = 4$ and $y = 2, 4, 6, 8, 10, \dots$ after inserting n in $6n+5$ this leads to the numbers 65, 143, 221, 299, 377, ... as you can see this is every 78th number starting at 65. Note, that $78=42+36$ and $65=35+30$.

For $x = 6$ and $y = 2, 4, 6, 8, 10, \dots$ after inserting n in $6n+5$ this leads to the numbers 95, 209, 323, 437, 551, ... as you can see this is every 114th number starting at 95. Note, that $114=42+2\cdot 36$ and $95=35+2\cdot 30$.

This pattern continues indefinitely and can be represented by $(35 + 30k) + (42 + 36k)m$ with $k = 0, 1, 2, 3, \dots$ and $m = 0, 1, 2, 3, \dots$

All nonprimes of type $6n+5$ can thus be represented as zeros of the following parametric sine function:

$$f_3(x) = \sin\left(\left(\frac{\pi}{42+36t}\right)(x - 35 - 30t)\right)$$

The allowed range for x in the parametric sine function $f_3(x)$ ranges from $35+30t$ to infinite, and for t , $t = 0, 1, 2, 3, 4, \dots$

If x and y are interchanged, the same numbers are produced, but in a different order. Therefore, the interchange of the variables x and y can be ignored in this context. Nevertheless, I would like to point out that swapping x and y also leads to regular columns of numbers, which can be represented as zeros of the parametric sine function: $f_4(x) = \sin\left(\left(\frac{\pi}{30+36t}\right)(x - 35 - 42t)\right)$. The allowed range for x reaches in $f_4(x)$ from $35+42t$ to infinite and for t applies $t = 0, 1, 2, 3, 4, \dots$

2. Under what conditions are numbers of type $6n+1$ not prime and what regularity do these follow?

$6n+1$ is not a prime number if: $(6n+1) : a = b \Leftrightarrow a \cdot b = 6n+1 \Leftrightarrow \frac{a \cdot b - 1}{6} = n \quad a, b, n \in \mathbb{N}$ and $a, b > 1$

Following the proof idea from above, a and b must be odd natural numbers to ensure divisibility by 2. For the divisibility by 3, it is necessary that a times b has either the form $(3x+1)(3y+1)$ or the form $(3x-1)(3y-1)$ because

$$\frac{(3x+1)(3y+1)-1}{3} = \frac{9xy+3x+3y+1-1}{3} = \frac{9xy+3x+3y}{3} = 3xy+x+y \quad \text{and}$$

$$\frac{(3x-1)(3y-1)-1}{3} = \frac{9xy-3x-3y+1-1}{3} = \frac{9xy-3x-3y}{3} = 3xy-x-y \quad \text{for all } x, y \in \mathbb{N}$$

Therefore, for a nonprime number of type $6n+1$, the following must hold:

$$\text{either } \frac{(3x+1)(3y+1)-1}{6} = n \text{ or } \frac{(3x-1)(3y-1)-1}{6} = n$$

for all $n \in \mathbb{N}$ and $x, y \in \text{even natural numbers} \geq 2$.

By using the values for x and y the same way as above, the first of those two terms produces the following columns of numbers:

For $x = 2$ and $y = 2, 4, 6, 8, 10, \dots$ after inserting n in $6n+1$ this leads to the numbers 49, 91, 133, 175, 217, ... as you can see this is every 42^{nd} number starting at 49.

For $x = 4$ and $y = 2, 4, 6, 8, 10, \dots$ after inserting n in $6n+1$ this leads to the numbers 91, 169, 247, 325, 403, ... as you can see this is every 78^{th} number starting at 91. Note, that $78=42+36$ and $91=49+42$.

For $x = 6$ and $y = 2, 4, 6, 8, 10, \dots$ after inserting n in $6n+1$ this leads to the numbers 133, 247, 361, 475, 589, ... as you can see this is every 114^{th} number starting at 133. Note, that $114=42+2\cdot 36$ and $133=49+2\cdot 42$.

This pattern continues indefinitely and can be represented by $(49 + 42k) + (42 + 36k)m$ with $k = 0, 1, 2, 3, \dots$ and $m = 0, 1, 2, 3, \dots$.

This first part of the nonprime numbers of type $6n+1$ can therefore be represented as zeros of the following parametric sine function:

$$f_5(x) = \sin\left(\left(\frac{\pi}{42+36t}\right)(x - 49 - 42t)\right)$$

The allowed range for x in the parametric sine function $f_5(x)$ ranges from $49+42t$ to infinite, and for t , $t = 0, 1, 2, 3, 4, \dots$

The second term produces the following columns of numbers:

For $x = 2$ and $y = 2, 4, 6, 8, 10, \dots$ after inserting n in $6n+1$ this leads to the numbers 25, 55, 85, 115, 145, ... as you can see this is every 30^{th} number starting at 25.

For $x = 4$ and $y = 2, 4, 6, 8, 10, \dots$ after inserting n in $6n+1$ this leads to the numbers 55, 121, 187, 253, 319, ... as you can see this is every 66^{th} number starting at 55. Note, that $66=30+36$ and $55=25+30$.

For $x = 6$ and $y = 2, 4, 6, 8, 10, \dots$ after inserting n in $6n+1$ this leads to the numbers 85, 187, 289, 391, 493, ... as you can see this is every 102^{nd} number starting at 85. Note, that $102=30+2\cdot 36$ and $85=25+2\cdot 30$.

Also this pattern continues indefinitely. It can be represented by $(25 + 30k) + (30 + 36k)m$ with $k = 0, 1, 2, 3, \dots$ and $m = 0, 1, 2, 3, \dots$.

This second part of nonprime numbers of type $6n+1$ can therefore be represented as zeros of the following parametric sine function:

$$f_6(x) = \sin\left(\left(\frac{\pi}{30+36t}\right)(x - 25 - 30t)\right)$$

The allowed range for x in the parametric sine function $f_6(x)$ ranges from $25+30t$ to infinite, and for t , $t = 0, 1, 2, 3, 4, \dots$

To make all prime numbers appear in the correct order, only the functions $f_1(x)$, $f_2(x)$, $f_3(x)$, $f_5(x)$ and $f_6(x)$ have to be entered into a common coordinate system. The following figures are made with MatheGrafix 11. The zeros of the characteristic green functions show all potential prime numbers of type $6n+5$ or type $6n+1$. Every green zero ($f_1(x)$ and $f_2(x)$), which is not crossed by the graph of one of the "forbidding" parametric sine functions ($f_3(x)$, $f_5(x)$, $f_6(x)$), represents a prime number. The software used does not allow a parameter-dependent definition set. Therefore, these two figures have the same rigid definition range and because of this, different parametric settings. Increasing the

parameters stepwise is necessary when using this software because the “forbidding” parametric sine functions logically cross out wrong numbers when the functions are under their definition range.

Fig. 1: prime numbers 107, 109, 113, 127, 131, 137

Setting in figure 1:

$$f_1(x): x \in [0; 5000]$$

$$f_2(x): x \in [0; 5000]$$

$$f_3(x): x \in [35; 5000] \quad t=0,1,2,3,4,5,6$$

$$f_5(x): x \in [49; 5000] \quad t=0,1,2,3,4,5$$

$$f_6(x): x \in [25; 5000] \quad t=0,1,2,3,4,5$$

Fig. 2: prime numbers 439, 443, 449, 457, 461, 463, 467

Setting in figure 2:

$$f_1(x): x \in [0; 5000]$$

$$f_2(x): x \in [0; 5000]$$

$$f_3(x): x \in [35; 5000] \quad t=0,1,2,\dots,9,10$$

$$f_5(x): x \in [49; 5000] \quad t=0,1,2,\dots,9,10$$

$$f_6(x): x \in [25; 5000] \quad t=0,1,2,\dots,9,10$$

3. A consequential possibility to prove whether a natural number larger than 3 is prime or not

1. Proof of whether a natural number A under investigation is of type $6n+5$ or of type $6n+1$. If $(A-5) : 6$ equals a natural number, then A is of type $6n+5$. If $(A-1) : 6$ equals a natural number, then A is of type $6n+1$. If both calculations do not equal a natural number, then the number under investigation is not of type $6n+5$ or of type $6n+1$. Such a natural number larger than 3 cannot be prime.

2. a) If the investigated number A is of type $6n+5$, calculate if $f_3(x) = \sin\left(\left(\frac{\pi}{42+36t}\right)(A - 35 - 30t)\right)$ has a zero at this number A. Then, A is not a prime number. To do this, it is enough to prove if $\frac{A-35-30t}{42+36t} = b$ can be solved with a natural number value for b and t (including $b=0$ and $t=0$).

- b) If the investigated number A is of type $6n+1$, calculate if $f_5(x) = \sin\left(\left(\frac{\pi}{42+36t}\right)(A - 49 - 42t)\right)$ or $f_6(x) = \sin\left(\left(\frac{\pi}{30+36t}\right)(A - 25 - 30t)\right)$ has a zero at this number A. If one of these two functions shows a zero at this number A, then A is not prime.

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Figures

Figure 1

prime numbers 107, 109, 113, 127, 131, 137

Figure 2

prime numbers 439, 443, 449, 457, 461, 463, 467

Supplementary Files

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- [OrdnungNichtprimzahlen.xlsx](#)

Setting in figure 1:

$$f_1(x): x \in [0; 5000]$$

$$f_2(x): x \in [0; 5000]$$

$$f_3(x): x \in [35; 5000] \quad t=0,1,2,3,4,5,6$$

$$f_5(x): x \in [49; 5000] \quad t=0,1,2,3,4,5$$

$$f_6(x): x \in [25; 5000] \quad t=0,1,2,3,4,5$$

Setting in figure 2:

$$f_1(x): x \in [0; 5000]$$

$$f_2(x): x \in [0; 5000]$$

$$f_3(x): x \in [35; 5000] \quad t=0,1,2,\dots,9,10$$

$$f_5(x): x \in [49; 5000] \quad t=0,1,2,\dots,9,10$$

$$f_6(x): x \in [25; 5000] \quad t=0,1,2,\dots,9,10$$