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Evaluation and selection of urban public transport paths based on Entropy weight TOPSIS and Hamilton cycle

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ABSTRACT The public transportation system of cities, including subway and public transportation, is becoming more and more perfect. With the rapid development of urban public transport, the path selection considering multiple factors has become a new problem. Based on the optimization model, we take urban public transport operators and travelers as the objects, and use the entropy weight TOPSIS method to comprehensively evaluate the feasible lines and paths between multiple OD pairs. Besides, the optimal path to traverse all nodes was solved by Hamilton cycle problem algorithm, which can also provide reference for both operators and travelers. According to the latest urban public transport data in 2021, we select Beijing, China as the empirical research object. This paper chooses the existing public transport network of Beijing to verify, and selects the optimal path of 5 nodes and 10 paths to traverse all nodes.

INDEX TERMS Optimal path; Entropy weight TOPSIS method; Hamilton cycle; Cost and service quality

I. INTRODUCTION

Nowadays, with the rapid development of urban public transportation system, there are many complicated lines in the city. People's modes of transportation have changed from single choice to multiple choices. Although people's travel needs have been met, the choice of travel mode is not only to meet the purpose of travel, but also to gradually enjoy the process of travel. On the other hand, traffic congestion is becoming more and more serious in large and medium-sized cities, and people often spend a lot of time on the road. At the same time, vehicles emit a lot of pollution gases, and people begin to pay attention to emission reduction. Accordingly, large cities are constantly limiting the number of trips and encouraging public transport. As a result, the selection and combination of existing lines and the optimal path selection under multiple nodes become an interesting research.

Yang Hai put forward the suggestion of optimizing bus line allocation and passenger traffic allocation at the same time. They considered using the minimum spanning tree to analyze the impact of different demands on the line, and took the Hong Kong Metro Line as an example to get the optimization results of minimizing the operation cost and maximizing the service quality[1]. Canca D considered service providers and users at the same time in order to minimize operating costs and select the most convenient frequency and capacity, and transport passengers from origin to destination while minimizing the average travel time[2]. Wang L proposed a two-level optimization model. The design of the top layer minimized the total operating cost and unserved passengers. And the design of the bottom layer made the weighted passengers flow allocation a mixed integer program, so as to maximize the service passengers and minimize the total travel time of all passengers. The model had been illustrated and verified in Taiwan High-speed Railway and Beijing-Shanghai High-speed Railway[3]. Chen Qun established a two-tier model, which designed the upper layer to minimize the total travel time, the total length of the rail transit network and the number of changes, and the lower layer to optimize the layout of the urban rail transit network[4]. Xu Jinxing studied the relationship between the Hamilton cycle problem in graph theory and the traveling salesperson problem, and used the Hamilton cycle shortest path algorithm in graph theory to obtain an approximate solution to the traveling salesperson problem[5]. Jun F used a fluorescent labeling strategy to solve the NP problem. By fixing the DNA molecule in the solution space of the problem on a carrier, a method for solving the DNA surface calculation model of the Hamilton cycle problem was given. And the superiority of parallel computing capability is verified[6]. Zhang Yihua analyzed the driver's preference for path selection based on the principle of ant path finding, and took the driver's expectation value as a constraint. They used sensitivity analysis to determine the influence of different factors on the optimal path selection[7]. Yao Jiao analyzed the influencing factors of the city's optimal path selection under emergency conditions, and used the TOPSIS method to establish an optimal path selection model. They used an improved ant colony algorithm to increase the number of targets, effectively reducing the time of vehicles in emergency rescue[8].

According to the previous literature, the general principles of public transport trip selection are: minimizing the operating cost and maximizing the service quality. Due to the urban policy and traffic development, it is often not feasible to modify the bus lines according to the optimization results. Taking into account cost and service quality factors such as line length, number of transfers, we create a new traffic optimization model. After processing the data, we get the OD matrix, and the entropy weight TOPSIS method is used to evaluate it. Besides, the optimal path to traverse all nodes was solved by Hamilton cycle problem algorithm, which can also provide reference for both operators and travelers.

The structure of the paper is as follows. The background of public transportation evaluation and selection is introduced in Sec.1. We consider the factors that affect the cost of operators and the quality of traveler service, and establish a new traffic optimization model in Sec.2. The OD matrix is obtained based on the partial bus network diagram of Beijing, and then the entropy weight TOPSIS is used for comprehensive evaluation in Sec.3. The Hamilton cycle problem algorithm is used to solve the optimal travel path selection based on multiple factors under multiple nodes in Sec.4, and the results show that the path efficiency and resident efficiency are effectively improved. The conclusion and prospect are given in Sec.5.

II. Optimization model

For managers, Cost minimization is related to the length of line and the quality of line service. The length of the line objectively determines the main part of the cost, while the quality of the line service (that is, the line running time and the frequency of the line) determines the rest of the cost.

The purpose of optimizing passenger transportation choice is to find a transportation path with the highest quality of service for passengers. In actual travel, passengers will make travel choices according to their own needs and preferences. The maximization of passenger service quality can be expressed by passenger travel time. Travel time is generally divided into waiting time, in-car time and transfer time. Therefore, we can regard the maximization of passenger service quality as the minimization of the sum of the three times.

For waiting time, we take buses and subways as examples, which are the most common means of transportation in cities. For passengers, traveling in traffic can be regarded as a queuing process. When passengers depart from the departure station, the average waiting time of passengers obeys the Uniform distribution on $(0, T)$, and the average waiting time is:

$$E[X(t)] = \frac{T}{2}$$

If passengers depart from a station in the middle, the waiting time of passengers follows Poisson process. According to the train frequency, the average arrival time of vehicles can be determined. The number of waiting events in the interval with time length t will obey Poisson distribution with parameter $\lambda t (\lambda > 0)$, that is, for all $t_1, t_2 \geq 0$, there are:

$$P\{X(t+s) - X(s) = k\} = \frac{(\lambda t)^k}{k!} e^{-\lambda t} \quad k=0,1,2,3,\dots$$

The event of passenger waiting time is called Poisson process with parameter λ , and

$$E[X(t)] = \lambda t$$

λ is the intensity or arrival rate of the passenger queuing process. According to the intensity, we can determine the average waiting time for passengers to travel.

For in-car time, it is the most time consumed by passengers, and it is often one of the important factors that passengers care about. Travel time is related to the length of the path. The in-car time can be expressed by the length of the path. From departure to destination there will be many modes and lines for passengers to choose, but only one path can be selected by passengers at last. The choice of the path has considered the choice of transportation mode and line.

For transfer time, when passengers choose at least two traffic lines, the transfer time comes naturally. The size of transfer time can be approximated as the number of transfers. The number of transfers depends on the number of lines selected by passengers. At the same time, passengers should not blindly pursue to reduce the number of transfers unilaterally, because the result of this is to choose longer paths, which increases the in-car time, even more unfavorable for passengers to travel.

III. Comprehensive evaluation based on Entropy weight TOPSIS method

The TOPSIS method is a commonly used comprehensive evaluation method, which can make full use of the information of the original data, and its results can accurately reflect the gap between the evaluation plans. The basic process of the method is to normalize the data, convert all the indicators into maximum indicators, use the cosine method to determine the optimal plan and the worst plan, and calculate the evaluation object and the best plan and the worst plan. The distance of the plan is to obtain the relative closeness of each evaluation object to the optimal plan, which is used as the basis for evaluating the pros and cons.

Entropy weight method is an objective method of assignment. In the specific application process, the entropy weight method uses information entropy to calculate the entropy weight of each indicator according to the variation of each indicator, and then corrects the weight of each indicator to obtain a more objective indicator weight.

According to the report of China's urban rail transit operation data in 2020. Among the 43 cities, Beijing has 24 lines in operation, ranking first; 727 km in operation, ranking second; 2292.654 million passenger trips, ranking third; all three indicators are at the top level. Therefore, choosing the development of public transport in Beijing as the research background is representative and cutting-edge for other ordinary cities. On the basis of some public transport network maps in Beijing, five nodes, namely capital International Airport, Beijing West Railway Station, Zhongguancun square, Olympic Park and Happy Valley, are selected to comprehensively evaluate the OD and optimal route selection under multi node. As shown in Figure 1, the five nodes cover the main urban areas of Beijing, all of which are crowded areas. Taking the OD pair of Olympic Park and Zhongguancun square as an example, the OD data of relevant cost and service quality are collected, and uses the entropy weight TOPSIS method for comprehensive evaluation.

Figure 1 Department map of Beijing

Table 1 Basic data of the path from Olympic Park to Zhongguancun square

Serial number	Travel path	Line length/km	Running time/hour	Waiting time/minute	Path length/km	Number of transfers
A	line8–line10	9.2	18.0	2.0	10.4	1
B	145	8.7	16.5	9.0	9.3	0
C	450	8.4	16.8	12.0	9.6	0
D	line15–355	8.5	17.1	1.5	9.3	1
E	386	7.7	16.5	7.0	9.2	0
F	86–307	7.0	17.0	5.0	7.8	1

According to the data in Table 1, using python software to calculate the weight values based on the entropy method: 0.004, 0.002, 0.223, 0.003, 0.768. Combined with the TOPSIS method for comprehensive evaluation to get Figure 2 and Table 2, according to the sort size, the optimal path between Olympic Park and Zhongguancun square is determined as path D, which is line15–535.

Table 2 Evaluation results of TOPSIS method

	Line length	Running time	Waiting time	Path length	Number of transfers	Positive ideal solution	Negative ideal solution	Composite score index	Sort
Path A	0.3617	0.3847	0.5683	0.3597	0.3203	0.1167	0.2236	0.5729	2
Path B	0.3825	0.4197	0.1263	0.4022	0.4804	0.2983	0.1412	0.3213	4
Path C	0.3964	0.4122	0.0947	0.3896	0.4804	0.3132	0.1404	0.3095	5
Path D	0.3915	0.4049	0.7577	0.4022	0.3203	0.1405	0.3131	0.6903	1
Path E	0.4321	0.4197	0.1624	0.4066	0.4804	0.2812	0.1440	0.3381	3
Path F	0.4754	0.4073	0.2273	0.4796	0.3203	0.2871	0.0634	0.1808	6

Figure 2 Radar chart of TOPSIS method evaluation results

Collect the cost and service quality O–D information of 10 paths between the capital International Airport, Beijing West Railway Station, Zhongguancun square, Olympic Park and Happy Valley, and use the entropy weight TOPSIS method to obtain the optimal path between the remaining O–D. There are nine optimal paths in Figure 3: Zhongguancun square and Happy Valley, Zhongguancun square and capital International Airport, capital International Airport and Beijing West Railway Station, capital International Airport and Happy Valley, Beijing West Railway Station and Olympic Park, Beijing West Railway Station and Zhongguancun square, Olympic Park and capital International Airport, Happy Valley and Beijing West Railway Station, Happy Valley and Olympic Park. The corresponding paths are path C line10–41, path A line10–Beijing Subway Capital Airport Express, path E Airport bus line4–387, path A Beijing Subway Capital Airport Express–line10–line7, path G line9–92, path A line9–line4, path A line15–Airport Road 3, path A line7, and path E line7–141.

Figure 3 The optimal path evaluation result diagram of the entropy weight TOPSIS method

IV. Optimal path selection based on Hamilton cycle algorithm

The shortest path is the path from a vertex to another vertex with the least weight sum of edges. There are many methods to find the shortest path in the literature in the past, including: depth-first algorithm, breadth-first algorithm, floyd shortest path method, Bellman-Ford method and so on. But these algorithms are all the optimal paths from a certain point to the rest of the graph. For the shortest path problem that traverses all nodes and returns to the origin, we call it the Hamilton cycle problem, and the optimal path can be obtained with the help of the algorithm of this problem. In this paper, the "shortest path" is the "optimal path" of cost and service quality based on entropy weight TOPSIS method, and the corresponding weight between each edge in the figure is the comprehensive score index of each O-D path. Change the weight in the figure to the comprehensive path score based on the entropy weight TOPSIS method to obtain the basic traffic map.

Figure 4 Basic traffic map

In order to facilitate the acquisition of the optimal path for traversing all nodes, we perform the following processing on the comprehensive score index between O-D in Figure 4:

$$x_{ij} = 100 - p_{ij} * 100$$

Where p_{ij} is the comprehensive score index of each O-D in Figure 4, and x_{ij} is the minimization index of converting the comprehensive score index into the percentage system. Get an adjacency matrix based on cost and service quality, as shown in Table 3.

Table 3 Adjacency matrix based on cost and service quality

	1	2	3	4	5
1	0	40	1	2	4
2	40	0	31	1	2
3	1	31	0	5	1
4	2	1	5	0	38
5	4	2	1	38	0

Using C++ programming software to solve the optimal path of traversing all nodes, The optimal path is obtained as shown in Figure 5: 1-3-5-2-4-1, starting from Beijing West Railway Station, we pass Olympic Park, capital International Airport, Zhongguancun square, Happy Valley, and finally return to Beijing West Railway Station. The total score index is 7.

```

Please enter the number of vertices and edges:
5 10
Please enter side and side length:
1 2 40
1 3 1
1 4 2
1 5 4
2 3 31
2 4 1
2 5 2
3 4 5
3 5 1
4 5 38
The adjacency matrix is
0 40 1 2 4
40 0 31 1 2
1 31 0 5 1
2 1 5 0 38
4 2 1 38 0
The Hamilton cycle length of the graph is:
7
Hamilton cycle path is
1-3-5-2-4-1
-----
Process exited after 4.297 seconds with return value 0
Please press any key to continue

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Figure 5 The results of the Hamilton cycle algorithm

V. Conclusion

This paper proposes an optimal path selection method for traversing all nodes under the condition of optimal bus line operating cost and passenger travel service quality. The problem of selecting the optimal path is transformed into the problem of minimizing line length, running time, waiting time, in-car time and transfer time. After that, a scoring mechanism is added to. The entropy weight TOPSIS method is used to transform the optimization results into objective data, and the optimal path between O-D is obtained. Finally, the Hamilton cycle problem algorithm is used to get the optimal path.

This paper uses capital International Airport, Beijing West Railway Station, Zhongguancun square, Olympic Park and Happy Valley, as the five basic nodes of the transportation network diagram to explore the feasible paths between each O-D. By balancing the line length, running time, waiting time, in-car time and transfer time to get the best path map under 5 nodes. The optimal travel path traversing all nodes was obtained by solving the Hamilton cycle problem algorithm. In urban traffic, operating companies may face financial pressure, industry competition and other issues; Travelers may also face many problems, such as multiple choices and diversified travel purposes. It is not only these factors that need to be considered when optimizing the bus line and passenger transport selection. When these factors change slightly, the optimal travel path will change. Therefore, operators and travelers can make different travel decisions based on different factors in actual operation.

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Figures

Figure 1

Department map of Beijing Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.

Figure 2

Radar chart of TOPSIS method evaluation results

Figure 3

The optimal path evaluation result diagram of the entropy weight TOPSIS method

Figure 4

Basic traffic map

Please enter the number of vertices and edges:

5 10

Please enter side and side length:

1 2 40

1 3 1

1 4 2

1 5 4

2 3 31

2 4 1

2 5 2

3 4 5

3 5 1

4 5 38

The adjacency matrix is

0 40 1 2 4

40 0 31 1 2

1 31 0 5 1

2 1 5 0 38

4 2 1 38 0

The Hamilton cycle length of the graph is:

7

Hamilton cycle path is

1-3-5-2-4-1

Process exited after 4.297 seconds with return value 0

Please press any key to continue

Figure 5

The results of the Hamilton cycle algorithm