Extension of the TOPSIS Method for Decision-making Problem Under Dual Hesitant Fuzzy Language Environment

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Abstract: In order to comprehensively and actually describe the evaluation process, the dual hesitant fuzzy linguistic (DHFL) set is introduced in this paper, which includes more decision-making information, such as fuzzy state, hesitant process and language information. Specifically, some basic concepts of DHFL set are illustrated and a new distance measure for DHFL information is proposed, which is suitable for overcoming the irrational traditional methodology upon the general distance measure and basic probability concepts. Then, technique for order preference by similarity to ideal solution (TOPSIS) method is extended in dual hesitant fuzzy language environment, a novel TOPSIS method using the DHFL set is presented. Finally, the sensitivity analysis is performed to verify the feasibility and stability of the developed method, then the advantages of the proposed method are also confirmed by detailed comparative analysis.

Keywords: Decision-making, Dual hesitant fuzzy linguistic set, Sensitivity analysis, TOPSIS

1. Introduction

After the presentation of TOPSIS method (Hwang and Yoon 1981), it has been widely applied in multi-criteria decision making (MCDM) problems. As a distance measure based classical method, the basic idea of TOPSIS is to set two reference points as benchmark: the positive ideal solution (PIS) point and negative ideal solution (NIS) point, then, the optimal solution finally selected needs to satisfy two conditions at the same time, that is, the closest distance to PIS and the farthest distance to NIS. Motivated by the fuzzy set theory (Zadeh 1965), many researchers successfully integrated it with TOPSIS to processing the uncertainty information generated in the decision-making process (Chen 2000; Jahanshahloo et al. 2006; Robinson and Nabil 2016; Wang and Chang 2007; Wang and Elhag 2006). Meanwhile, to enhance the utility of TOPSIS in fuzzy environment, various approaches and theories were proposed for optimizing the key parts of TOPSIS, such as PIS, NIS and distance measurement (Chu and Lin 2009; Ewa and Tomasz 2015; Kuo et al. 2007; Mahdavi et al. 2008; Wang and Lee 2009; Wang and Lee 2007).

However, with the increasing complexity of the environment, it becomes more difficult to make a correct final decision, so that the TOPSIS is extended in diversified fuzzy environments by some researchers to solve the MCDM problem. For instance, by extending the TOPSIS to the intuitionistic fuzzy environment, a new MCDM method is proposed to evaluate smart phones, which can eliminate the uncertainty and describe the preferences of decision makers (DMs) (Gülçin and Sezin 2016). Biswas et al. (2016) developed an innovative TOPSIS method, which uses the single-valued neutrosophic set. Additionally, Joshi and Kumar (2016) presented an interval-valued intuitionistic hesitant fuzzy TOPSIS method considering the correlation among the decision criteria. Wang and Chen (2017) integrated the interval-valued intuitionistic fuzzy (IVIF) sets and LP method with the extended TOPSIS, to overcome the shortcomings of previous methods used to cope with the MADM problems in IVIF environment.

As the research continues to deepen, more realistic situations are considered. Lourenzutti and Krohling (2016) presented a GMo-RTOPSIS approach, which can support the DMs expressing their personal views fully and flexibly in group MCDM under a dynamic environment. Moreover, to enhance

the comprehensiveness and rationality of group MCDM process, Hatami and Kangi (2017) introduced three types of fuzzy TOPSIS methods for handling the imprecise information. On the basis of hesitant fuzzy correlation coefficient, Sun et al., (2018) presented a new TOPSIS method to deal with the negative-value information. Sajjad et al., (2018) extended the TOPSIS approach through the combination of Choquet integral-based distance and IVPFCIG operator for MAGDM problems, while Hajek and Froelich (2019) presented an IVIFCM-TOPSIS method, which can model the interactions among imprecise criteria for MCGDM problems. Furthermore, some novel TOPSIS methods are extended to hesitant Pythagorean fuzzy set (Liang and Xu 2017), spherical fuzzy sets (Kutlu Gündoğdu and Kahraman 2019), interval-valued hesitant fuzzy N-soft set (Akram and Adeel 2019), interval-valued spherical fuzzy sets (Gündoğdu and Kahraman 2019), ordered fuzzy numbers (Kacprzak 2019), and etc.

Recently, decision making under linguistic environment has become a hotspot, and there are a series of papers focus on this research (Wei et al. 2016; Yang and Ju 2014; Zhang et al. 2019a; Zhang et al. 2019b; Zhang et al. 2018). But to make a right decision, there are many other factors that need to be considered. Obviously, because of the complex socioeconomic environment and vague human thinking, hesitant and uncertain information is usually appeared when making decisions, so that the DMs may could not precisely express their decisions by using linguistic expressions, and the criteria weights and the preference values of DMs are frequently ambiguous, which cannot be represented by crisp numerical value of the classical TOPSIS methods. With the purpose of expressing the preferences of the DMs more comprehensively, the dual hesitant fuzzy linguistic set (DHFLS) is introduced to evaluate linguistic terms (Yang and Ju 2015). Compared with other sets, the DHFLS has both consider the influence of the membership hesitancy degree as well as the non-membership hesitancy degree, which is an efficient tool to comprehensively describe various types of uncertainty. Therefore, DHFLS could better indicate the evaluation information in MCDM problems. Aiming at eliminate the uncertainty caused by ambiguous information and describe the preferences of DMs better, a MCDM method with mutually supportive arguments under DHFL environments is constructed in this paper.

The remainder of this article is arranged as follows: In Sect. 2, some basic definitions of DHFLS, distance measures and dual hesitant language fuzzy MCDM problem are briefly introduced. In Sect. 3, the procedure to solve the MCDM problem in DHFL environment using the proposed method is described in details. In Sect. 4, the sensitivity analysis, comparative study and illustrative example are used to prove the rationality of the presented method. Finally, concluding remarks and suggestions of further research are made in Sect. 5.

2. Hesitant fuzzy language set and decision making problem

2.1. Concept and definition description

Some basic definitions and notations concerning DHFLS, distance measures and dual hesitant language fuzzy MCDM problem are introduced in this section.

Definition 1 (Yang and Ju 2014). Suppose X is a reference set, then a DHFLS on X can be expressed as follows:

$$H = \left\{ \left\langle x, s_{\theta(x)}, h_{(x)}, g_{(x)} \right\rangle \middle| x \in X \right\} \tag{1}$$

Where $s_{\theta(x)}$ denotes the linguistic variable, ϑ is called a DHFL element (DHFLE), $h(x)$ and

$g(x)$ represent the hesitant possible membership/non-membership degrees to $s_{\theta(x)}$, respectively. In

addition, to make the expression more concise, the 3-tuples $\vartheta(x) = \left\langle s_{\theta(x)}, h_{(x)}, g_{(x)} \right\rangle$ is simplified to

$$\mathcal{G} = \left\langle s_\theta, h, g \right\rangle.$$

Definition 2. Suppose $\mathcal{G} = \left\langle s_\theta, h, g \right\rangle$ is a DHFLE, the score function and accuracy function of $\mathcal{G}$ are given as:

$$S(\mathcal{G}) = \sum_{\gamma \in h} \sum_{\eta \in g} \frac{\theta \times (\gamma - \eta)}{\xi \times \#h \times \#g} \quad \text{and} \quad P(\mathcal{G}) = \sum_{\gamma \in h} \sum_{\eta \in g} \frac{\theta \times (\gamma + \eta)}{\xi \times \#h \times \#g} \tag{2}$$

Here $\#h$ ($\#g$) is the number of values in h (g). ξ is the cardinality of $S = \left\{ s_0, s_1, L, s_{\xi-1} \right\}$ (Zadeh 1975).

Definition 3 (Yang and Ju 2014). Let $\mathcal{G}_k = \left\langle s_{\theta_k}, h_k, g_k \right\rangle$ and $\mathcal{G}_l = \left\langle s_{\theta_l}, h_l, g_l \right\rangle$ represent two DHFLEs, the comparison order of DHFLEs has the following situations: (1) if $S(\mathcal{G}_k) > S(\mathcal{G}_l)$, then $\mathcal{G}_k$ is superior to $\mathcal{G}_l$, expressed as $\mathcal{G}_k \, f \, \mathcal{G}_l$; (2) if $S(\mathcal{G}_k) = S(\mathcal{G}_l)$, then (i) if $P(\mathcal{G}_k) = P(\mathcal{G}_l)$, then $\mathcal{G}_k$ is equivalent to $\mathcal{G}_l$, expressed as $\mathcal{G}_k \approx \mathcal{G}_l$; (ii) if $P(\mathcal{G}_k) > P(\mathcal{G}_l)$, then $\mathcal{G}_k$ is superior than $\mathcal{G}_l$, expressed as $\mathcal{G}_k \, f \, \mathcal{G}_l$.

2.2. Distance measures

As we know, the main idea of the existing methods (Chen et al. 2011; Torra 2010; Xia and Xu 2011) is to compare the number of elements in the hesitant fuzzy sets firstly. If the result of the comparison is not equal, then the maximum/minimum degree of the membership/non-membership is added to the set with fewer number of elements several times until the number in the two sets is same. However, there are two problems with this approach: (1) Adding the maximum/minimum evaluation value, which greatly highlights the subjectivity of the DMs; (2) Judgement and determination of the DMs' risk attitude is a hard work. For the purpose of computing the distance between two DHFL variables, a new distance measure for DHFL information is introduced and is suitable for overcoming the irrational traditional methodology upon the well-known distance measure and basic probability concepts, which are computed directly from DHFL variables and any maximum/minimum value does not need to be added to the evaluation set.

Definition 4. Suppose $\mathcal{G}_k = \left\langle s_{\theta_k}, h_k, g_k \right\rangle$ and $\mathcal{G}_l = \left\langle s_{\theta_l}, h_l, g_l \right\rangle$, then the normalized Euclidean distance for two DHFLEs are described by:

$$d(\mathcal{G}_k, \mathcal{G}_l) = \frac{1}{2\xi} \left(\sum_{k=1}^{\#h_k} \sum_{l=1}^{\#h_l} \left| \frac{\theta_k \gamma_k - \theta_l \gamma_l}{\#h_k \, \#h_l} \right|^2 + \sum_{k=1}^{\#g_k} \sum_{l=2}^{\#g_l} \left| \frac{\theta_k \eta_k - \theta_l \eta_l}{\#g_k \, \#g_l} \right|^2 \right)^{1/2} \tag{3}$$

Where $\#h_k$, $\#h_l$, $\#g_k$ and $\#g_l$ are the numbers of values in h_k, h_l, g_k and g_l respectively, such that $\gamma_k \in h_k, \gamma_l \in h_l, \eta_k \in g_k, \eta_l \in g_l$. The distance $d(\mathcal{G}_k, \mathcal{G}_l)$ between $\mathcal{G}_k$ and $\mathcal{G}_l$ satisfies: (1) $0 \le d(\mathcal{G}_k, \mathcal{G}_l) \le 1$; (2) $d(\mathcal{G}_k, \mathcal{G}_l) = 0$, if only if $\mathcal{G}_k = \mathcal{G}_l = \left\langle s_\theta, \{\gamma\}, \{\eta\} \right\rangle$; (3)

$$d(\vartheta_k, \vartheta_l) = d(\vartheta_l, \vartheta_k).$$

2.3. Dual hesitant language fuzzy MCDM problem

To describe the MCDM process under DHFL information in detail, suppose $\mathbf{A} = \{A_1, A_2, L, A_m\}$ is the alternatives and $\mathbf{C} = \{c_1, c_2, L, c_n\}$ be the criteria with the weight $\mathbf{W} = (w_1, w_2, ..., w_n)^T$, in which $w_j \in [0,1]$, $\sum_{j=1}^{n} w_j = 1$. The DHFLE $\vartheta_{ij} = \langle s_{\theta_{ij}}, h_{ij}, g_{ij} \rangle (i = 1, 2, L, m, j = 1, 2, L, n)$ is applied to estimate the preference information of alternative A_i concerning criterion c_j by DMs, where $s_{\theta_{ij}}$ is the assessment information in linguistic form, h_{ij} (g_{ij}) denotes the set of the possible degrees that alternative A_i satisfy (not satisfy) criterion c_j.

Let the DHFLE ϑ_{ij} denotes the evaluation grade of $A_i \in \mathbf{A}$ under the criterion $c_j \in \mathbf{C}$. By using the following DHFL decision matrix, an MCDM problem with DHFLEs can be represented succinctly:

$$\mathbf{H} = \begin{array}{c} \\ A_1 \\ A_2 \\ M \\ A_m \end{array} \begin{array}{cccc} c_1 & c_2 & L & c_n \\ \left[\begin{array}{cccc} \vartheta_{11} & \vartheta_{12} & L & \vartheta_{1n} \\ \vartheta_{21} & \vartheta_{22} & L & \vartheta_{2n} \\ M & M & O & M \\ \vartheta_{m1} & \vartheta_{m2} & L & \vartheta_{mn} \end{array} \right] \end{array} \qquad (4)$$

3. Extension of the TOPSIS under DHFL environment

The TOPSIS method will be extended to the DHFL environment for handling the hesitant and uncertain information in this section. Subsequently, according to the comprehensive indexes, all alternatives are ranked, the specific steps are as follows:

Step 1. Determine the PIS and NIS based on DHFL information.

$$\vartheta^+ = (\vartheta_1^+, \vartheta_2^+, \vartheta_3^+, L L, \vartheta_n^+), \quad \vartheta^- = (\vartheta_1^-, \vartheta_2^-, \vartheta_3^-, L L, \vartheta_n^-) \qquad (5)$$

Where $\vartheta_j^+ = \max_i (\vartheta_{ij}) = \langle \max_i s_{\theta_{ij}}, \max_i h_{ij}, \min_i g_{ij} \rangle$, $\vartheta_j^- = \min_i (\vartheta_{ij}) = \langle \min_i s_{\theta_{ij}}, \min_i h_{ij}, \max_i g_{ij} \rangle$.

Step 2. Use equations (6) and (7) to calculate the weighted Hamming distance of each alternative to PIS and NIS, respectively.

$$d_i(\vartheta_i, \vartheta^+) = \sum_{j=1}^{n} w_j d_{ij}(\vartheta_{ij}, \vartheta_j^+) \qquad (6)$$

$$d_i(\vartheta_i, \vartheta^-) = \sum_{j=1}^{n} w_j d_{ij}(\vartheta_{ij}, \vartheta_j^-) \qquad (7)$$

Step 3. According to the ideal solution determined in step 1, the relative closeness of each alternative is calculated. Take alternative A_i as an example, the formula is as follows:

$$\lambda_i(a_i) = d(\vartheta_i, \vartheta^-) / (d(\vartheta_i, \vartheta^-) + d(\vartheta_i, \vartheta^+)) \qquad (8)$$

Step 4. Rank all alternatives A_i and choose the optimal one(s) according to $\lambda_i(A_i)$.

Step 5. Give a sensitivity analysis of the solution.

4. Illustrative example

4.1. Illustration of the presented approach

As we all know, maintenance services, as a backup for repairable products, can effectively improve the customer satisfaction of particular manufacturing companies (Davies and Isaac 1999). In fact, maintenance services have been regarded as an important part of the product. In order to successfully achieve the operational goals, choosing a service agent with good performance is an extremely significant decision making problem (Liao et al. 2018). Suppose that there are five service agents $\mathbf{A} = \{A_1, A_2, A_3, A_4, A_5\}$ are available for assessment, and seven benefit criteria must be considered comprehensively when evaluating the five alternatives: service attitude (c_1), response speed (c_2), maintenance quality (c_3), level of technical consultation (c_4), informatization level (c_5), rationality of the charge (c_6), firm size (c_7), with the weight vector $\mathbf{W} = (0.35, 0.10, 0.05, 0.10, 0.05, 0.15, 0.20)$.

Because the information that can be obtained about the alternative service agents is incomplete and vague, experts can only make an intuitive assessment based on the DHFL set. According to the linguistic term set $\mathbf{S} = \begin{Bmatrix} s_0 = None; \ s_1 = VeryLow; \ s_2 = Low; \ s_3 = AlmostMedium; \\ s_4 = Medium; s_5 = AlmostHigh; s_6 = High; s_7 = VeryHigh; s_8 = Perfect \end{Bmatrix}$, the result is shown in Table 1.

In what follows, the proposed method is utilized to choose the most desirable one.

Step 1. For each criterion, the PIS and NIS are determined, which is presented in Table 2.

Table 2. PIS and NIS

Criterion	c_1	c_2	c_3	c_4
$\mathcal{S}_j^+$	$\langle 0.75, \{0.9\}, \{0.1\} \rangle$	$\langle 0.75, \{0.8\}, \{0.1\} \rangle$	$\langle 0.5, \{0.6\}, \{0.3\} \rangle$	$\langle 0.5, \{0.6\}, \{0.1\} \rangle$
$\mathcal{S}_j^-$	$\langle 0.5, \{0.5\}, \{0.3\} \rangle$	$\langle 0.25, \{0.3\}, \{0.6\} \rangle$	$\langle 0.125, \{0.1\}, \{0.7\} \rangle$	$\langle 0.125, \{0.1\}, \{0.7\} \rangle$

Criterion	c_5	c_6	c_7	
$\mathcal{S}_j^+$	$\langle 0.625, \{0.8\}, \{0.1\} \rangle$	$\langle 0.5, \{0.7\}, \{0.1\} \rangle$	$\langle 0.75, \{0.9\}, \{0.1\} \rangle$	
$\mathcal{S}_j^-$	$\langle 0.25, \{0.1\}, \{0.6\} \rangle$	$\langle 0.375, \{0.3\}, \{0.6\} \rangle$	$\langle 0.375, \{0.3\}, \{0.6\} \rangle$	

Step 2. Using the formulas (6) and (7) to calculate the separation measures, which are distances of each individual decision from PIS and NIS, respectively.

Table 1. DHF decision matrix

Attribute	c_1	c_2	c_3	c_4
A_1	$\langle s_6, \{0.9, 0.8, 0.7\}, \{0.1, 0\} \rangle$	$\langle s_6, \{0.7, 0.8, 0\}, \{0.1, 0.2\} \rangle$	$\langle s_1, \{0.1, 0.2, 0\}, \{0.6, 0.7\} \rangle$	$\langle s_1, \{0.1, 0.2, 0\}, \{0.6, 0.7\} \rangle$
A_2	$\langle s_5, \{0.6, 0.7, 0, 0\}, \{0.1, 0.2, 0\} \rangle$	$\langle s_2, \{0.3, 0.4, 0, 0\}, \{0.5, 0.6, 0\} \rangle$	$\langle s_4, \{0.4, 0.6, 0, 0\}, \{0.3, 0.4, 0\} \rangle$	$\langle s_4, \{0.6, 0.5, 0.4, 0\}, \{0.1, 0.2, 0.3\} \rangle$
A_3	$\langle s_6, \{0.7, 0.9, 0\}, \{0.1, 0\} \rangle$	$\langle s_5, \{0.6, 0.5, 0.8\}, \{0.1, 0.2\} \rangle$	$\langle s_2, \{0.2, 0.3, 0.4\}, \{0.5, 0.6\} \rangle$	$\langle s_2, \{0.3, 0.4, 0\}, \{0.5, 0.6\} \rangle$
A_4	$\langle s_4, \{0.5, 0.6, 0.7\}, \{0.1, 0.2, 0.3\} \rangle$	$\langle s_5, \{0.8, 0.5, 0\}, \{0.2, 0.1, 0\} \rangle$	$\langle s_2, \{0.3, 0.2, 0\}, \{0.5, 0.6, 0\} \rangle$	$\langle s_2, \{0.2, 0.3, 0.4\}, \{0.4, 0.5, 0.6\} \rangle$
A_5	$\langle s_6, \{0.7, 0.8, 0\}, \{0.1, 0.2\} \rangle$	$\langle s_3, \{0.3, 0.6, 0\}, \{0.4, 0.3\} \rangle$	$\langle s_1, \{0.1, 0.3, 0\}, \{0.5, 0.6\} \rangle$	$\langle s_3, \{0.4, 0, 0\}, \{0.4, 0.6\} \rangle$

Attribute	c_5	c_6	c_7
A_1	$\langle s_5, \{0.6, 0.8, 0\}, \{0.1, 0.2\} \rangle$	$\langle s_4, \{0.5, 0.6, 0\}, \{0.3, 0.2\} \rangle$	$\langle s_6, \{0.7, 0.8, 0.9\}, \{0.1, 0\} \rangle$
A_2	$\langle s_3, \{0.3, 0.5, 0, 0\}, \{0.2, 0.3, 0\} \rangle$	$\langle s_3, \{0.3, 0.4, 0.5, 0.6\}, \{0.2, 0.3, 0.4\} \rangle$	$\langle s_3, \{0.3, 0.5, 0, 0\}, \{0.4, 0.5, 0\} \rangle$
A_3	$\langle s_4, \{0.5, 0.6, 0\}, \{0.2, 0.3\} \rangle$	$\langle s_3, \{0.4, 0.5, 0\}, \{0.5, 0\} \rangle$	$\langle s_4, \{0.6, 0.5, 0.4\}, \{0.2, 0.3\} \rangle$
A_4	$\langle s_4, \{0.6, 0.5, 0\}, \{0.2, 0.3, 0\} \rangle$	$\langle s_4, \{0.5, 0.6, 0.7\}, \{0.1, 0.2, 0.3\} \rangle$	$\langle s_4, \{0.5, 0.7, 0\}, \{0.1, 0.3, 0\} \rangle$
A_5	$\langle s_2, \{0.1, 0.4, 0.5\}, \{0.4, 0.6\} \rangle$	$\langle s_3, \{0.3, 0.4, 0.5\}, \{0.4, 0.6\} \rangle$	$\langle s_4, \{0.4, 0.6, 0.7\}, \{0.1, 0.6\} \rangle$

$$d\left(\vartheta_1,\mathscr{I}^b\right)=0.0312,\quad d\left(\vartheta_2,\mathscr{I}^b\right)=0.0838,\quad d\left(\vartheta_3,\mathscr{I}^b\right)=0.0575,\quad d\left(\vartheta_4,\mathscr{I}^b\right)=0.0654,$$

$$d\left(\vartheta_5,\mathscr{I}^b\right)=0.0763,\quad d\left(\vartheta_1,\mathscr{I}^b\right)=0.0890,\quad d\left(\vartheta_2,\mathscr{I}^b\right)=0.0378,\quad d\left(\vartheta_3,\mathscr{I}^b\right)=0.0718,$$

$$d\left(\vartheta_4,\mathscr{I}^b\right)=0.0453,\quad d\left(\vartheta_5,\mathscr{I}^b\right)=0.0579.$$

Step 3. For each individual decision, the relative closeness to ideal solution is calculated.

$$\lambda_1\left(A_1\right)=0.7406,\quad \lambda_2\left(A_2\right)=0.3110,\quad \lambda_3\left(A_3\right)=0.5553,\quad \lambda_4\left(A_4\right)=0.4093,\quad \lambda_5\left(A_5\right)=0.4314$$

Step 4. In terms of the values of the relative closeness, rank all candidates in descending order:

A_1 f A_3 f A_5 f A_4 f A_2. The optimal alternative is A_1.

4.2. Sensitivity analysis

In order to highlight the practicability and superiority of the presented approach, focusing on the weights, a sensitivity analysis of the solution to the MCDM problem and a comprehensive comparative study are introduced in this section. According to the original evaluation information, the evaluation criteria are objectively weighted. When the weight changes, how the priority order changes? Through sensitivity analysis, it can be determined that the potential change of the weight of the evaluation criterion will lead to deviation of the decision result. This is the key to effectively use the model and implement the quantitative decision-making (Simanaviciene and Ustinovichius 2010). Thus, the sensitivity analysis of the weight of evaluation criterion is carried out by perturbation method, that is, when the weight is slightly disturbed, the priority order will accordingly change.

Here, w_j is the initial weight of evaluation criterion c_j, which is denoted as $w_j'=\varsigma w_j$ after disturbance. Where $0\le w_j'\le 1$, the variation interval of parameters ς is $0\le\varsigma\le 1/w_j$. Because of the normalization of weights, the other weights correspondingly change due to the change of w_j, and denoted as $w_k'=\phi w_k$, $k\ne j$, $k=1,2,\mathrm{L}\,,m$, which is satisfy $w_j'+\sum_{k\ne j,k=1}^{m}w_k'\Rightarrow\varsigma w_j+\phi\sum_{k\ne j,k=1}^{m}w_k=1$.

By calculating the weights, $\phi=\left(1-\varsigma w_j\right)/\left(1-w_j\right)$ can be get. For the weight w_j of each evaluation criterion, the corresponding priority order can be obtained using the TOPSIS method when different parameters ς were taken. The weights of the seven evaluation criteria in the example were respectively disturbed, where ς is evaluated in order at $[0,2]$ and the interval between values of ς is 0.05, then a total of 266 experiments are conducted to obtain the sensitivity analysis results.

As can be seen from figure 1, Q_i of candidate A_1 is the smallest in 266 experiments. Candidate A_2 ranked fourth in 8 experiments, so the sensitivity of candidate A_2 concerning c_1 is higher than other potential candidates. Candidate A_3 ranked third in 10 experiments, therefore, the sensitivity of candidate A_3 concerning c_1 is relatively higher than other potential candidates. Candidate A_4 ranked second in 10 tests and third in 41 experiments, and candidate A_5 ranked fourth in 51 experiments. Thus, compared with other candidates, the sensitivities of A_4 and A_5 concerning $\{c_1,c_2,c_6\}$ are relatively higher. When the weights of $\{c_3,c_4,c_5\}$ are disturbed, the change of the

weights of chosen criteria will not affect the result of decision-making, which is also A_1 f A_3 f A_5 f A_4 f A_2 , where the symbol ' f ' expresses 'prior to'. In conclusion, the proposed method in this paper for dealing with the MCDM problems is relatively insensitive to the change of weight information, and the optimal candidate is always A_1 .
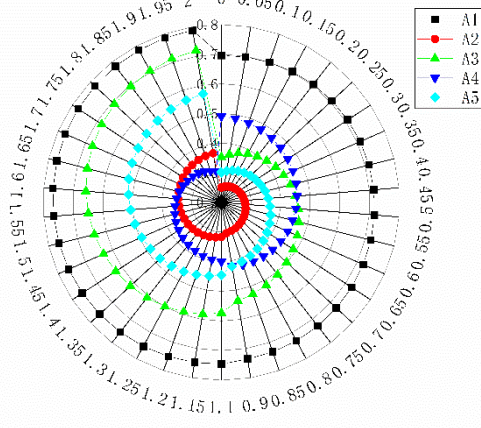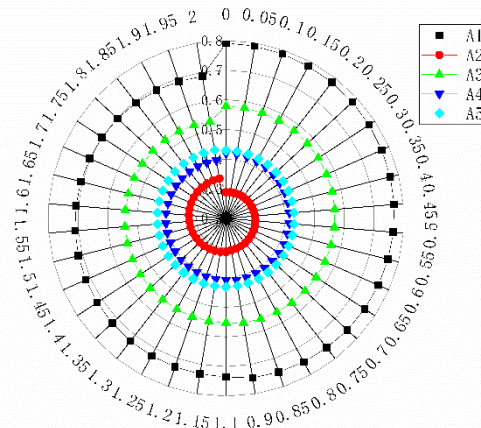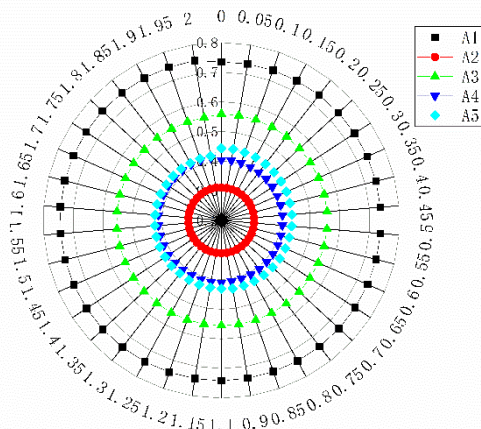

(1) The initial weight is w_1
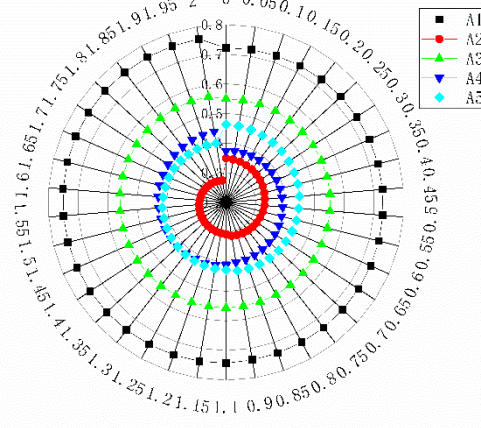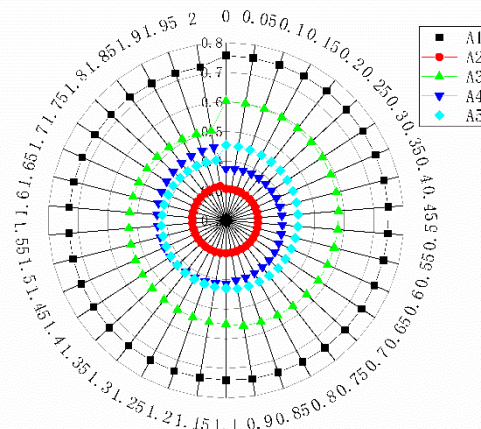

(2) The initial weight is w_2
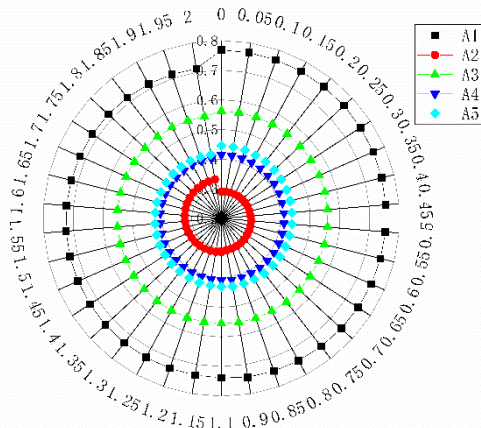
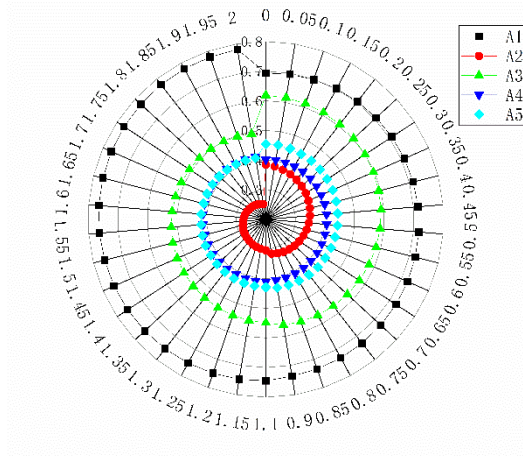(3) The initial weight is w_3

(4) The initial weight is w_4

(5) The initial weight is w_5

(6) The initial weight is w_6

(7) The initial weight is w_7

Figure 1. **Sensitivity analysis results by TOPSIS method**

5. Conclusion

Considering that DHFLS is more suitable than other fuzzy sets for dealing with fuzzy and hesitant information in complex environments, to solve the MCDM problem with DHFL information, an DHFLS based innovative TOPSIS method was proposed. The new method is straight forward and easy to implement on computer, where the form of attribute data is the DHFL numbers. The basic concepts of DHFL set and extended definition of hamming distance are introduced. In order to prove the efficiency of the presented approach, an example is used in this paper. Additionally, a sensitivity analysis of the final ranking results to the weights on the TOPSIS method is studied to test the stability of proposed method. In future, MCDM problems involving other types of fuzzy hesitant information can be further studied, and to provide more effective ways for DMs, the decision-making methods such as TODIM and VIKOR can also be applied in the DHFL environment.

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Ethical approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of interest

The authors declare no conflict of interest.

Authorship contributions

Introduction, Q.Z.; Hesitant fuzzy language set and decision making problem, Q.Z.; Extension of the TOPSIS under DHFL environment, Z.N.; Illustrative example Z.N. and G.W.; Conclusion, G.W..

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