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The effect of Fractional damping and time-delayed feedback on the stochastic resonance of asymmetric SD oscillator

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Abstract

This paper proposes the stiffness nonlinearities and asymmetric SD (smooth and discontinuous) oscillators under time-delay feedback control with a fractional damping. With the effect of displacement and velocity feedback, the oscillator can be adjusted to the desired vibration state and then the stochastic resonance (SR) is achieved. This article discusses the contribution of various system parameters and time-delay feedback to SR, especially which induced by fractional damping. It should be noted that this paper provides effective guidance for fault diagnosis and weak signal detection, energy harvesting, vibration isolation and vibration reduction.

Keywords:

SD oscillators; fractional damping; time-delay; stochastic resonance

1. Introduction

Fault diagnosis, signal detection and vibration control have been active fields of research over the past decades. There are various methods for weak signal detection and feature extraction for early mechanical faults, two effective methods as follow:

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The first one is detecting weak signals and extracting key features such as amplitude and frequency, by using SR or it is rich dynamical behaviour transition with the help of filter which constructed by nonlinear dynamic systems. [1–6] has been done an excellent in weak signal detection by SR. The paper [7] has made a comprehensive summary of the principle and limitations of SR. Besides, the work on fault diagnosis and signal detection by Lu et al. [8] has given a complete summary of such method. For signal detection using the dynamical behaviours transfer, these papers [9–11] make a preliminary discussion with duffing oscillator. This paper is based on the methods of TVICM and TV that proposed by us [12, 13] Furthermore, we obtain a whole set of procedures for signal detection and feature extraction.

The second method is using Deep Neural Networks (DNN) for fault diagnosis on the basis of certain fault samples and most of them are fault classification [14–16].

The combination of two methods is an inevitable trend in the future. It is well-known that strengthening the effective components in the pre-processed signal before diagnosis and detection, which is the goal that all methods want to achieve. SR is an effective method at present, it can transfer the energy of noise to the effective signal under the state of resonance and lay a good foundation for further work.

Many methods are proposed to vibration control, which can be divided into three categories as engineering measures: reducing disturbance, preventing resonance and taking vibration isolation [17–21]. For example, many researchers have investigated based on the vibration control technology of the time-delay feedback [22–29]. The phenomenon of time delay in finite signal transmission is common [30–34], which may induce greatly changes in the dynamic characteristics of the system. Thus, by studying the effect of time delay, we can not only understand the operating mechanism of the system, but also obtain unexpected gains by actively making use of it [35–38]. For example, the effectively control of time delay on the vibration of SD oscillator [39] lays the foundation for the vibration energy harvesting. The stochastic perturbation is unavoidable in vibration control, so the study on control the vibration of the system reasonably by adjusting the random parameters (the noise and so on) is more and more important. Such as by adjusting the values of the parameters, the system can reach the state of SR.

As we all know, SR is a physical phenomenon, which exists in nonlinear systems. The original work on SR was done by Benzi et al. [40, 41], in which the term was coined, was in the context of simulating the transition

of Earth's climate between ice ages and periods of relative warmth with a
45 period of about 100,000 years in the past 700,000 years. Nicolis.C and Nico-
lis.G [42] proposed to use the bistable potential climate model to simulate the
two states of quaternary glacial and warm climate period, and established
the corresponding stochastic differential equation, using adiabatic approxi-
mation conditions to solving differential equations, and then drew the same
50 conclusion as Benzi.

The SD oscillator with nonlinear stiffness, which was proposed by Cao et
al. [43]. The nonlinear stiffness of SD oscillator [44–49] makes its dynamic
behavior specially abundant, even under the circumstances of single degree
of freedom. The phenomena of snap-through buckling in engineering was re-
55 vealed by Thompson and Hunt [50], who put forward that the SD oscillator
was developed based on the shallow arch model. It is necessary for the en-
gineering practice needs to study on nonlinear dynamics of oscillator under
nonlinear stiffness time-delay feedback control [39]. Moreover, the principal
resonance of the quasi-zero-stiffness SD oscillator is studied, and the reso-
60 nance of negative-stiffness SD oscillator is analyzed. The optimal time delay
of changing the control intensity can be obtained according to the proposed
optimization standard.

In this paper, the SD oscillator is further modified on the basis [39].
The SD oscillator with asymmetric horizontal stiffness is proposed and frac-
65 tional order damping with Caputo derivative is introduced. There are many
fractional derivatives in the literature of fractional calculus, such as Caputo
[51], Riemann-Liouville and Grünwald-Letnikov. Due to Caputo fractional
derivative can accurately describe and reflect actual physical problems and
non-locality plays a significant role in various fields, more and more research
70 studies complete with it, so do we. In addition, in order to construct a filter
based on SD oscillator for fault diagnosis and signal detection, the SR of
the nonlinear stiffness SD oscillator subjected to time delay feedback, asym-
metric parameters and fractional damping is studied under the influence of
harmonic forcing and Gaussian white noise.

75 This paper is organized as follows: In section 2, the stochastic delay
differential equation (SDDE) with fractional order is given to describe the
nonlinear SD oscillator, in order to analyze it was reduced to the Langevin
equation after dimensionless processing. In section 3, the effects of time
delay feedback control on the nonlinear SD oscillator and the contribution of
80 fractional order damping and asymmetric parameter in the equation to SR
are discussed. In section 4, conclusion and discussion are provided and put

forward the prospect of the future work.

2. Time delay-controlled asymmetrical SD oscillator driven by fractional derivatives with stiffness nonlinearities

85 In 2006, Cao [52] proposed a SD (smooth and discontinuous) oscillator, and studied its complex dynamical behavior, then found the "strange hand" attractor (SD attractor). Subsequently, a large number of scholars have studied the abundant dynamical behavior of SD oscillator [53–55].

Based on the SD oscillator [56], the dynamic model of asymmetrical SD oscillator, which is driven by fractional derivatives with stiffness nonlinearities and controlled by delay feedback, is established, seen as Fig. 1.

Figure 1: Asymmetrical SD oscillator considered the gravity of the loaded mass coupled with both time-delayed displacement and velocity feedbacks

The equation of motion of the system has the following form

$$\begin{aligned}
 M\ddot{X} + CD^q X + K_1 X + K_2 X \left(1 - \frac{L_1}{\sqrt{X^2 + l_1^2}}\right) + K_2 X \left(1 - \frac{\rho L_1}{X^2 + l_2^2}\right) - K_1 \xi + Mg \\
 = F \cos(\Omega T) + F_{th} + \Upsilon_d X(T - \sigma) + \Upsilon_r \dot{X}(T - \sigma)
 \end{aligned} \quad (1)$$

where X is the displacement of the loaded mass M , K_1 is a vertical linear stiffness spring and C is the coefficient of a viscoelastic damping; K_2 is the

95 stiffness of a pair of oblique springs, which produces the nonlinear stiffness mechanism, and unstretched length L_1 reveals the asymmetry of the system. $L_2 = \rho L_1$ and the length l_1, l_2 while compressed the direction in the horizontal; ξ represents the free length of the oblique of spring over the origin, it represents the midpoint of the pivots of the supporting ends of the oblique
100 springs; Mg is the gravity of the loaded mass; $\Upsilon_d X(T - \sigma) + \Upsilon_r \dot{X}(T - \sigma)$ is the signal of delayed feedback control, where Υ_d is the displacement feedback intensities and Υ_r is velocity feedback intensities; $\sigma > 0$ is the time-delay feedback; The purpose of designing mechanical actuator is to realize delayed feedback control; F is amplitude of harmonic exciting force and F_{th} is the environmental base excitation; Ω is frequency of harmonic exciting force; $D^q x$
105 is the Caputo fractional derivative, which has rich physical meaning, such as viscous liquid damper i.e.,

$$D^q x = \frac{1}{\Gamma(1-q)} \int_0^t \frac{\dot{x}(\varpi)}{(t-\varpi)^q} d\varpi, \quad 0 < q \leq 1, \quad (2)$$

where $t \in [0, d]$, as the initial torque of a physical oscillator is $t = 0$, and $\Gamma(\cdot)$ is a Gamma function;

110 For brevity, the oblique springs are horizontal, at the same time the unforced mass in the position of static equilibrium that $X = 0$, the constant force $K_1 \xi - Mg$ can be neglected, and define dimensionless processing

$$\begin{aligned} x &= \frac{X}{L_1} & \alpha_1 &= \frac{l_1}{L_1} & \alpha_2 &= \frac{l_2}{L_2} & L_2 &= \rho L_1 & t &= \omega_1 T & \omega_1^2 &= \frac{K_1}{M} & \gamma &= \frac{\omega_2^2}{\omega_1^2} \\ 2c &= \frac{C}{M\omega_1} & \omega_2^2 &= \frac{K_2}{M} & f &= \frac{F}{ML\omega_1^2} & \omega &= \frac{\Omega}{\omega_1} & \eta(t) &= \frac{F_{th}}{ML_1\omega_1^2} \\ \mu &= \frac{\Upsilon_d}{M\omega_1^2} & \nu &= \frac{\Upsilon_r}{M\omega_1} & \tau &= \omega_1 \sigma \end{aligned} \quad (3)$$

and by the transformation of variables, we can obtain the equation

$$\begin{aligned} \ddot{x} + 2cD^q x + x + \gamma x \left(1 - \frac{1}{\sqrt{x^2 + \alpha_1^2}}\right) + \gamma x \left(1 - \frac{1}{\sqrt{(\frac{x}{\rho})^2 + \alpha_2^2}}\right) \\ = f \cos(\omega t) + \eta(t) + \mu x(t - \tau) + \nu \dot{x}(t - \tau) \end{aligned} \quad (4)$$

where $\eta(t)$ is Gaussian white noise, which satisfies

$$E[\eta(t)] = 0, \quad E[\eta(t)\eta(t-s)] = 2\kappa\delta(s) \quad (5)$$

115 where κ denotes the intensities of Gaussian white noise $\eta(t)$, and $\kappa^2 = 2\pi K$ (K is a constant power spectral density), $\delta(s)$ is the Dirac function, and s is the correlation time of Gaussian white noise $\eta(s)$.

Approximation of fractional derivatives [57] and displacement and velocity delay feedback with the assumption that τ is finite [58, 59]

$$\begin{aligned} D^q x &= \omega^q \cos\left(\frac{\pi q}{2}\right)x + \omega^{q-1} \sin\left(\frac{\pi q}{2}\right)\dot{x}, \quad 0 < q \leq 1 \\ x(t - \tau) &= x \cos(\omega\tau) - \frac{\dot{x} \sin(\omega\tau)}{\omega} \\ \dot{x}(t - \tau) &= x\omega \sin(\omega\tau) + \dot{x} \cos(\omega\tau) \end{aligned} \quad (6)$$

120 where, ω is the dominant frequency of the oscillation [56].

Then we can get the reduced equation

$$\begin{aligned} \ddot{x} + a\dot{x} + bx + \gamma x \left(1 - \frac{1}{\sqrt{x^2 + \alpha_1^2}}\right) + \gamma x \left(1 - \frac{1}{\sqrt{\left(\frac{x}{\rho}\right)^2 + \alpha_2^2}}\right) \\ = f \cos(\omega t) + \eta(t) \end{aligned} \quad (7)$$

where

$$\begin{aligned} a &= 2c\omega^{q-1} \sin\left(\frac{\pi q}{2}\right) + \frac{\mu}{\omega} \sin(\omega\tau) - \nu \cos(\omega\tau) \\ b &= 2c\omega^q \cos\left(\frac{\pi q}{2}\right) - \mu \cos(\omega\tau) - \nu\omega \sin(\omega\tau) + 1 \end{aligned} \quad (8)$$

The Langevin equation after the reduction is

$$\begin{cases} \dot{x} = y, \\ \dot{y} = -ay - bx - \gamma x \left(1 - \frac{1}{\sqrt{x^2 + \alpha_1^2}}\right) - \gamma x \left(1 - \frac{1}{\sqrt{\left(\frac{x}{\rho}\right)^2 + \alpha_2^2}}\right) \\ \quad + f \cos(\omega t) + \eta(t). \end{cases} \quad (9)$$

125 The restoring force of the dimensionless asymmetric SD oscillator dynamic model with stiffness nonlinearity is given as follow

$$G(x) = bx + \gamma x \left(1 - \frac{1}{\sqrt{x^2 + \alpha_1^2}}\right) + \gamma x \left(1 - \frac{1}{\sqrt{\left(\frac{x}{\rho}\right)^2 + \alpha_2^2}}\right) \quad (10)$$

The deterministic potential of this system is

$$V(x) = \left(\frac{b}{2} + \gamma\right)x^2 - \gamma\sqrt{x^2 + \alpha_1^2} - \rho\gamma\sqrt{x^2 + (\rho\alpha_2)^2} \quad (11)$$

and the equivalent stiffness of this system can be described as

$$K(x) = b + 2\gamma - \frac{\gamma}{\sqrt{x^2 + \alpha_1^2}} - \frac{\rho\gamma}{\sqrt{x^2 + (\rho\alpha_2)^2}} + \frac{\gamma x^2}{(x^2 + \alpha_1^2)^{\frac{3}{2}}} + \frac{\rho\gamma x^2}{(x^2 + (\rho\alpha_2)^2)^{\frac{3}{2}}} \quad (12)$$

3. Delay-fractional derivative-controlled stochastic resonance

In this section, we use the joint probability density of velocity and displacement to discuss the influence of time delay, fractional order and other parameters on the SNR and seek for the maximum SNR which is stochastic resonance.

3.1. Joint quasi-stationary probability density function P_e and effective potential energy U_e

According to the adiabatic approximation theory, the FPK equation is described by Eq.(9) takes the form

$$\begin{aligned} \frac{\partial}{\partial t}P(x, y, t) + \frac{\partial}{\partial x}(yP(x, y, t)) + \frac{\partial}{\partial y}[-ay - bx - \gamma x(1 - \frac{1}{\sqrt{x^2 + \alpha_1^2}}) - \gamma x \\ (1 - \frac{1}{\sqrt{(\frac{x}{\rho})^2 + \alpha_2^2}})]P(x, y, t) + \frac{\partial}{\partial y}(f \cos(\omega t))P(x, y, t) + \frac{\partial^2}{\partial y^2}P(x, y, t) = 0. \end{aligned} \quad (13)$$

From Eq.(13), $P_e(x, y, t)$ (where x represents the displacement and y is velocity) is the joint quasi-stationary probability density function, which is given following

$$P_e(x, y, t) = N \exp\left[-\frac{U_e(x, y, t)}{\pi\kappa}\right], \quad (14)$$

where N represents a normalization constant, the effective potential energy is denoted by $N = (\int \exp(P_e)dx)^{-1}$ and $U_e(x, y, t)$ has given

$$U_e(x, y, t) = a\left(\frac{y^2}{2} + \frac{1}{2}bx^2 + \gamma x^2 - \gamma\sqrt{x^2 + \alpha_1^2} - \gamma\rho\sqrt{x^2 + (\rho\alpha_2)^2} - xf \cos(\omega t)\right) \quad (15)$$

Figure 2: Stochastic Hopf-Bifurcation with different parameter τ

Stochastic Hopf-bifurcation will arise through analysing the shape of probability density function P_e varies with parameters τ and q , as shown in the Figs.2 and 3. Furthermore, considering the effect of time delay τ , the value of q is assumed to be 0.46 in Fig.2; $\tau = \frac{\pi}{4}$ is hypothesised when analyze the influence of the order of the fractional order q in Fig.3, other deterministic parameters are set as $c = 0.2$, $\gamma = 0.9$, $\alpha_1 = 0.2$, $\alpha_2 = 0.1$, $f = 0.05$, $\omega = 1$, $\mu = 0.5$, $\nu = 0.4$, $\rho = 0.5$ and $\kappa = 0.1$. The stochastic D-bifurcation is represented in figure (a) through figure (b); The stochastic P-bifurcation is shown from figure (b) to figure (c), both in Figs.2 and 3.

The three-dimensional surfaces of the effective potential energy U_e as functions of x and q (or $\rho, \tau, \mu, \nu, \kappa$) are shown in Fig.4. From Fig.4(a) and (c), it could be notice that with the increase of the order q of fractional derivative and the time delay τ , the potential barriers increases obviously at first and then decreases. The potential barriers are basically the same as the increasing of displacement feedback strength μ , asymmetric proportion ρ and velocity feedback intensity ν , shown from Fig.4(b), (d) and (e).

Figure 3: Stochastic Hopf-bifurcation with different parameter q

To put it simply, based on the effective potential of the FPK equation, the physical meaning of noise-delay and fractional-assisted frequency hopping mechanism is explained. In the next section, we will analyze and extract a function which plays the role of activation energy to identify the SNR.

3.2. Signal-to-noise ratio

For further analysis of signal-to-noise ratio(SNR), with the condition of $(1 + 2\gamma)\alpha_1\alpha_2 < \gamma(\alpha_1 + \alpha_2)$ and $\gamma \neq -\frac{1}{2}$, three singular points $x_{\pm}, x_0$ of the asymmetrical SD oscillator are obtained when lack of the delay feedback control [60]. The eigenvalues of the linearization matrix at the equilibria $x_{\pm}$ and x_0 are $\beta_{1,2}(\pm)$ and $\lambda_{1,2}(0)$, respectively.

Research on SNR from the view of Kramers rate. We can calculate the propagation rate of noise-driven motion in the Eq.(1) with the help of the famous Kramers rate [61]. For the exact expression of escape rate, for reference [62]. Fundamentally, we can start with the Kolomogorovs backward equation, which is equivalent to (15). Eventually result in the escape rate

(a) $\rho = 0.5, \tau = 1.5, \mu = 0.5, \nu = 0.4, \kappa = 0.1$

(b) $q = 0.46, \tau = 1.5, \mu = 0.5, \nu = 0.4, \kappa = 0.1$

(c) $q = 0.46, \rho = 0.5, \mu = 0.5, \nu = 0.4, \kappa = 0.1$

(d) $q = 0.46, \rho = 0.5, \tau = 1.5, \nu = 0.4, \kappa = 0.1$

(e) $q = 0.46, \rho = 0.5, \tau = 1.5, \mu = 0.5, \kappa = 0.1$

(f) $q = 0.46, \rho = 0.5, \tau = 1.5, \mu = 0.5, \nu = 0.4$

Figure 4: Effective potential energy U_e vs displacement x and $q, \rho, \tau, \mu, \nu, \kappa$. The other parameters are $c = 0.2, \gamma = 0.9, \alpha_1 = 0.2, \alpha_2 = 0.1, f = 0.05, \omega = 1$.

$R_{\pm}(t)$ for

$$R_{\pm}(t) = \frac{\sqrt{\beta_1 \beta_2}}{2\pi} \sqrt{-\frac{\lambda_1}{\lambda_2}} \exp\left[\frac{U_e(x_{\pm s}, t) - U_e(x_0, t)}{\pi \kappa}\right], \quad (16)$$

The mean-first-passage time (MFPT) over the potential barrier is calculated from the following definition:

$$\text{MFPT}_{R,L} = \frac{1}{R_{\pm}(t)}. \quad (17)$$

Let us substitute Eq.(15) into Eq.(16), then the probability transition rate can be obtained as

$$R_{\pm}(t) = \frac{\sqrt{\beta_1\beta_2}}{2\pi} \sqrt{-\frac{\lambda_1}{\lambda_2}} \exp\left[\frac{a}{\pi\kappa} \left(\frac{1}{2}bx_{\pm s}^2 + \gamma x_{\pm s}^2 - \gamma\sqrt{x_{\pm s}^2 + \alpha_1^2} - \gamma\rho\sqrt{x_{\pm s}^2 + (\rho\alpha_2)^2} - x_{\pm s}f\cos(\omega t) - \gamma\alpha_1 - \gamma\rho^2\alpha_2\right)\right] \quad (18)$$

$R_{\pm}(t)$ can be expressed in the form of Taylor series

$$R_{\pm}(t) = R_0(1 \mp x_{\pm s}f\cos(\omega t) + \frac{1}{2}x_{\pm s}^2f^2\cos^2(\omega t) \pm \dots) \quad (19)$$

where

$$R_0 = \frac{\sqrt{\beta_1\beta_2}}{2\pi} \sqrt{-\frac{\lambda_1}{\lambda_2}} \exp\left[\frac{a}{\pi\kappa} \left(\frac{1}{2}b|x_{\pm s}|^2 + \gamma|x_{\pm s}|^2 - \gamma\sqrt{|x_{\pm s}|^2 + \alpha_1^2} - \gamma\rho\sqrt{|x_{\pm s}|^2 + (\rho\alpha_2)^2} - \gamma\alpha_1 - \gamma\rho^2\alpha_2\right)\right] \quad (20)$$

Further, the output spectrum of the system can be obtained

$$S(\Omega) = S_1(\Omega) + S_2(\Omega) \quad (21)$$

where $S_1(\Omega)$ is the signal of the output power spectra and $S_2(\Omega)$ is noise of the output power spectra, and their expressions can be written as follows

$$\begin{aligned} S_1(\Omega) &= \frac{\pi x_{\pm s}^2 (R_0 a |x_{\pm s}| f)^2}{2(R_0^2 + \omega^2)} [\delta(\Omega - \omega) + \delta(\Omega + \omega)] \\ S_2(\Omega) &= \left[1 - \frac{(R_0 a |x_{\pm s}| f)^2}{2(R_0^2 + \omega^2)}\right] \frac{2x_{\pm s}^2 R_0}{2(R_0^2 + \Omega^2)} \end{aligned} \quad (22)$$

According to the definition of output power SNR, the SNR of fraction-delay-controlled asymmetric SD oscillator can be obtained as

$$\text{SNR} = \frac{\int_0^\infty S_1(\Omega) d\Omega}{S_2(\Omega = \omega)} = \frac{\pi (R_0 a |x_{\pm s}| f)^2 (R_0^2 + \omega^2)}{4R_0(R_0^2 + \omega^2) - 2R_0(R_0 a |x_{\pm s}| f)^2} \quad (23)$$

Figure 5: The SNR as a function of fractional order q or time delay τ for various asymmetric ratio ρ . The other parameter values are $\gamma = 0.25, \alpha_1 = 0.21, \alpha_2 = 0.79, \omega = 0.5, c = 0.1, f = 0.1, \kappa = 0.01, \mu = 0.5, \nu = 0.1$

185 REMARK: There is a minor error in formula (41) in the Ref.[56], it should be $\frac{\pi(R_0\zeta_1|x_{\pm s}|f)^2(R_0^2+\omega^2)}{4R_0(R_0^2+\omega^2)-2R_0(R_0\zeta_1|x_{\pm s}|f)^2}$. But this small error will not have a great impact on the SNR analysis, as shown in Figs. 13 and 14 of Ref[56], it does not affect the shape of the SNR.

In Fig.5(a), under the particular parameter, the SNR decreases first, then increases, and finally decreases with the increase of fractional order q . When the SNR reaches the peak, the stochastic resonance (SR) occurs in the system, which indicates that the order change of the fractional order q will affect the occurrence and disappearance of SR, our purpose is to adjust the order q so that the system achieves maximum SR. We find that $q = 0$ is also an extreme point, there is no damping term $\dot{x}$ in the system, that is, the system has no energy loss; And with the increase of q , it can be seen that the damping term plays a certain role from formula (6), which can be interpreted as the energy consumption of the system, resulting in the reduction of SNR. An interesting phenomenon worth mentioning is that when the order q increases again, the SNR does not decrease but rises to a peak value. This will mean that by adjusting the order q , the time delay-controlled asymmetrical SD oscillator driven by fractional derivatives can achieve ideal SR, laying a good foundation for the detection of weak signals.

Next, we can choose a fractional order $q = 0.7$ and analyze the SNR changes with time delay τ . It can be seen that the appropriate time delay τ will make the SNR reach the peak, shown as Fig5(b). In addition, the effect of asymmetric ratio ρ of asymmetric SD oscillator on SNR is also considered, presenting a non-monotone change. When time delay $\tau = 0.1, \rho = 0.3$ is the

Figure 6: SNR vs. joint fractional order q and noise intensity κ with $\tau = 0.1, \mu = 0.5, \nu = 0.1$

most appropriate, when $q = 0.7, \rho = 0.2$ is optimal. In a word, q, τ and ρ jointly restrict the change of SNR.

With the parameter of Fig.5, according to Eq.(23), the SNR versus joint fractional order q and noise intensity κ is displayed in Fig.6.

Figure 7: SNR vs. joint fractional order q and noise intensity κ with different τ, μ , and ν $\gamma = 0.25, \alpha_1 = 0.21, \alpha_2 = 0.79, \omega = 0.5, c = 0.1, f = 0.1, \kappa = 0.01$

In Fig.7(a, b and c), under the particular parameter, the SNR increases

Figure 8: SNR vs. joint time delay τ and noise intensity κ with $q = 0.7$ varies with different μ or ν .

with the increase of time delay τ . As Fig.7(c) shown that the maximum
 215 time delay τ will make the SNR reach the peak. The SR phenomenon occurs
 at the maximum of the SNR. Besides the SNR can also reach the peak
 under the change of displacement feedback strength μ and velocity feedback
 intensity ν in Fig.7(f and g). We can see the parameter of τ has the most
 obvious influence on the SNR, that's because the time delay τ affects the
 220 effect of the system on the non-linearization of the input signal. We find
 that there are many adjustable parameters for the stochastic resonance(SR)
 in the system. In Fig.8 we have studied the joint effects of displacement
 feedback strength μ and velocity feedback intensity ν on the SNR. Our aim
 is to use the mathematical optimal control method to find the appropriate
 225 parameters to make the SNR reach the maximum value.

3.3. Suggestion for application: weak signal detection and vibration energy harvesting

3.3.1. Weak signal detection

When add the harmonic weak signal containing Gaussian white noise $r \cos(\omega_1 t)$ to the original system and use the use the method of chaotic state "transient vacancy" (namely "TV" method) to detect the frequency of weak signal, system will be period state if $\omega = \omega_1$, and if $\omega \neq \omega_1$ system will be in chaotic state. We can presume that the SR of the system contributes to the periodic state and at that time it is obviously to observe.

Adjusting the parameters $\gamma = 0.9, \alpha_1 = 0.2, \alpha_2 = 0.1, \mu = 0.5, v = 0.4, \rho = 0.5, k = 0.1, c = 0.2$, when $q = 0.7$ and $\omega = \omega_1 = 1$ the system reaches the state of SR. Comparing Fig.9(a and b), on the one hand, it obvious that when $q = 0.7$ the periodic state is clearer than $q \neq 0.7$, which makes the periodic state is easier to identify. On the other hand, when $q = 1$, only with enough the chaotic threshold f , the periodic state can be identified, see the left hand of the blue line which $f = 0.3$ in Fig.9(a). But in Fig.9(b), when $q = 0.7$, the value of f can be little when the system reaches the periodic state, that is to say, weak signal detection more quickly and lower time cost when $q = 0.7$ than $q = 1$. Therefore, the fractional order on weak signal detection can be obviously reflected than the integer order, besides it is more efficient and takes less time.

The discovery of stochastic resonance(SR) has completely subverted people's perception of noise. Noise has always been considered to destroy the orderly behavior of the system and reduce the system performance, but now we find that the noise can be used to improve the output SNR of the system. When the system is adjusted to the state of SR, the noise energy is transferred to the weak signal, the SNR reach the maximum value.

In Fig.10, the effect of noise intensity κ on the periodic state of the system have been investigated. Fixed the parameters as $q = 0.7, \gamma = 0.25, \alpha_1 = 0.21, \alpha_2 = 0.79, \mu = 0.5, v = 0.1, \rho = 0.2, c = 0.1$. In Fig.10(b), when $\kappa = 0.04$ and $\omega = \omega_1$, the system reaches the SR state, at that time the periodic state is clearer than the others values of ω . In the left hand of the blue line $f = 0.3$, in Fig.10(b) the periodic state of the system can be easily observed. We can see from Fig.10(a and b) that the increase in κ leads to the increase in the SNR. This illustrates that in the state of SR, the energy of the noise can be transferred to the signal which to be detected.

The effects of time-delay τ , you can refer to [39], which have been exten-

(a) $q=1$, $\omega = 1$ and fix $\omega = \{0.9, 0.95, 1, 1.05, 1.1, 1.15, 1.2\}$ from top to bottom, it shows when $\omega = \omega_1 = 1$ the system first reaches the periodic state.

(b) $q=0.7$, $\omega = 1$ and fix $\omega = \{0.9, 0.95, 1, 1.05, 1.1, 1.15, 1.2\}$ from top to bottom, it is obvious that when $\omega = \omega_1 = 1$ the system first reaches the periodic state.

Figure 9: The periodic state of the system with different parameter q .

sively studied by the author.

3.3.2. Vibration energy harvesting

265 As for vibration energy harvesting, the SR effect [48, 63?] should be effectively utilized. However, the development of practical SR precise control methods is a further challenge that needs to be overcome. In this paper, the application of SR with fractional damping and time-delay feedback control in asymmetric SD oscillator is comprehensively analyzed and a mathematical
270 model is established, which plays a key role in improving energy harvesting. The equation of motion controlled by the model is

$$\begin{aligned} M\ddot{X} + CD^q x + \frac{dV(X)}{dX} + \theta\hat{V} &= \Upsilon_d X(T - \delta) + \Upsilon_v \dot{X}(T - \delta) - M\ddot{Y} \\ C_p \ddot{\hat{V}} + \frac{\hat{V}}{R} &= \theta \dot{X} \end{aligned} \quad (24)$$

It should be noted that θ is a linear electromechanical coupling coefficient; $\ddot{Y}$ represents the base acceleration and C_p represents the internal piezoelectric capacitance; $\hat{V}$ is the voltage measured on an equivalent resistive load, R ; $V(X)$ is the function of the deterministic potential energy of mechanical
275 asymmetric SD oscillator.

$$\frac{d(V(X))}{d(X)} = (b + 2\gamma)X - \gamma X \frac{1}{\sqrt{X^2 + \alpha_1^2}} - \rho\gamma X \frac{1}{\sqrt{X^2 + \rho\alpha_2^2}} \quad (25)$$

In Fig.11, from the perspective of three-dimensional on the SNR as function of noise intensity and time-delayed feedback. Obviously, the SR can be enhanced by adjusting the value of time delay τ , fractional order q and
280 asymmetric parameter. The results show that under the optimal time delay, fractional order and asymmetric proportion ρ , the theoretical value of energy harvesting is optimized and the significant improvement of energy collection is particularly important in real life and the production activity.

4. Conclusions and discussions

285 In this paper, fractional order damping and time-delay feedback are given to control the vibration of asymmetric SD oscillators with stiffness nonlinearities. And we mainly study the noise-induced SR of the controlled nonlinear stiffness SD oscillator and the effect of fractional damping on the SNR. Based

(a) $\kappa = 0.01$, $\omega = 1.2$ and fix $\omega = \{1.1, 1.15, 1.2, 1.25, 1.3, 1.35, 1.4\}$ from top to bottom, it shows when $\omega = \omega_1$ the system first reaches the periodic state.

(b) $\kappa = 0.04$, $\omega = 1.2$ and fix $\omega = \{1.1, 1.15, 1.2, 1.25, 1.3, 1.35, 1.4\}$ from top to bottom, it is obvious that when $\omega = \omega_1$ the system first reaches the periodic state.

Figure 10: The periodic state of the system with different parameter κ .

Figure 11: 3D plots of the SNR vs. joint noise intensity κ and time delay τ with $\rho = 0.25, \gamma = 0.25, \alpha_1 = 0.21, \alpha_2 = 0.79, \omega = 0.5, c = 0.1, f = 0.1, q = 0.7, \mu = 0.1, \nu = 0.1$

on the theoretical research of this article, we have obtain the following interesting conclusions:

(i) Under the small noise intensity κ . With the time delay τ increasing, it can be find that the potential barriers has suffered a sharp increases first, then decreases slightly, finally increases again. Thus it is non-monotonic about the changes of the barriers. According to Fokker-Planck-Kolmogorov equation (FPK equation), the SNR expression of an asymmetric SD oscillator with double-well potential is obtained. Besides, we have investigated the effect of harmonic forcing and Gaussian white noise on the SR characteristics of the controlled SD oscillator. The design of asymmetric and fractional order damping on the system increases the adjustable parameters, which enhances the value of SNR than the previous [39].

(ii) The design of fractional order damping is added in this paper. It can be clearly seen from the study that the maximum point of SNR is moved to the different values of the fractional order q . Fractional order damping can not only make the SR phenomenon of nonlinear stiffness SD oscillator weak, but also can enhance it. This phenomenon occurs in nonlinear stiffness SD oscillators, etc. We first proposed that the SR in the oscillator can be induced by fractional order damping due to the occurrence of noise.

(iii) The main motivation is to gain enough the effective signal to enhance the value of the output SNR, by constructing the stiffness nonlinearities and

310 asymmetric SD oscillator which has rich dynamic behavior. When the system reach the state of SR, SNR is the maximum. Laying a good foundation for signal detection and fault diagnosis using the correspondence method, which is our pursued direction. Besides, our research can be applied to vibration control and vibration energy harvesting, etc.

315 (iv) We adopted the Caputo fractional derivative in this article, it is suitable for lots of physical problems and many research studies have been done based on the Caputo fractional derivative. The definitions of fractional derivative are various but not unified at present.

(v) In this paper, we assumed that the extraneous noise is independent
320 of the variable $\dot{x}$, but it is also interesting to consider the situation of internal noise. For example, noise related to the memory effect of fractional damping. In fact, the damping originates from the internal noise, which will undoubtedly complicate the study and it is the pursued direction for us.

Declarations

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Conflicts of interest

330 The authors declare that they have no conflict of interest.

Availability of data and material

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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