Estimating the Lithospheric Flexure of a Plate with Non-uniform Flexural Rigidity: A Quantitative Modelling Approach

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1 **Estimating the lithospheric flexure of a plate with non-uniform** 2 **flexural rigidity: A quantitative modelling approach**

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Abstract

Lithospheric deformation is a fundamental process in plate tectonics. It is therefore critical to determine how the lithosphere responds to geological loads to better understand tectonic processes. The lithosphere can be modelled as the flexure of a thin, elastic plate over long-term ($>10^5$ yr) geological timescales. The partial differential equation for the flexure of an orthotropic plate is used indirectly to calculate theoretical admittance and coherence, which are then compared against the observed admittance and coherence to invert for the non-uniform flexural rigidity (or effective elastic thickness, T_e) of the plate. However, the process for accurately recovering variable lithospheric flexure remains unresolved, as the classical lithospheric model may overestimate the deflection of the plate. Here we adopt the classic lithospheric model with applied external and internal loads at the surface and Moho, respectively, and assume that the compensation material is denser than the mantle material beneath the Moho. The lithospheric flexure errors are derived mainly from the T_e and Moho recovery errors in this lithospheric model. Synthetic modelling is then performed to analyse the influence of the T_e and Moho errors. The analysis of synthetic modelling shows that: 1) the inverted flexure values are insensitive to the T_e errors, and the T_e error-induced flexure errors exhibit a rippling pattern; 2) the Moho error-induced flexure errors occur mainly in the low T_e regions, and the Moho disturbance is alleviated by both an attenuation coefficient of the form $-(\rho_F - \rho_m)/(\rho_m - \rho_c)$ and the lithospheric strength as the Moho disturbance propagates into the lithospheric flexure error; and 3) the lithospheric flexure errors are dominated by the T_e and Moho errors in the high and low T_e regions, respectively.

Keywords: Loading of the earth; Effective elastic thickness; Lithospheric flexure; Error analysis

Introduction

Our understanding of the mechanics of the Earth's crust and lithosphere have been greatly advanced since the 1970s, via numerous modelling, including experimental examinations of the behaviour of physical bodies (e.g., Dorman & Lewis 1970, 1972; Lewis & Dorman 1970a, 1970b) and derivations of the mathematics that describe lithospheric processes (e.g., McNutt 1980; Forsyth 1985; Burov & Diament 1992; Banks *et al.* 2001; Braitenberg *et al.* 2002; McKenzie 2003; Kirby & Swain 2009; Zhang *et al.* 2018). These studies followed the principle that the lithosphere can be idealised if the main structural features are considered, thereby allowing simplified models to capture the key structures and geodynamics of the lithosphere. However, no information on the internal structure of the lithosphere has been used in these investigations, and its mechanical properties, such as the flexure of the tectonic plates, may be incorrectly estimated (Ribe 1982).

The lithospheric strength of tectonic plates reflects their resistance to vertical deformation in response to geological loads over long-term ($>10^5$ yr) geological timescales (Watts & Burov 2003). This assumption allows the lithosphere to be modelled as the flexure of a thin, elastic plate via the partial differential equation for the flexure of an orthotropic plate (Timoshenko & Woinowsky-Krieger 1959). This equation is usually used indirectly to calculate theoretical admittance and coherence, which are then compared against observed ones to invert for the non-uniform flexural rigidity (or effective elastic thickness, T_e) of the plate (Kirby 2014). Numerous studies have used T_e to analyse lithospheric rheology and deformation

(Pérez-Gussinyé *et al.* 2009; Chen *et al.* 2015; Ji *et al.* 2017, 2020; Lu *et al.* 2020). However, this idealised term does not refer to an existing thickness or physical layer within the Earth, but instead corresponds to the thickness of an ideal elastic plate that undergoes the same deformation as the lithosphere under the same loads (Watts 2001). As T_e is a periphrastical parameter for understanding lithospheric rheology and deformation, directly modelling lithospheric flexure should provide key insights into the nature of tectonic evolution and dynamics.

Here we use the classic lithospheric model with applied external and internal loads at the surface and Moho, respectively, and assume that the compensation material is denser than the mantle material beneath the Moho (Fig. 1). The density contrast between the compensation material and mantle material beneath the Moho is set to 200 kg/m^3 , based on the density contrast between the uppermost and basal lithospheric mantle (Kaban *et al.* 2016). This is still an idealised model that considers the vertical density variation in the lithosphere. Furthermore, we obtain T_e variations via the fan wavelet coherence method (Kirby & Swain 2004, 2008) and the Moho topography via a constrained gravity inversion method (Wu *et al.* 2017). We then employ the finite-difference (FD) method to model lithospheric flexure. The errors associated with the lithospheric flexure estimation in such a lithospheric model are mainly derived from the T_e and Moho errors. Synthetic modelling is then performed to analyse the influence of the T_e and Moho errors on the lithospheric flexure estimation.

Fig. 1. Conceptual mechanical model of the lithosphere and asthenosphere.

Methodology

The key to calculating lithospheric flexure via the FD method is to obtain accurate spatial distributions of the Moho and T_e . The Moho depth is usually recovered via gravity inversion (Wu *et al.* 2017). There are currently two main methods for mapping the spatial variations in T_e based on the coherence between the Bouguer gravity anomaly and topography/bathymetry. One is the multitaper method (McKenzie & Fairhead 1997; Pérez-Gussinyé *et al.* 2004), and the other is fan wavelet coherence method (Kirby & Swain 2004, 2008). The characteristic flexural wavelength is correlated with the strength of the lithosphere, and the flexural amplitude varies with both T_e and the magnitude of the load (Watts 2001). These variations make it difficult to choose the multitaper window size to recover T_e . Signals of features on a stronger lithosphere can be truncated when the selected window size is too small, resulting in an artificially low T_e . Conversely, T_e can be blended across multiple features, with a potential bias toward the largest

feature in the window, when the selected window size is too large (Pérez-Gussinyé *et al.* 2004, 2008; Kalnins & Watts 2009). The fan wavelet coherence method can alleviate these issues by allowing the T_e features to be located at different wavelengths, and is employed to estimate T_e in this study.

Gravity inversion for Moho topography

Joint inversions that involve gravity measurements and other geophysical constraints are generally employed to reduce the non-uniqueness of the best-fit models (Chappell & Kusznir 2008; Bai *et al.* 2014; Ji *et al.* 2018). Here we perform a three-dimensional gravity inversion that is constrained by seismic observations (Wu *et al.* 2017) to recover an accurate Moho topography. The Moho topography that corresponds to the complete Bouguer gravity anomaly can be expressed as $h(\mathbf{r}) = h_e(\mathbf{r}) + \Delta h(\mathbf{r})$, where $h_e(\mathbf{r})$ is the initial interface. The difference between the Moho and initial interface can then be obtained by substituting this difference into the gravity inversion equation (Oldenburg 1974):

$$F[\Delta h(\mathbf{r})] = -\frac{F[\Delta g]}{2\pi G \rho} e^{kz_0} - \sum_{n=2}^{\infty} \frac{k^{n-1}}{n!} F[\{h_e(\mathbf{r}) + \Delta h(\mathbf{r})\}^n] - F[h_e(\mathbf{r})], \quad (1)$$

where $F[]$ is the Fourier transform, Δg is the complete Bouguer gravity anomaly, G is Newton's gravitational constant, ρ is the density contrast across the interface, $\mathbf{k}$ is the wave vector of the transformed function, $\mathbf{r}$ is the entire x-y plane and z_0 is the reference depth of the density interface.

We largely follow the procedure outlined in Wu *et al.* (2017), except for the

implementation of Eq. (1) to determine the interface difference during each iteration of the inversion, to ensure that both the Moho depth and Bouguer gravity anomaly (Bg) satisfy the inversion equation (Oldenburg 1974) after the Moho topography is recovered. The final Moho topography is obtained iteratively until the interface difference between adjacent iteration steps meets the error threshold.

Fan wavelet coherence method

The wavelet method for calculating the coherence between the topography and gravity data was first proposed by Stark *et al.* (2003), who employed a Gaussian window function as the transform kernel to estimate T_e in southern Africa. The continuous wavelet transform offers the ability to obtain the local phase and amplitude of the topography and gravity anomalies, whereas the Fourier transform requires wavenumber averaging of the spectral estimates. The Gaussian wavelet fails to reproduce the Fourier power spectrum exactly (Kirby 2005). Two-dimensional Morlet wavelets, which are Gaussian-modulated complex exponential functions, can reproduce the Fourier spectrum (Kirby & Swain 2004). The spatial resolution of T_e can be improved by adjusting the central wavenumber of the Morlet wavelet (Kirby & Swain 2011). Only the square of the real component of the wavelet coherence needs to be employed to estimate T_e (Kirby & Swain 2008, 2009). The squared real coherency can be expressed as a function of scale s and position x :

$$\gamma_{obs}^2(s, x) = \frac{\left| \text{Re} \left\langle B_{sx\theta} H_{sx\theta}^* \right\rangle_{180} \right|^2}{\left\langle B_{sx\theta} B_{sx\theta}^* \right\rangle_{180} \left\langle H_{sx\theta} H_{sx\theta}^* \right\rangle_{180}}, \quad (2)$$

where $B_{sx\theta}$ and $H_{sx\theta}$ represent the complex wavelet coefficients of the Bouguer gravity anomaly and topography, respectively, the * indicates complex conjugation, and the angular brackets denote azimuthal averaging performed over the entire 180° azimuthal range. The predicted squared-real coherency of the load-deconvolution method is (Kirby & Swain 2009, 2011):

$$\gamma_{pre}^2(s, x) = \frac{(\mu_T \kappa_T + \mu_B \kappa_B f^2 r^2)^2}{(\mu_T^2 + \mu_B^2 f^2 r^2)(\kappa_T^2 + \kappa_B^2 f^2 r^2)}, \quad (3)$$

where:

$$\begin{cases} \kappa_B = \Delta\rho_2 / \varphi \\ \kappa_T = 1 - \Delta\rho_1 / \varphi \\ \mu_B = 2\pi G \Delta\rho_2 (1 - \Delta\rho_2 / \varphi) e^{-kZ_m} \\ \mu_T = 2\pi G \Delta\rho_2 (1 - \Delta\rho_1 / \varphi) e^{-kZ_m} \\ \varphi = Dk^4 / g + (\rho_m - \rho_f) \\ \Delta\rho_1 = \rho_c - \rho_f \\ \Delta\rho_2 = \rho_m - \rho_c \\ D = \frac{ETe^3}{12(1 - \sigma^2)} \end{cases},$$

$r = (\rho_c - \rho_f) / (\rho_m - \rho_c)$, ρ_c and ρ_m represent the crust and mantle densities, respectively, ρ_f represents either the air density (on land) or equivalent topography and seawater density (in the ocean), Z_m is the Moho depth, g is the gravitational acceleration, E is the Young's modulus, σ is the Poisson's ratio, and $k \equiv |\mathbf{k}|$ is the wavenumber modulus, which can be replaced by the equivalent Fourier wavenumber in the fan-wavelet method (Kirby & Swain 2006). The final T_e distributions are obtained by finding the T_e that provides the minimum

misfit between the observed and predicted coherencies at each grid point.

Thin plate flexure

For quick reference, we provide the key equations for estimating thin plate flexure, which can be easily calculated via the approach of Kirby & Swain (2008). We consider an elastic slab model that involves only the initial external (at the surface) and internal (at the Moho) loads, which are represented as $\left[(\rho_c - \rho_f)gh_i\right]$ and $\left[(\rho_m - \rho_c)gw_i\right]$, respectively. Four end-member models are used here to illustrate the response of an elastic plate to an initial load with a deflection amplitude v (Fig. 2). The topography can be realistically considered the superposition of these four models if we place the subsurface load at the Moho. The final deflection amplitude v can then be expressed as:

$$v = h - h_i = w - w_i. \quad (4)$$

Furthermore, we substitute the initial loads with Eq. (4) in the partial differential equation for the flexure of an orthotropic plate (Timoshenko & Woinowsky-Krieger 1959; Stark *et al.* 2003):

$$D\nabla\nabla v + 2\frac{\partial D}{\partial x}\frac{\partial}{\partial x}\nabla v + 2\frac{\partial D}{\partial y}\frac{\partial}{\partial y}\nabla v + \nabla D\nabla v - (1 - \sigma) \cdot \left\{ \frac{\partial^2 D}{\partial x^2} \frac{\partial^2 v}{\partial y^2} - 2\frac{\partial^2 D}{\partial x \partial y} \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 D}{\partial y^2} \frac{\partial^2 v}{\partial x^2} \right\} \quad (5)$$
$$= -(\rho_c - \rho_f)gh - (\rho_m - \rho_c)gw$$

where all variables are functions of the (x, y) position and $\nabla = (\partial^2 / \partial x^2) + (\partial^2 / \partial y^2)$ is the Laplace operator. The solution to Eq. (5) via the FD method is shown in Appendix A.

Before flexure

After flexure

Fig. 2. Schematic cross sections illustrating the response of an elastic plate to four end-member models. (a) Concave topography with an initial external (surface) load only. (b) Concave topography with an initial internal (subsurface) load only. (c) Convex topography with an initial external (surface) load only. (d) Convex topography with an initial internal (subsurface) load only.

A larger Winkler foundation

A foundation that acts with a force proportional to the deflection at every point is known as a Winkler foundation. An elastic foundation is usually expressed as $\rho_m g v$ in linear elastic plate theory. However, the deflection predicted via linear theory is always too high; this non-linearity decreases with increasing plate thickness, such that plates with $T_e > 20$ km are only slightly non-linear (Ribe 1982). This overestimated deflection should be suppressed in ocean settings since T_e is generally low in these settings. Here we extend the Banks *et al.* (2001) model, whereby the internal load is confined to the crust, such that the entire lithosphere is modelled with a lithospheric elastic foundation with higher density (ρ_f) than that of the uppermost layer of the mantle (ρ_m). Equation (5) is then changed to:

$$D \nabla \nabla v + 2 \frac{\partial D}{\partial x} \frac{\partial}{\partial x} \nabla v + 2 \frac{\partial D}{\partial y} \frac{\partial}{\partial y} \nabla v + \nabla D \nabla v - (1 - \sigma) \cdot \left\{ \frac{\partial^2 D}{\partial x^2} \frac{\partial^2 v}{\partial y^2} - 2 \frac{\partial^2 D}{\partial x \partial y} \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 D}{\partial y^2} \frac{\partial^2 v}{\partial x^2} \right\} + (\rho_f - \rho_m) g v = -(\rho_c - \rho_f) g h - (\rho_m - \rho_c) g w, \quad (6)$$

and the first term in Eq. (A2) is changed to:

$$v_{i,j} : (3 - 2\sigma) (D_{i-1,j} + D_{i+1,j} + D_{i,j-1} + D_{i,j+1}) + 8(1 + \sigma) D_{i,j} + (\rho_f - \rho_m) g v \lambda^4. \quad (7)$$

The main advantages of this model are:

1. The overestimated deflection of the lithosphere predicted via linear theory can be suppressed;
2. The density of the mantle usually increases with depth, so the model fits real data better in the framework of lithospheric-scale deflection;
3. The main inversion function (Eq. (5)) in such a model can be converted to Eq. (6), which can be considered as a forward equation with a small Winkler foundation, thereby ensuring that stable calculation results are always obtainable.

Synthetic modelling

Synthetic modelling is commonly used to test the accuracy of the wavelet method (Stark *et al.* 2003; Kirby and Swain, 2004, 2008). Random fractal initial loads and T_e models are used as inputs to the flexural equation, which outputs the final surface topography (h) and Moho topography (w). We follow the same approach in our synthetic modelling. We employ Eq. (6) in the inversion since it is a thorough rearrangement of the flexural equation (Eq. (11)) in Kirby and Swain (2009), with the final topography (h) and a random fractal initial internal load (w_i) as inputs.

The equivalent topography, T_e and Moho topography, are required to estimate lithospheric flexure. The equivalent topography can be specifically calculated using a global elevation model, sediment thickness model and sediment thickness–density conversion formula (e.g., Sclater & Christie 1980; Ji *et al.* 2020), such that this random fractal initial load (h_i) and flexure (v) derived topography (h) can be used in the synthetic modelling. The influences of both the recovered T_e errors and inverted Moho errors on the FD-derived lithospheric flexure are investigated to determine how these inputs shape the model results.

Model Generation

We follow the synthetic modelling procedure of Macario *et al.* (1995), and use the mid-point displacement method (Peitgen & Saupe 1988) to generate two-dimensional fractal

Brownian surfaces that represent the initial external (surface) and internal (subsurface) loads. A fractal dimension of 2.5 is used for the surface (Fig. 3a) and subsurface loads (Fig. 3b), as well as the T_e model (Fig. 3c). The grids consist of 255×255 points at a 20-km spacing and are scaled to possess a realistic amplitude. Furthermore, the T_e grid is low-pass filtered with a cut-off wavelength of 150 km. We choose a central wavenumber of 5.336 in the subsequent fan wavelet coherence inversion since we place greater importance on the T_e accuracy than its spatial resolution. We do not set a constant F (loading ratio), as in Kirby and Swain (2008). Although this approach may go against Forsyth's fourth assumption about the proportionality of the power spectral densities of the initial loads (Simons & Olhede 2013), the feasibility of the T_e inversion with multiple varying F interfaces via the fan wavelet coherence method has already been tested by Kirby and Swain (2009). The topography (Fig. 3d), Moho (Fig. 3e) and lithospheric flexure models (Fig. 3f) are obtained by applying the FD method to Eq. (6). The Moho interface is then used to obtain the forward Bouguer gravity anomaly (Fig. 4), with the relevant parameters listed in Table 1. The Moho inherits high-frequency information from the initial subsurface. However, the modelled Bouguer gravity anomaly may suffer from signal degradation due to upward continuation of the signal since the observation surface is much higher than the Moho interface, resulting in attenuation of this high-frequency information.

Table 1. Symbols and employed values for the model constants (revised from Kirby & Swain 2008).

Constant	Symbol	Value	Units
Young's modulus	E	100	GPa
Newtonian gravitational constant	G	6.67259×10^{-11}	$\text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}$
Poisson's ratio	ν	0.25	
Acceleration due to	g	9.80	$\text{m} \cdot \text{s}^{-2}$

gravity			
Mean Moho depth	z_m	40	km
Crustal density	ρ_c	2750	$\text{kg}\cdot\text{m}^{-3}$
Mantle density	ρ_m	3200	$\text{kg}\cdot\text{m}^{-3}$
Compensation			
Mantle density	ρ_F	3400	$\text{kg}\cdot\text{m}^{-3}$

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Fig. 3. (a) initial surface (h_i), (b) initial subsurface (w_i), (c) model, (d) topography after flexure, (e) Moho after flexure, and (f) lithosphere flexure.

Fig. 4. Forward-modelled Bouguer gravity anomaly (Bg).

Effect of T_e errors

The fan wavelet coherence method is employed to invert for T_e using the generated surface topography, Moho topography and forward-modelled Bouguer anomaly grids (Fig. 5a), and to calculate its associated errors (Fig. 5b). This T_e inversion recovers well the main T_e features in the T_e model. The errors are randomly distributed across the entire map, but the trend of the local errors is consistent with the T_e gradient belts. This consistency is due mainly to the resolution of the transition wavelets, as well as two other adverse factors: (1) the decoupling assumption and (2) the assumption of uniform T_e in the loading equations (Kirby & Swain 2008).

We then used the generated topography, Moho topography and inverted T_e grids to estimate (Eq. (6)) the lithospheric flexure (Fig. 6a) and the associated errors (Fig. 6b). The long-wavelength component of the two flexure maps is basically the same (Figs 3f and 6a), and the amplitudes of the flexure errors (Fig. 6b) are up to one order of magnitude smaller than the T_e errors (Fig. 5b). This means that the inverted flexure is insensitive to the T_e errors. The T_e

error-induced flexure errors also exhibit a rippling pattern due to the deflection of a higher T_e plate possessing a lower amplitude and slower amplitude decay rate, although such a rippling pattern is truncated and/or deformed by the attitudes of T_e and total load. Furthermore, the range of the rippling pattern is affected by the T_e values and load amplitude, although it has a higher correlation with the T_e values in this synthetic modelling study, as the rippling pattern is obviously broader in the high T_e regions.

Fig. 5. (a) Inverted T_e grid. (b) T_e errors due to the inversion.

Fig. 6. (a) Lithospheric flexure using the inverted T_e grid. (b) Lithospheric flexure errors for the model inversion using the inverted T_e grid.

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271 *Effect of Moho errors*

272 Bouguer gravity anomaly reflects the long-wavelength Moho information. It follows that
273 the short-wavelength component of the flexure-derived Moho will be the main source of error
274 (Fig. 7b) when we use the Bouguer anomaly to invert for the Moho topography (Fig. 7a).
275 Although the maximum Moho error is 6 km, the Moho inversion yields reliable results as the
276 root-mean-square error is only 0.91 km. Furthermore, the even distribution of the Moho errors
277 provides insights into how the Moho inversion errors impact the flexure errors.

278 We also used the generated topography, T_e model and recovered (inverted) Moho
279 topography to estimate (Eq. (6)) the lithospheric flexure (Fig. 8a) and the associated errors (Fig.
280 8b). A comparison of the Moho and flexure results yields the following findings. 1) The Moho
281 inversion errors are distributed evenly across the entire region, whereas the main flexure errors
282 occur in the low T_e regions. 2) The Moho errors tend to induce flexure errors with similar
283 magnitudes but opposite signs (see the white rectangles in Figs 7b and 8b). 3) There seems to be
284 a linear attenuation coefficient of the form $-(\rho_F - \rho_m)/(\rho_m - \rho_c)$ between the large flexure
285 errors and corresponding Moho errors. 4) The plate still acts like a low-pass filter to the Moho
286 disturbance, even in the low T_e regions, with an increase in the intensity of filtering where there
287 is an increase in the T_e values.

288 Although the flexure error in the low T_e regions, e.g., oceanic lithosphere, is sensitive to the
289 Moho disturbance, this method is still suitable for investigations of the oceanic lithosphere. The

reference depth of the oceanic Moho is shallow, which may allow us to retain more short-wavelength information, such that more accurate inversion results can be obtained. And the Moho disturbance is attenuated by an attenuation coefficient as well as the lithosphere strength as it propagates into the lithospheric flexure error. In addition, the FD method-derived flexure is suited for Airy isostasy theory, even in the extremely low T_e regions.

Fig. 7. (a) Inverted Moho topography with the forward-modelled Bouguer anomaly (Bg). (b) Moho errors.

Fig. 8. (a) Lithospheric flexure using the inverted Moho values. (b) Lithospheric flexure errors for the model inversion using the inverted Moho values.

Integrated effects of the T_e and Moho errors

Neither T_e nor the Moho can be recovered with absolute accuracy using real datasets. We therefore investigated the integrated effects of the T_e and Moho errors on the estimated lithospheric flexure errors to explicate the sources of the flexure errors. We used the generated topography, recovered T_e and inverted Moho topography to invert (Eq. (6)) for the lithospheric flexure (Fig. 9a) and calculate their associated errors (Fig. 9b). The long-wavelength component of the lithospheric flexure is still well recovered, indicating that the FD method is stable when an integrated disturbance is applied. The lithospheric flexure errors are dominated by the T_e errors in the high T_e regions (Figs 6b and 9b). Correspondingly, the flexure errors are dominated by the Moho errors in the low T_e regions, although the amplitudes of the flexure errors have been

somehow changed in the low T_e regions (Figs 8b and 9b).

Fig. 9. (a) Lithospheric flexure using the inverted Moho and T_e values. (b) Lithospheric flexure errors from the model inversion using the inverted Moho and T_e values.

Conclusions

Here we employed the classic lithospheric model with an applied external load at the surface and internal load at the Moho, and assumed that the compensation material was denser than the mantle material beneath the Moho, in an attempt to estimate lithospheric flexure. The resultant lithospheric model is closer to the actual lithospheric structure, with effective suppression of the overestimated lithospheric deflection predicted via linear theory, and conversion of the main lithospheric flexure inversion function (Eq. (5)) into a forward equation with a small Winkler foundation (Eq. (6)). This lithospheric model, which based on T_e estimates

from the fan wavelet coherence method and gravity-derived Moho topography, yields a finite-difference solution of the partial differential equation that recalibrates the lithospheric flexure with high accuracy. Our main conclusions are as follows:

1. The inverted flexure is insensitive to T_e errors, and the T_e error-induced flexure errors exhibit a rippling pattern.
2. The Moho error-induced flexure errors mainly occur in the low T_e regions. The Moho disturbance is attenuated by an attenuation coefficient of the form $-(\rho_F - \rho_m)/(\rho_m - \rho_c)$ and by the lithospheric strength as Moho disturbance propagates into the lithosphere flexure error.
3. The flexure errors in the low and high T_e regions are dominated by the Moho and T_e errors, respectively.

Availability of Data and Materials The datasets that were used, analysed and produced during the current study are available from the corresponding author upon reasonable request.

Competing Interests

The authors declare that they have no competing interests.

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351 **Authors' Contributions**

352 MX processed and interpreted the data, produced the figures and wrote the main manuscript text.

353 ZW assisted in the gravity inversion. FJ assisted in the application of the fan wavelet coherence

354 method. AR, CL and JG contributed to the Lithospheric model discussion and manuscript

355 revisions.

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473
474

475 **Appendix A: Finite-difference method**

476 The FD solution in Eq. (5) is mainly from Chen (2013), but we use the final loads instead of
477 the initial loads on the right side of the equation. Equation (5) can be further simplified to:

$$\begin{aligned} 478 \quad & \nabla(D\nabla v) - (1 - \sigma) \cdot \left\{ \frac{\partial^2 D}{\partial x^2} \frac{\partial^2 v}{\partial y^2} - 2 \frac{\partial^2 D}{\partial x \partial y} \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 D}{\partial y^2} \frac{\partial^2 v}{\partial x^2} \right\} \\ & = -(\rho_c - \rho_f)gh - (\rho_m - \rho_c)gw \end{aligned} \quad (A1)$$

479 The differential operations in Eq. (A1) are approximated using central difference operators.

480 A schematic diagram of the central difference grid (Gahli & Neville 2017) is shown in Fig. A1,
481 whereby a square grid is adopted, such that $dx = dy$. The corresponding central difference
482 operators for the first-order, second-order and Laplace differential operations are shown in Fig.
483 A2.

484 The approximation to the differential operations is then undertaken with the central
485 difference operators, followed by formula expansion and combination. The following
486 coefficients of the indicated $v_{i,j}$ terms for an isotropic plate can then be obtained:

$$\begin{cases}
v_{i,j} : (3 - 2\sigma)(D_{i-1,j} + D_{i+1,j} + D_{i,j-1} + D_{i,j+1}) + 8(1 + \sigma)D_{i,j} \\
v_{i+1,j} : -2(1 + \sigma)D_{i,j} - 4D_{i+1,j} - (1 - \sigma)(D_{i,j-1} + D_{i,j+1}) \\
v_{i-1,j} : -2(1 + \sigma)D_{i,j} - 4D_{i-1,j} - (1 - \sigma)(D_{i,j-1} + D_{i,j+1}) \\
v_{i,j+1} : -2(1 + \sigma)D_{i,j} - 4D_{i,j+1} - (1 - \sigma)(D_{i-1,j} + D_{i+1,j}) \\
v_{i,j-1} : -2(1 + \sigma)D_{i,j} - 4D_{i,j-1} - (1 - \sigma)(D_{i-1,j} + D_{i+1,j}) \\
v_{i+1,j+1} : D_{i+1,j} + D_{i,j+1} + A \\
v_{i+1,j-1} : D_{i+1,j} + D_{i,j-1} - A \\
v_{i-1,j+1} : D_{i-1,j} + D_{i,j+1} - A \\
v_{i-1,j-1} : D_{i-1,j} + D_{i,j-1} + A \\
v_{i+2,j} : D_{i+1,j} \\
v_{i-2,j} : D_{i-1,j} \\
v_{i,j+2} : D_{i,j+1} \\
v_{i,j-2} : D_{i,j-1}
\end{cases} \quad (A2)$$

where $A = \frac{1 - \sigma}{8}(D_{i+1,j+1} - D_{i-1,j+1} - D_{i+1,j-1} + D_{i-1,j-1})$, $dx = dy = \lambda$, and all of the coefficients must be divided by λ^4 .

The datasets of the study area can be meshed into $M \times N$ grids, and $M \times N$ deflection values need to be calculated. Linear algebraic equations that incorporate the coefficients in Eq. (A2) can be used to calculate the deflection values as follows:

$$R\vec{V} = -(\rho_c - \rho_f)g\vec{h} - (\rho_m - \rho_c)g\vec{w}, \quad (A3)$$

where $\vec{V}$ is the deflection vector (a column vector with $M \times N$ rows), R is the difference coefficient matrix (a matrix with $M \times N$ columns and $M \times N$ rows), and $\vec{h}$ and $\vec{w}$ are the final surface and subsurface topography vectors (column vectors with $M \times N$ rows), respectively. We employed periodic boundary conditions during the calculations, and 24 kinds of boundary conditions that should be considered (Eq. A2 and Fig. A1). Furthermore, compression of the

499 sparse matrix R is recommended since it contains very few nonzero elements.

500

501

502 **Fig. A1.** Schematic diagram of the central difference grid.

503

504 **Fig. A2.** Schematic diagram of the node operation for the central difference operators. (a)

505 First-order differential operations. (b) Second-order differential operations. (c) Second-order
506 mixed partial differential operations. (d) Laplace differential operations. Null nodes indicate zero
507 coefficients.
508

Figures

Figure 1

Conceptual mechanical model of the lithosphere and asthenosphere.

Figure 2

Schematic cross sections illustrating the response of an elastic plate to four end-member models. (a) Concave topography with an initial external (surface) load only. (b) Concave topography with an initial internal (subsurface) load only. (c) Convex topography with an initial external (surface) load only. (d) Convex topography with an initial internal (subsurface) load only.

Figure 3

(a) initial surface (h_i), (b) initial subsurface (w_i), (c) model, (d) topography after flexure, (e) Moho after flexure, and (f) lithosphere flexure.

Figure 4

Forward-modelled Bouguer gravity anomaly (Bg).

Figure 5

(a) Inverted T_e grid. (b) T_e errors due to the inversion.

Figure 6

(a) Lithospheric flexure using the inverted T_e grid. (b) Lithospheric flexure errors for the model inversion using the inverted T_e grid.

Figure 7

(a) Inverted Moho topography with the forward-modelled Bouguer anomaly (Bg). (b) Moho errors.

Figure 8

(a) Lithospheric flexure using the inverted Moho values. (b) Lithospheric flexure errors for the model inversion using the inverted Moho values

Figure 9

(a) Lithospheric flexure using the inverted Moho and T_e values. (b) Lithospheric flexure errors from the model inversion using the inverted Moho and T_e values.

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