Prediction and Compensation Strategy of Contour Error in Multi-Axis Motion System

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Prediction and Compensation Strategy of Contour Error in Multi-axis Motion System

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Abstract

The contour error of the machined parts is an important index to evaluate the accuracy of CNC machining. Considering multi-axis servo control system, a predictive compensation strategy for contour error is presented in this paper. First, the offline identification method is adopted to establish the transfer function of each motion system. In this case, the relationship between the interpolation command and the feedback position of the grating is determined for the selected machine tool. Thus, before machining, the trajectory information of each motion axis can be predicted according to the interpolation instruction and the transfer relationship. It can be converted into the contour trajectory of the part through kinematic analysis, so as to predict the contour error. Finally, the predicted contour error is compensated into the command trajectory to ensure the contour accuracy of the part to be machined. Compared to existing methods, our method can effectively reduce the trial-produced cycle of parts and avoid unnecessary waste of processing materials. Moreover, without increasing the complexity of the control system and greatly reducing the machining efficiency, the dynamic error caused by the dynamic characteristics of the shaft is reduced. Taking the starfish pattern and the contour line of the impeller as the milling processing experiment case, the contour error after compensation is greatly reduced compared with the contour error processed by the original command. Therefore, the predictive compensation method proposed in this paper can significantly improve the machining accuracy. In addition, it also has application value for trial production of complex curved parts.

Keywords Contour error System identification CNC machining Error compensation

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1 Introduction

Multi-axis numerical control machining method is a typical representative of the development of CNC technology. It refers to the simultaneous coordinated movement of multiple axes under the control of a computer numerical control system (CNC) to improve the machining accuracy, quality and efficiency of free-form surfaces, which has a wide range of applications in the field of precision machining and manufacturing[1]. The key performance index of a multi-axis motion control system is the accuracy of contour motion. Especially during the machining of high-speed and large-curvature parts, the dynamic performance mismatch between each axis will be enlarged as the speed and curvature increase. Therefore, the research on the high-precision machining of multi-axis, high-speed and variable-curvature parts has important theoretical significance and remarkable engineering application value. According to the existing research, the main reasons that lead to the contour accuracy of the machined parts include

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the quasi-static error represented by the geometrical size of the machine tool structural components, the material instability, the clamping positioning error and the thermal deformation, the dynamic error caused by the servo lag and the dynamic adaptation[2]. Among them, the dynamic error is caused in the movement, and changes with the change of trajectory and speed. The influence often exceeds the influence of quasi-static error on the accuracy of part contour for multi-axis in the high-speed, complex trajectory machining. Therefore, this paper will uniformly consider the dynamic error that affects the contour accuracy of the machined part as the contour error, predict and compensate it.

The contour error is defined as the vector from the actual position point of the coordinated motion of each motion axis to the vertical foot of the expected contour curve planned by the interpolator. The size of its vector modulus is the standard to measure the quality of multi-axis synchronous machining. Scholars from various countries have adopted different solutions to improve the contour accuracy of the multi-axis motion system and ensure the processing efficiency. A considerable number of scholars start from the design of the controller, and propose control strategies for indirect control and direct control of contour error respectively. The representative indirect control methods are zero phase error tracking controller[3], disturbance observer[4], model predictive control[5], which are to improve the followability of each axis, so as to broaden the frequency response bandwidth of each motion axis.

However, the load inertia of each motion axis is different, and it is difficult to coordinate the best dynamic characteristics of each axis. Therefore, professor Koren[6] proposed a cross-coupled control, which can balance the tracking control effect of each axis to achieve the purpose of directly controlling the contour error. In the beginning, most of the cross-coupling controllers(CCC) were based on PID control[7]. With the development of modern control theory, many experts and scholars applied fuzzy control[8, 9], adaptive control[10] and robust control[11] methods to achieve feedback control of contour error. In addition, the combination of single-axis controller changes and cross-coupling control has become the current research development trend[12].

At the same time, the structure of the contour error feedback controller has also changed. Yang[13] proposed a two-layer CCC mechanism, which divides the three-dimensional space contour error into two layers of control. At the bottom layer, a two-axis CCC controller was used to control the XY plane contour, and the top layer was designed separately to control the three-dimensional contour error. Each layer could be designed individually, which made the design of the contour error controller more flexible. However, the direct feedback control of the contour error made the whole system have a strong

coupling response, which reduced the stability of the system. Secondly, the tool path existed in the form of discrete points after numerical control interpolation, so the servo layer didn't save all the information of the trajectory. At the same time, in order to realize the versatility and stability of the CNC system and the servo system, the CNC manufacturers have carried out relatively independent design and system packaging respectively, so the users are difficult to design the controller according to their needs. Therefore, the method of reducing contour error from the perspective of controller design is difficult to be applied and popularized.

Some scholars who study numerical control interpolation consider contour error as a constraint condition in speed planning[14]. Jia[15] established the relationship model of feed rate, trajectory curvature and tracking error, and realized theoretical error constraint by limiting the feed rate. Dong[16] simplified the drive system to a second-order system model, and took the contour error as a constraint condition of real-time interpolation to ensure that the error didn't exceed the set constraint limit. In general, the above methods increase the processing accuracy at the cost of efficiency and increase a large amount of computational burden. From the perspective of system parameter correction, Wu[17] sequentially tuned the control parameters in the position loop, speed loop, and current loop to minimize the mismatch of the dynamic characteristics of each axis, so as to obtain the improvement of the contour accuracy. But this method required a large number of experiments, the difficulty would increase as the number of motion axis increased.

This paper will propose an offline predictive compensation method to determine the discrete transfer function of each motion axis through system identification, aiming to obtain the dynamic characteristics of each axis. Input the interpolation instruction of the part to be machined into the discrete transfer function, convert it into the workpiece coordinate system through kinematics to predict the contour error caused by the dynamic mismatch of each motion axis, and compensate the error to the interpolated discrete trajectory before the actual machining. The method verified by experiments can effectively reduce the contour error of variable curvature parts in the machining process. Especially in the state of lack of processing experience, for the first trial-cut parts, it can effectively improve the pass rate of parts machining and avoid the waste of materials. Meanwhile, it can also avoid the fact that the process personnel adopt conservative parameters during on-site processing, and meet the requirements of part accuracy at a very low speed.

The rest of this article is as follows. The second section introduces the identification method of the feed system transfer function of the machine tool. The third section is to evaluate and screen the identification model to determine the best prediction model for each axis position. The fourth

Fig.2 Translational axis transfer model and rotational axis transfer model

section is to solve the predicted contour error through the instruction trajectory of the pre-machined part and the predicted actual trajectory. Predicted error is compensated. The fifth section verifies the effectiveness of the proposed offline prediction and compensation method for contour error through comparative experiments. Our conclusion is in the sixth section.

2 Identification method of transfer function of feed axis of machine tool

System identification is to determine a model equivalent to the tested system from a set of given model classes on the basis of input and output data. The parameter model identification method is the most frequently used in the system identification. Based on the presumption of the model structure, the parameters of the model are determined by minimizing the error quasi-measurement function between the model and the object. The basic principle is shown as Fig.1.

Fig.1 Basic principles of system identification

At the current time, the system output prediction $\hat{y}$ is calculated according to the estimate parameter $\hat{\theta}(k-1)$ at the last moment, the current and historical input $u(k)$ and output $y(k)$.

$$\hat{y}(k) = \Phi^T(k) \hat{\theta}(k-1) \quad (1)$$

$$\Phi(k) = [-y(k-1), L, -y(k-n_a), u(k-d), L, u(k-d-n_b)] \quad (2)$$

Prediction error is as follows:

$$e(k) = y(k) - \hat{y}(k) \quad (3)$$

Then the prediction error $e(k)$ is fed back to the identification algorithm, and the estimated parameter $\hat{\theta}(k)$ is calculated at the k time. Through the iterative loop, until the minimum value of the criterion function is reached, which means that the model estimated parameters $\hat{\theta}(\infty)$ is obtained, and the model output is also the best approximation to the system output under the criterion. Therefore, in order to achieve the accurate prediction of each axis position information, the following is introduced in turn from the structure of the identification model, the determination of the model order and the design of the identification signal.

2.1 Prediction model structure analysis

In a multi-axis system, each axis is independently controlled, and each motion axis works in a limited frequency range under servo control. Therefore, the high-frequency flexible model part can be ignored in the dynamic mathematical model of the control system. The low-order linear link can be retained. The general structure of the control of the feed axis is shown in Fig.2. The G code is processed by the CNC system to obtain the interpolation point. Through the three-loop control, the motor is controlled to move, driving the mechanical system. Therefore, this paper utilizes real-time

acquisition of the interpolated instruction position signal and grating feedback signal of the CNC system to identify the relationship between them for realizing the prediction of the contour error of the part before machining.

According to Wold decomposition theorem, this paper will use the Prediction-Error Identification Method (PEM) to abstract the identified link into the following discrete transfer function expression:

$$y_k = G(z^{-1})u_k + H(z^{-1})e_k \quad (4)$$

Where u_k , y_k in the Eq. (4) respectively represent the interpolation instruction position of the model input and the grating feedback position of the model output at the time k . $G(z^{-1})$ is the linear model, $H(z^{-1})$ is the noise model, e_k is the Gaussian white noise signal. $G(z^{-1})$ and $H(z^{-1})$ from different PEM model is equipped with different structure. According to the characteristics of the feed system, three PEM models are pre-selected in this paper: extended auto-regressive model(ARX), extended auto-regressive moving average model(ARMAX), output-error model(OE), shown as

Fig.3 Current closed loop control system diagram

table 1.

Table1 PEM model structure

PEM	Model structure	$G(z^{-1})$	$H(z^{-1})$
ARX	$A(z^{-1})y_k = B(z^{-1})u_k + e_k$	$\frac{B(z^{-1})}{A(z^{-1})}$	$\frac{1}{A(z^{-1})}$
ARMAX	$A(z^{-1})y_k = B(z^{-1})u_k + C(z^{-1})e_k$	$\frac{B(z^{-1})}{A(z^{-1})}$	$\frac{C(z^{-1})}{A(z^{-1})}$
OE	$F(z^{-1})y_k = B(z^{-1})u_k + F(z^{-1})e_k$	$\frac{B(z^{-1})}{F(z^{-1})}$	1

2.2 Model order determination

According to the Wold decomposition theorem[18], any discrete stationary time series can be decomposed into the sum of two uncorrelated stationary series, one of which is a deterministic series and the other is a random series. At the same time, the deterministic sequence can be expressed as a linear function, which is $G(z^{-1})$ in the Eq. (4), and its expanded form is as follows:

$$G(z^{-1}) = \frac{\sum_{i=0}^p b_i z^{-i}}{1 + \sum_{j=1}^q a_j z^{-j}} \quad (5)$$

Where b_i and a_j are the coefficients of the numerator and denominator of the discrete transfer function, which can be obtained by identification using the least square method[19]. p and q are respectively the order of the numerator and denominator of the discrete transfer function. In previous applications, the commonly used method is to start from the smallest order, and keep increasing the order until the accuracy of the model meets the requirements. Some scholars use the BIC criterion of the model to find the optimal order within the specified range, but the result can only be used as a reference for the model order determination.

Different from other studies, $G(z^{-1})$ is the dynamic model of the servo feed system with a clear physical meaning in this paper. Therefore, the order of p and q can be determined

by establishing the closed-loop transfer function of the theoretical position system.

First, the establishment starts in sequence from the inner ring to the outer ring. The innermost part is the current loop, which usually consists of a PI-controlled regulator, filter link, inverter equivalent link and armature winding link, as shown in Fig.3. Where T_{oi} is the current feedback filter time constant, T_{inv} and K_{inv} are the time constant and gain of the inverter equivalent inertia link. L_q and R_s are the armature winding inductance and resistance, K_{pi} and τ_i are the proportional coefficient and integral time constant of the PI controller. After approximating the inertia link in Fig.3, it is shown in the following Fig.4. The approximate condition is that the cut-off frequency of the open-loop frequency characteristic of the current loop ω_{ci} needs to be satisfied $\omega_{ci}^2 \leq 1 / (9T_{oi}T_{inv})$.

Fig.4 Current closed loop system model after approximate treatment of small inertia link

Where the small inertia link is combined to obtain the time constant of the inertia link $T_1 = T_{oi} + T_{inv}$, $K_1 = k_{pi}k_{inv} / \tau_i$, which is the equivalent gain. $T_2 = L_q / R_s$ is the electrical time constant and $K_2 = 1 / R_s$ is the electrical gain. Generally, the zero point of the controller is canceled with the large time

Currently, the common identification signals include: step signal, square wave signal, sine signal, Chirps signal, white noise, etc. But the power of these signal is basically concentrated in a certain frequency band, which can't be evenly distributed in the whole frequency band. Therefore,

Fig.5 Transfer function diagram of position closed loop system

constant pole of the control object, that is $\tau_i = T_2$, the transfer function of the final current loop is:

$$G_i(s) = \frac{K_1 K_2}{T_1 s^2 + s + K_1 K_2} \approx \frac{K_1 K_2}{s + K_1 K_2} = \frac{K_I}{s + K_I} \quad (6)$$

The simplified current loop is brought into the velocity loop, and the final position transfer relationship is as follows.

Where T_{os} is the time constant of the speed feedback filter, J is the equivalent moment of inertia of the controlled object, K_s and τ_s is the proportional and integral time

$$G_p(s) = \frac{P_r}{P_r^*} = \frac{K_I K_I K_s \tau_s s^2 + (K_I K_I K_s + K_I K_I K_p K_s \tau_s) s + K_I K_I K_p K_s}{T_{os} \tau_s J s^5 + (T_{os} K_I J \tau_s + J \tau_s) s^4 + K_I J \tau_s s^3 + K_s K_I K_I \tau_s s^2 + (K_I K_I K_s + K_I K_I K_p K_s \tau_s) s + K_I K_I K_p K_s} \quad (7)$$

constant of the speed loop PI controller. $F(s)$ is the feedforward of the speed loop, which is theoretically composed of a differential link with a coefficient of 1[20]. K_p is the proportional coefficient of the position loop. The closed-loop transfer relation of the simplified position system is shown as Eq. (7).

It can be seen that the order of the numerator and denominator of the transfer function of the continuous system are 2 and 5 respectively. When converted into the discrete domain by the backward difference method, it can be incorporated into the equation $s = (1 - z^{-1}) / T$, where T is the sampling period. The final discrete transfer function does not affect the order of numerator and denominator. Therefore, the value of linear functions p and q of the deterministic sequence is 2 and 5 respectively.

2.3 Design method of identifying signal

In the system identification process, different input signals will affect the identification effectiveness. The identification signal should have enough frequency domain information to completely cover the frequency band of the identified system. The signal should not be too large in power and amplitude, which should avoid the identification object working in the nonlinear region.

this paper adopts the inverse M sequence signal in the pseudo-random binary sequence signal under the condition that the normal tolerance range of the servo electromechanical drive and the sufficient bandwidth spectrum range can maintain continuous equal power energy input.

$$\mathbf{m}(k) = C_1 m(k-1) \oplus C_2 m(k-2) \oplus \dots \oplus C_n m(k-n) \quad (8)$$

$$\mathbf{im}(k) = \mathbf{m}(k) \oplus S(k) \quad (9)$$

In the Eq. (8), Eq. (9), the feedback weight coefficient is $\{C_i | C_i \in \{0,1\}\}$. n is the bits number of the shift register, and

the M sequence is $\{\mathbf{m}(k)\} = \{m_1 m_2 \dots m_k\}$. $S(k)$ is a square wave sequence with a period of 2 bits and the element of 0 or 1. The inverse M sequence signal overcomes the static disturbance to the identification system from the DC component in the M-sequence. Besides, it has satisfying correlation characteristics and the power is evenly distributed on each frequency point, as shown in Fig.6.

Fig.6 Time domain diagram of inverse M series signals

The characteristic parameters of inverse M-sequence are the number of signal bits n , signal amplitude a , the minimum time interval of state changes T_d .

This paper takes the feed system of the KMC800 machine tool as the research object, and the maximum linear velocity of the feed axis is $v_{\max} = 48m/min$. The sampling frequency of CNC system software is $f_s = 500Hz$. The bandwidth of the position loop of each axis according to the frequency sweep test is $B < 100Hz$. In order to facilitate the observation of the frequency characteristics of the signal and eliminate the leakage error, Eq. (10) can be obtained.

$$T_f = T_{im} \quad (10)$$

Where T_f is the length of the sampling time, T_{im} is the cycle period of the reverse M sequence, and the relationship between the number of shift registers n and the cycle period is as follows:

$$T_{im} = 2T_d(2^n - 1) \quad (11)$$

Then,

$$\frac{N_f}{f_s} = 2T_d(2^n - 1) \quad (12)$$

f_s is the sampling frequency and N_f is the number of samples. Generally speaking, even number of samples will reduce the process of Fourier transform. The cycle period of the inverse M cycle is an even number, so for the convenience, you can set $2q(2^n - 1)$, then $T_d = q(1/f_s)$. Where q is adjustment coefficient.

The signal amplitude a mainly affects the power spectrum amplitude of the signal, which is mainly subject to physical limitations.

$$0 < a < \frac{l_{\max}}{2} \quad (13)$$

Where $l_{\max}$ is the maximum range of working. In order to prevent the system from entering the non-linear region, the value of a is chosen to be 30mm in this paper. At the same time, in order to realize that the signal bandwidth B_{im} covers the bandwidth of the identified object, therefore:

$$B_{im} = 0.433 \times 2 \times (2^n - 1) \times f_{im} \quad (14)$$

$$B_{im} \geq B \quad (15)$$

The frequency resolution ratio f_{im} is set to 10Hz, so a 4-bit register can meet the experiment requirement.

3 Evaluation and screening of identification models

In this section, the above design signals are input to the feed system of the KMC800 machine tool in the form of G codes, which will ignore the influence of CNC interpolation process on signal characteristics. Collecting the actual grating feedback data of each feed axis, the discrete transfer function of the corresponding machine tool X, Y, and Z feeds can be

obtained according to three PEM models. Meanwhile, the order of numerator and denominator of discrete transfer function are derived from section 2.2. Because the accuracy of the identification model directly affects the prediction of the contour accuracy, it is necessary to select according to the evaluation index. The test signal in time domain is shown in Fig.7.

Fig.7 Time domain test signal

From the time domain point of view, a time domain signal is input into the actual system and the identified model at the same time. By comparing the different identified models, the coefficient of determination and the mean square error between the predicted position value and the position value output by the actual system are obtained. Finally, the identification model which is closest to the actual system is selected. R^2 , MSE calculation expression is as follows:

$$R^2 = \frac{\sum_{i=1}^n (\hat{y}_i - \bar{y})}{\sum_{i=1}^n (y_i - \bar{y})} \quad (16)$$

$$MSE = \frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2 \quad (17)$$

Among them, $\hat{y}_i$ is the predicted value, y_i is the actual value, and $\bar{y}$ is the average of the actual value. The predication error of the discrete transfer function of the feed system identification is shown in the following table.

Table2 The feed system identification error

Logical axis	Model structure	$1 - R^2$	MSE
X	ARX	9.042×10^{-9}	5.542×10^{-6}
	ARMAX	4.063×10^{-9}	2.490×10^{-6}
	OE	3.601×10^{-9}	2.207×10^{-6}
Y	ARX	4.801×10^{-9}	2.292×10^{-6}
	ARMAX	4.335×10^{-9}	2.069×10^{-6}
	OE	3.849×10^{-9}	1.837×10^{-6}
Z	ARX	7.708×10^{-9}	4.675×10^{-6}
	ARMAX	5.788×10^{-9}	3.510×10^{-6}
	OE	1.274×10^{-5}	7.772×10^{-3}

Finally, according to the evaluation index, the discrete transfer functions of X, Y, and Z axis identification are determined as following equation.

4 Contour error prediction and compensation

represent the expected positions of the previous interpolation

$$G_x(z^{-1}) = \frac{-0.06175 + 0.2272 z^{-1} - 0.1531 z^{-2}}{1 - 3.07 z^{-1} + 4.096 z^{-2} - 2.91 z^{-3} + 1.005 z^{-4} - 0.1077 z^{-5}} \quad (18)$$

$$G_y(z^{-1}) = \frac{-0.01029 + 0.04974 z^{-1} - 0.03546 z^{-2}}{1 - 3.83 z^{-1} + 6.106 z^{-2} - 5.053 z^{-3} + 2.166 z^{-4} - 0.3844 z^{-5}} \quad (19)$$

$$G_z(z^{-1}) = \frac{-0.108 + 0.3155 z^{-1} + 0.0194 z^{-2}}{1 - 0.515 z^{-1} - 0.2634 z^{-2} - 0.5258 z^{-3} + 0.591 z^{-4} - 0.05991 z^{-5}} \quad (20)$$

$$C_z(z^{-1}) = 1 + 2.03 z^{-1} + 1.802 z^{-2} + 0.5988 z^{-3} \quad (21)$$

In the last section, through identifying the mathematical model of the selected machine tool feed system, the actual position of the machine tool feed axis during the machining process can be predicted. The discrete data points according

cycle and the next interpolation cycle, where $n = (m-1) / 2$. The length of the storage area m is user-defined.

After the first search, the instruction position Pc_r closest to

Fig.8 Internal structure of the buffer area

Fig.9 Three cases of searching the location of foot point

to the interpolation period are regarded as the expected trajectory. The vector from the predicted actual position point to the vertical foot of the expected contour curve is obtained. Therefore, determining the foot point is the key. Next, an offline contour error solution method will be introduced. This paper will use two search strategies to determine the footsteps. The first is to establish a storage area with a length of m , and then search for the instruction position of the closest point to the actual point in the area. The definition of the buffer area is shown in Fig.8.

In Fig.8, Pe_i is the actual position point at time i . Pc_i is the expected interpolation position at time i , Pc_{i-1} and Pc_{i+1}

the actual tool point Pe_a is determined. Through the second search, the position of the foot point Po_a closest to the actual point Pe_a on the path of the discrete interpolation point is searched, and the second search is carried out according to the situation listed below. When $|Pc_i Pe_a| = 0$, Pc_r is the foot point Po_a , when $|Pc_i Pe_a| \neq 0$, since the foot point closest to the actual point does not necessarily fall on $Pc_i Pc_{i+1}$, it is necessary to judge the relative position of points Pc_{i+1} , Pc_{i-1} , Pe_a , as shown in Fig.9. Among them, because the interpolation period is very short, the first-order linear approximation fit can be used between the two interpolation points.

As shown in Fig.9, according to the angle bisector theorem, the nearest foot point is determined by the relationship with $\angle\delta$ and $\angle\beta$, where $\angle\delta, \angle\beta \in [0^\circ, 180^\circ]$. As shown in Fig.9 (a), when $\angle\delta < \angle\beta/2$, the nearest foot point is on $P_c P_{c+1}$, which $\overline{P_c P_o_a}$ is the projection of the vector $\overline{P_c P_e_a}$ along $P_c P_{c+1}$. When $\angle\delta > \angle\beta/2$, the nearest foot point is on $P_c P_{c-1}$, which $\overline{P_c P_o_a}$ is the projection of the vector $\overline{P_c P_e_a}$ along $P_c P_{c-1}$. As shown in Fig.9 (b). When $\overline{P_c P_o_a}$ is exactly on the angular bisector of $\angle\beta$, the distance from P_e_a to both sides is

Fig.10 Milling experiment of contour error prediction and compensation

Fig.11 Predicted contour error and actual contour error comparison diagram and contour error thermal map

equal, as shown in Fig.9 (c). Our goal focuses on solving the contour error, so the third case can be merged into the two cases, and finally the calculation process of the above case is unified as:

$$P_o_a = \begin{cases} P_c + \frac{\overline{P_c P_e_a} \cdot \overline{P_c P_{c+1}}}{\overline{P_c P_{c+1}}} & \angle\delta \leq \angle\beta/2 \\ P_c + \frac{\overline{P_c P_e_a} \cdot \overline{P_c P_{c-1}}}{\overline{P_c P_{c-1}}} & \angle\delta > \angle\beta/2 \end{cases} \quad (22)$$

If P_{c-1} , P_{c+1} and P_c coincide, the foot point can be considered as P_c . After two searches, the foot point closest to the actual position is finally obtained. The contour error is $\epsilon = \overline{P_e_a P_o_a}$ and the value of the contour error is $|\overline{P_e_a P_o_a}|$. Therefore, the contour error of prediction can be avoided in the actual machining process. The predicted contour error value can be compensated to the interpolation point at the corresponding position before machining. The value of the interpolation after compensation Pn_a is shown in Eq. (23).

$$\overline{OPn_a} - \overline{OP_c} = \overline{P_c Pn_a} = \epsilon \quad (23)$$

5 Experimental verification and analysis of contour error prediction and compensation

KMC600 UMT turning-milling vertical machining center was selected as the machine tool. The starfish pattern with a large curvature variation and the profile of integral impeller

with a wide range of applications were selected as the parts to be machined, as shown in Fig.10. According to the identification model of feed system selected in section 3 and the contour error solution method introduced in section 4, the predicted contour error is shown in Fig.11.

Fig.12 Enlarged view from local position

Fig.13 Machining parts and contour error comparison

From Fig.11, the predicted contour error value is consistent with the actual true contour error value. It can be seen from the data that most of the predicted contour error values can accurately express the contour error of the part in the actual machining process, except for a few points. Then the predicted contour error is used as the compensation vector to compensate to the interpolation point of the response position. To make the discussion clear, the compensation position map is partially enlarged, and the enlarged position is consistent with Fig.11, as shown in Fig.12. Input the compensated interpolation point into the CNC system for machining. The contour error value after

machining is shown as below. The contour error defined at this time is the distance between the compensated machining tool point and the theoretical contour of the original design. In order to illustrate the effectiveness of compensation, this

paper compared the contour errors of parts machined with the original interpolation points. As shown in Fig.13. It can be seen from Fig.13 that the contour error after compensation is obviously reduced. After calculation, the maximum value of the contour error before compensation is 0.0135 mm and the maximum value of the contour error after compensation is 0.0078mm. The average error value is reduced by 61.76% after compensation. Therefore, the proposed method can effectively reduce the contour error. Through predicting and compensating the contour error before machining, the machining accuracy of the part can be obviously improved.

Integral impeller machining is a typical representative of 5-axis motion control, so this paper takes the integral impeller as the part to be machined. The transfer function of the A axis and the C axis is:

$$G_a(z^{-1}) = \frac{-0.02258 + 0.06362 z^{-1} - 0.03866 z^{-2}}{1 - 4.162 z^{-1} + 7.163 z^{-2} - 6.362 z^{-3} + 2.918 z^{-4} - 0.5545 z^{-5}} \quad (24)$$

$$G_c(z^{-1}) = \frac{-0.05127 + 0.1485 z^{-1} - 0.09092 z^{-2}}{1 - 3.725 z^{-1} + 5.924 z^{-2} - 5.017 z^{-3} + 2.252 z^{-4} - 0.4276 z^{-5}} + 0.000525 \quad (25)$$

$$\begin{cases} w_x = (z + l_{yaz} + l) \sin(q_a) \sin(q_c) - (y + l_{yay}) \cos(q_a) \sin(q_c) \\ \quad + (x + l_{acx} + l_{yax}) \cos(q_c) - l_{acy} \sin(q_c) + l_{cwx} \\ w_y = (y + l_{yay}) \cos(q_a) \cos(q_c) - (z + l_{yaz} + l) \sin(q_a) \cos(q_c) \\ \quad + (x + l_{acx} + l_{yax}) \sin(q_c) + l_{acy} \cos(q_c) + l_{cwy} \\ w_z = (z + l_{yaz} + l) \cos(q_c) + (y + l_{yay}) \sin(q_a) + l_{acz} + l_{cwz} \end{cases} \quad (26)$$

Fig.14 The contour error comparison diagram and the contour error thermal diagram

Fig.15 Local blade machining drawing and Comparison diagram of contour error before and after compensation

In particular, the movement range of the C-axis is $[0^\circ, 360^\circ]$, so the M signal identification is adopted. One DC component

needs to be subtracted to effectively describe the C-axis movement process.

For the convenience of explanation, a contour line of the blade is extracted for display, and the contour error is shown

in Fig.14 according to the above method and after kinematic transformation as Eq. (26)[21-22]. Where $(w_x, w_y, w_z)^T$ is

position coordinate under the workpiece coordinate system and $(x, y, z, q_a, q_c)^T$ is kinematic position of machine tool coordinate. $(l_{cwz}, l_{cwz}, l_{cwz})^T$, $(l_{yax}, l_{yay}, l_{yaz})^T$, $(l_{acx}, l_{acy}, l_{acz})^T$ are respectively offset distance in three directions according to T_C^W , T_Y^A , T_A^C coordinate systems. l is the tool length.

It can be seen from Fig.15 that the accuracy of the 5-axis contour prediction is obviously lower than that of the 3-axis machining prediction. The reason is that the small prediction errors of the A-axis and C-axis will be magnified after kinematic transformation. Therefore, the prediction accuracy of the A-axis and C-axis will directly affect the prediction of the final contour error. According to the above, the compensation effect is as follows.

After compensation, the contour error value of a blade profile can be basically stabilized at about 0.0024mm, which can be reduced by 31.43% compared with the contour error value of the blade profile processed by the original command. Especially at the edge of the blade, the contour error is significantly reduced.

6 Conclusion

This paper introduces a contour error prediction and compensation method. Before machining, contour errors can be predicted and compensated based on the identification model of selected machine tool and planned interpolation trajectory. While avoiding the iterative machining of new parts, the proposed method can effectively solve the problems of stability and compatibility of real-time online control algorithms. The experimental results show that the proposed method improves the contour accuracy effectively and equipped with the following characteristics:

- (1) This study has determined the order of the identification model through theoretical derivation. Since the mechanical structure of each axis is different, its model characteristics are also inconsistent. A model evaluation process is adopted to avoid that a single model identification is insufficient to express the dynamic performance of the actual system.
- (2) This study used two-search-strategies to determine the foot points of the contour error. After searching the command position of the closest point to the actual point in the storage area, the second search in the specified range is performed to obtain the contour error of the continuous small line segment, reducing the computational burden.
- (3) The prediction compensation method proposed in this study is based on the model identification. To a large extent, the accuracy of the compensated contour depends on the accuracy of the model identification. Therefore, this method is suitable for the first trial cut parts and the machining of high-precision parts in batches on the machine tool with accurate model identification.

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Declarations

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Figures

Figure 1

Basic principles of system identification

Figure 2

Translational axis transfer model and rotational axis transfer model

Figure 3

Current closed loop control system diagram

Figure 4

Current closed loop system model after approximate treatment of small inertia link

Figure 5

Transfer function diagram of position closed loop system

Figure 6

Time domain diagram of inverse M series signals

Figure 7

Time domain test signal

Figure 8

Internal structure of the buffer area

Figure 9

Three cases of searching the location of foot point

Figure 10

Milling experiment of contour error prediction and compensation

Figure 11

Predicted contour error and actual contour error comparison diagram and contour error thermal map

Figure 12

Enlarged view from local position

Figure 13

Machining parts and contour error comparison

Figure 14

The contour error comparison diagram and the contour error thermal diagram

Figure 15

Local blade machining drawing and Comparison diagram of contour error before and after compensation