Water Shortage Risk Considering Encounter Situation of Water Supply in Multi-Resource Water City

Ming Dou (✉ dou_ming@163.com)

Zhengzhou university <https://orcid.org/0000-0002-3745-8774>

Chen An

Zhengzhou University

Jianling Zhang

Zhengzhou University

Guiqiu Li

Zhengzhou University

Research Article

Keywords: Multi-source water supply, Copula function, Encounter probability, Urban water shortage

Posted Date: June 7th, 2021

DOI: <https://doi.org/10.21203/rs.3.rs-472110/v1>

License: This work is licensed under a Creative Commons Attribution 4.0 International License.

[Read Full License](#)

Water shortage risk considering encounter situation of water supply in multi-resource water city

Ming Dou ^{a, b*}, Chen An ^a, Jianling Zhang ^c, Guiqiu Li ^b

^a School of Water Conservancy Science and Engineering, Zhengzhou University, Zhengzhou, 450001, China

^b School of Ecology and Environment, Zhengzhou University, Zhengzhou, 450001, China

^c Construction and Administration Bureau of Middle Route Project of South-to-North Water Diversion, Nanyang, 473000, China

*Corresponding Author: Ming Dou

E-mail address: dou_ming@163.com

School of Water Conservancy Science and Engineering

Zhengzhou University

No. 100 Kexue Road,

Zhengzhou 450001, Henan, China

Abstract: For multi-source water supply city, water supply varies due to different inflow conditions, which brings more uncertainty to water resources management. Therefore, it is necessary to scientifically evaluate and give countermeasures in advance. Based on the data sequence of Location, Yellow River and South-to-North Water Transfer in Zhengzhou from 1989 to 2018, the joint probability distribution model is constructed by using the optimized marginal distribution function and Copula function. The encounter probability is calculated, and the risk analysis is carried out according to the water shortage of each user under different encounters. The results show:①The probability of asynchronous is 85.96%, it is 71.92% higher than the probability of synchronous, which indicates that the three water sources complement each other well; ② Local water supply has the greatest impact on the risk of water shortage in Zhengzhou. Local water should be used reasonably and efficiently, and the external water source should be used as a supplement; ③The maximum encounter probability is 13.58%, all users except domestic water are in a water shortage state, the most serious water shortage in the industry is (-2.049, -1.089) billion m³. The minimum encounter probability is 1.14%, all users have different degrees of water shortage, the shortage of domestic water is (-3.037, 0.779) billion m³second only to industrial water shortage, the reservoir should be jointly dispatched and the groundwater exploitation should be increased to maintain the security of the urban water supply system and ensure the normal water use of social life.

Keywords: Multi-source water supply; Copula function; Encounter probability; Urban water shortage

1 Introduction

With the acceleration of urbanization, reducing the risk of urban water shortage and improving the guaranteeing rate of water supply is an important guarantee for the healthy operation of the social economy. The construction and development of water conservancy projects make cities whose local water resources cannot meet the development needs, and can alleviate the local water shortage problem through water diversion, water diversion, and other multi-source water supply measures. Currently, 343 inter-regional (or basin) water transfer projects have been completed in 39 countries abroad (Yang 2003). In 1957, the United States started construction of a water transfer project to solve the water shortage and urban development needs of the central and southern regions of California, with the Father River, Sacramento River, San Francisco River as its sources. Snowy Mountain Scheme in Australia is one of the inter-basin and inter-regional water diversion projects in the world, involving two local states and one special district. The project diverts water from the eastern part to the western arid areas for irrigation and power generation and provides a guarantee for urban water use.

Water shortage risk mainly refers to the water shortage risk of regional water resources system due to the uncertainty of water supply sources under specific spatial and temporal conditions. Multi-source water supply effectively improves the guaranteeing rate of urban water supply, but it also presents new challenges in water resources management (Zhang et al., 2015; Ren et al., 2016; Li et al., 2017), in some special cases, there will still be a poor synchronous situation of water sources, which makes the urban

water supply seriously affected. A case in Ningxia province, northwest China, the Yellow River is the main source of water diversion, suffered a severe dry year in 2015, while local rainfall decreased by 21% compared with the previous year, causing serious economic losses to the local area due to the poverty of both water sources. Therefore, for multi-water supply cities, it is necessary to analyze the water supply the situation of different water sources and study the risk of water shortage under various encounter scenarios, which is of great significance for urban water supply security.

In recent years, climate change has made precipitation in China show a trend of ' increasing in the north and decreasing in the south ' (Kong et al., 2017), the water diversion project has a significant impact on water supply sources, demand distribution, and groundwater storage (Andrade 2011). By studying the joint distribution probability of multi-source water supply sequences, the probability of the high or low water encountering of different water supply sources is quantitatively studied, and the corresponding emergency measures are customized to reduce losses in advance. Previously, more and more scholars have tried to develop various optimization methods to incorporate the uncertainty of water resources into water management, such as inaccurate two-stage stochastic programming and stochastic multi-objective optimization model (Andrew et al., 2008; Jafarzadegan et al., 2008; Nazemi et al., 2013; Borgomeo et al., 2014; Dai et al., 2016) and the use of stochastic simulation techniques to establish water dispatching risk analysis model and give a quantitative description of the risk (Wang et al., 2001). With the first application of the Coupla function in hydrology by De Michele and Salvadori, the door was opened to the application of the Copula function in the

hydrological community (De Michele et al., 2003; Favre et al., 2004), based on binary Copula function simulation, variable point identification and mean shift time series reconstruction method, the risk characteristics of joint water shortage of Dongjiang River in the water receiving area and Xijiang River in the water source area of the West-to-East Water Transfer Project in the Pearl River Delta are studied (Xu et al., 2016); Copula function was used to study the compensating effects of two external water sources on the urban water supply of Zhengzhou(Zhang et al., 2019).

Zhengzhou is located in the hinterland of the central plains, there are three kinds of urban water supply sources: South-to-North Water Transfer (SD), Yellow River Water (YD) and Local Water (LD). Affected by the change of water supply situation of different water sources, water conservancy project scheduling and interval water use (Chen 2013), there are also inter-annual differences in the water supply capacity of the two major outer basins of Zhengzhou (South-to-North Water Transfer Project and the Yellow River Diversion Project), which further affects the water supply security of Zhengzhou. Copulas are increasingly used in joint probability analysis (Li et al., 2015; Lian et al., 2015; Zhang et al., 2015), compared with other methods, the marginal properties and dependence structures of random variables can be studied separately. Therefore, taking the pattern of multi-source water supply in Zhengzhou as the breakthrough point, a multi-source joint water supply encounter probability model based on the Copula function is established. The occurrence probability of multi-dimensional variables can be revealed by using a trivariate joint probability distribution, and the comprehen-

sive impact of multi-source water on water resources shortage risk assessment is investigated (Xue et al., 2010), which fully reflects the water shortage risk under various supply and demand conditions, and calculates the water shortage situation of users according to the three-source encounter scenario, so as to serve the water resources management and social and economic construction in Zhengzhou.

2 Materials and methods

2.1 Materials

2.1.1 Study area

Zhengzhou is the capital of Henan Province, China, located in the central north of Henan Province, $112^{\circ}42' \sim 114^{\circ}14'$ east longitude and $34^{\circ}16' \sim 34^{\circ}58'$ north latitude, bordered by the Yellow River to the north, the Song Mountains to the west, and the vast Yellow and Huai Plain to the southeast. On December 15, 2014, the South-North Water Transfer Project was officially opened to the public, changing the water supply pattern of Zhengzhou from "single water source (Yellow River water and local water)" to "multiple water sources (Yangtze River water, Yellow River water and local water)". According to the water supply source, the water supply project of Zhengzhou includes local water supply project and external water supply project. The Yellow River creates a terrain of overhanging rivers on the ground because the lower reaches of the river are higher than the two sides. At the same time, the main canal of the middle route of the South-to-North Water Transfer Project also runs through Zhengzhou from south to

north, accepting the North Water Transfer from Hanjiang River, an important tributary of the Yangtze River. Since the terrain of Zhengzhou is lower than that of the main Huaihe River, the whole city is in the Huaihe River Basin. Therefore, the phenomenon of simultaneous acceptance of water supply from the Yellow River, the Yangtze River and the Huaihe River is formed.

The main canal of the South-to-North Water Transfer Middle Route Project is 731km long in Henan Province. The water receiving range of Henan Province involves the Yangtze River, Huaihe River, Yellow River and Haihe River. According to *The Water Scheduling Scheme for The First Phase of The Middle Route of The South-to-North Water Transfer Project* (trial), the source of water supply is from the Danjiangkou Reservoir in the Hanjiang River, the average annual water transfer of Taocha is 9.5 billion m³, deducting 600 million m³ of water distributed by the irrigation area of Yindan, and the total water distribution at the main canal outlet of Henan Province is 2.994 billion m³. According to Henan provincial people's government [2011] No.66, the allocation of water in Zhengzhou, the annual allocation of water is 0.54 billion m³.

The water consumption index of the water receiving area in Henan Province is 5.54 billion m³, including 3.567 billion m³ in the mainstream of the Yellow River and 1.973 billion m³ in the tributaries of the Yellow River. The total water diversion capacity of the Yellow River Diversion Project in Zhengzhou is 75.5m³/s. According to the *Refinement Scheme of Total Quantity Control Index of Yellow River Water Permission in Henan Province*, the water intake permission index of the Yellow River mainstream allocated to Zhengzhou is 0.42 billion m³.

In the current situation, the water inflow of YD, LD, and SD are different, which may lead to water shortage in Zhengzhou. In this paper, the uncertainty of water resources combination is considered as the main source of uncertainty in water resources allocation (Xue et al., 2010). By analyzing the joint probability of three water sources under different rich and poor conditions, the risk probability of different water shortage phenomena in Zhengzhou can be quantitatively described, which provides a scientific basis for Zhengzhou to deal with unexpected situations. As shown in **Fig. 1**.

[Insert Fig.1 here]

2.1.2 Available data

The Yellow River Water Supply Allocation Plan adopts the annual natural runoff of the Yellow River from 1919 to 1975, which is distributed to 9 provinces (districts), and the index of water receiving area in Henan Province is 14.9 %. According to the *Refinement Scheme of Total Quantity Control Index of Yellow River Water Permission in Henan Province* released in 2009, the water intake license index of Zhengzhou only accounts for 7.58% of the index of Henan Province. The sequence of runoff data of Huayuankou Hydrological Station used in this study are based on the *Hydrological Yearbook of the People's Republic of China statistical data*, represented by the Huayuankou Hydrological Station, and the series of measured runoff data from 1989 to 2018 is used as the basic data for YD analysis. Considering the uncertainty of the upstream inflow, the water supply sequence in Zhengzhou from 1989 to 2018 is obtained after equal proportion adjustment according to the rule that “the water is allocated at

same rate regardless of available water volumes”.

The amount of local water resource available in Zhengzhou is calculated based on the amount of surface water resource and underground water resource generated by local precipitation. The amount of surface water resource is mainly calculated based on the river runoff of several rivers in Zhengzhou, and several rivers (Jialu River and Shuangji River) are originated in the west. According to the hydrological station data on these rivers, the hydrological sequence is calculated and then converted to the whole basin according to the hydrological analogy method. The number of groundwater resource is calculated in accordance with the groundwater recharge and runoff conditions, and the water balance method is used to calculate the amount of groundwater resource in each hydrogeological partition. From the surface water resource and underground water resource obtained from 1989 to 2018, the amount of water resources that cannot be used (such as the minimum ecological water demand of the river and the flood discharge in the flood season) is deducted, and the available water resources are obtained (Hu et al., 2010).

The research period of *General Report on Feasibility Study of Middle Route Project of South-to-North Water Diversion (2005)* is 1956-1997. The index of water receiving area in Henan Province is 37.0%, and the index of water distribution in Zhengzhou is 18.0%. The series of runoff data of Danjiangkou Reservoir used in this study are based on *The Statistical Data of the Hydrological Yearbook of the People's Republic of China*, and the series of measured runoff data of 1989-2018 from the upstream of Danjiangkou Reservoir are used as the basic data for the analysis of the incoming water, which is

adjusted according to the rule that " the water is allocated at the same rate regardless of available water volumes " to obtain the water supply series of Zhengzhou. The water supply series of SD from 1989 to 2018 was obtained.

The annual data series of YD, LD and SD in Zhengzhou from 1989 to 2018, as shown in **Fig. 2**. According to *The Statistical Data of Zhengzhou Water Resources Bulletin* and related departments, the water supply sequence and water source structure of different users are obtained. The average water demand for life, agriculture, industry and ecology in Zhengzhou in the past five years is 0.564 billion m³, 0.461 billion m³, 0.49 billion m³ and 0.293 billion m³, respectively. The total water demand is 1.808 billion m³.

[Insert Fig.2 here]

2.2 Methodology

Water supply in Zhengzhou consists of three major components together, including YD, SD and LD. Multi-source water supply not only improves the guarantee rate of water supply but also brings high risks. The model research focuses on the calculation of water shortage risk in multi-source water supply cities. The process of joint risk probability for multi-water supply city is shown in **Fig. 3**. Firstly, the three-source water supply source sequence of Zhengzhou from 1989 to 2018 is used as input, and the appropriate marginal distribution is selected for the construction of joint distribution. Through the optimized bivariate and trivariate joint distribution, the joint water supply interval and encounter probability of Zhengzhou under different high and low water

encounters are calculated. The water supply quantity of each user in Zhengzhou is divided proportionally and the average water consumption in the past five years to obtain the water shortage quantity of each user, and the water shortage risk in each encounter scenario is analyzed.

[Insert Fig.3 here]

2.2.1 Copula function-based analysis model for encounter

Copula function is an association function with different single variable marginal distributions, and does not change the dependent structure of variables. The information contained in the original data will not be distorted during the conversion process, so it can construct the joint probability distribution of any single variable marginal distribution (Guo et al., 2008).

(1) Establishment of the marginal distribution

Firstly, the marginal distribution function of the water supply series of each water source should be determined. The marginal distributions are mainly selected from the gamma distribution (GAM), lognormal distribution (LOGN), Pearson type III curve (P-III), and extreme value distribution (GEV). As shown in **Table 1**.

[Insert Table 1 here]

(2) Joint probability distribution

According to the data, the water supply sequence of three sources from 1989 to 2018 is obtained, assume that X 、 Y and Z denote the sequences of YD, LD and ND,

respectively, that are relevant in the joint water supply event. Their marginal distribution functions are $u = F(x), v = F(y), w = F(z)$, respectively, and the expressions of their joint bivariate and trivariate distribution probabilities are shown (1) and (2) (Xie et al., 2010).

$$F_{UV}(u, v) = \int_{-\infty}^u \int_{-\infty}^v f(u, v) du dv = C(u, v) \quad (1)$$

$$F_{UVW}(u, v, w) = \int_{-\infty}^u \int_{-\infty}^v \int_{-\infty}^w f(u, v, w) du dv dw = C(u, v, w) \quad (2)$$

The Copula function theory is used to establish the joint probability distribution among the water supply sources, which is used to quantitatively analyze the joint distribution probability under multi-source water supply. The bivariate and trivariate models (Niu 2010; Hou 2010; Song et al., 2012) are shown in **Table 2**.

[Insert Table 2 here]

The goodness-of-fit test of the Copula function mainly analyzes whether the selected Copula function can appropriately describe the correlation among variables. This paper mainly uses the K-S test statistic D to test the goodness-of-fit of the Copula function. The goodness-of-fit evaluation is an important criterion for screening the joint distribution probability of the Copula function. The commonly used goodness-of-fit evaluation methods include graphical evaluation analysis, RMSE, AIC and BIC, and the Copula functions that pass the goodness-of-fit test will continue to be selected according to the goodness-of-fit evaluation index. The principle and solution process of the above methods can be found in the related literature, and will not be repeated here (Xie et al., 2010; Niu 2010).

(3) Encounter probability calculation of Copula joint distribution

According to the relevant research results, high or low state classification in China is limited by frequency and the corresponding value of frequency $P_f=37.5\%$ and $P_k=62.5\%$ (Lu 2000; Cao 2015). The corresponding inflow is X_{pf} and X_{pk} , respectively, then the abundance year $X > X_{pf}$, the withering year $X \leq X_{pk}$, and the normal water year $X_{pk} < X \leq X_{pf}$. The bivariate combined encounter probability of YD, LD and SD can be deduced as (3) (4) (5) and the trivariate combined encounter probability can be deduced as (6).

$$P(X_1 < X \leq X_2, Y_1 < Y \leq Y_2) = C(u_2, v_2) - C(u_1, v_2) - C(u_2, v_1) + C(u_1, v_1)$$

$$(3) \quad P(Y_1 < Y \leq Y_2, Z_1 < Z \leq Z_2) = C(v_2, w_2) - C(v_1, w_2) - C(v_2, w_1) +$$

$$C(v_1, w_1) \quad (4)$$

$$P(X_1 < X \leq X_2, Z_1 < Z \leq Z_2) = C(u_2, w_2) - C(u_1, w_2) - C(u_2, w_1) +$$

$$C(u_1, w_1) \quad (5)$$

$$P(X_1 < X \leq X_2, Y_1 < Y \leq Y_2, Z_1 < Z \leq Z_2)$$

$$= C(u_2, v_2, w_2) - C(u_1, v_2, w_2)$$

$$- C(u_2, v_1, w_2) - C(u_2, v_2, w_1) + C(u_1, v_1, w_2) + C(u_1, v_2, w_1)$$

$$+ C(u_2, v_1, w_1)$$

$$- C(u_1, v_1, w_1)$$

$$(6)$$

Where, X_l , Y_l and Z_l are the minimum water inflow corresponding to the three water sources; X_l , Y_l and Z_l are the maximum water inflow corresponding to the three water sources; u_l , u_2 , v_l , v_2 , w_l , w_2 are the value of the marginal distribution function corresponding to X_l , X_2 , Y_l , Y_2 , Z_l , Z_2 , respectively; among them, there are

9 kinds of binary encounter combinations and 27 kinds of ternary encounter combinations, with the same rich type, the same normal type and the same poor type being synchronous and the others being asynchronous.

2.2.2 Urban water shortage risk analysis

Urban water shortage risk analysis is to analyze the water shortage and distribution of urban users. As a multi-water supply city, Zhengzhou has different proportions of water supply from each water source to each user. According to the ratio, the water supply of each user is calculated, and its water consumption minus the corresponding water supply is the water shortage of the user. Usually, urban water shortage is divided into domestic water shortage, agricultural water shortage, industrial water shortage and ecological water shortage, and the calculation process is shown in equation (7).

$$Q[P_{ij}] = E_j - [a_j X_{1i} + b_j Y_{1i} + c_j X_{1i}, a_j X_{2i} + b_j Y_{2i} + c_j X_{2i}] \quad (7)$$

Where, $Q[P_{ij}]$ is the water shortage interval for user j in encounter scenario i , E_j indicates the average water consumption of user j in the past five years, a_j , b_j , c_j denotes the percentage of water supply from different sources for user j , X_{1i} , Y_{1i} , Z_{1i} denote the minimum available water supply for user j in encounter scenario i ; X_{2i} , Y_{2i} , Z_{2i} denote the maximum available water supply for user j in encounter scenario i .

3 Results and discussions

3.1 Marginal distribution of YD, LD and SD

It is assumed that the marginal distribution of YD, LD and SD in Zhengzhou from 1989 to 2018 may follow gamma distribution (GAM), lognormal distribution (LOGN), Pearson type III curve (P-III) and extreme value distribution (GEV). The maximum likelihood method is used for estimating the statistical parameters of the marginal distribution, and the K-S statistic is used for verifying whether the assumed marginal distribution is reasonable. The significant level $\alpha = 0.05$, when $n=30$, the corresponding quantile of K-S statistics is $D_0=0.24$. If the statistical value $D < D_0$ of the determined K-S, the assumed marginal distribution is the fitting value.

[Insert Table 3 here]

The AIC, BIC, RMSE, and K-S statistic D for the marginal distributions of YD, LD, and SD are given in **Table 3**. The results show that all of the three hypothesized marginal distributions pass the statistical test except for the gamma distribution of LD and the extreme distribution of YD for which the statistic D cannot pass the K-S test, that is, the statistics $D < D_0$ of the K-S test. Among them, the minimum AIC of LD is -191.3208, the minimum BIC is -188.5184, and the minimum RMSE is 0.0373, so the extreme value distribution is selected as its marginal distribution; The minimum AIC of YD is -163.7869, the minimum BIC is -160.9845, and the minimum RMSE is 0.0590, so the lognormal distribution is selected as its marginal distribution; The minimum AIC of SD is -199.4430, the minimum BIC is -196.6406, and the minimum RMSE is

0.0325, so lognormal distribution is selected as its marginal distribution. As shown in **Fig.4**, the correlation coefficient between the empirical distribution and the assumed marginal distribution of the sample is more than 95 %, which shows that the assumed marginal distribution of the YD, LD, and SD is reasonable, and the parameters they choose can be adopted.

[Insert Fig.4 here]

3.2 Selection of Copula function

The bivariate and trivariate combinations of YD, LD, and SD in Zhengzhou from 1989 to 2018 are carried out. The Clayton Copula, Frank Copula and Gumbel Copula are used to fit the bivariate combination. The Gaussian Copula and t-Copula function are used to fit the trivariate combination, and secondly, the Copula functions suitable for this combination is selected through the goodness-of-fit evaluation.

3.2.1 Fitting test

According to the theoretical frequencies obtained from the joint distribution model of bivariate Copula and trivariate Copula functions and the empirical frequencies obtained by sequence calculation, the statistics D of different Copula function joint distribution combinations are calculated and the results are shown in **Table 4**. The quantile value corresponding to the significance level of K-S test $\alpha = 0.05$ and $n = 30$ is 0.24. It can be seen from the table that the statistics D of Clayton Copula in the bivariate combination of YD and LD is greater than 0.24, the statistics D of Gumbel Copula in the bivariate combination of LD and SD is greater than 0.24, and the statistics D of the joint

distribution of other functions is less than 0.24, which can be calculated. In order to select the most suitable Copula function type for this combination mode, the next step of goodness-of-fit evaluation is required.

[Insert Table 4 here]

3.2.2 Evaluation of goodness-of-fit

AIC criterion method, BIC criterion RMSE method are used to further evaluate the goodness-of-fit of each Copula function for quantitative analysis. The goodness-of-fit of the theoretical joint probability value of the Copula function to the empirical joint probability value was also used to determine the optimal Copula function.

[Insert Table 5 here]

According to the criterion that the smaller the values of RMSE, AIC and BIC, the better the Copula joint probability distribution function is fitted, it can be seen from **Table 5** that for the bivariate combination (YD, LD) under the Frank Copula function, the minimum AIC and BIC are -193.6408 and -190.8385, and the minimum RMSE is 0.0356, the Frank Copula joint probability model with the better fitting degree is selected to calculate the joint distribution probability; for the bivariate combination (YD, ND) under the Clayton Copula function, the minimum AIC and BIC are -195.2270 and -192.4246, and the minimum RMSE is 0.0349, the Clayton Copula joint probability model with the better fitting degree is selected to calculate the joint distribution probability; for the bivariate combination (ND, LD) under the Clayton Copula function, the minimum AIC and BIC are -212.4743 and -209.6719, and the minimum RMSE is

0.0262, the Clayton Copula joint probability model with the better fitting degree is selected to calculate the joint distribution probability; for the trivariate combination (YD, LD, ND) under the Gaussian Copula function, the minimum AIC and BIC are -189.2766 and -189.8754, respectively, and the minimum RMSE is 0.0386. Therefore, the Gaussian Copula joint probability model with a good fit was chosen to calculate the joint distribution probability.

The empirical joint distribution probability values of the bivariate combination and the trivariate combination are calculated according to the joint empirical probability distribution. Combined with the theoretical joint distribution probability values calculated by the joint distribution model of Copula functions optimized by each combination, the two are drawn into a scatter plot, and the correlation coefficients are above 95 %. As shown in **Fig. 5**.

[Insert Fig.5 here]

As can be seen, the optimal Copula function fits better for the bivariate combinations and the trivariate combination, which indicates that the preferred Copula function type is reasonable.

3.3 Analysis of the joint probability of rich and poor

In this paper, the uncertain factors of water supply sources are considered, and this uncertainty is expressed as the joint probability between each water supply. Firstly, the levels of each water source are classified according to the marginal distribution, the results are shown in **Table 6**.

[Insert Table 6 here]

The Copula functions of different bivariate and trivariate combinations are optimized, and the joint probability distribution model is established based on the data series of YD, LD, and SD from 1989 to 2018 in Zhengzhou, Henan Province, China. The joint probability distribution diagram and contour (surface) diagram of the bivariate and trivariate combinations are plotted.

[Insert Fig.6 here]

Fig. 6 shows that there is a close correlation among YD, LD, and SD. **Fig6(a)** illustrates the joint probability distribution diagram of the joint water supply of LD and YD. Therefore, given the combination of various LD and YD, the joint distribution probability can be obtained. It can be seen that with the increase of LD or YD, the value of the joint distribution probability becomes larger, and when it is between 0.1 and 0.2, the isoline spacing changes significantly. **Fig6(b)** illustrates the joint probability distribution diagram of SD and YD. Therefore, given the combination of various SD and YD, the joint distribution probability can be obtained. It can be seen that with the increase of SD or YD, the value of the joint distribution probability becomes larger, and when it is between 0.1 and 0.4, the isoline spacing changes significantly. **Fig6(c)** illustrates the joint probability distribution diagram of LD and SD. Therefore, given the combination of various LD and SD, the joint distribution probability can be obtained. It can be seen that with the increase of LD or SD, the value of the joint distribution probability becomes larger, and when it is between 0.1 and 0.3, the isoline spacing changes significantly. **Fig6(d)** illustrates the joint probability distribution diagram of YD, LD and SD,

so the joint distribution probability of each combination of three water sources can be obtained. It can be seen that with the increase of YD, LD, and SD, the joint distribution probability value increases, and when it is between 0.1 and 0.3, the isoline spacing changes significantly. Whether YD, LD, and SD or combined water supply, the joint probability tends to increase with the increase of water supply.

The optimized Copula function is used to calculate the frequencies of nine bivariate combinations. Among them, the high-high type, the normal-normal type, the low-low type is synchronous, the rest is asynchronous. The results are shown in **Table 7**.

[Insert Table 7 here]

The optimal Copula function is used to calculate the frequencies of 27 trivariate combinations. Among them, the high-high-high type, the normal-normal-normal type, the low-low-low type is synchronization, the rest is asynchronous. The results are shown in **Table 8**.

[Insert Table 8 here]

The calculation shows that it is $P(X \leq X_{pk}, Y > Y_{pf}) = 0.2312$ under the condition of YD and LD, $P(Y > Y_{pf}, Z > Z_{pf}) = 0.1406$ under the condition of LD and SD, and $P(X \leq X_{pk}, Y > Y_{pf}, Z > Z_{pf}) = 0.0003$ under the condition of three water sources combined supply. The results show that the trivariate joint encounter probability is significantly less than the bivariate joint encounter probability. Therefore, if only considering the bivariate joint distribution probability in water supply, it is difficult to correctly describe the statistical characteristics of water supply. Considering the trivariate joint distribution probability can more comprehensively express the water supply characteristics of Zhengzhou, and further quantitatively explain the joint probability of different

combinations under any water supply situation. Among them, the synchronous frequency of the three water sources is 14.04%, the asynchronous frequency of the three water sources is 85.96%, and the asynchronous frequency of the three water sources is greater than the synchronous frequency of the three water sources 71.92 %. There is a complementary relationship among the three water sources. Among the combination types, the frequency of high-low-normal type was the highest (13.58%), while that of low-high-high type was the lowest (0.03%). Therefore, using the trivariate joint distribution probability can more reasonably understand the joint water supply encounter probability of Zhengzhou, and provide technical guidance for the water supply planning.

3.4 Analysis of urban water shortage situation

The analysis of the encounter is from the perspective of water resources, rather than from the perspective of disasters, that is, poor encountering may not be a drought, rich encountering may not be a flood (Song et al., 1994; Qiao et al., 1997). Therefore, the analysis of the encountering of multi-source water supply should be combined with the urban water demand, so as to analyze the urban water shortage.

As shown in **Table 8**, the probability of encountering the three water sources with the same abundance is 10.64%, and the water supply volume interval is (1.775, 2.894) billion m³, while the probability of encountering the same depletion is 1.14%, and the water supply volume interval is (0.445, 1.409) billion m³. The maximum difference of water supply volume between the two cases is 2.449 billion m³, and the water supply volume of the three water sources with the same depletion is very likely to be insufficient. In addition, the joint probability of normal-high-normal type and low-high-low

type are both 3.29 %, but the joint available water supply was (1.604, 2.251) billion m³ and (1.282, 2.081) billion m³, respectively. Although the two cases had the same probability, the maximum difference in water supply was 0.482 billion m³. The joint probability of synchronous normal type and low-normal-normal type are both 2.28%. The combined water supply was (1.409, 1.775) billion m³ and (1.284, 1.705) billion m³, respectively. The maximum difference in water supply was 0.296 billion m³.

[Insert Fig.7 here]

As can be seen from **Fig.7**, under the conditions that the YD and the SD are both wet years, S1~S10~S19 represent the three states of LD respectively, and the maximum variation of water supply is 1.956 billion m³; in the state where both YD and LD are both wet years, S1~S4~S7 represent the three states of SD respectively, and the maximum variation of water supply is 1.417 billion m³; in the state where both LD and SD are both wet years, S1~S2~S3 represent the three states of YD respectively, and the maximum variation of water supply is 1.314 billion m³; among them, the variation of S1~S10~S19 is greater than S1~S4~S7 than S1~S2~S3. Zhengzhou mainly uses LD as the water supply source and external water transfer as a supplement, and the rational allocation between the three water sources to achieve efficient use of water resources.

According to the domestic, agricultural, industrial and ecological water supply under 27 scenarios and the average water demand of each user in the past five years, the water shortage situation of each user in Zhengzhou in 27 scenarios is calculated, where negative values indicate water shortage and positive values indicate water surplus.

[Insert Fig.8 here]

Combined with the joint distribution probability and **Fig. 8**, it can be seen that S12 is the scenario with the largest encounter probability and it is 13.58 %, the surplus of domestic water is (0.151, 0.579) billion m³ and the remaining users are in a state of water shortage. In S12, the water shortage in the industry being (-0.205, -0.109) billion m³, as an industrial city, Zhengzhou can preferentially allocate the surplus part of domestic water supply to industrial water to reduce the risk of water shortage in Zhengzhou. The encounter probability of S1 is only second to that of S12 and S21, in this scenario, the probability of synchronous poverty of three water sources is 10.62 %, and the possibility of the surplus of each user is the largest, the surplus of domestic water is (0.242, 0.774) billion m³, the interval of agricultural water shortage is (-0.007, 0.285) billion m³, the interval of industrial water shortage is (-0.093, 0.138) billion m³, and the interval of ecological water shortage is (-0.056, 0.070) billion m³. Although the remaining users have the possibility of water shortage, the surplus of domestic water can meet the total amount of water shortage of the remaining users. At the same time, excess water can be stored in the reservoir as a backup water source, so as to improve the water supply guarantee rate of Zhengzhou. Under S27, the three water sources are poor, and each user has different degrees of water shortage. The largest gap of industrial water is (-4.262, -1.761) billion m³, and the water shortage of domestic water is only second to the industrial water (-3.037, 0.779) billion m³. The water shortage interval of agricultural water is (-3.550, -1.003) billion m³, and the ecological water shortage interval is (-0.564, -0.701) billion m³. Although the probability of encountering S27 is only

1.14 %, the supply and demand balance of urban water supply system is seriously damaged, and it is easy to cause large losses, at this time, the reservoirs should be transferred to joint dispatch and increase the amount of groundwater extraction to maintain the security of the urban water supply system and ensure the normal use of water for social life.

In summary, for Zhengzhou, the use of YD and SD can return some of the squeezed agricultural water to agriculture, alleviate the contradiction of urban water squeezing agricultural water, improve agricultural production conditions and control groundwater overdraft. The local water source should be combined with the South-to-North Water Transfer and the Yellow River water source to give full play to the role of regulating reservoirs and realize the optimal allocation and rational utilization of water resources. The multi-source urban water supply encounter model established in this paper can quantitatively analyze the encounter probability and water shortage interval of multi-source urban water supply, reflect the statistical characteristics of each water source, improve the guarantee rate of urban water supply and reduce the risk of water shortage.

4 Conclusions

(1) Using Gaussian-Copula function, a trivariate joint probability distribution of YD, LD, and SD is established. The empirical distribution of samples is consistent with the marginal distribution and has a high correlation coefficient. This shows that Gaussian-Copula function can explain the statistical characteristics of YD, LD, and SD.

(2) The established joint probability distribution indicates the occurrence probability of different water source combinations. It can effectively estimate the risk of water shortage in Zhengzhou under different water supply conditions. In addition, the tri-variate joint distribution probability of YD, LD, and SD can better reflect the characteristics of multi-source water supply than the bivariate joint distribution probability in Zhengzhou.

(3) The asynchronous frequency of water supply in YD, LD, and SD in Zhengzhou, is greater than the synchronous frequency. The inconsistent change of water inflow causes the uneven spatial and temporal distribution of water resources. It shows that the three water supply sources are complementary, which is beneficial to the joint water supply of urban multi-water sources and can effectively reduce the risk of water supply shortage in scanty water.

(4) Local water in Zhengzhou occupies a larger share of the water supply system, supplemented by water from the Yellow River and water from South-to-North Transfer. Rational use of local water resources and efficient allocation of external water sources can alleviate the water shortage in Zhengzhou.

These important results provide new ideas for risk assessment of water shortage in Zhengzhou: Considering the joint distribution probability of different combinations of three water sources, the risk of different water shortage grades can be described and quantitatively evaluated. Through the joint operation of three waters, rational allocation, and mutual supplement, all kinds of water sources can be effectively used to improve the guaranteeing rate of urban water supply.

Acknowledgments

Funding: This work was supported by the National Natural Science Foundation of China [No. 51879239], and the Program for key Science & Technology projects in Universities of Henan Province [No. 21A570008].

Funding

This work was supported by the National Natural Science Foundation of China [No. 51879239], and the Program for key Science & Technology projects in Universities of Henan Province [No. 21A570008].

Authors Contributions

Chen An: Analyze data, and write drafts; Ming Dou: Research and investigation, methods, draft revision; Jianling Zhang: Experimental Conjecture Verification, Method; Guiqiu Li: Error checking, draft revision

Competing Interests

The authors declare no competing interests.

Availability of data and materials

Not applicable

Ethical Approval

Not applicable

Consent to Participate

Consent

Consent to Publish

Consent

References

Andrade, J. G. P. D., Francobarbosa, P. S., Souza, L., Makino, D. L. (2011). Interbasin water transfers: the brazilian experience and international case comparisons. *Water Resources Management*, 25(8), 1915-1934.

<https://doi.org/10.1007/s11269-011-9781-6>

Andrew, Higgins, Ainsley, ArcherStefan, Hajkowicz. (2008). A stochastic non-linear programming model for a multi-period water resource allocation with multiple objectives. *Water Resources Management*, 22(10), 1445-1460.

<https://doi.org/10.1007/s11269-007-9236-2>

Borgomeo, E., Hall, J. W., Fung, F., Watts, G., Colquhoun, K., Lambert, C. (2015). Risk-based water resources planning: incorporating probabilistic nonstationary climate uncertainties. *Water Resources Research*, 50(8), 6850-6873.

<https://doi.org/10.1002/2014WR015558>

Cao Gentang. (2015). Research on computer integrated application system based on information management of water conservancy and hydropower survey and design. *Electronic test* (04), 100-102.

<https://doi.org/10.16520/j.cnki.10008519.2015.04.040>

Chen Lu. Application of Copula function theory in multivariate hydrological analysis and calculation [M]. Wuhan: Wuhan University Press, 2013: 28.

Dai, C., Cai, Y. P., Lu, W. T., Liu, H., Guo, H. C. (2016). Conjunctive water use optimization for watershed-lake water distribution system under uncertainty: a case

study. Water Resources Management, 30(12), 4429-4449.

<https://doi.org/10.1007/s11269-016-1430-7>

D Michele, C. (2003). A generalized pareto intensity-duration model of storm rainfall exploiting 2-copulas. Journal of Geophysical Research Atmospheres, 108(D2).

<https://doi.org/10.1029/2002JD002534>

Lu Kejiang. (2000). The determination and implementation of seasonal water price in irrigation areas. Rural water conservancy and hydropower in China, 000 (010), 9-

10. <https://doi.org/10.3969/j.issn.1007-2284.2000.10.005>

Favre, A. C., Adlouni, S. E., Perrault, L. (2004). Multivariate hydrological frequency using copulas. Water Resources Research, 40(12), 1-12.

<https://doi.org/10.1029/2003WR002456>

Guo, Yan, Xiao, Fang progress of Copula function in multivariate hydrological analysis and calculation. Hydrology (03), 1-7.

<https://doi.org/10.3969/j.issn.1000-0852.2008.03.001>

Hou Yunyun, Song Song Songbai, Zhao Lina, Wang Jianfeng(2010). Multivariate flood frequency calculation based on copula function. Journal of Northwest A & F University: Natural Science Edition (02), 227-236.

Hu Caihong, Wu Zening, Gao Jun, Chai Xianjun (2010). Study on regional water availability. Geography of arid areas (Chinese version) (03), 92-98.

<https://doi.org/10.13826/j.cnki.cn65-1103/x.2010.03.012>

Jafarzadegan, K, Abed-Elmdoust, A, Kerachian, R (2014). A stochastic model for optimal operation of inter-basin water allocation systems: a case study. Stochastic

Environmental Research & Risk Assessment, 28(6), 1343-1358.

<https://doi.org/10.1007/s00477-013-0841-8>

Kong Feng, et al (2017). A comparative study on the spatial pattern of variation trend and fluctuation characteristics of rainfall in different months and durations in China (1961-2015). Journal of Capital Normal University (Natural Science Edition) (06), 85-95. <https://doi.org/10.19789/j.1004-9398.2017.06.015>

Li Y, Gu W, Cui W, et al., (2015). Exploration of copula function use in crop meteorological drought risk analysis: a case study of winter wheat in Beijing, China. Natural Hazards (2). <https://doi.org/10.1007/s11069-015-1649-2>

Li Y, Cui Q, Li C, Wang X, Cai Y, Cui G, Yang Z(2017) An improved multi-objective optimization model for supporting reservoir operation of China's South-to-North Water Diversion Project. Sci Total Environ 575:970–981. <https://doi.org/10.1016/j.scitotenv.2016.09.165>

Lian, J, He, W, Ma, C, Xu, K (2015). Guarantee rate of freshwater in a river mouth intruded by saltwater with respect to the joint impact of runoff and tide. Journal of Hydroinformatics, 17(6), 917-929. <https://doi.org/10.2166/hydro.2015.038>

Nazemi, A, Wheeler, H. S, Chun, K. P, Elshorbagy, A (2013). A stochastic reconstruction framework for analysis of water resource system vulnerability to climate-induced changes in river flow regime. Water Resources Research, 49(1), 291–305. <https://doi.org/10.1029/2012WR012755>

Niu Junyi (2010). Study on safety management of water environment system in water supply hub project. (Doctoral thesis, Tianjin University).

<https://doi.org/10.7666/d.y1874680>

Qiao Jirong, Lin Jinrui, Zhang Yan. (1997). Long-term characteristics of drought and flood changes in the middle line of South-to-North Water Diversion. *Climate and environment studies*, 2 (4), 339-346.

<https://doi.org/10.3878/j.issn.1006-9585.1997.04.03>

Ren, C. F., Li, R. H., Zhang, L. D., Guo, P. (2016). Multiobjective stochastic fractional goal programming model for water resources optimal allocation among industries. *Journal of Water Resources Planning and Management*, 142(10), 04016036.

[https://doi.org/10.1061/\(ASCE\)WR.1943-5452.0000681](https://doi.org/10.1061/(ASCE)WR.1943-5452.0000681)

Song Huizhu, Zhang Shifa (1994). Preliminary analysis of regional drought statistical characteristics. *Advances in water science*, 5 (001), 18 - 25.

Song Songbai, (2012). Copula function and its application in hydrology. Science Press.

Tu, Chen, Diao, Zhao, Du. (2016). Copula runoff model of the Pearl River Delta and water shortage risk analysis of the West-to-East Water Transfer Project. *Journal of Agricultural Engineering*, 32 (18), 162-168.

<https://doi.org/10.11975/j.issn.1002-6819.2016.18.022>

Wang, Zhu, Hou. (2001). Risk analysis of water regulation in the lower Yellow River. *Journal of Hehai University: Natural Science Edition* (02), 71–74.

<https://doi.org/10.3321/j.issn:1000-1980.2001.02.016>

Xie Zhonghua. (2010). MATLAB statistical analysis and application: 40 case studies. Beijing University of Aeronautics and Astronautics Press.

639 Xueping, Gao, Yin Zhu, Liu, Bowen, Sun (2018). Water shortage risk assessment con-
 640 sidering large-scale regional transfers: a copula-based uncertainty case study in
 641 lunan, china. Environmental Science & Pollution Research.
 642 <https://doi.org/10.1007/s11356-018-2408-1>
 643 Yang Lixin (2003). Summarization of foreign water diversion projects. Research on
 644 water conservancy development, 3 (006), 55 - 56 .
 645 <https://doi.org/10.3969/j.issn.1671-1408.2003.06.016>
 646 Zhang, Q, Xiao, M, Singh, V. P. (2015). Uncertainty evaluation of copula analysis of
 647 hydrological droughts in the east river basin, china. Global & Planetary Change,
 648 129(jun.), 1-9. <https://doi.org/10.1016/j.gloplacha.2015.03.001>
 649 Zhang Qian, Wu Zening, Lv Cuimei, Guo Xi, Gao Shen. (2019). The compensation
 650 characteristics of external water supply in Zhengzhou based on copula function.
 651 The People ' s Yellow River, 041 (004), 37-41.
 652 <https://doi.org/10.3969/j.issn.1000-1379.2019.04.009>

Fig.1 Overview of the study area

Fig. 2 The annual data series of Yellow River water, South-to-North Water Transfer and local water in Zhengzhou

Fig. 3 Flow Chart

Fig. 4 Empirical Distribution and Theoretical Distribution Fitting of Marginal Distribution

Fig. 5 Fitting graphs of theoretical joint frequency and empirical joint frequency of each combination under the optimized Copula

Fig. 6 Bivariate and trivariate joint probability distribution and contour map

Fig.7 Comparison of the variation of the three water sources

Fig.8 Water shortage of each user under various combinations of wet and dry conditions

691

Table 1 Marginal distribution function model

Type	Marginal distribution function
GAM	$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$
LOGN	$f(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma x} \exp\left[-\frac{1}{2\sigma^2} (\ln x - \mu)^2\right] & , x > 0 \\ 0 & , x \leq 0 \end{cases}$
P-III	$f(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} (x - a_0)^{\alpha-1} e^{-\beta(x-a_0)}$
GEV	$f(x) = \sigma^{-1} \exp\left\{\frac{x-\mu}{\sigma} - \exp\left(\frac{x-\mu}{\sigma}\right)\right\}, -\infty < x < +\infty, \sigma > 0$

692

$\Gamma(\alpha)$ is the gamma function, α is shape parameter, β is scale parameter, a_0 is

693

location parameter, $\alpha > 0$, $\beta > 0$. $\alpha = \frac{4}{C_s^2}, \beta = \frac{2}{\bar{x} \times C_v \times C_s}, a_0 = \bar{x} \left(1 - \frac{2C_v}{C_s}\right)$, $\bar{x}$ is

694

means, C_v is coefficients of variation, C_s is coefficients of skewness; in GEV distri-

695

bution function, x is the plurality of the distribution, μ is location parameter, σ is

696

scale parameter.

697

698

Table 2 Copula function joint probability distribution model

Type		Functions
Bivariate distribution	Frank	$C(u, v, \alpha) = -\frac{1}{\alpha} \ln \left[1 + \frac{(e^{-\alpha u} - 1)(e^{-\alpha v} - 1)}{e^{-\alpha} - 1} \right],$ $(\alpha \in (-\infty, 0) \cup (0, \infty))$
	Clayton	$C(u, v, w) = (u^{-\alpha} + v^{-\alpha} - 1)^{-\frac{1}{\alpha}}, (\alpha > 0)$
	Gumbel	$C(u, v, \alpha) = \exp\{-[(-\ln u)^\alpha + (-\ln v)^\alpha]^{\frac{1}{\alpha}}\}, (\alpha > 1)$
Trivariate distribution	Guassian	$C(u, v, w; \rho) = \Phi_\rho(\Phi^{-1}(u), \Phi^{-1}(v), \Phi^{-1}(w))$
	t-copula	$C(u, v, w; \rho, k) = t_{\rho, k}(t_k^{-1}(u), t_k^{-1}(v), t_k^{-1}(w))$

α is the parameter describing the relationship of 2 and 3 variables; ρ is a 2-order
 3-order symmetric positive definite matrix with all elements on the diagonal being 1;
 Φ_ρ denotes the distribution function of the bivariate and trivariate standard normal dis-
 tributions with correlation coefficient matrix ρ ; Φ^{-1} denotes the inverse function of
 the standard normal distribution; $t_{\rho, k}$ denotes the distribution function of the standard
 bivariate and trivariate distributions with correlation coefficient matrix ρ and degrees
 of freedom k , t_k^{-1} denotes the inverse function of the bivariate and trivariate t dis-
 tribution with degree of freedom k .

711

Table 3 Marginal distribution function and statistic D

		AIC	BIC	RMSE	D
LD	GAM	-153.1868	150.3844	0.0703	0.3130
	LOGN	-136.0467	-133.2443	0.0936	0.1797
	P-III	-152.0946	-149.2922	0.0716	0.1495
	GEV	-191.3208	-188.5184	0.0373	0.0853
YD	GAM	-153.0903	-150.2880	0.0705	0.2019
	LOGN	-163.7869	-160.9845	0.0590	0.1785
	P-III	-162.6328	-159.8304	0.0601	0.1727
	GEV	-115.7323	-112.9299	0.1313	0.2793
SD	GAM	-184.8859	-182.0832	0.0415	0.1116
	LOGN	-199.4430	-196.6406	0.0325	0.0891
	P-III	-196.8717	-194.0694	0.0340	0.0873
	GEV	-139.1674	-136.3650	0.0889	0.1923

712

713

714

Table 4 Different types of Copula function joint probability distribution fitting test statistics D

Type	Combination			
	(YD, LD)	(YD, SD)	(LD, SD)	(YD, LD, SD)
Clayton Copula	0.3407	0.1123	0.0783	---
Frank Copula	0.1171	0.1030	0.1024	---
Gumbel Copula	0.0999	0.1245	0.4599	---
Gaussian Copula	---	---	---	0.1061
t-Copula	---	---	---	0.1708

Table 5 Evaluation of the goodness-of-fit of the joint probability distribution of different Copula functions

Method	Type	Combination			
		(YD, LD)	(YD, SD)	(LD, SD)	(YD, LD, SD)
AIC	ClaytonCopula	---	-195.2270	-212.4743	---
	Frank Copula	-193.6408	-191.9822	-202.8437	---
	Gumbel Copula	-178.3046	-191.3451	---	---
	GaussianCopula	---	---	---	-189.2766
	t-Copula	---	---	---	-145.9761
BIC	Clayton Copula	---	-192.4246	-209.6719	---
	Frank Copula	-190.8385	-191.1798	-200.0413	---
	Gumbel Copula	-175.5022	-188.5427	---	---
	Gaussian Copula	---	---	---	-189.8754
	t-Copula	---	---	---	-146.5749
RMSE	Clayton Copula	---	0.0349	0.0262	---
	Frank Copula	0.0356	0.0356	0.0307	---
	Gumbel Copula	0.0643	0.0372	---	---
	Gaussian Copula	---	---	---	0.0386
	t-Copula	---	---	---	0.0793

725

Table 6 High or low state classification of YD, LD and SD

Frequency	YD (10^8m^3)	LD (10^8m^3)	SD (10^8m^3)
62.5%	2.8307	7.2337	4.0301
37.5%	3.5249	9.1846	5.0365

726

727

728

729

Table 7 Probability of joint distribution of bivariate combinations

		LD			YD		
		$Y \geq Y_{pf}$	$Y_{pk} < Y \leq Y_{pf}$	$Y \leq Y_{pk}$	$Y \geq Y_{pf}$	$Y_{pk} < Y \leq Y_{pf}$	$Y \leq Y_{pk}$
SD	$Z \geq Z_{pf}$	0.1406	0.0938	0.1406	0.2069	0.1015	0.0666
	$Z_{pk} < Z \leq Z_{pf}$	0.0938	0.0624	0.0938	0.1015	0.0772	0.0713
	$Z \leq Z_{pk}$	0.1406	0.0938	0.1406	0.0666	0.0713	0.2371
	$X \geq X_{pf}$	0.0569	0.0869	0.2312	---	---	---
YD	$X_{pk} < X \leq X_{pf}$	0.0869	0.0762	0.0869	---	---	---
	$X \leq X_{pk}$	0.2312	0.0869	0.0569	---	---	---

730

731

732

733 **Table 8** Synchronous and asynchronous encounter situations and encounter probabili-
734 ties of YD-LD-SD

Encounter situation	Sce-nario	YD-LD-SD	Encounter probability representation	Water supply capacity	SEP (%)	AEP (%)
Synchronous	S1	High-high-high	$P(X \geq X_{pf}, Y \geq Y_{pf}, Z \geq Z_{pf})$	(17.75,28.94)	10.62	14.04
	S14	Normal-normal-normal	$P(X_{pk} < X \leq X_{pf}, Y_{pk} < Y \leq Y_{pf}, Z_{pk} < Z \leq Z_{pf})$	(14.10,17.71)	2.28	
	S27	Low-low-low	$P(X \leq X_{pk}, Y \leq Y_{pk}, Z \leq Z_{pk})$	(4.45,14.09)	1.14	
Asynchronous	S2	Normal-high-high	$P(X_{pk} < X \leq X_{pf}, Y \geq Y_{pf}, Z \geq Z_{pf})$	(17.05,26.04)	3.41	85.96
	S3	Low-high-high	$P(X \leq X_{pk}, Y \geq Y_{pf}, Z \geq Z_{pf})$	(15.79,25.34)	0.03	
	S4	High-high-normal	$P(X \geq X_{pf}, Y \geq Y_{pf}, Z_{pk} < Z \leq Z_{pf})$	(16.74,25.41)	2.29	
	S5	Normal-high-normal	$P(X_{pk} < X \leq X_{pf}, Y \geq Y_{pf}, Z_{pk} < Z \leq Z_{pf})$	(16.04,22.51)	3.29	
	S6	Low-high-normal	$P(X \leq X_{pk}, Y \geq Y_{pf}, Z_{pk} < Z \leq Z_{pf})$	(14.79,21.81)	3.80	
	S7	High-high-low	$P(X \geq X_{pf}, Y \geq Y_{pf}, Z \leq Z_{pk})$	(14.77,24.40)	1.38	
	S8	Normal-high-low	$P(X_{pk} < X \leq X_{pf}, Y \geq Y_{pf}, Z \leq Z_{pk})$	(14.08,21.50)	1.99	
	S9	Low-high-low	$P(X \leq X_{pk}, Y \geq Y_{pf}, Z \leq Z_{pk})$	(12.82,20.81)	3.29	
	S10	High-normal-high	$P(X \geq X_{pf}, Y_{pk} < Y \leq Y_{pf}, Z \geq Z_{pf})$	(15.80,24.17)	3.94	
	S11	Normal-normal-high	$P(X_{pk} < X \leq X_{pf}, Y_{pk} < Y \leq Y_{pf}, Z \geq Z_{pf})$	(15.11,21.27)	2.31	
	S12	Low-normal-high	$P(X \leq X_{pk}, Y_{pk} < Y \leq Y_{pf}, Z \geq Z_{pf})$	(13.84,20.58)	3.13	
	S13	High-normal-normal	$P(X \geq X_{pf}, Y_{pk} < Y \leq Y_{pf}, Z_{pk} < Z \leq Z_{pf})$	(14.79,20.65)	1.68	
	S15	Low-normal-normal	$P(X \leq X_{pk}, Y_{pk} < Y \leq Y_{pf}, Z_{pk} < Z \leq Z_{pf})$	(12.84,17.05)	2.28	
	S16	High-normal-low	$P(X \geq X_{pf}, Y_{pk} < Y \leq Y_{pf}, Z \leq Z_{pk})$	(12.82,19.64)	3.07	
	S17	Normal-normal-low	$P(X_{pk} < X \leq X_{pf}, Y_{pk} < Y \leq Y_{pf}, Z \leq Z_{pk})$	(12.12,16.74)	3.03	
	S18	Low-normal-low	$P(X \leq X_{pk}, Y_{pk} < Y \leq Y_{pf}, Z \leq Z_{pk})$	(10.87,16.04)	3.28	
	S19	High-low-high	$P(X \geq X_{pf}, Y \leq Y_{pk}, Z \geq Z_{pf})$	(9.37,22.22)	6.13	
	S20	Normal-low-high	$P(X_{pk} < X \leq X_{pf}, Y \leq Y_{pk}, Z \geq Z_{pf})$	(8.68,19.32)	4.43	
	S21	Low-low-high	$P(X \leq X_{pk}, Y \leq Y_{pk}, Z \geq Z_{pf})$	(7.42,18.63)	3.50	
	S22	High-low-normal	$P(X \geq X_{pf}, Y \leq Y_{pk}, Z_{pk} < Z \leq Z_{pf})$	(8.37,18.70)	13.58	
	S23	Normal-low-normal	$P(X_{pk} < X \leq X_{pf}, Y \leq Y_{pk}, Z_{pk} < Z \leq Z_{pf})$	(7.67,15.80)	2.15	
	S24	Low-low-normal	$P(X \leq X_{pk}, Y \leq Y_{pk}, Z_{pk} < Z \leq Z_{pf})$	(6.42,15.10)	1.05	
	S25	High-low-low	$P(X \geq X_{pf}, Y \leq Y_{pk}, Z \leq Z_{pk})$	(6.40,17.69)	10.81	
	S26	Normal-low-low	$P(X_{pk} < X \leq X_{pf}, Y \leq Y_{pk}, Z \leq Z_{pk})$	(5.70,14.79)	2.11	

735

Figures

Figure 1

Overview of the study area. Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.

Figure 2

The annual data series of Yellow River water, South-to-North Water Transfer and local water in Zhengzhou

Figure 3

Flow Chart

Figure 4

Empirical Distribution and Theoretical Distribution Fitting of Marginal Distribution

Figure 5

Fitting graphs of theoretical joint frequency and empirical joint frequency of each combination under the optimized Copula

Figure 6

Bivariate and trivariate joint probability distribution and contour map

Figure 7

Comparison of the variation of the three water sources

Figure 8

Water shortage of each user under various combinations of wet and dry conditions