Vectorial Association of Linearly Oriented Residuals: Application to Swarm High-latitude Magnetic Field Measurements

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1 Vectorial Association of Linearly Oriented Residua (VALOR): 2 Application to Swarm high-latitude magnetic field measurements

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Abstract

We introduce the methodological framework *VALOR* for the analysis of field-aligned current (FAC) sheets based on dual-spacecraft magnetic field observations. *VALOR* generalizes existing approaches in two respects. First, it takes advantage of the primary observables at both spacecraft, namely, the full magnetic field vectors, instead of derived scalar quantities like single- or dual-spacecraft FAC densities. Second, rather than being restricted to linear correlation measures, *VALOR* allows to define a custom-made statistical measure of association between the signals at both spacecraft, that is chosen to be particularly sensitive to small-scale fluctuations for this first demonstration. Additionally, *VALOR* offers to consider the linear orientation of the magnetic field residual vectors, i.e., the sheets' polarization, in order to infer their orientation and to accentuate the association measure. This paper illustrates the method on the basis of an exemplary auroral oval crossing on March 1, 2019 by the lower pair of ESA's multi-spacecraft mission Swarm. Explicit comparisons to competing methods are performed to give credence to the results of the case study while the general capability of *VALOR* is demonstrated on the basis of a statistical study comprising around 9700 auroral oval crossings from 2014–2020. One suitable area of application is the analysis of meso- to small-scale (tens of km to below one km) auroral arc systems in combination with optical data from modern all-sky imagers. Conceptually, *VALOR* is neither bound to the auroral zone nor to Swarm and can be applied by any multi-spacecraft geospace mission with suitable instrumentation and orbit configuration.

Keywords

Correlation analysis, field-aligned currents, current sheet polarization, Swarm, auroral oval, magnetosphere-ionosphere coupling, magnetic field perturbation

Introduction

Electromagnetic magnetosphere-ionosphere (M-I) coupling processes are optically manifested through auroral phenomena in the high-latitude ionosphere, with their occurrence frequency maximizing in oval-shaped region around the geomagnetic poles. Auroral emissions exhibit a variety of colors, shapes, and dynamics. Among the best studied forms are elongated linear structures called auroral arcs where

thickness indicates the spatial scale of the underlying physical process. Statistical studies reveal a wide spectrum of arc scales ranging from about hundred meters (Maggs and Davis 1968) to several hundreds of kilometers, comparable to the width of the auroral oval. While narrow or small-scale auroral arcs with scales in the range of hundred meters to a kilometer are typically very dynamic and short-lived (Sandahl et al 2008), auroral arcs in the mesoscale range of a few ten kilometers are both rather frequent (Knudsen et al 2001), and can be relatively stable with lifetimes in the order of minutes. As shown by Partamies et al (2010), optical instrument characteristics can strongly influence the measured scale distribution.

Auroral emissions are closely associated with electric currents flowing along geomagnetic field lines (e.g., Lysak 1990; Paschmann et al 2002; Vogt 2002). Such field-aligned currents (FACs) transmit energy and momentum between remote regions in the near-Earth space environment, and thus serve as important coupling agents facilitating global-scale plasma convection and magnetospheric substorms. The spatial scale L of FACs serves as an indicator for the type of M-I coupling, with quasi-static coupling associated with large scales $L \gtrsim 100$ km (Lyons 1980; Lotko et al 1987), and Alfvénic coupling with intermediate scales $L \gtrsim 10$ km (Vogt and Haerendel 1998).

Auroral FACs are routinely measured using magnetic field instruments on Earth-orbiting satellites such as Magsat, Freia, FAST, Cluster, Champ, ST5, and Swarm. For recent reviews of studies on auroral FACs and their spatial scale distributions, see the introductory sections of Blăgău and Vogt (2019) and Bunescu et al (2019). Apart from the dual-spacecraft gradient technique applied to the low-pass filtered and thus large-scale magnetic field data of the lower pair of Swarm satellites (Ritter and Lühr 2006; Ritter et al 2013; Vogt et al 2013, 2020), FAC scale analyses rest on finite magnetic field differences along the tracks of single spacecraft (Lühr et al 1996; Ritter and Lühr 2006). The scalar FAC density estimate can then be differentiated with regard to spatial scales by means of appropriate filters (Forsyth et al 2017) or transformation techniques (Stasiewicz and Potemra 1998). The multiscale FAC analyzer introduced by Bunescu et al (2015, 2017, 2019) produces wavelet-like scalograms not only for the FAC density but also for the planarity and the orientation of FAC structures.

In the case of multi-spacecraft missions such as ST5 or Swarm, scale-dependent patterns in FAC density profiles measured at different locations can be statistically compared to infer their degree of association. The most popular association measure is linear correlation, employed in the auroral FAC context to study the scale dependence of day-night differences (Gjerloev et al 2011), the temporal variability of FAC structures (Lühr et al 2015), the validity of the single-spacecraft FAC estimator (Forsyth et al

2017), characteristics and controlling parameters (McGranaghan et al 2017), and auroral current sheet orientations (Yang et al 2018).

This paper introduces the methodological framework *VALOR* (*Vectorial Association of Linearly Oriented Residua*) to extend the toolbox of multi-spacecraft correlation analysis techniques that have so far been applied to Swarm observations of auroral FAC structures. Although VALOR builds on general multi-spacecraft analysis theory (Vogt et al 2011, 2013; Vogt 2014), we have in mind the specific constellation of Swarm satellites Alpha (SwA) and Charlie (SwC). We are particularly interested in the study of intermediate to small spatial scales down to a few kilometers, hopefully involving also measurements at a higher resolution than one Hz. To this end, VALOR is designed to extract scale information with as little assumptions about filter shapes or scale-dependent basis functions as possible. Instead of using the derived Level-2 scalar single-spacecraft or dual-spacecraft FAC densities, VALOR is based on the full Level-1b magnetic field vector to construct maps of suitable statistical association measures. VALOR is not restricted to the class of linear correlation techniques, and can explicitly consider signal polarization at an auroral current sheet, i.e., the directional preference of the magnetic field vector, to infer the sheet orientation.

We proceed as follows. In the *Methods* section, the VALOR approach is developed and illustrated on the basis of an exemplary auroral oval crossing by the lower pair of Swarm spacecraft. The section closes with an explanation of how the data base for the multi-event study is compiled. The *Results* section presents a detailed evaluation of the case study followed by the statistical results. The same structure is kept throughout the *Discussion & Outlook* section where we explicitly compare VALOR to related methods and give an outlook on future work before we present our *Conclusions*.

Methods

To provide context and a reference analysis technique, linear cross-correlation analysis is summarized before the VALOR methodology is explained. The general VALOR framework for vectorial time series is presented, and then we discuss how polarization information comes into play. The rest of the section is concerned with data selection and preprocessing in preparation of our statistical study of auroral crossings.

Linear correlation analysis

Linear (cross-)correlation analysis rests on the linear correlation coefficient $r = r(X, Y_\tau)$ between one signal $X = X(t)$ and a lagged version of another signal $Y_\tau = Y(t + \tau)$. In the context of Swarm auroral FAC density studies, Forsyth et al (2017) pointed out that r quantifies only the degree of linear dependence between the two signals. Additionally, they considered the slope $m = m(X, Y_\tau)$ of the regression line to study if the amplitudes of the signals are comparable, and argued that both $r \approx 1$ and $m \approx 1$ are required for a good match of two FAC density profiles.

As shown in Appendix A, the two quality parameters r and m can be combined into a single measure of signal association, namely, the mean squared deviation (MSD) of X and Y_τ :

$$\text{MSD}(X, Y_\tau) = \langle |X(t) - Y_\tau(t)|^2 \rangle_t = (\Delta X)^2 (1 - r^2) + (\Delta Y_\tau)^2 (1 - m)^2 + (\bar{X} - \bar{Y}_\tau)^2. \quad (1)$$

Here, $\langle \bullet \rangle_t$ and $\bar{\bullet}$ both denote time averaging, and $(\Delta X)^2$ and $(\Delta Y_\tau)^2$ are the variances of the two signals. Hence for the MSD to be small, both r and m must be close to one, and the time series means $\bar{X}$ and $\bar{Y}_\tau$ must not differ much. The third term cancels if centered time series $X_c = X - \bar{X}$ and $Y_{\tau,c} = Y_\tau - \bar{Y}_\tau$ are considered instead (see Appendix A). With the MSD being close to zero for a good match the inverse MSD as a function of τ is expected to display a clear peak at optimum lag τ_0 . This idea is picked up by the VALOR methodology with respect to the construction of an association measure as a function of both t and τ .

The VALOR analysis framework

A straightforward generalization of the MSD from the scalar to the vectorial case gives $\text{MSD}(\mathbf{X}, \mathbf{Y}_\tau) = \langle \|\mathbf{X}(t) - \mathbf{Y}_\tau(t)\|^2 \rangle_t$ where $\|\bullet\|$ is the standard vector norm. For the sake of symmetry the lagged term $\mathbf{Y}_\tau(t)$ is replaced by a pattern function $\mathbf{P} = \mathbf{P}(t|\tau)$ constructed from both signals. VALOR can then be understood as the special dual-satellite case of the general multi-spacecraft analysis framework suggested by Vogt (2014, Chapter 5) for estimating boundary parameters from vectorial magnetic time series. Here, $\mathbf{X}(t)$ and $\mathbf{Y}(t)$ refer to the ionospheric magnetic perturbations ($\mathbf{b}_\sigma$) registered at SwA ($\sigma = 1$) and SwC ($\sigma = 2$) as they traverse the auroral ovals. The perturbations are extracted from the vector field magnetometer data ($\mathbf{B}_\sigma$, Level-1b, MAG[A/C]_LR_1B) recorded at 1 Hz resolution:

$$\mathbf{b}_\sigma(t) = \mathbf{B}_\sigma(t) - \mathbf{B}^*(\mathbf{r}_\sigma(t)) \quad \sigma = 1, 2. \quad (2)$$

The ambient magnetic field ($\mathbf{B}^*$) is modeled as a superposition of contributions from the core, the lithospheric and the magnetospheric magnetic fields and evaluated at the spacecraft positions ($\mathbf{r}_\sigma$) using predictions from CHAOS-7 (Finlay et al 2020).

The magnetic perturbations during one such auroral oval crossing on March 1, 2019 around 02:00 UTC are shown in Figure 1a where they are given in the geographic North (N) - East (E) - Center (C) local Cartesian coordinate frame (NEC). The two spacecraft record very similar signatures with the SwC time series being lagged by the difference in equatorial crossing times ($\tau_{\text{eq}} = -6$ s) for the purpose of this illustration. The distinct disturbance patterns evident in the East components (orange/red) indicate that SwA and SwC cross two roughly East-West oriented current sheets on their outbound (or equatorward) orbits: first a downward (or earthward) and then an upward directed FAC in accordance with the canonical Region-1/Region-2 morphology of large-scale auroral FACs (Iijima and Potemra 1976; He et al 2012, 2014). This is clearly seen in the bipolar FAC density profiles shown in Figure 1b (derivation in Ritter et al 2013). Obviously, the small-scale variability contained in the Level-1b single-spacecraft estimators (j_σ , FAC[A/C]TMS_2F, blue) is largely subdued in the Level-2 dual-spacecraft estimator (j_{12} , FAC-TMS_2F, orange) following the low-pass filtering (cutoff period at 20 s) of $\mathbf{b}_\sigma$ prior to its calculation. Since the j_σ estimators are more closely related to along-track differences of $\mathbf{b}_\sigma$ than to $\mathbf{b}_\sigma$ themselves, the pattern function $\mathbf{P}$ is constructed as an average of temporal derivatives $\dot{\mathbf{b}}_\sigma$ (obtained using centered finite differencing)

$$\mathbf{P}(t|\tau) = \frac{1}{2} \left[\dot{\mathbf{b}}_1(t) + \dot{\mathbf{b}}_{2,\tau}(t) \right], \quad (3)$$

which provides a parametric family of reference perturbation signatures in the centered spacecraft frame. Here, the time series are restricted to a window $t = [-t_{\text{max}}, \dots, t_{\text{max}} = 150 \text{ s}]$ selected based on a probabilistic interpretation of the normalized j_{12} magnitude (Fig. 1c, details in the end of the section).

Considering a range of SwC orbital lags with respect to SwA, $\tau = [-\tau_{\text{max}}, \dots, \tau_{\text{max}} = 20 \text{ s}]$, the pattern function is evaluated over a $2t_{\text{max}} - 2\tau_{\text{max}} + 1 = 261 \text{ s} \times 41 \text{ s}$ sampling domain.

The $t|\tau$ -sampling domain is queried with respect to the optimum spacecraft lag (τ_0) as well as the center (t_0) and spatial scale (L) of the FAC signature. These tasks are conveniently addressed by evaluating the scalar association measure

$$\mathcal{C}(t|\tau) = \frac{\|\mathbf{P}(t|\tau)\|^2}{\text{MSD}(\dot{\mathbf{b}}_1, \mathbf{P})} = \frac{\|\mathbf{P}(t|\tau)\|^2}{\left\langle \|\dot{\mathbf{b}}_1(t) - \mathbf{P}(t|\tau)\|^2 \right\rangle_t}. \quad (4)$$

139 According to our preceding deliberation τ_0 is the element of the τ -sampling domain at which $\mathcal{C}$ maxi-
 140 mizes:

$$\tau_0 = \arg \max_{\tau} \mathcal{C}(t|\tau) \quad (5)$$

141 Statistically robust estimates of t_0 and L , i.e., ones with limited dependency on the specific form of
 142 $\mathbf{P}(t|\tau_0)$, are obtained by interpreting the normalized association measure at optimum lag $\mathcal{C}(t|\tau_0)$ as
 143 probability density function (PDF) with quartiles Q1, Q2 (median) and Q3:

$$t_0 = \text{Q2} \quad (6)$$

$$L = \text{IQR} \cdot \|\mathbf{v}\| = (\text{Q3} - \text{Q1}) \cdot \|\mathbf{v}\| \quad (7)$$

$$\mathbf{v} = -\frac{1}{2} [\mathbf{v}_1(t_0) + \mathbf{v}_{2,\tau_0}(t_0)] \quad (8)$$

144 Here, IQR is the interquartile range and $\mathbf{v}$ is the FAC velocity in the centered spacecraft frame calculated
 145 under the assumption that the FAC doesn't move with respect to the ground. The spacecraft velocities
 146 $\mathbf{v}_\sigma$ are obtained from centered finite differences of the respective positions $\mathbf{r}_\sigma$.

147 Note that the VALOR framework is not dependent on a particular choice of association measure. The
 148 one introduced here (Eqs. 3, 4) uses the magnetic perturbations at SwA ($\sigma = 1$) as a reference and is
 149 tailored to the identification of small-scale FAC characteristics.

150 Magnetic polarization at FAC sheets

151 In addition to being capable of processing vectorial time series and selecting a wider class of association
 152 measures than standard linear correlation techniques, VALOR may explicitly address the current sheet's
 153 orientation by estimating a triplet of unit vectors $\{\hat{\mathbf{q}}, \hat{\mathbf{n}}, \hat{\mathbf{p}}\}$. Here, $\hat{\mathbf{q}} = \mathbf{q}/\|\mathbf{q}\|$ is the ambient magnetic
 154 field direction in the centered spacecraft frame obtained from a magnetic field model $\mathbf{B}^*$ (see Eq. 2) as
 155 follows:

$$\mathbf{q}(t|\tau) = \frac{1}{2} [\mathbf{B}^*(\mathbf{r}_1(t)) + \mathbf{B}^*(\mathbf{r}_{2,\tau}(t))] \quad (9)$$

156 In an ideal planar FAC sheet geometry, magnetic perturbation vectors are perpendicular to both $\hat{\mathbf{q}}$ and
 157 the sheet normal unit vector $\hat{\mathbf{n}}$, thus $\hat{\mathbf{p}} = \hat{\mathbf{q}} \times \hat{\mathbf{n}}$ is the direction of the associated linear polarization of
 158 the magnetic perturbation vector. Candidate unit vectors $\hat{\mathbf{n}}_c$ and $\hat{\mathbf{p}}_c$ are constructed from differences in
 159 SwA and SwC spacecraft locations $\Delta\mathbf{r}_{12}$ according to (see Fig. 2)

$$\hat{\mathbf{n}}_c(t|\tau) = \left(\frac{\Delta\mathbf{r}_{12}(t|\tau) \times \hat{\mathbf{q}}(t|\tau)}{\|\Delta\mathbf{r}_{12}(t|\tau) \times \hat{\mathbf{q}}(t|\tau)\|} \right) \quad \text{with} \quad \Delta\mathbf{r}_{12}(t|\tau) = \mathbf{r}_{2,\tau}(t) - \mathbf{r}_1(t), \quad (10)$$

$$\hat{\mathbf{p}}_c(t|\tau) = \hat{\mathbf{q}}(t|\tau) \times \hat{\mathbf{n}}_c(t|\tau). \quad (11)$$

We expect the association measure (Eq. 4) to further accentuate when the current sheet polarization is taken into account in the construction of an improved pattern function $\mathbf{P}_p$. To this end, $\mathbf{P} = \mathbf{P}(t|\tau)$ is projected onto the candidate polarization vector $\hat{\mathbf{p}}_c$ to yield

$$\mathbf{P}(t|\tau)_p = [\mathbf{P}(t|\tau) \cdot \hat{\mathbf{p}}_c(t|\tau)] \hat{\mathbf{p}}_c(t|\tau) . \quad (12)$$

The anticipated merit of this projection is to reduce contributions from incorrect candidate polarization directions to the association measure. Once the final triplet $(\hat{\mathbf{q}}, \hat{\mathbf{n}}_0, \hat{\mathbf{p}}_0)$ has been selected from the candidates based on (t_0, τ_0) , the FAC's spatial scale *in the direction of $\hat{\mathbf{n}}_0$* can be calculated as

$$L_n = \text{IQR} \cdot \| [\mathbf{v} \cdot \hat{\mathbf{n}}_0(t_0|\tau_0)] \hat{\mathbf{n}}_0 \|, \quad (13)$$

using the updated IQR and leaving $\mathbf{v}$ (Eq. 8) unchanged.

Preprocessing for multi-event analysis

Apart from the demonstration of VALOR's performance on the basis of a sample event a proper assessment of the method also requires its evaluation in a statistical sense. Theoretically, given four auroral oval crossings per orbit with an orbital period of ~ 93.3 minutes for Swarm, we can expect $\sim 135 \cdot 10^3$ crossing events within the six year long analysis time span from April 2014 to April 2020. Here, we apply VALOR to a reduced set of suitable auroral FAC signatures observable in the 1 Hz low resolution data as a first step.

First, we extract the polar segments from all orbits during the analysis span by restricting the spacecrafts' quasi-dipole (QD, see Laundal and Richmond 2017) latitude to $|\theta_m| > 50^\circ$. Then, we require that (1) the polar orbit segments pass close to either one of the QD poles, $\max(|\theta_m|) > 88^\circ$, in order to ensure quasi-perpendicular passages through the current sheets and thus small inclinations α (cf., Fig. 2b), (2) full records of the dual-spacecraft FAC density product j_{12} are available along the selected segments in order to specify the VALOR analysis windows (details below). Finally, the remaining orbit segments are split at $\max(|\theta_m|)$ into inbound (poleward) and outbound (equatorward) orbital arcs. While the first requirement accounts for a reduction of $\sim 86\%$ with respect to the theoretically determined number of events the second one halves the remaining record. This leaves 9709 ($\sim 7\%$) auroral oval crossings in the final set of which the sample event in Figure 1 is event #7842.

We standardize the selected orbital windows prior to the application of VALOR (Eqs. 2–13) on the basis of the j_{12} time series. Their normalized magnitudes constitute PDFs along the arcs with the associated

medians (Q2) and IQRs being robust initial estimators of the FAC signatures' centers (t_{0i}) and length scales (L_i), respectively (as in Eqs. 6–8). Given a median IQR of 86 s ($\tilde{L}_i \sim 650$ km) across all 9709 events, we decide to select 300 s long windows centered on t_{0i} (just 23 events with IQR > 300 s) which are now free to extend below $|\theta_m| = 50^\circ$. Following the scope of event comparability the subsequent VALOR analysis is performed in the global Cartesian geographic coordinate frame (NEC→GEO; see Laundal and Richmond 2017, p. 29) in which the magnetic perturbations $\mathbf{b}_\sigma$ are set to zero whenever the event windows extend beyond the QD poles in order to avoid confusion of consecutive events.

Lastly, note that the selection scheme outlined here allows not only for typical single-peak magnetic perturbation profiles (as evident in event #7842) but also for different noisy profiles including sigmoidal (single FAC crossing) and multi-peak signatures ($> two crossings$). This needs to be taken into account when interpreting VALOR's results.

Results

In this section we first present the VALOR results for the sample event #7842 (2019/03/01 01:58:49–02:03:49 UT) before we put them into perspective in the context of the statistical results obtained on the basis of all 9707 selected FAC crossings.

The sample event

According to the preprocessing the analysis window is centered on $t_{0i} = 02:01:19$ UT, when SwA is located at $\theta_m = 66.3^\circ$ QD latitude and MLT = 21.655 magnetic local time (see Laundal and Richmond 2017, p. 51), and the initial FAC scale length is $L_i \sim 626$ km (IQR = 82 s, see Fig. 1c).

These estimates are updated in the VALOR framework on the basis of the association measure (Eq. 4) as illustrated in Figure 3. The plot compares three association measures based on different kinds of input: Scalar j_σ instead of $\dot{\mathbf{b}}_\sigma$ ($\mathcal{C}_s$, panels a & d), vector $\dot{\mathbf{b}}_\sigma$ ($\mathcal{C}_v$, panels b & e), and the final combination of vector $\dot{\mathbf{b}}_\sigma$ and $\hat{\mathbf{p}}_c$ ($\mathcal{C}_p$, panels c & f). We acknowledge that the consideration of polarization information helps to constrain the association measure with respect to τ as expected (Fig. 3b vs. 3c). At the same time we notice that $\mathcal{C}(t|\tau)$ maximize at τ_0 that diverge notably between the “scalar” case on the one hand and the “vector only” and “polarization” cases on the other (dashed horizontal lines, Fig. 3a vs. 3b & 3c). For the latter the corresponding $\mathcal{C}(t|\tau_0)$ -based PDFs are better focused around the medians (t_0 , dashed vertical lines) yielding smaller IQRs and thereby spatial scales (Fig. 3d vs. 3e & 3f). Using both

polarization and vector information in the final setup VALOR gives a $\mathcal{C}(t|\tau_0=-11\text{ s})_p$ derived median 15 s after t_{0i} ($t_0=02:01:34$ UT), corresponding to $\theta_m = 65.48^\circ$ and MLT = 21.72 in centered spacecraft coordinates, as well as a scale length $L_n \sim 345$ km (IQR = 49 s, Fig. 3f) roughly half the size of L_i . If not stated otherwise we will refer to this setup in the following.

Regarding the geometry the sample event presents a favorable case as the spacecraft tangential plane roughly coincides with the $\hat{\mathbf{q}}_\perp$ -plane (dihedral angle $\beta = \angle\{\hat{\mathbf{r}}, \hat{\mathbf{q}}\} = 167^\circ$, see Fig. 2b). The latter is approximated by the X-Y-plane of the mean field-aligned coordinate system (MFA) in Figure 4a, where the Z-axis is aligned with the mean ambient magnetic field ($\bar{\mathbf{B}}^*$) at the centered spacecraft locations from start (s) to end (e), and the Y-axis points towards local magnetic East. Lines of MLT (20, 22) and QD latitude (60° , 70°) provide spatial orientation in accordance with Figure 1.

Apparently, the pattern function $\mathbf{P}(t|\tau_0=-11\text{ s})_p$ (colored arrows) is polarized roughly along local magnetic West ($\gamma = \angle\{[0, 1, 0], \hat{\mathbf{p}}_{0\text{MFA}}\} = 163^\circ$) which is consistent with a relatively small inclination ($\alpha = \angle\{\hat{\mathbf{v}}_{\hat{\mathbf{q}}_\perp}, \hat{\mathbf{n}}_0\} = 19^\circ$) as imposed via the event selection scheme. The sign reversal evident shortly after reaching t_0 (red dot) is indicative of the transition from the downward to the upward oriented current sheet (c.f., Fig. 1a,b). Note that the agreement between t_0 and the position of maximum pattern amplitude (27 nT/s) is not generally implied by the VALOR technique but rather a peculiarity of the sample event arising from the asymmetry of the underlying magnetic perturbation signatures. Given the common bi-modal current structure, t_0 may be interpreted as a proxy for the auroral oval center. Accordingly, L_n refers to the inter-FAC distance as a typical length scale of the auroral oval.

Furthermore, event #7842 presents a suitable sample case because SwA and SwC pass almost centrally through the field of view (FOV; gray ellipse in Fig. 4a) of the THEMIS All-Sky-Imager (ASI) at Kuujjuaq, Canada (58.1° geographic latitude; 291.5° longitude). The ASI camera takes 256×256 pixel resolved white light images of the night sky at 3 s cadence (Mende et al 2008). In order to summarize the recorded auroral activity one pixel wide North-South slices at the FOV's center are stacked in time with 6 s cadence forming a keogram (Fig. 4b). During event #7842 the keogram shows a distinct auroral structure (marked by blue lines) which moves southward at roughly 0.64 km/s. This is smaller than 10% of the centered spacecraft velocity ($\|\mathbf{v}\| = 7.63$ km/s) so that the assumption on the FAC's stationarity is justified in this case.

242 The set of selected events 04/2014–03/2020

243 The VALOR results for the 9709 preselected FAC crossings are summarized in form of nine histograms in
 244 Figure 5. The sample event (dashed lines) is representative of FACs registered around midnight that are
 245 typically centered at lower latitudes ($\theta_m \sim 69^\circ$) than those registered around noon ($\theta_m \sim 78^\circ$; Fig. 5a).
 246 The larger event counts at noon and midnight together with the relatively equal sampling of MLTs
 247 (Fig. 5b) indicate that the distribution is more narrowly confined in latitude compared to dawn and dusk
 248 where current sheets are spread between $\theta_m \sim 62^\circ$ and 85° . Note that these positions are based on
 249 VALOR-derived optimum spacecraft lags which follow a normal distribution around $\bar{\tau}_0 = -8$ s (Fig. 5c,
 250 gray) that notably differs from the distribution formed by the offsets in equatorial crossing times (τ_{eq} ,
 251 black).

252 Regarding the geometrical considerations the statistical results show that the favorable condition of
 253 a relatively small dihedral angle holds in general ($\beta \leq 17^\circ$, Fig. 5d). This lends credence to the
 254 estimated orientations according to which the FAC sheets are closely oriented along the magnetic East-
 255 West direction (median $\tilde{\gamma} = 17^\circ$, Fig. 5e) and are thus crossed quasi-perpendicularly ($\tilde{\alpha} = 17^\circ$, Fig. 5f)
 256 as expected. The angle intervals $[0^\circ - 90^\circ)$ and $[90^\circ, 180^\circ)$ are overlain as stacked histograms (cyan,
 257 magenta in Fig. 5d–f) and reveal asymmetries caused by the hemispheric differences in magnetic field
 258 morphology and a relative oversampling of northern-hemispheric events (70%). However, the distribution
 259 of inclinations is fairly symmetric (Fig. 5f) owing to the even sampling of events on inbound and
 260 outbound orbits. For instance, there are about as many East-West polarized, northern hemispheric
 261 sheets encountered on outbound orbits (“ $\hat{\mathbf{p}}$ twds W, out N” like sample event) as on inbound ones.

262 The consideration of the polarization (Eq. 12) in the association measure results in a general reduction of
 263 the FAC spatial scales by ~ 40 km to $\tilde{L}_n = 227$ km compared to the “scalar” (L_s) and “vector only” (L_v)
 264 cases (Fig. 5g, gray vs. blue & red). As expected, this is significantly less than $\tilde{L}_i = 656$ km obtained
 265 from the j_{12} based preprocessing (black). The resolution power of the latter is limited to $L_i \geq 150$ km
 266 because, in contrast to VALOR, the underlying magnetic field residuals are low-pass filtered (cf., Fig. 1b).
 267 The obtained statistics depend on the chosen association measure (Fig. 5h, Eq. 4) in that different FAC
 268 spatial scales provoke systematically different responses. This becomes evident when the histograms are
 269 split into small ($L_n < 150$ km), medium ($150 \leq L_n \leq 300$ km) and large ($L_n \geq 300$ km) scale subclasses
 270 (Fig. 6, not stacked): Large scales are most prevalent at dawn/dusk MLT sectors with relatively small
 271 similarities ($\tilde{\mathcal{C}}_p = 7$) while small scales dominates at noon/midnight with larger similarities ($\tilde{\mathcal{C}}_p = 41$) and

the medium scales fall in-between (Fig. 6b, g, h). In that sense the sample event is a typical medium to large scale hybrid.

However, the amplitude of the association measure at the selected center points does not readily lend itself to a quantitative assessment of the results' validity, i.e., large-scale structures are not per se less eligible for the VALOR analysis. In fact, one possibility to assess the fundamental assumption of sheet-like current geometries is via the “planarity” measure N , i.e., the mean ratio of variances of $\mathbf{b}$ projected onto the $\hat{\mathbf{p}}_0$ and $\hat{\mathbf{n}}_0$ directions (Fig. 5i). Truly planar structures should give planarity measures significantly larger than one which holds for the majority of cases ($\tilde{N} = 18.6$; $20\% \leq 5$). Here, it is the large-scale fraction that scores best although by a smaller margin (Fig. 6i).

Discussion and Outlook

Following the logic of the previous section our discussion is structured into two parts: First, we directly compare VALOR to linear correlation and regression analysis as well as Minimum Variance Analysis by means of the sample event. We then highlight to what extent our statistical results reflect the known characteristics of multiscale auroral FACs and comment on the influence of our event selection scheme. Finally, we give thought to the future development and application of the VALOR method.

In the beginning we have demonstrated how VALOR emerges from the combination and generalization of scalar association measures defined in both linear cross-correlation (r) and regression analysis (m). Following Forsyth et al (2017) we have evaluated both parameters for the sample event in order to substantiate the capability of VALOR (Fig. 7). The reference analysis indicates pronounced similarity ($r > 0.9 \vee 0.9 < m < 1.1$) of the 40 – 50 s bandpass filtered single-spacecraft current densities j_σ for two distinct time periods (around 02:00:00 and 02:02:30 UT) consistent with the consecutive crossing of two large-scale ($\sim 300 - 380$ km) FAC sheets (cf., Fig. 1b). On the other hand VALOR detects relatively high levels of association in the FAC transition zone ($t_0 = 02:01:34$ UT, auroral oval center) which persist on a comparable spatial scale ($L_n = 345$ km). The difference in the positioning results from VALOR's sensitivity to aligned small-scale amplitude fluctuations in $\dot{\mathbf{b}}_\sigma$ (large $\|\mathbf{P}(t, \tau_0)\|$) and the fact that *one* position is calculated in the presence of *two* current sheets. Actually, if one applies VALOR separately to the periods before and after t_0 the resulting sheet centers (and widths) are consistent with the reference analysis (Fig. 8). This experiment also shows that different optimum spacecraft lags (-4 s and -10 s) can be estimated for adjacent FAC crossings ($\tau_0 = -11$ s). The ability to adapt to different crossing

geometries (cf., Fig. 2) is a profound advantage of VALOR compared to the reference technique which, given that it operates on scalar j_σ , assumes that all FACs are crossed perpendicularly ($\alpha = 0$) so that the differences in SwA and SwC equatorial crossing times τ_{eq} are used as proxies for τ_0 (Forsyth et al 2017, see also Fig. 5c, black). Here, we used the VALOR-determined τ_0 for the reference analysis in order to facilitate the comparison.

Next, we cross-check the inclination obtained for the sample event with a constraint ($\bar{\mathbf{B}}^* \cdot \hat{\mathbf{n}} = 0$) Minimum Variance Analysis (MVA; Sonnerup and Scheible 1998), a technique commonly used to infer the orientation of auroral FACs (e.g., Vogt et al 2020, section 4.4.1). The MVA-inferred normal and polarization directions essentially coincide with the X- and Y- axes of the MFA coordinate frame (Fig. 9) meaning that the inclination is considerably smaller ($\alpha_{\text{MVA}}=3.3^\circ$) than what is obtained with VALOR ($\alpha = 19^\circ$). Also, the MVA eigenvalue ratio is much higher ($\lambda_{\text{max}}/\lambda_{\text{min}}=100$) than the planarity measure we've constructed (N , Fig. 5i). We don't think that MVA's negligence of the ambient magnetic field variability along the orbits ($\angle(\mathbf{B}, \bar{\mathbf{B}}^*) = 6.6^\circ \pm 3.8^\circ$) is the sole cause for these differences. For now, we attribute them to the spatial resolution power of the respective methods with MVA targeting the orientation of the entire sheet structure while VALOR extracts the local geometry around t_0 .

The purpose of the statistical study is to demonstrate that VALOR can be readily applied to a vast range of auroral oval crossings in a meaningful and largely automated way. We find that the presented results reflect the current knowledge of auroral FAC characteristics in the following sense. First, the MLT asymmetry in center quasi-dipole latitudes (Fig. 5a) is consistent with the ellipsoidal shape of the auroral ovals being squeezed towards and stretched away from the magnetic poles at noon and midnight, respectively (Xiong et al 2014; Xiong and Lühr 2014). Second, the presence of a systematic relation between scale length and MLT, i.e., large-scale FACs (≥ 300 km) dominate at dawn/dusk while small-scale FACs (≤ 150 km) dominate at noon/midnight (Fig. 5b, g) is corroborated by McGranaghan et al (2017). Lastly, we find some evidence that the low-latitude FACs are better ordered along the auroral oval (smaller γ) than the high-latitude ones (additional file 1, Fig. 1) although we do not bisect the orbit segments into Region-1 and Region-2 latitudinal parts as do Yang et al (2018).

Regarding the event selection process it is worth to deliberate on the suitability of j_{12} as decisive measure. While its smoothness compared to the single-spacecraft estimates helps the automated determination of the analysis windows, it is not available in the so-called exclusion zones near the geographic poles (lat. $> 86^\circ$) due to the degeneracy of the underlying virtual four-point configuration to a quasi-linear array.

“Loosing” auroral oval crossings due to the exclusion zones could be avoided by estimating current density based on a least-squares approach (e.g., Blăgău and Vogt 2019) that is conveniently formulated in the RASADA framework (Vogt et al 2020). The other critical point is the absence of a minimum amplitude requirement for j_{12} which exposes the results to potentially unreliable contributions from signals just above instrument noise. We’ve tested the influence of FAC density on the results and conclude that low density FACs preferably occur at dawn/dusk with large characteristic scales while high density FACs are prevalent at noon/midnight with smaller scales, respectively (additional file 1, Fig. 2). Excluding the low-intensity FACs would thus lead to an amplification of the already noticeable oversampling at noon/midnight.

Moving forward the application of VALOR to SwA and SwC 50 Hz high-resolution magnetic field observations suggests itself in order to adequately probe the regime of mesoscale auroral arcs (few tens of km) or even small-scale (below one km width) dynamic structures with sufficient East-West elongation (~ 200 km). New camera technologies allow to routinely image these aurorae (e.g., Kataoka et al 2021) but their relation to large-scale “background” FACs is not yet fully understood (e.g., Wu et al 2020). With respect to this problem VALOR can be optimized in terms of the specific similarity measure used (e.g., changing the average time scale of MSD in the denominator). Regardless of the application case an important task is to define an easily interpretable quality measure that indicates how well the tuple (t_0, τ_0) can be isolated, e.g., by adopting the concept of topographic prominence. A complementary and independent way to judge VALOR’s validity is to properly incorporate concurrent optical data (e.g., from the EMCCD cameras in the MIRACLE network). Of course, VALOR is technically neither limited to the auroral zone nor to Swarm. It can be applied to any multi-spacecraft geospace mission with suitable instrumentation and orbit configuration, e.g., the Magnetospheric Multiscale mission in the plasma sheet boundary layer (Nakamura et al 2018).

Conclusions

We have introduced the methodological framework *VALOR* for the investigation of field-aligned current (FAC) sheets based on dual spacecraft magnetic field observations. The method was illustrated by means of a sample event (March 1, 2019, 01:58:49-02:03:49 UT), consisting of two consecutive FAC sheet crossings (down and up), regarding which we conclude the following:

- Relatively high levels of vectorial association are found for an optimum spacecraft lag of -11 s and

center time 02:01:34 UT at which the polarization helps to constrain the lag while the vectorial nature of the magnetic input data helps to constrain the center position (Fig. 3).

- The center position falls in between the individual FAC traversals and within the field of view of the Kuujuaq all-sky camera that concurrently observes a relatively stable auroral arc (southward drifting at $\sim 10\%$ of spacecraft velocity, Fig. 4). Given the bi-modal nature of the pattern function an interpretation of the center position as the auroral oval center is justified.
- The reference analysis, i.e., linear correlation & regression according to Forsyth et al (2017), does not indicate large overall similarity at t_0 but rather small- to meso-scale (< 305 km) fluctuations of matching amplitude (Fig. 7) to which we ascribe VALOR's choice of t_0 . The VALOR determined characteristic length scale $L_n = 345$ km is corroborated by the reference analysis.
- The sample event presents a typical case of a low-latitude, evening-sector, large-scale auroral current sheet (Fig. 5, 6) with average inclination ($\alpha = 19^\circ$) and convincingly large planarity. We attribute the differences between VALOR and MVA to different inherent resolution powers (VALOR: small-scale, MVA: large-scale) of the respective methods.

The fact that VALOR can reproduce known characteristics of auroral FACs (i.e., MLT vs. quasi-dipole latitude occurrence pattern, systematic relation between scale length and magnetic local time) lends credence to the overall capability of the approach. The statistical analysis delivered a comprehensive data base of FAC characteristics that we plan to explore in future studies.

Declarations

Availability of data and materials

The datasets used and/or analysed during the current study are available from the following repositories.

Swarm data and derived products are available from <https://earth.esa.int/eogateway/missions/swarm/data>.

The CHAOS-7 geomagnetic field model is hosted at <http://www.spacecenter.dk/files/magnetic-models/CHAOS-7>.

Keograms from the THEMIS All-Sky imagers are accessible via <http://themis.ssl.berkeley.edu>.

Competing interests

The authors declare that they have no competing interests.

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Authors' contributions

LP designed the study, derived the results, and lead the writing of the manuscript. JV developed the VALOR approach and participated in the writing of the manuscript. AB contributed to the design of the study and performed a reference MVA analysis. NS implemented the linear correlation/regression analysis. All authors read and approved the final manuscript.

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Appendix A. The Mean Squared Deviation in terms of Pearson's correlation coefficient and the slope of a linear regression line

We consider two real-valued vectors (time series), $X = \{x_1, x_2, \dots, x_N\}$ and $Y = \{y_1, y_2, \dots, y_N\}$, and choose the inner product

$$\langle X|Y \rangle = \frac{1}{N} \sum_n x_n y_n. \quad (\text{A.1})$$

Then, we get the following expressions for

- a unit vector: $\langle e|e \rangle = 1$ for $e = \{1, 1, \dots, 1\}$,
- the mean: $\bar{X} = \langle X|e \rangle$ (scalar) and $X_{\parallel} = \langle X|e \rangle e$ (vector),
- the variance: $\Delta X^2 = \|X_c\|^2 = \langle X_c|X_c \rangle$ with $X_c = X - X_{\parallel}$,
- Pearson's corr. coeff.: $r(X, Y) = \cos \angle(X_c, Y_c)$,
- Mean Squared Deviation: $\text{MSD}(X, Y) = \|X - Y\|^2 = \langle X - Y|X - Y \rangle$.

410 The MSDs of X , Y on the one hand and X_c , Y_c on the other are related as follows:

$$\begin{aligned}
\text{MSD}(X, Y) &= \|X - Y\|^2 = \|X_{\parallel} + X_c - (Y_{\parallel} + Y_c)\|^2 \\
&= \frac{1}{N} \sum_n (\bar{X}e_n + x_{c,n} - \bar{Y}e_n - y_{c,n})^2 \\
&= \frac{1}{N} \left[N(\bar{X} - \bar{Y})^2 + N\|X_c - Y_c\|^2 + 2N(\bar{X} - \bar{Y}) \overbrace{\langle X_c - Y_c | e \rangle}^{=0} \right] \\
&= (\bar{X} - \bar{Y})^2 + \|X_c - Y_c\|^2 = (\bar{X} - \bar{Y})^2 + \text{MSD}(X_c, Y_c).
\end{aligned} \tag{A.2}$$

411

$$\begin{aligned}
\text{MSD}(X_c, Y_c) &= \|X_c - Y_c\|^2 = \frac{1}{N} \sum_n (x_{c,n} - y_{c,n})^2 \\
&= \langle X_c | X_c \rangle + \langle Y_c | Y_c \rangle - 2\langle X_c | Y_c \rangle \\
&= \Delta X^2 + \Delta Y^2 - 2r(X, Y)\Delta X \Delta Y.
\end{aligned} \tag{A.3}$$

412 Given that $r(X, Y)$ can also be interpreted as the standardized slope m of the linear regression line

$$413 \quad X = mY + \delta X,$$

$$r(X, Y) = \frac{\langle X_c | Y_c \rangle}{\Delta X \Delta Y} = \frac{\Delta Y}{\Delta X} m \iff r \Delta X = m \Delta Y, \tag{A.4}$$

414 we can rewrite $\text{MSD}(X, Y)$ as:

$$\begin{aligned}
\text{MSD}(X, Y) &= \Delta X^2(1 - r^2) + \Delta Y^2(1 - m^2) + (\bar{X} - \bar{Y})^2 \\
&= \Delta X^2 - \Delta X^2 r^2 + \Delta Y^2 - 2m \Delta Y^2 + m^2 \Delta Y^2 + (\bar{X} - \bar{Y})^2 \\
&= \Delta X^2 - \cancel{\Delta X^2 r^2} + \Delta Y^2 - 2r(X, Y) \Delta X \Delta Y + \cancel{r^2 \Delta X^2} + (\bar{X} - \bar{Y})^2 \quad \square
\end{aligned} \tag{A.5}$$

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Figure legends

Fig. 1: Data for sample event vs. time, magnetic local time (MLT) and quasi-dipole latitude (QDLat). **a)** Vectorial magnetic field perturbations measured at Swarm Alpha (subscript A, light colors) and Charlie (subscript C, dark colors) pointing towards N (North), E (East) and C (Center). **b)** Current density according to the Swarm Level-1b single-spacecraft ($j_{A,C}$, blue) and Level-2 dual-spacecraft (j_{AC} , orange) products. **c)** Probability density (black) and cumulative density (red) functions derived from j_{AC} with median position (t_{0i} , red dashed) and interquartile range (IQR, gray shaded). Black dashed lines mark the final 300 s long event window boundaries.

Fig. 2: Sketch of observation geometry.

Sketch of a northern hemispheric Field-Aligned Current sheet (FAC, light gray, aligned with ambient magnetic field $\hat{q}$) being crossed by Swarm Alpha (SwA) and Charlie (SwC) on orbit segments forming the tangential plane ($\hat{r}_\perp$, blue) under inclination α . **a)** Special (simple) case where the tangential plane coincides with $\hat{q}_\perp$. The association delivers an optimum lag τ_0 at center time t_0 which isolates the polarization direction ($\hat{p}_0$, red) from the candidates ($\hat{p}_c$) and determines the sheet normal ($\hat{n}_0$, red) along which the spatial scale (L_n) is measured. **b)** General case where the tangential plane is rotated away from $\hat{q}_\perp$ by the dihedral angle $\beta = \angle(\hat{r}, \hat{q})$. Note that $\hat{p}_0$ and $\hat{n}_0$ always lie in the $q_\perp$ -plane

regardless of the specific viewing geometry.

Fig. 3: Comparison of association measures for the sample event.

A comparison of association measures $\mathcal{C}(t|\tau)$ for sample event #7842 based on single-spacecraft FAC densities j_σ ($\mathcal{C}_s$), temporal derivatives of magnetic field perturbations $\dot{\mathbf{b}}_\sigma$ ($\mathcal{C}_v$) and a combination of $\dot{\mathbf{b}}_\sigma$ and the polarization candidates $\hat{\mathbf{p}}_c$ ($\mathcal{C}_p$, preferred option). **a)-c)** Maps of $C(t, \tau)$ normalized to [0,1] with identified optimum lag τ_0 indicated by dashed lines. **d)-f)** Association profiles at optimum lag $C(t, \tau_0)$ normalized such that the integral under the curve is one (probability densities, black) and corresponding cumulative densities (red) with median (gray dashed) and interquartile range (IQR, gray shaded).

Fig. 4: VALOR results for the sample event and optical evidence of the associated auroral arc.

a) VALOR orientation results for sample event #7842 in the X-Y-plane of the mean field-aligned coordinate system. Gray arrows represent the projected ambient magnetic field, contour lines indicate quasi-dipole latitude (vertical) and magnetic local time (horizontal). Colored arrows represent the projected pattern function (Eq. 12) at optimum spacecraft lag along the centered spacecraft locations between event start and end (s, e, black dots). Normalized black arrows show the identified normal and polarization directions at the sheet's center (t_0 , red dot). Note that the ambient field is down-scaled with respect to the pattern function. The gray ellipse marks the 9°-wide field of view of the THEMIS All-Sky Imager (ASI) at Kuujjuaq, Canada, at 110 km altitude. **b)** Keogram (geographic latitude vs. universal time) constructed from the central North-South slices of the images taken by the Kuujjuaq ASI over a 30-minutes period. Colored lines mark the event boundaries (blue) and center times (preprocessing: orange; VALOR: red) of event #7842. The shown keogram combines segments from two originals spanning 00:01-00:02 UT and 00:02-00:03 UT.

Fig. 5: Statistical VALOR results.

Results for 9709 events (04/2014-04/2020). The 2-D histogram in panel a) and the gray bar plots in panels b)-i) indicate VALOR “polarization” results (cf., Fig. 3c, f). The angles in panels d)-f) describe the FAC sheets’ orientation in space and are explained in Fig. 2b and Fig. 3a. Here, the colored step functions are stacked and represent two subsets covering the interval $[0^\circ, 90^\circ)$ (cyan) and $[90^\circ, 180^\circ)$ (magenta). Additional step functions in black, blue and red (not stacked) refer to results obtained either in the preprocessing or from the “scalar” and “vector only” methods, respectively (cf., Fig. 3a, b,

d, e). Dashed lines mark the corresponding bins for the sample event #7842.

Fig. 6: Statistical VALOR results in dependence of spatial scale.

Histograms from Fig. 5 (gray) split into three subsets of spatial scales (not stacked).

Fig. 7: Linear correlation and regression analysis applied to the sample event.

Running Pearson’s correlation coefficient (panel a) and the slope of the linear regression (panel b) between the filtered single-spacecraft FAC densities from Swarm A and C (lagged by $\tau_0 = -11$ s). The window lengths correspond to the smallest periods in the filters which cover the small (23–152 km; blue), medium (152–305 km; yellow/orange) and large (>305 km; red) spatial scale regimes. Horizontal dashed lines indicate $r = 0.9$, $m = 0.9$ and $m = 1.1$ levels and red background marks periods at which $r > 0.9 \wedge 0.9 < m < 1.1$ holds for the 40–50 s filtered signals. Vertical dashed lines indicate the VALOR-determined center time t_0 and gray background marks the corresponding FAC length scale (IQR). The design of this plot follows that of Forsyth et al (2017).

Fig. 8: VALOR results for the sample event split into two analysis windows.

Same as Fig. 4a but here the event was split at t_0 and VALOR was applied again to each segment separately. First segment: $t_{01} = 02:00:36$ UT, $\tau_{01} = -4$ s, $L_{n1} = 385$ km, $\alpha_1 = 14^\circ$, $\beta_1 = 169^\circ$, $\gamma_1 = 164^\circ$. Second segment: $t_{02} = 02:02:17$ UT $\tau_{02} = -10$ s, $L_{n2} = 230$ km, $\alpha_2 = 14^\circ$, $\beta_2 = 166^\circ$, $\gamma_2 = 168^\circ$.

Fig. 9: Magnetic hodogram for sample event.

The blue line tracks the spacecraft centered magnetic field perturbation $\mathbf{b} = 0.5 [\mathbf{b}_1(t) + \mathbf{b}_{2,\tau_0}(t)]$ in the plane perpendicular to the mean ambient magnetic field $\bar{\mathbf{B}}^*$ from start (s) to end (e). The X- and Y-axes are aligned with the eigenvectors $\mathbf{x}_1 (= \hat{\mathbf{p}}_{\text{MVA}})$ and $\mathbf{x}_2 (= \hat{\mathbf{n}}_{\text{MVA}})$ to the eigenvalues $\lambda_1 > \lambda_2$ of the data covariance matrix according to a constraint $(\bar{\mathbf{B}}^* \cdot \hat{\mathbf{n}} = 0)$ Minimum Variance Analysis. The VALOR results (black arrows) estimated at t_0 are shown for reference.

Additional files

- Additional file 1, “Additional_file_1.pdf”, includes two additional figures with legends.

Figures

Figure 1

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Figure 2

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Figure 3

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Figure 4

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Figure 5

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