Method of variable separation for investigating exact solutions and dynamical properties of the time-fractional Fokker-Planck equation

Xinsong Yang (✉ xinsongyang@163.com)

Sichuan University - Wangjiang Campus: Sichuan University <https://orcid.org/0000-0003-3599-5020>

Weiguo Rui

Chongqing Normal University - University Town Campus: Chongqing Normal University

Fen Chen

Chongqing Normal University - University Town Campus: Chongqing Normal University

Research Article

Keywords: Time-fractional Fokker-Planck equation, Variable separation method, Mittag- Leffler function, Exact solution and dynamical property, Anomalous diffusion

Posted Date: May 4th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-467496/v1>

License: This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Abstract

Based on the idea of variable separation, the time-fractional Fokker-Planck equation with external force field is studied by using the property of Mittag-Leffler function and some special algorithm skills. In the cases of various external potential functions such as linear potential, harmonic potential, logarithmic potential, exponential potential, and quartic potential, exact solutions and dynamical properties of the above mentioned equation is investigated. The some interesting dynamical behaviors and phenomena are discovered. The profiles of some representative exact solutions are illustrated by 3D-graphs.

Full Text

Due to technical limitations, full-text HTML conversion of this manuscript could not be completed. However, the manuscript can be downloaded and accessed as a PDF.

Figures

(a) 3D-graph of solution (2.17)

(b) 2D-graph of solution (2.17) at $t = 2$

(c) 3D-graph of the solution (2.18)

(d) 2D-graph of the solution (2.18) at $t = 2$

Figure 1

The dynamical profiles of the solutions (2.17) and (2.18); (a) & (b): $na = 0.6$, $Ka = 0.44$, $\alpha = 0.75$, $\omega = 0.3$.
(c) & (d): $na = 0.4$, $Ka = 0.3$, $\alpha = 0.75$, $\omega = 0.4$:

(a) 3D-graph of the solution (2.22)

(b) 2D-graph of the solution (2.22) at $t = 2$

Figure 2

The dynamical profiles of the solution (2.22); the parameters are taken as $na = 0.8$, $Ka = 0.6$, $\alpha = 0.25$, $\omega = 0.3$, $C1 = C2 = 0.3$ in drawing.

(a) 3D-graph as $\lambda < 0$

(b) 2D-graph at $t = 2$ as $\lambda < 0$

(c) 3D-graph as $\lambda > 0$

(d) 2D-graph at $t = 2$ as $\lambda > 0$

Figure 3

The dynamical profiles of the solution (2.32) under the case $\lambda < 0$ and $\lambda > 0$.

(a) 3D-graph of the solution (2.36)

(b) 2D-graph of the solution (2.36) at $t = 2$

(c) 3D-graph of the solution (2.38)

(d) 2D-graph of the solution (2.38) at $t = 2$

Figure 4

The dynamical profiles of the solutions (2.36) and (2.38).

(a) 3D-graph of the solution (2.43)

(b) 2D-graph of the solution (2.43) at $t = 2$

(c) 3D-graph of the solution (2.45)

(d) 2D-graph of the solution (2.45) at $t = 2$

Figure 5

The dynamical profiles of the solutions (2.43) and (2.45) under $C2 = 0$.

(c) 3D-graph of the solution (2.52)

(d) 2D-graph of the solution (2.52) at $t = 2$

Figure 6

The dynamical profiles of the solution (2.52).