Sliding Mode Observer-based Control of Teleoperation System With Uncertain Dynamics and Kinematics

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Abstract This paper concentrates on the control issue of nonlinear teleoperators in the presence of uncertain dynamics and kinematics. An observer-based control framework is introduced to compensate for the unfavorable effects arising from the uncertainties. The employment of the proposed sliding mode observers provide control system with the ability of finite-time estimation errors convergence, upon which, it is demonstrated that the bilateral teleoperators are stable and both of position and velocity tracking can be achieved with uncertain dynamics in joint space. Due to the practical requirement of driving the end-effectors to perform specific tasks, the control law which can ensure position coordination with uncertain dynamics and kinematics in task space is subsequently developed. The Lyapunov method is applied to demonstrate the stability of the closed-loop system. Simulation results are provided to testify the performance of the suggested algorithm.

Keywords Bilateral teleoperators · Dynamic/kinematic uncertainties · Sliding-mode observer

1 Introduction

Teleoperation system constitutes a considerable benchmark problem in remote or complex environments, such as aerospace, drilling, surgical operations, etc [1–7]. Bilateral teleoperation system, as illustrated in Fig. 1, generally consisting of local and remote manipulators and exchanging information via a communication channel, enables the robotic system to accomplish intricate missions precisely and accurately just like human do. Motivated by practical demands and theoretical challenges, the issue of stability analysis and control of teleoperation system has received sustaining interest over the past decades.

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Many remarkable results have been obtained for control of teleoperation system [8–16]. The passivity-based control innovatively proposed in [8] has been the cornerstone of the teleoperation system control, which is further developed in [9] to address the problem of steady-state position as well as force tracking. In [10], position tracking is addressed for teleoperators with a simple PD-like and a scattering-based controller. In an attempt to realize position tracking of bilateral teleoperators with communication delays and without measuring velocity, a second-order dynamical controller is proposed in [11]. In these works, control strategies are usually developed for joint position coordination. From a robot point of view, the missions for bilateral teleoperators are usually specified in task space in practical application. Therefore, it is also imperative to exploit advanced control approaches in the work space on the end-effector. Some attempts have been made to develop the control algorithms for task-space bilateral teleoperators [12–15]. In [13], an adaptive control is designed for nonlinear teleoperators in the presence of actuator uncertainty and quantized input. In [14], a transpose Jacobian control strategy is developed by employing passivity based technique with taking communication time delays into consideration. The control approach introduced for a semi-autonomous teleoperator documented in [15] is capable of ensuring position and velocity tracking between the local and remote robots subjected to dynamic uncertainties and communication delays while the redundancy of the remote manipulator is also addressed. Despite the significant progress of the bilateral control, common assumptions in most of the aforesaid algorithms are that the teleoperator dynamics and kinematics are perfectly known, which reduces the robustness and effectiveness of the controllers since uncertainties may give rise to undesirable performance or unpredictable responses. Therefore, uncertainties including dynamic uncertainties and kinematic uncertainties should be considered in the development of telecontrol schemes.

Kinematic parameters of teleoperation system, which usually characterized by a group of parameters including the length and grasping angle of objects that the manipulators hold, are difficult to be measured accurately and the varied or unknown circumstances may lead to further kinematic uncertainties. In addition, the perfect knowledge of the teleoperator dynamics may be unavailable or obtained costly resulting from the model uncertainties. To remove such restriction, many scholars have committed themselves to the control of robotic system with uncertainties and some pioneering works can be seen in [17–23]. Among the uncertainty attenuation methods, sliding mode control (SMC) has been widely studied due to its robustness to uncertainties [24–31]. To mention a few, in [25], a multiple model-based SMC is proposed to realize tracking control of the robot system with large-scale parametric uncertainties. Combining neural network with SMC, the manipulator system with the constraint uncertainty in dynamics and kinematics is further discussed in [26]. To enhance the efficiency and accuracy of the controllers, an exponential tracking control for manipulators in [27] ensures a globally exponentially stability with uncertain dynamics and kinematics by applying a sliding mode observer. In [28], to eliminate the obstruction of chattering effect caused by the SMC, the time-delay estimation technique is employed to develop an adaptive sliding-mode control (ASMC) scheme, which can be with great potential for the practical application. Moreover, a second-order sliding mode controller is studied in [29] to design a motion control algorithm for robot manipulators. However, most of the aforementioned research is confined to the set-point control or tracking control which is applicable to single robot instead of teleoperation system. Actually, teleoperation control construction is a challenging topic and the major impediment lies in how to regulate the teleoperators without a common set-point or a desired trajectory. Moreover, external forces from the human and the environment contribute to the further difficulty of control design and aforesaid control algorithms are no longer feasible.

To exploit effective control approach of multi-robot system, the adaptive control techniques are used to deal with the uncertainties in [32–36]. In [32], an adaptive task-space synchronization approach is introduced to achieve the weighted average consensus among the uncertain networked robotic systems. In [33], by establishing the adaptive control laws, the stability and position tracking is guaranteed in teleoperation system with uncertain dynamics, kinematics and asymmetric constant communication delays. Based on [5], the adaptive control scheme in [34] is extended to the case that the gravity is non-zero and the errors of tracking positions and velocities are utilized to construct the adaptation laws for nonlinear teleoperators. The authors in [35] address the issue of nonlinear teleoperation system which is both dynamically and kinematically uncertain and the exact knowledge of the human operator and the environment is also unavailable. It should be underscored that the existing adaptive controllers are generally developed based on linear parameterization properties, which may lead to massive and tedious computation to obtain a regressor matrix. This would undermine the practicability and validity of the controller. On the other hand, in SMC or ASMC-based control schemes, most approaches only guarantee the estimation errors of uncertainties converge to zero asymptotically, which indicates that the estimation convergence time is not minimized. To obtain high-efficiency control in teleoperation system, how to avoid the computational complexity caused by linear parameterization properties and how to tackle uncertainties with high-efficiency still remain an open problem.

In this paper, the formulation of a sliding mode observer-based telecontrol approach oriented to deal with the dynamic and kinematic uncertainties is addressed. The main contributions of this article are threefold.

1) To deal with the constraint uncertainties of dynamics and kinematics of the teleoperation system, two sliding mode observers are proposed. One observer is employed to cope with the uncertain dynamics and the other is to tackle the uncertain kinematics. With the proposed observers, the controllers can be extended from joint space to task space with faciity, which are derived by employing the estimation signals.

2) Compared with the adaptive controllers in [5, 32–36], the major superiority of the proposed control scheme lies in the computational-reduction operation. The linear parameterization properties are no more required and complicated computation for obtaining the regressor matrix can be avoided, extending the appli-

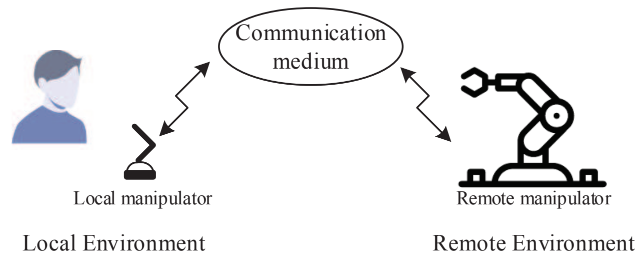


Fig. 1: Bilateral teleoperation.

cation of the teleoperation system to a more practical field.

3) Take the uncertain dynamics and kinematics into consideration simultaneously in teleoperation system, in comparison with the estimation scheme in [37–39], the suggested sliding mode observers in this study can guarantee finite-time estimation. Moreover, the proposed controllers ensure that the tracking errors between local and remote robots converge to the arbitrary set close to the origin within finite time, with the faster convergence time realized.

2 Preliminaries

Let $\boldsymbol{v}$ denote a vector that $\boldsymbol{v} = [v_1 \ v_2 \ \cdots \ v_n]^T \in R^n$, $\|\cdot\|$ represents its standard Euclidean norm or induced norm. Denote $\alpha \in R$ as a positive scalar, $\lfloor \boldsymbol{v} \rceil^\alpha = [|v_1|^\alpha \mathrm{sgn}(v_1) \ |v_2|^\alpha \mathrm{sgn}(v_2) \ \cdots \ |v_n|^\alpha \mathrm{sgn}(v_n)]^T \in R^n$ and $\mathrm{sgn}(\boldsymbol{v}) = [\mathrm{sgn}(v_1) \ \mathrm{sgn}(v_2) \ \cdots \ \mathrm{sgn}(v_n)]^T \in R^n$, where $\mathrm{sgn}(\cdot)$ represents the signum function. For the sake of clarity of the equations in this paper, the argument of time-dependent signals will be omitted sometimes, e.g. $\dot{q} \equiv \dot{q}(t)$.

Lemma 1 *Let $\gamma_1, \gamma_2, ..., \gamma_n$ and ρ be positive constants with $0 < \rho < 2$, then the following inequality holds [40]:*

$$(\gamma_1^2 + \gamma_2^2 + ... + \gamma_n^2)^\rho \leq (\gamma_1^\rho + \gamma_2^\rho + ... + \gamma_n^\rho)^2 \tag{1}$$

2.1 Dynamic Model

Consider an n-link robot system with revolute joints, the Euler-Lagrange equation of the system can be represented as

$$H(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) = \tau(t) \tag{2}$$

where $q(t) \in R^{n\times 1}$ and $\dot{q}(t) \in R^{n\times 1}$ are the vectors of joint position and joint velocity, respectively. $H(q) \in R^{n\times n}$ is the inertia matrix, $C(q, \dot{q}) \in R^{n\times n}$ is the Coriolis and centrifugal matrix, $g(q) \in R^{n\times 1}$ denotes the gravitational torque, and $\tau(t) \in R^{n\times 1}$ denotes the control torque. It should be noted that the robot system has the following dynamical properties.

Property 1 The inertia matrix $H(q)$ is symmetric and uniformly positive definite. Moreover, $H(q)$ is bounded by

$$c_1\|x\|^2 \leq x^T H(q) x \leq c_2\|x\|^2, \qquad \forall x \in R^n \tag{3}$$

where $c_1 \in R$ and $c_2 \in R$ are positive scalars.

Property 2 The Coriolis and centrifugal matrix $C(q, \dot{q})$ can be properly selected to ensure that $\dot{H}(q) - 2C(q, \dot{q})$ be skew-symmetric, where $\dot{H}(q)$ denotes the time derivative of $H(q)$.

2.2 Problem Formulation in Joint Space

The local and remote manipulators are described as a pair of n-link robots in joint space. The dynamical equations including nonlinear dynamics as well as the human operators and environment interactions are represented as

$$\begin{aligned} H_a(q_a)\ddot{q}_a + C_a(q_a, \dot{q}_a)\dot{q}_a + g_a(q_a) + F_{c,a}(\dot{q}_a) + U_a(q_a) \\ = \tau_a + F_a \end{aligned} \tag{4}$$

$$\begin{aligned} H_b(q_b)\ddot{q}_b + C_b(q_b, \dot{q}_b)\dot{q}_b + g_b(q_b) + F_{c,b}(\dot{q}_b) + U_b(q_b) \\ = \tau_b - F_b \end{aligned} \tag{5}$$

where the subscript $i = \{a, b\}$ denotes the local manipulator and the remote manipulator, respectively. $U_i(q_i)$ are the total uncertain dynamics, $F_{c,i}(\dot{q}_i)$ are viscous/static friction torques and $F_i(t) \in R^{n\times 1}$ are the forces applied on the manipulators.

Remark 1 The mathematical models in (4), (5) describe a class of teleoperation systems considering uncertain dynamics. The total dynamical uncertainties $U_i(q_i)$ considered in this paper are comprised of uncertain inertia $\Delta H_i(q_i)$, uncertain Coriolis-centrifugal matrix $\Delta C_i(q_i, \dot{q}_i)$, and the uncertain time-varying disturbances $d_i(t)$, namely, $U_i(q_i) = \Delta H_i(q_i)\ddot{q}_i + \Delta C_i(q_i, \dot{q}_i)\dot{q}_i - d_i(t)$.

Observing the dynamic models in (4), (5), we can denote the gravitational torques $g_i(q_i)$, friction torques $F_{c,i}(\dot{q}_i)$ and uncertain dynamics $U_i(q_i)$ as the lumped unknown torques $\Theta_i(q_i)$, i.e., $g_i(q_i) + U_i(q_i) + F_{c,i}(\dot{q}_i) = \Theta_i(q_i)$. In order to exploit the estimation laws of the unknown dynamics $\Theta_i(q_i)$, we make the following assumption to discuss the upper bounds of $\Theta_i(q_i)$.

Assumption 21 *For any manipulator, we can obtain $\Theta_i(q_i)$ are bounded for all q_i, and $\|g_i(q_i) + U_i(q_i) + F_{c,i}(\dot{q}_i)\| = \|\Theta_i(q_i)\| \leq c_{3,i}$, where $c_{3,i} \in R$ are positive constants.*

In this work, the tracking errors between the local and the remote robots are denoted as

$$e_a(t) = q_b(t) - q_a(t); \quad e_b(t) = q_a(t) - q_b(t) \tag{6}$$

Similar to [33], the state synchronization of teleoperation system in joint space is defined as

$$\lim_{t\to\infty} \|e_a(t)\| = \lim_{t\to\infty} \|e_b(t)\| = 0 \tag{7}$$

$$\lim_{t\to\infty} \|\dot{e}_a(t)\| = \lim_{t\to\infty} \|\dot{e}_b(t)\| = 0 \tag{8}$$

2.3 Problem Formulation in Task Space

Without loss of generality, the local and remote manipulators are considered to be robotic manipulators with identical degree of freedom. Their corresponding nonlinear dynamics in task space are given by

$$H_a(q_a)\ddot{q}_a + C_a(q_a, \dot{q}_a)\dot{q}_a + g_a(q_a) + F_{c,a}(\dot{q}_a) + U_a(q_a) = \tau_a^d + J_a^T(q_a)f_a \tag{9}$$

$$H_b(q_b)\ddot{q}_b + C_b(q_b, \dot{q}_b)\dot{q}_b + g_b(q_b) + F_{c,b}(\dot{q}_b) + U_b(q_b) = \tau_b^d - J_b^T(q_b)f_b \tag{10}$$

where $f_i(t) \in R^n$ denote the forces from the human and the environment, and $J_i(q_i) \in R^{m\times n}$ are the Jacobian matrices. Generally speaking, the external forces are exerted on the end-effector of the manipulators, and $J_i^T(q_i)f_i(t)$ denote the transformation of the external forces from task space to joint space by the Jacobian matrices.

Let $X_i(t) \in R^m (m \le n)$ be the generalized end-effector position vectors in task space,

$$\begin{cases} X_a = r_a(q_a) & \dot{X}_a = J_a(q_a)\dot{q}_a \\ X_b = r_b(q_b) & \dot{X}_b = J_b(q_b)\dot{q}_b \end{cases} \tag{11}$$

where the subscript $i = \{a, b\}$ denotes the local manipulator and remote manipulator, respectively. $r_i(q_i) \in R^m$ denotes the nonlinear mapping reflecting the relationship from joint space to work space. Note that the parameters of Jacobian matrices are unknown when there exist the kinematic uncertainties in teleoperation system.

Suppose that the nominal Jacobian matrices and the uncertain Jacobian matrices are denoted as $\bar{J}_i(q_i) \in R^{m\times n}$ and $\Delta J_i(q_i) \in R^{m\times n}$, respectively. Hence, $J_i(q_i)$ can be divided into the nominal term and the uncertain term

$$\bar{J}_i(q_i) + \Delta J_i(q_i) = J_i(q_i) \tag{12}$$

where $\bar{J}_i(q_i)$ are known and $\Delta J_i(q_i)$ are uncertain. To facilitate the work on controller design, we make an assumption of $J_i(q_i)$.

Assumption 22 *For the teleoperation system, the singularities associated with the nominal Jacobian matrices $\bar{J}_i(q_i)$ are always avoided. Meanwhile, the uncertain kinematics $\Delta J_i(q_i)$ will always have the upper bounds denoted as the positive constants $c_{4,i}$, i.e., $\|\Delta J_i(q_i)\| \le c_{4,i}$.*

The position tracking errors between the local manipulator and remote manipulator are defined as

$$\varepsilon_a(t) = X_b(t) - X_a(t); \quad \varepsilon_b(t) = X_a(t) - X_b(t) \tag{13}$$

3 Observer-based bilateral controller Design

In this section, we investigate the observer-based control approach for achieving tracking control of teleoperator system with uncertain kinematics and dynamics. Two controllers are designed for teleoperation system in joint space and task space, respectively. First, an observer-based estimation law is developed to deal with the uncertain dynamics and the control law is designed in joint space. Then, considering uncertain kinematics, another observer is designed to realize tracking control of teleoperator in task space. With application of the observers and control laws, the tracking errors between local manipulator and remote manipulator will be asymptotically stabilized.

3.1 Control Design in Joint Space

First, new variables are defined as $z_{1,i} = q_i, z_{2,i} = \dot{q}_i$, then the dynamic models (4), (5) can be rearranged as

$$\dot{z}_{1,i} = z_{2,i} \tag{14}$$

$$H_i(z_{1,i})\dot{z}_{2,i} = \tau_i - C_i(z_{1,i}, z_{2,i})z_{2,i} - \Theta_i(q_i) \tag{15}$$

where $z_{1,i} = [z_{1,a}, z_{1,b}]^T, z_{i,2} = [z_{2,a}, z_{2,b}]^T$.

The sliding mode manifold is defined as

$$\chi_i = z_{2,i} - \delta_i \tag{16}$$

where δ_i is constructed as

$$\dot{\delta}_i = H_i^{-1}(z_{1,i})(K_i\chi_i + \tau_i - C_i(z_{1,i}, z_{2,i})\delta_i + b_{1,i}\mathrm{sgn}(\chi_i) + b_{2,i}\chi_i^{p_1/q_1}) \tag{17}$$

where $\mathrm{sgn}(\cdot)$ is the signum function, $K_i, b_{1,i}, b_{2,i}$ are positive constants, and p_1, q_1 are positive odd integers with $p_1 < q_1$.

In order to estimate the dynamic uncertainties $\Theta_i(q_i)$, the nonlinear estimation laws based on a nonlinear observer technique are proposed as

$$\Theta_{r,i} = -K_i\chi_i - C_i(z_{1,i}, z_{2,i})\chi_i - b_{1,i}\mathrm{sgn}(\chi_i) - b_{2,i}\chi_i^{p_1/q_1} \tag{18}$$

Next, a continuous and smooth function $\phi(x)$ is proposed as

$$\phi(x) = \begin{cases} |x|^{\sigma}\mathrm{sgn}(x), & \text{if } |x| \geq \lambda \\ \iota_1 x + \iota_2 x^2 \mathrm{sgn}(x), & \text{if } |x| < \lambda \end{cases} \tag{19}$$

where σ, λ are positive constants with $\sigma < 1$, $\iota_1 = (2-\sigma)\lambda^{\sigma-1}$ and $\iota_2 = (\sigma-1)\lambda^{\sigma-2}$.

The vector $\Phi(e_i) \in R^n$ is defined as

$$\Phi(e_i) = \left[\phi(e_{i1}), \phi(e_{i2}), ..., \phi(e_{in})\right]^T \tag{20}$$

where $e_{ij}(j = 1, 2, ..., n)$ denotes the jth element of the vector e_i.

With the function $\Phi(x)$ and setting $\sigma = p_2/q_2$, the virtual inputs are defined as

$$\hat{\dot{q}}_a = \dot{q}_b(t) + k_{1,a}\Phi(e_a) + (k_{2,a}+1)s_a \tag{21}$$

$$\hat{\dot{q}}_b = \dot{q}_a(t) + k_{1,b}\Phi(e_b) + (k_{2,b}+1)s_b \tag{22}$$

where $k_{1,i}, k_{2,i}$ are positive constants, p_2, q_2 are positive odd integers with $p_2 < q_2$, and we define $s_i(t)$ as

$$s_a = \hat{\dot{q}}_a - \dot{q}_a, \quad s_b = \hat{\dot{q}}_b - \dot{q}_b \tag{23}$$

Then, the observer-based controllers are exploited as

$$\begin{aligned} \tau_i =& H_i(q_i)\frac{d(\hat{\dot{q}}_i)}{dt} + C_i(q_i, \dot{q}_i)\hat{\dot{q}}_i + k_{3,i}\lfloor s_i\rceil^{p_2/q_2} \\ &+ \Theta_{r,i} - k_{2,i}e_i \end{aligned} \tag{24}$$

where $k_{3,i}$ are positive constants.

Remark 2 The function $\phi(x)$ plays an essential role in avoiding the singularity problem. To explain how the function $\phi(x)$ works, we may define the virtual inputs without the employment of function $\phi(x)$ as:

$$\hat{\dot{q}}_{ar} = \dot{q}_b(t) + k_{1,a}\lfloor e_a\rceil^{p_2/q_2} + (k_{2,a}+1)s_a \tag{25}$$

$$\hat{\dot{q}}_{br} = \dot{q}_a(t) + k_{1,b}\lfloor e_b\rceil^{p_2/q_2} + (k_{2,b}+1)s_b \tag{26}$$

Differentiating $\hat{\dot{q}}_{ir}$ with respect to time, it follows that $d(\hat{\dot{q}}_{ir})/dt$ may produce a term related to $\mathrm{diag}\big(|e_i|^{\frac{p_2}{q_2}-1}\big)$ from $\lfloor e_i\rceil^{\frac{p_2}{q_2}}$, where $\mathrm{diag}\big(|e_i|^{\frac{p_2}{q_2}-1}\big)$ denotes a diagonal matrix as:

$$\mathrm{diag}\big(|e_i|^{\frac{p_2}{q_2}-1}\big) = \mathrm{diag}\big(|e_{i1}|^{\frac{p_2}{q_2}-1}, |e_{i2}|^{\frac{p_2}{q_2}-1}, ..., |e_{in}|^{\frac{p_2}{q_2}-1}\big)$$

where $e_{ij}(j = 1, 2, ..., n)$ denotes the jth element of the vector e_i. Since $d(\hat{\dot{q}}_{ir})/dt$ contains a negative fractional power $\frac{p_2}{q_2} - 1$, this may lead to the singularity problem [41] for $\dot{e}_i \neq 0$ when $e_i = 0$. To remove the singularity of $d(\hat{\dot{q}}_{ir})/dt$, the function $\phi(x)$ is then introduced in (19). The derivative of $\phi(x)$ with respect to x can be obtained as:

$$s^{\sigma}(x) = \begin{cases} \sigma|x|^{\sigma-1}, & \text{if } |x| \geq \lambda \\ \iota_1 + 2\iota_2|x|, & \text{if } |x| < \lambda \end{cases} \tag{27}$$

As $s^{\sigma}(x, \lambda^{+}) = \sigma\lambda^{\sigma-1}$ and $s^{\sigma}(x, \lambda^{-}) = \iota_1 + 2\iota_2|\lambda| = \sigma\lambda^{\sigma-1}$, we can learn that the function $\phi(x)$ is continuous and smooth. Therefore, the replacement of $\lfloor e_i\rceil^{p_2/q_2}$ by the function $\phi(e_i)$ can make the control free of singularity.

Theorem 1 *For a joint-space teleoperator with the transformed systems (14), (15) without external forces such that $F_a = F_b = 0$, the estimation laws (18) can exactly estimate the dynamic uncertainties $\Theta_i(q_i)$ and the corresponding estimation errors $\Theta_{e,i}(q_i) = \Theta_{r,i} - \Theta_i(q_i)$ converge to zero in finite time. Additionally, with the observer-based controllers (24), the teleoperation system (4), (5) achieve the position and velocity tracking with the tracking errors converge to the arbitrary set close to the origin within finite time.*

Proof Consider the Lyapunov function candidate for the transformed system (14), (15)

$$V_1(t) = \frac{1}{2}\sum_{i\in\{a,b\}} \chi_i^T H_i(z_{1,i})\chi_i \tag{28}$$

The derivative of $V_1(t)$ is given by

$$\dot{V}_1(t) = \sum_{i\in\{a,b\}} \{\frac{1}{2}\chi_i^T \dot{H}_i(z_{1,i})\chi_i + \chi_i^T H_i(z_{1,i})(\dot{z}_{2,i} - \dot{\delta}_i)\} \tag{29}$$

Using the skew symmetry property of $(1/2)\dot{H}_i(z_{1,i}) - C_i(z_{1,i}, z_{2,i})$ and substituting equation (15) and (17) into $\dot{V}_1(t)$, the equation (29) follows that

$$\begin{aligned} \dot{V}_1(t) =& \sum_{i\in\{a,b\}} \{\frac{1}{2}\chi_i^T \dot{H}_i(z_{1,i})\chi_i + \chi_i^T\big(-\Theta_i(q_i) \\ &- C_i(z_{1,i}, z_{2,i})z_{2,i} + C_i(z_{1,i}, z_{2,i})\delta_i \\ &- b_{1,i}\mathrm{sgn}(\chi_i) - b_{2,i}\chi_i^{p_1/q_1} - K_i\chi_i\big)\} \\ =& \sum_{i\in\{a,b\}} \{\chi_i^T\big(-\Theta_i(q_i) - b_{1,i}\mathrm{sgn}(\chi_i) - b_{2,i}\chi_i^{p_1/q_1} \\ &- K_i\chi_i\big)\} \end{aligned} \tag{30}$$

Employing the inequality $\|\Theta_i(q_i)\| \leq c_{3,i}$ and Property 2.1, choosing $b_{1,i} > c_{3,i}$, we can obtain

$$\begin{aligned} \dot{V}_1(t) \leq& \sum_{i\in\{a,b\}} \{c_{3,i}\|\chi_i\| - b_{1,i}\|\chi_i\| - b_{2,i}\chi_i^T\chi_i^{p_1/q_1} \\ &- K_i\|\chi_i\|^2\} \\ \leq& \sum_{i\in\{a,b\}} \{-b_{2,i}\chi_i^T\chi_i^{p_1/q_1} - K_i\|\chi_i\|^2\} \\ \leq& \sum_{i\in\{a,b\}} \{-\frac{K_i}{c_{2,i}}V_1(t) - b_{2,i}\left(\frac{2}{c_{2,i}}\right)^{\frac{p_1+q_1}{2q_1}} V_1^{\frac{p_1+q_1}{2q_1}}(t)\} \end{aligned} \tag{31}$$

As p_1 and q_1 are positive odd, we can infer that p_1+q_1 are positive even such that $V_1^{\frac{p_1+q_1}{2q_1}}(t) \geq 0$. Then, solving (31) yields

$$\begin{aligned}&\ln\left\{\sum_{i\in\{a,b\}}\left(b_{2,i}\left(\frac{2}{c_{2,i}}\right)^{\frac{p_1+q_1}{2q_1}}+\frac{K_i}{c_{2,i}}V_1^{\frac{q_1-p_1}{2q_1}}(t)\right)\right\} \\ \leq&\ln\left\{\sum_{i\in\{a,b\}}\left(b_{2,i}\left(\frac{2}{c_{2,i}}\right)^{\frac{p_1+q_1}{2q_1}}+\frac{K_i}{c_{2,i}}V_1^{\frac{q_1-p_1}{2q_1}}(0)\right)\right\} \\ &-\sum_{i\in\{a,b\}}\frac{K_i(q_1-p_1)}{2q_1c_{2,i}}t\end{aligned} \tag{32}$$

It is thus obtained from (32) that $V_1(t)\equiv 0$ for all $t\geq T_f$, where

$$T_f = \sum_{i\in\{a,b\}}\frac{2q_1c_{2,i}}{K_i(q_1-p_1)}\ln\frac{\sum_{i\in\{a,b\}}b_{2,i}\left(\frac{2}{c_{2,i}}\right)^{\frac{p_1+q_1}{2q_1}}+V_1^{\frac{q_1-p_1}{2q_1}}(0)}{\sum_{i\in\{a,b\}}b_{2,i}\left(\frac{2}{c_{2,i}}\right)^{\frac{p_1+q_1}{2q_1}}} \tag{33}$$

Therefore we can get $\chi_i\equiv 0$ by the time $t=T_f$, and it leads to $\dot{\chi}_i\equiv 0$ for all $t\geq T_f$. Considering the estimation errors as $\Theta_{e,i}(q_i)=\Theta_{r,i}-\Theta_i(q_i)$ and substituting (15) as well as (18) results in

$$\Theta_{e,i}(q_i) = H_i(z_{1,i})\dot{\chi}_i \tag{34}$$

It means $\Theta_{e,i}(q_i)\equiv 0$ for $t\geq T_f$. At this point, the unknown dynamics $\Theta_i(q_i)$ will be estimated by $\Theta_{r,i}$ in finite time.

Next, we choose the Lyapunov function candidate for the teleoperation system (4), (5) as

$$V_2(t) = \frac{1}{2}\sum_{i\in\{a,b\}}\left\{e_i^Te_i+s_i^TH_i(q_i)s_i\right\} \tag{35}$$

The derivative of $V_2(t)$ yields as

$$\begin{aligned}\dot{V}_2(t) =& s_a^TH_a(q_a)\dot{s}_a+\frac{1}{2}s_a^T\dot{H}_a(q_a)s_a \\ &+s_b^TH_b(q_b)\dot{s}_b+\frac{1}{2}s_b^T\dot{H}_b(q_b)s_b \\ &+e_b^T\{\dot{q}_a(t)-\dot{q}_b(t)\}+e_a^T\{\dot{q}_b(t)-\dot{q}_a(t)\}\end{aligned} \tag{36}$$

Substituting (21)-(22) into $\dot{V}_2(t)$ and two cases are discussed due to the $\phi^\sigma(x)$ defined in (19):

1) For the case of $|e_{ij}|\geq\lambda$, we can obtain that $\Phi(e_i)=\lfloor e_i\rceil^{p_2/q_2}$, with the designed observer-based control laws (24), using the skew symmetry property of $(1/2)\dot{H}_i(q_i)-C_i(q_i,\dot{q}_i)$ and substituting equations (4), (5), (18) into $\dot{V}_2(t)$, we get

$$\begin{aligned}\dot{V}_2(t) =& -k_{1,a}e_a^T\lfloor e_a\rceil^{p_2/q_2}-k_{1,b}e_b^T\lfloor e_b\rceil^{p_2/q_2} \\ &-s_a^T\left\{k_{3,a}\lfloor s_a\rceil^{p_2/q_2}+\Theta_{r,a}-\Theta_a(q_a)\right\} \\ &-s_b^T\left\{k_{3,b}\lfloor s_b\rceil^{p_2/q_2}+\Theta_{r,b}-\Theta_b(q_b)\right\}\end{aligned} \tag{37}$$

As $\Theta_i(q_i)$ can be exactly estimated by term $\Theta_{r,i}$ and the estimation errors can be zero for all $t\geq T_f$, which means $\Theta_i(q_i)\equiv\Theta_{r,i}$ for all $t\geq T_f$. Hence, equation (37) can be written as

$$\begin{aligned}\dot{V}_2(t)\leq& -k_{1,a}\lfloor e_a\rceil^{\frac{p_2+q_2}{q_2}}-k_{1,b}\lfloor e_b\rceil^{\frac{p_2+q_2}{q_2}} \\ &-k_{3,a}\lfloor s_a\rceil^{\frac{p_2+q_2}{q_2}}-k_{3,b}\lfloor s_b\rceil^{\frac{p_2+q_2}{q_2}} \\ \leq& -\min\{k_{1,i},k_{3,i}\}\sum_{i\in\{a,b\}}\left(\|e_i\|^{\frac{p_2+q_2}{q_2}}+\|s_i\|^{\frac{p_2+q_2}{q_2}}\right)\end{aligned} \tag{38}$$

for $t\geq T_f$.

Employing Lemma 2.1 yields

$$\|e_i\|^{\frac{p_2+q_2}{q_2}}+\|s_i\|^{\frac{p_2+q_2}{q_2}}\geq\left(\|e_i\|^2+\|s_i\|^2\right)^{\frac{p_2+q_2}{2q_2}} \tag{39}$$

Thus, we have

$$\dot{V}_2(t)\leq -\min\{k_{1,i},k_{3,i}\}\sum_{i\in\{a,b\}}\left(\|e_i\|^2+\|s_i\|^2\right)^{\frac{p_2+q_2}{2q_2}} \tag{40}$$

for $t\geq T_f$.

By virtue of (35) and utilizing Property 2.1, we obtain $V_2(t)$ has the upper bound as

$$V_2(t)\leq\frac{1}{2}\sum_{i\in\{a,b\}}\left\{\max\{1,c_{1,i}\}\left(\|e_i\|^2+\|s_i\|^2\right)\right\} \tag{41}$$

Combining (40) and (41) leads to

$$\left(\|e_i\|^2+\|s_i\|^2\right)^{\frac{p_2+q_2}{2q_2}}\geq\left(\frac{2}{\max\{1,c_{1,i}\}}\right)^{\frac{p_2+q_2}{2q_2}}V_2^{\frac{p_2+q_2}{2q_2}}(t) \tag{42}$$

As a result, we can get

$$\dot{V}_2(t)\leq -L_1V_2^{\frac{p_2+q_2}{2q_2}}(t),\quad |e_{ij}|\geq\lambda,\quad t\geq T_f \tag{43}$$

where $L_1=\sum_{i\in\{a,b\}}\{\min\{k_{1,i},k_{3,i}\}\left(\frac{2}{\max\{1,c_{1,i}\}}\right)^{\frac{p_2+q_2}{2q_2}}\}$. Solving (43) leads to $V_2(t)\equiv 0$ for all $t\geq T_r$ with T_r chosen as

$$T_r = T_f+\frac{q_2V_2^{\frac{q_2-p_2}{q_2}}(T_f)}{L_1(q_2-p_2)} \tag{44}$$

2) For the case of $|e_{ij}| < \lambda$ and after finite time $T_f, \dot{V}_2(t)$ can be written as

$$\begin{aligned}\dot{V}_2(t) \leq & - k_{1,a}\sum_{j=1}^{n}(\iota_1 e_{aj}^2 + \iota_2|e_{aj}|^3) - k_{3,a}\sum_{j=1}^{n}|s_{aj}|^{\frac{p_2+q_2}{q_2}} \\ & - k_{1,b}\sum_{j=1}^{n}(\iota_1 e_{bj}^2 + \iota_2|e_{bj}|^3) - k_{3,b}\sum_{j=1}^{n}|s_{bj}|^{\frac{p_2+q_2}{q_2}}\end{aligned} \tag{45}$$

where s_{ij} represents the jth element of the vector s_i.

Here, we can reach a conclusion that the dynamic uncertainties can be tackled within finite time. In addition, the tracking control of position as well as velocity for teleoperation system is realized, which indicates the high performance and accuracy of bilateral teleoperation.

Remark 3 As discussed in [32–36], linearity-in-parameters property is a basic operation to ensure decomposition of the coupling terms into a form of regressor matrix and parameter vector, which may lead to the superfluous occupation of the limited computational resources. Moreover, it is sometimes unfulfillable to compute and obtain this regressor matrix. For the sake of avoiding this drawback, the sliding mode observers are employed to deal with the uncertain dynamics $\Theta_i(q_i)$ in teleoperation system. From the proof of Theorem 3.1, we can obtain that the uncertain dynamics $\Theta_i(q_i)$ can be precisely estimated by (18) within finite time $t = T_f$ in joint space. The estimation laws (18) are not based on the linear parameterization properties and no efforts will be made to compute the regressor matrix.

Remark 4 The proposed observer-based controllers (24) guarantee the convergence of position and velocity tracking errors between the local and the remote robots in joint space, which indicates the state synchronization for teleoperation system is realized. When tracking errors satisfy that $|e_{ij}| \geq \lambda$, the controllers ensure that $|e_{1,ij}|$ converge to an arbitrary set centered on the origin (whose radius is denoted by λ) within finite time T_r. When the absolute value of tracking errors reduce to λ, then they go to zero asymptotically. To some extent, the settling time of the teleoperation system can be shortened to reach better performance.

3.2 Control Design in Task Space

In this section, we extend the observer-based architecture to ensure position coordination for teleoperation system in task space. For the sake of realizing the position tracking between the remote and the local manipulators, we develop an observer-based controller with the consideration of uncertain dynamics and kinematics. The stability of the closed-loop system is then demonstrated.

To deal with the kinematic uncertainties, consider the following observers:

$$\dot{\widehat{X}}_i(t) = \bar{J}_i(q_i)\dot{q}_i - c_{4,i}\|\dot{q}_i\|\mathrm{sgn}(\widetilde{X}_i) - k_{4,i}\lfloor\widetilde{X}_i\rceil^{p_3/q_3} \tag{46}$$

where $k_{4,i}$ are positive constants and p_3, q_3 are positive odd integers with $p_3 < q_3$. $\widetilde{X}_i(t)$ are defined as

$$\widetilde{X}_i(t) = \widehat{X}_i(t) - X_i(t) \tag{47}$$

where $\widehat{X}_i(t)$ are the estimates of $X_i(t)$.

With the function $\Phi(x)$ defined in (19) and setting $\sigma = p_4/q_4$, the virtual inputs are defined as

$$\hat{\dot{q}}_a^d = [\bar{J}_a(q_a)]^{-1}\{\dot{\widehat{X}}_b + k_{4,a}\Phi(\varepsilon_a) - c_{4,a}\|\dot{q}_a\|\mathrm{sgn}(\widetilde{X}_a)\} \tag{48}$$

$$\hat{\dot{q}}_b^d = [\bar{J}_b(q_b)]^{-1}\{\dot{\widehat{X}}_a + k_{4,b}\Phi(\varepsilon_b) - c_{4,b}\|\dot{q}_b\|\mathrm{sgn}(\widetilde{X}_b)\} \tag{49}$$

where p_4, q_4 are positive integers with $p_4 < q_4$. S_a and S_b are defined as

$$S_a = \dot{q}_a - \hat{\dot{q}}_a^d, \quad S_b = \dot{q}_b - \hat{\dot{q}}_b^d \tag{50}$$

In the light of Theorem 3.1 and the observer (46), the task-space control laws for teleoperation system are proposed as

$$\begin{aligned}\tau_i^d = & - k_{5,i}\lfloor S_i\rceil^{p_4/q_4} + \Theta_{r,i} + H_i(q_i)\frac{d(\hat{\dot{q}}_i^d)}{dt} + C_i(q_i,\dot{q}_i)\hat{\dot{q}}_i^d \\ & + \bar{J}_i(q_i)\varepsilon_i\end{aligned} \tag{51}$$

where $k_{5,i}$ are positive constants.

Theorem 2 *Consider the teleoperation system described by dynamics (9), (10) and kinematics (11) without external forces, employing the estimation laws (18) and sliding-mode observers (46), with the application of the controllers (51), the dynamic uncertainties and kinematic uncertainties can be tackled within finite time. Further, the task-space position tracking errors between the local manipulator and remote manipulator will asymptotically stabilized.*

Proof First, a Lapunov function candidate is chosen as

$$V_3(t) = \frac{1}{2}\sum_{i\in\{a,b\}}\widetilde{X}_i^T(t)\widetilde{X}_i(t) \tag{52}$$

Its time derivative can be obtained as

$$\begin{aligned}\dot{V}_3(t) = &\sum_{i\in\{a,b\}} \widetilde{X}_i^T(t)\{-c_{4,i}\|\dot{q}_i\|\mathrm{sgn}(\widetilde{X}_i)\\ &- k_{4,i}\lfloor \widetilde{X}_i\rceil^{p_3/q_3} + \Delta J_i(q_i)\dot{q}_i\}\\ \le &\sum_{i\in\{a,b\}} \{-c_{4,i}\|\dot{q}_i\|\|\widetilde{X}_i\| - k_{4,i}\|\widetilde{X}_i\|^{\frac{p_3+q_3}{q_3}}\\ &+ \|\Delta J_i(q_i)\|\|\dot{q}_i\|\|\widetilde{X}_i\|\}\end{aligned} \tag{53}$$

Due to $\|\Delta J_i(q_i)\| \le c_{4,i}$ in Assumption 3.1, it implies that

$$\begin{aligned}\dot{V}_3(t) \le &- \sum_{i\in\{a,b\}} k_{4,i}\|\widetilde{X}_i\|^{\frac{p_3+q_3}{q_3}}\\ = &- \sum_{i\in\{a,b\}} k_{4,i}2^{\frac{p_3+q_3}{2q_3}} V_3^{\frac{p_3+q_3}{2q_3}}(t)\end{aligned} \tag{54}$$

Since p_3 and q_3 are positive odd, leading to $V_3^{\frac{p_3+q_3}{q_3}}(t) \ge 0$. Solving (54), one obtains

$$\frac{2q_3}{q_3-p_3}\left\{V_3^{\frac{q_3-p_3}{2q_3}}(t) - V_3^{\frac{q_3-p_3}{2q_3}}(0)\right\} \le - \sum_{i\in\{a,b\}} k_{4,i}2^{\frac{p_3+q_3}{2q_3}} t \tag{55}$$

which implies that $V_3(t) \equiv 0$ for all $t \ge T_c$, where

$$T_c = \frac{\sum_{i\in\{a,b\}} q_3\|\widetilde{X}_i(0)\|^{\frac{q_3-p_3}{q_3}}}{\sum_{i\in\{a,b\}} k_{4,i}(q_3-p_3)} \tag{56}$$

Hence, it is shown that the observers (46) can ensure that $\Delta J_i(q_i)$ will be estimated within finite time in presence of the kinematic uncertainties in teleoperation system.

In what follows, the validity and good performance of the controllers (51) will be demonstrated theoretically. Let us define a positive definite Lyapunov function candidate for the teleoperation system (9), (10)

$$V_4(t) = \frac{1}{2}\sum_{i\in\{a,b\}} \{\varepsilon_i^T\varepsilon_i + S_i^T H_i(q_i) S_i\} \tag{57}$$

Taking the time derivative of $V_4(t)$, following the proof of Theorem 3.1 and substituting (51), applying Property 2.2 and substituting (48)-(49) yields

$$\begin{aligned}\dot{V}_4(t) =&\varepsilon_a^T\{\dot{\widetilde{X}}_b(t) - k_{4,a}\Phi(\varepsilon_a) - \Delta J_a(q_a)\dot{q}_a\\ &+ [-c_{4,a}\|\dot{q}_a\|\mathrm{sgn}(\widetilde{X}_a)]_{eq}\}\\ &\varepsilon_b^T\{\dot{\widetilde{X}}_a(t) - k_{4,b}\Phi(\varepsilon_b) - \Delta J_b(q_b)\dot{q}_b\\ &+ [-c_{4,b}\|\dot{q}_b\|\mathrm{sgn}(\widetilde{X}_b)]_{eq}\}\\ &+ S_a^T\{-k_{5,a}\lfloor S_a\rceil^{p_4/q_4} + \Theta_{r,a} - \Theta_a(q_a)\}\\ &+ S_b^T\{-k_{5,b}\lfloor S_b\rceil^{p_4/q_4} + \Theta_{r,b} - \Theta_b(q_b)\}\end{aligned} \tag{58}$$

where $[-c_{4,i}\|\dot{q}_i\|\mathrm{sgn}(\widetilde{X}_i)]_{eq}$ denote the equivalent value on the sliding surface $\widetilde{X}_i = \dot{\widetilde{X}}_i = 0$.

From the definition of $\Phi^\sigma(x)$ in (19), the following two cases are discussed:

1) For the case of $|\varepsilon_{ij}| \ge \lambda$, we can obtain that $\Phi(\varepsilon_i) = \lfloor\varepsilon_i\rceil^{p_4/q_4}$, thus

$$\begin{aligned}\dot{V}_4(t) =&- k_{4,a}\varepsilon_a^T\lfloor\varepsilon_a\rceil^{p_4/q_4} - k_{4,b}\varepsilon_b^T\lfloor\varepsilon_b\rceil^{p_4/q_4}\\ &- k_{5,a}S_a^T\lfloor S_a\rceil^{p_4/q_4} - k_{5,b}S_b^T\lfloor S_b\rceil^{p_4/q_4}\\ &+ S_a^T\{\Theta_{r,a} - \Theta_a(q_a)\} + S_b^T\{\Theta_{r,b} - \Theta_b(q_b)\}\\ &+ \varepsilon_a^T\dot{\widetilde{X}}_b(t) + \varepsilon_a^T\{[-c_{4,a}\|\dot{q}_a\|\mathrm{sgn}(\widetilde{X}_a)]_{eq}\\ &- \Delta J_a(q_a)\dot{q}_a\} + \varepsilon_b^T\{[-c_{4,b}\|\dot{q}_b\|\mathrm{sgn}(\widetilde{X}_b)]_{eq}\\ &- \Delta J_b(q_b)\dot{q}_b\} + \varepsilon_b^T\dot{\widetilde{X}}_a(t)\end{aligned} \tag{59}$$

By virtue of Theorem 3.1 and the first part of the proof of Theorem 3.2, we can learn that $\Theta_{r,i} = \Theta_i(q_i)$ for $t \ge T_f$ and $[-c_{4,i}\|\dot{q}_i\|\mathrm{sgn}(\widetilde{X}_i)]_{eq} = \Delta J_i(q_i)\dot{q}_i$ for $t \ge T_c$. In addition, we can obtain $\dot{\widetilde{X}}_i(t) = 0$ for $t \ge T_c$. Thus, $\dot{V}_4(t)$ can be written as

$$\begin{aligned}\dot{V}_4(t) \le &- k_{4,a}\varepsilon_a^T\lfloor\varepsilon_a\rceil^{p_4/q_4} - k_{4,b}\varepsilon_b^T\lfloor\varepsilon_b\rceil^{p_4/q_4}\\ &- k_{5,a}S_a^T\lfloor S_a\rceil^{p_4/q_4} - k_{5,b}S_b^T\lfloor S_b\rceil^{p_4/q_4}\\ \le &- \min\{k_{4,i}, k_{5,i}\}\big(\|\varepsilon_i\|^{\frac{p_4+q_4}{q_4}} + \|S_i\|^{\frac{p_4+q_4}{q_4}}\big)\end{aligned} \tag{60}$$

for $t \ge T_o$, with $T_o = \max\{T_f, T_c\}$.

Invoking Lemma 2.1, we obtain

$$\|\varepsilon_i\|^{\frac{p_4+q_4}{q_4}} + \|S_i\|^{\frac{p_4+q_4}{q_4}} \ge \big(\|\varepsilon_i\|^2 + \|S_i\|^2\big)^{\frac{p_4+q_4}{2q_4}} \tag{61}$$

Substituting (61) into (60) yields

$$\dot{V}_4(t) \le -\min\sum_{i\in\{a,b\}}\{k_{4,i}, k_{5,i}\}\big(\|\varepsilon_i\|^2 + \|S_i\|^2\big)^{\frac{p_4+q_4}{2q_4}} \tag{62}$$

with the condition of $t \ge T_o$.

From the definition of $V_4(t)$, we can obtain

$$V_4(t) \le \frac{1}{2}\sum_{i\in\{a,b\}}\Big\{\max\{1, c_{1,i}\}\big(\|\varepsilon_i\|^2 + \|S_i\|^2\big)\Big\} \tag{63}$$

Combining (62) and (63) leads to

$$\big(\|\varepsilon_i\|^2+\|S_i\|^2\big)^{\frac{p_4+q_4}{2q_4}} \ge \big(\frac{2}{\max\{1, c_{1,i}\}}\big)^{\frac{p_4+q_4}{2q_4}} V_4^{\frac{p_4+q_4}{2q_4}} \tag{64}$$

As a result, we can get

$$\dot{V}_4(t) \le -L_2 V_4^{\frac{p_4+q_4}{2q_4}}(t), \quad |\varepsilon_{ij}| \ge \lambda, \quad t \ge T_o \tag{65}$$

where $L_2 = \sum_{i\in\{a,b\}}\{\min\{k_{4,i}, k_{5,i}\}\big(\frac{2}{\max\{1,c_{1,i}\}}\big)^{\frac{p_4+q_4}{2q_4}}\}$. Solving (65) leads to $V_4(t) \equiv 0$ for all $t \geq T_k$ with T_k chosen as

$$T_k = T_o + \frac{q_4 V_4^{\frac{q_4-p_4}{q_4}}(T_o)}{L_2(q_4-p_4)} \tag{66}$$

2) For the case of $|\varepsilon_{ij}| < \lambda$ and after time T_o, we obtain

$$\begin{aligned}\dot{V}_4(t) \leq &- k_{4,a}\sum_{j=1}^{n}(\iota_1\varepsilon_{aj}^2 + \iota_2|\varepsilon_{aj}|^3) - k_{5,a}\sum_{j=1}^{n}|S_{aj}|^{\frac{p_4+q_4}{q_4}} \\ &- k_{4,b}\sum_{j=1}^{n}(\iota_1\varepsilon_{bj}^2 + \iota_2|\varepsilon_{bj}|^3) - k_{5,b}\sum_{j=1}^{n}|S_{bj}|^{\frac{p_4+q_4}{q_4}}\end{aligned} \tag{67}$$

where S_{ij} represents the jth element of the vector S_i.

As a result, after the global finite-time convergence of $|\varepsilon_{ij}| = \lambda$, the global asymptotic convergence of $\varepsilon_i = 0$ and $S_i = 0$ can be ensured by Lyapunov's direct method.

Remark 5 Based on Theorem 3.1, the observer-based control laws (51) for teleoperation system in task space are proposed in Theorem 3.2. To deal with the kinematically uncertain term $\Delta J_i(q_i)$ in (15), the sliding mode observers (46) are proposed. On the sliding surface $\widetilde{X}_i = \dot{\widetilde{X}}_i = 0$, it can be solved for the equivalent output injection $\Delta J(q_i)\dot{q}_i = [-c_{4,i}\|\dot{q}_i\|\mathrm{sgn}(\widetilde{X}_i)]_{eq}$, where $[-c_{4,i}\,\|\dot{q}_i\|\,\mathrm{sgn}(\widetilde{X}_i)]_{eq}$ denotes the equivalent value of $\Delta J(q_i)\dot{q}_i$.

3.3 Teleoperation System With External Force

In this section, the teleoperation system with uncertain dynamics and kinematics is studied when there are external forces exerted on the manipulator, i.e., $F_a(t), F_b(t)$ in (4), (5) and $f_a(t), f_b(t)$ in (9), (10) are nonzero. We develop the teleoperation system with external forces on the basis of foregoing Theorem 3.1 in joint space and Theorem 3.2 in task space, respectively.

Assumption 31 *Without loss of generality, the external forces exerted on the manipulators are assumed to satisfy passive maps [9], which results in*

$$-\int_0^t s_a F_a(t)d\tau \leq \kappa_a, \qquad \int_0^t s_b F_b(t)d\tau \leq \kappa_b \tag{68}$$

$$-\int_0^t S_a f_a(t)d\tau \leq K_a, \qquad \int_0^t S_b f_b(t)d\tau \leq K_b \tag{69}$$

where κ_a, κ_b and K_a, K_b are nonnegative constants.

Theorem 3 *When there are external forces exerted on the local manipulator and remote manipulator with the forces satisfying Assumption 3.1, then, the bilateral observer-based controllers (24), (51), the estimation laws (18) and the sliding-mode observers (46) ensure the tracking control for the teleoperation system (4)-(5) and (9)-(11) with uncertainties.*

Proof Choose the Lyapunov-like function candidate for joint space as $V_j(t)$ and task space as $V_t(t)$

$$\begin{aligned}V_j(t) =& V_2(t) + \kappa_a + \int_0^t s_a F_a(t)d\tau + \kappa_b - \int_0^t s_b F_b(t)d\tau \\ V_t(t) =& V_4(t) + K_a + \int_0^t S_a f_a(t)d\tau + K_b - \int_0^t S_b f_b(t)d\tau\end{aligned} \tag{70}$$

where $\kappa_a + \int_0^t s_a F_a(t)d\tau + \kappa_b - \int_0^t s_b F_b(t)d\tau \geq 0$ and $K_a + \int_0^t S_a f_a(t)d\tau + K_b - \int_0^t S_b f_b(t)d\tau \geq 0$ are guaranteed. In the light of Theorem 3.1 and Theorem 3.2, it can be derived that tracking control of teleoperation system can be guaranteed.

4 Simulation Results

The developed sliding mode observers and the control algorithms for the teleoperation system with uncertain dynamics and kinematics are verified via numerical examples in this section. The remote and local robots are considered to be a pair of 2-DOF manipulators with different link length. Let the joint angles be $q_i = [q_{i,1}, q_{i,2}]^T$, the angle velocities be $\dot{q}_i = [\dot{q}_{i,1}, \dot{q}_{i,2}]^T$. In the simulation, the physical parameters of the teleoperation system are given in Table 1 and the dynamic parameters are given as $d = [d_{i,1}, d_{i,2}, d_{i,3}, d_{i,4}]$, where

$$\begin{aligned}&d_{i,1} = I_{i,1} + m_{i,1}l_{i,2}^2 + I_{i,2} + m_{i,2}l_{i,3}^2 + m_{i,2}l_{i,1}^2,\\ &d_{i,2} = I_{i,2} + m_{i,2}l_{i,3}^2,\\ &d_{i,3} = m_{i,2}l_{i,1}l_{i,3}\cos(\epsilon_{i,1}), \quad d_{i,4} = m_{i,2}l_{i,1}l_{i,3}\sin(\epsilon_{i,1})\end{aligned}$$

Then, the elements of the inertia matrices $H_i(q_i)$, the Coriolis and centrifugal matrices $C_i(q_i, \dot{q}_i)$ and the

Table 1: Physical parameters of local and remote robots.

	$m_{i,1}$	$m_{i,2}$	$l_{i,1}$	$l_{i,2}$	$l_{i,3}$	$I_{i,1}$	$I_{i,2}$	$\epsilon_{i,1}$	$\epsilon_{i,2}$	$\epsilon_{i,3}$
Local robot	1	3	1	1/2	2/5	1/12	2/5	0	-7/12	9.81
Remote robot	1.82	3.14	0.84	1.28	2/5	1/7	1/4	0	-1/2	9.81

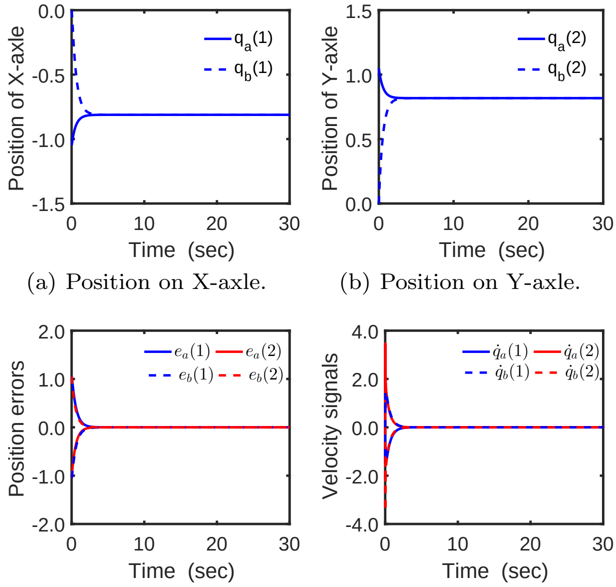
the gravitational torques $g_i(q_i)$ are

$$\begin{aligned}
H_i^{11} &= d_{i,1} + 2d_{i,3}\cos(q_{i,2}) + 2d_{i,4}\sin(q_{i,2}),\\
H_i^{12} = H_i^{21} &= d_{i,2} + d_{i,3}\cos(q_{i,2}) + d_{i,4}\sin(q_{i,2}),\\
H_i^{22} &= d_{i,2},\\
C_i^{11} &= \{-2d_{i,3}\sin(q_{i,2}) + 2d_{i,4}\cos(q_{i,2})\}\dot{q}_{i,2},\\
C_i^{12} &= \{-d_{i,3}\sin(q_{i,2}) + d_{i,4}\cos(q_{i,2})\}\dot{q}_{i,2},\\
C_i^{21} &= \{d_{i,3}\sin(q_{i,2}) - d_{i,4}\cos(q_{i,2})\}\dot{q}_{i,1},\\
C_i^{22} &= 0,\\
g_i^1 &= d_{i,3}\epsilon_{i,3}\cos(q_{i,1}+q_{i,2}) + d_{i,4}\epsilon_{i,3}\sin(q_{i,1}\\
&\quad + q_{i,2}) + (d_{i,1} - d_{i,2} + \epsilon_{i,2})\epsilon_{i,3}\cos(q_{i,1}),\\
g_i^2 &= d_{i,3}\epsilon_{i,3}\cos(q_{i,1}+q_{i,2}) + d_{i,4}\epsilon_{i,3}\sin(q_{i,1}+q_{i,2}).
\end{aligned}$$

Remark 6 In the practical employment of the proposed algorithm, it should be stressed that the adverse chattering effect caused by signum function in observers (18) and control laws (24),(51) may deteriorate the system performance from an engineering point of view. Hence, it is imperative to use a more practical method to alleviate the chattering problem. In this simulation, a continuous function $\beta/(|\beta|+\mu)$ is applied to replace the signum function $sgn(\beta)$, where μ denotes a small positive constant.

4.1 Simulation results in joint space

The initial joint configuration and velocity of the teleoperation system are set as $q_a(0) = [-\pi/3, \pi/3]^T, q_b(0) = [0,0]^T$, $\dot{q}_a(0) = [0,0]^T, \dot{q}_b(0) = [0,0]^T$. For the observers and controllers of teleoperation system, the following control gains are adopted: $K_a = K_b = 1700, b_{1,a} = b_{1,b} = 18, b_{2,a} = b_{2,b} = 3.5, p_1 = 17, q_1 = 23, k_{1,a} = k_{1,b} = 0.05, k_{2,a} = k_{2,b} = 6, k_{3,a} = k_{3,b} = 4, p_2 = 17, q_2 = 47$. Without external forces exerted on the manipulators, the simulations of X/Y positions, position errors and velocity signals in joint space are shown in Fig. 2. It is shown that the teleoperation system is stable, which indicates all signals in closed-loop system are bounded. As depicted in Fig. 2(a) and 2(b), the remote manipulator can stably track the position of the local manipulator and the tracking errors converge to zero quickly as shown in Fig. 2(c). In addition, Fig. 2(d) shows that the state synchronization is realized



(a) Position on X-axle. (b) Position on Y-axle.

(c) Position errors. (d) Velocity signals.

Fig. 2: Performance results of telecontrol scheme in joint space.

in the sense that position errors as well as velocity errors converge to zero. Hence, it is demonstrated that the state synchronization for the teleoperation system in joint space with uncertain dynamics is implemented successfully.

Fig. 3 illustrates the simulation results of sliding mode observers proposed for uncertain dynamics. It can be observed that the uncertain dynamics $\Theta_i(q_i)$ can be successfully estimated with the observers $\Theta_{r,i}(q_i)$. Fig. 4 presents the detailed response of $\Theta_a(q_a)$, which shows that the observer $\Theta_{r,a}(q_a)$ is capable to estimate the uncertain dynamics within finite time when uncertain dynamics are time-varying. Fig. 5 displays the steady response of the estimation error $\Theta_{e,a}(q_a)$, which demonstrates the high accuracy and good performance of the proposed observers. To this end, the conclusion in Theorem 3.1 is well manifested.

4.2 Simulation results in task space

To examine the performance of the designed observers and controllers for the teleoperation system in task s-

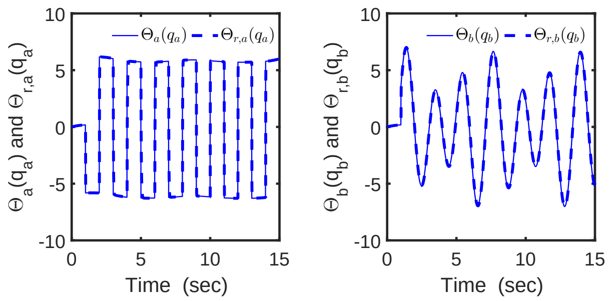


Fig. 3: Performance of sliding mode observers proposed for uncertain dynamics.

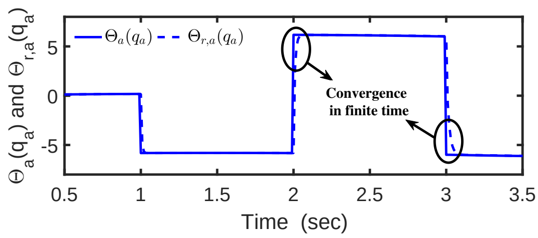


Fig. 4: Detailed response of $\Theta_{r,a}(q_a)$.

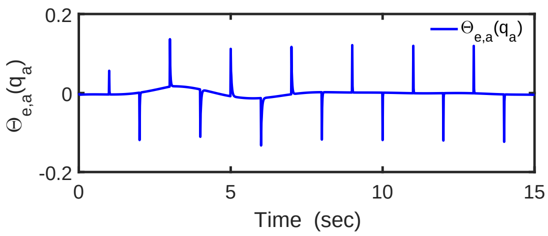


Fig. 5: Steady response of the uncertain dynamics estimation error $\Theta_{e,a}(q_a)$.

pace, the initial end-effector position is set as $X_a(0) = [-1, 1]^T, X_b(0) = [0, 0]^T$. It is assumed that the end-effector position of the teleoperation system can be obtained by position sensors in task space. The Jacobian matrices $J_i(q_i)$ are given as

$$J_i(q_i) =$$

$$\begin{bmatrix} -l_{i,1}\sin(q_{i,1}) - l_{i,2}\sin(q_{i,1} + q_{i,2}) & -l_{i,2}\sin(q_{i,1} + q_{i,2}) \\ l_{i,1}\cos(q_{i,1}) + l_{i,2}\cos(q_{i,1} + q_{i,2}) & l_{i,2}\cos(q_{i,1} + q_{i,2}) \end{bmatrix}$$

In this section, the teleoperation system is considered to move when the local operator applies a piecewise constant force on the local robot to change the state of the teleoperation system. By employing the controllers and observers proposed in Theorem 3.2, the numerical simulations for teleoperation system with uncertain dynamics and kinematics are conducted and the results are depicted in Fig. 6. In Fig.6, it is shown that the teleoperation system can achieve stability quickly with external force applied. To be specific, in Fig. 6(a) and 6(b), when there is no external force before $t = 5s$, the local and remote robots converge to each other and position errors $\epsilon_i(t)$ converge to zero. When human exerts force to local manipulator at time $t = 5 - 10s, 15 - 17s$ and $20 - 22s$, the local robot moves consequently and the position errors $\epsilon_i(t)$ increase first and then go to zero quickly, which proves that the controllers in Theorem 3.2 can guarantee the fast position tracking between local and remote robots.

Fig. 7 is shown to analyze the behavior of the designed sliding mode observers in (46). From the graph in Fig. 7(a), it is shown that the estimation errors $\widetilde{X}_i$ are able to converge to zero after a short period of time. Fig. 7(b) shows the detailed response of $\widetilde{X}_i$, which means X_i can be precisely estimated with the proposed observers. Steady-state behavior of the estimation errors $\widetilde{X}_i$ are shown in Fig. 7(c), and the estimation accuracy is smaller than 2e-3 mm. From the above graphs, it is proved that the proposed observers and controllers ensure the stability and tracking performance of the closed-loop teleoperation system subject to uncertain dynamics and kinematics.

4.3 Quantitative Analysis

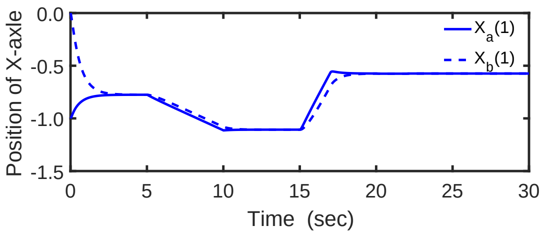
To further evaluate the performance of the sliding mode observer-based control (SMOBC), it is compared with the adaptive fuzzy control (AFC) [38] and nonlinear adaptive bilateral control (NAC) [39] for teleoperation system with dynamic and kinematic uncertainties. Two indices are selected to compare the performance of the controllers, which are defined as

$$|\varepsilon|_d = \frac{1}{m}\sum_{k=1}^{m}|\varepsilon_k|, \qquad |t|_d = \frac{1}{m}\sum_{k=1}^{m}|t_k| \tag{71}$$

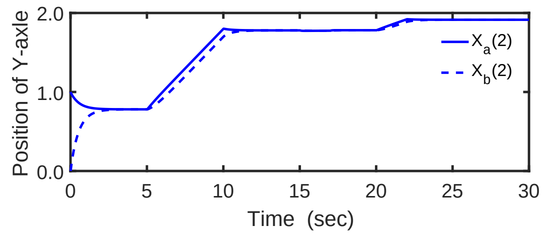
where m denotes the number of sampling steps of the simulations, then ε_k and t_k denote the position tracking error and the time to reach steady state in kth step, $k = 1, 2, ..., m$. The term $|\varepsilon|_d$ is utilized to evaluate the tracking accuracy between the local and remote robots, and $|t|_d$ acts as a numerical measure of settling time.

Simulated comparison is examined in teleoperation system by these three control methods under the same initial end-effector position. The external forces are considered to be zero for the sake of simplicity. The following two scenarios are considered.

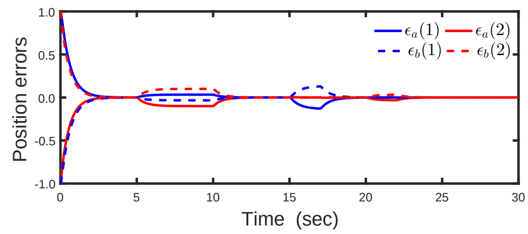
($S1$): The initial end-effector position is $X_a(0) = [1.1, -0.1732]^T, X_b(0) = [0, 0]^T$. The teleoperators undergo the dynamic uncertainty $\Theta_{s1} = 0.4$ for $t \geq 2$, while $\Theta_{s1} = 0$ for $t < 2$.



(a) Position on X-axle.

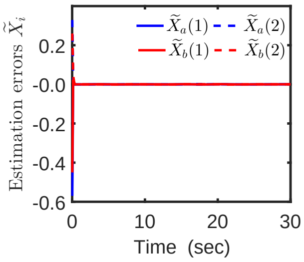


(b) Position on Y-axle.

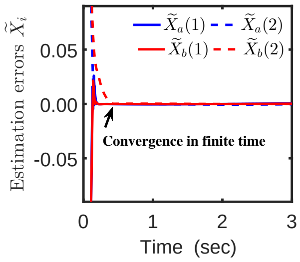


(c) Position errors.

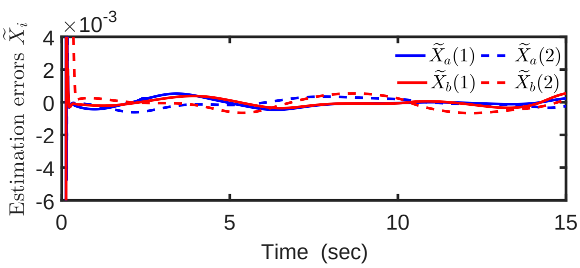
Fig. 6: Performance results of telecontrol scheme in task space.



(a) Estimation errors $\widetilde{X}_i$.



(b) Detailed response of the estimation errors $\widetilde{X}_i$.



(c) Steady-state behavior of the estimation errors $\widetilde{X}_i$.

Fig. 7: Performance of sliding mode observers proposed for uncertain kinematics.

($S2$): The initial end-effector position is $X_a(0) = [0.8, -0.4]^T, X_b(0) = [0, 0]^T$. The teleoperators undergo the dynamic uncertainty $\Theta_{s2} = 0.03sin(0.5\pi t + 0.7)$ for $t \geq 0$.

The comparative analysis are illustrated in Fig.8, where Fig. 8(a),(d) show the position errors for the AFC scheme and Fig. 8(b),(e) show the results for the NAC scheme, and the simulation results for the proposed S-MOBC are depicted in Fig. 8(c),(f),respectively. In case $S1$, as can be clearly seen from Fig. 8(a)-(c), compared with the AFC scheme and the NAC scheme, a better responses of position tracking occur in SMOBC scheme, in which case less oscillations of tracking errors and s-mooth transient performance are presented. In case $S2$, as shown in Fig. 8(d)-(f), where the teleoperation system suffers from the time-varying uncertain dynamics, it is shown that the AFC scheme and NAC scheme have less capability to approach the uncertain dynamics. Obviously, these results show the fact that the proposed SMOBC scheme can better satisfy the tracking performance of teleoperation system with strong robustness to uncertainties compared with the NAC scheme and the AFC scheme.

Moreover, the results about average tracking accuracy and the average settling time are listed in Table 2. From the perspective of tracking accuracy, the AFC scheme is inferior to the SMOBC and the NAC scheme due to the fact that fuzzy control rules depend on human factors to a large extent and it is hard to eliminate the steady state error by fuzzy control. If we evaluate these three approaches by settling time, it is found that the proposed SMOBC scheme outperforms the other t-wo, which is because linearity-in-parameters condition used in the AFC scheme and the NAC scheme require complicated computations to obtain the regressor matrix, which leads to the slow rate of the convergence. D-ifferent from the above schemes, the designed SMOBC scheme guarantee that the tracking errors converge to the arbitrary set close to the origin within finite time via avoiding applying linearity-in-parameters condition. Thus, based on the comparative simulation responses, the effectiveness of the proposed sliding mode observer-based controllers can be verified.

Table 2: Performance comparisons.

Indices	The control approaches		
	SMOBC	AFC	NAC
$\lvert\varepsilon\rvert_d(1)$(mm)	2.87e-3	0.245	4.56e-2
$\lvert\varepsilon\rvert_d(2)$(mm)	1.96e-3	0.229	4.37e-2
$\lvert t\rvert_d$(s)	9.46	23.5	18.63

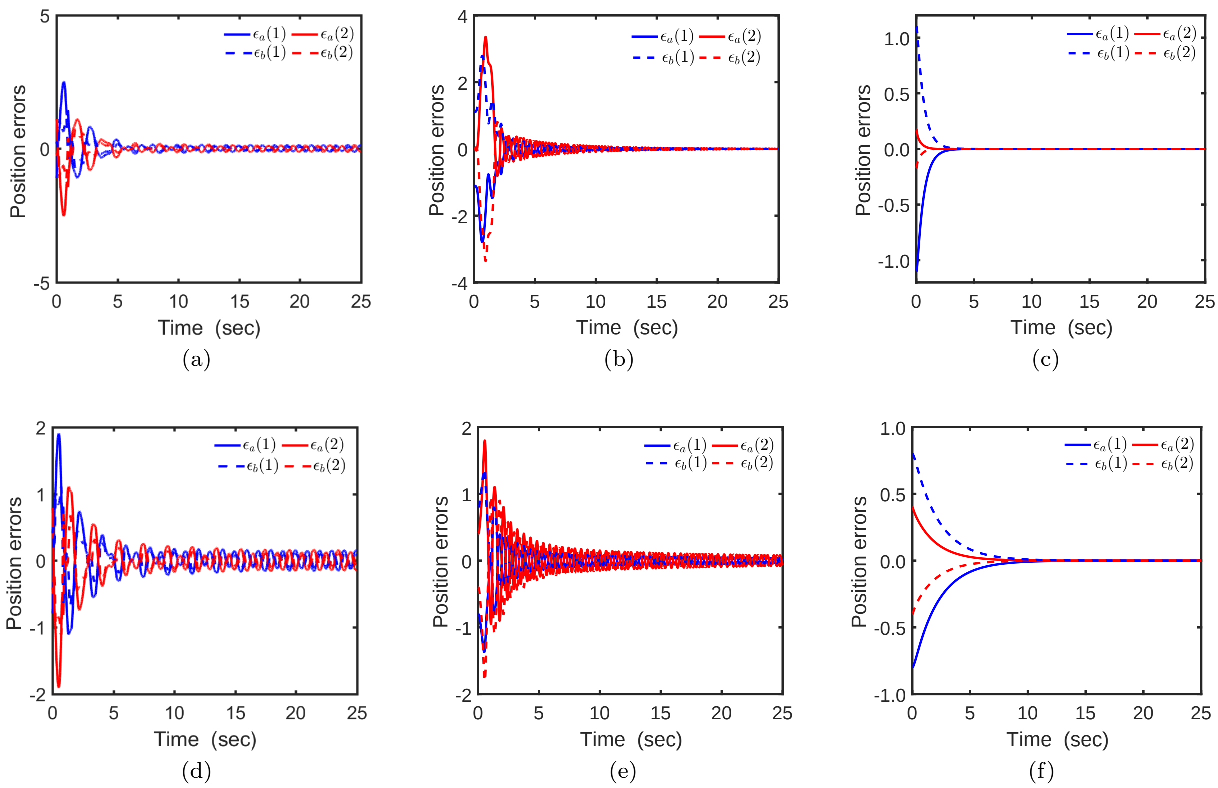


Fig. 8: Simulation of position errors under two scenarios. Results of AFC scheme: (a)position errors under S1; (d)position errors under S2. Results of NAC scheme: (b)position errors under S1; (e)position errors under S2. Results of SMOBC scheme: (c)position errors under S1; (f)position errors under S2.

5 Conclusion

This paper reported the problem of observer-based control for nonlinear teleoperators with uncertain dynamic and kinematic parameters was addressed. An observer-based control strategy was proposed for the purpose of tracking control of teleoperation system. The estimation errors can be guaranteed to converge to zero within finite time with the proposed observers. The closed-loop system was proven to be stable by utilizing the Lyapunov theory. Simulation results showed effectiveness of the proposed scheme by position tracking and uncertainty handling comparison with other two approaches.

Declarations

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Data Availability: No underlying data are included.

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Figures

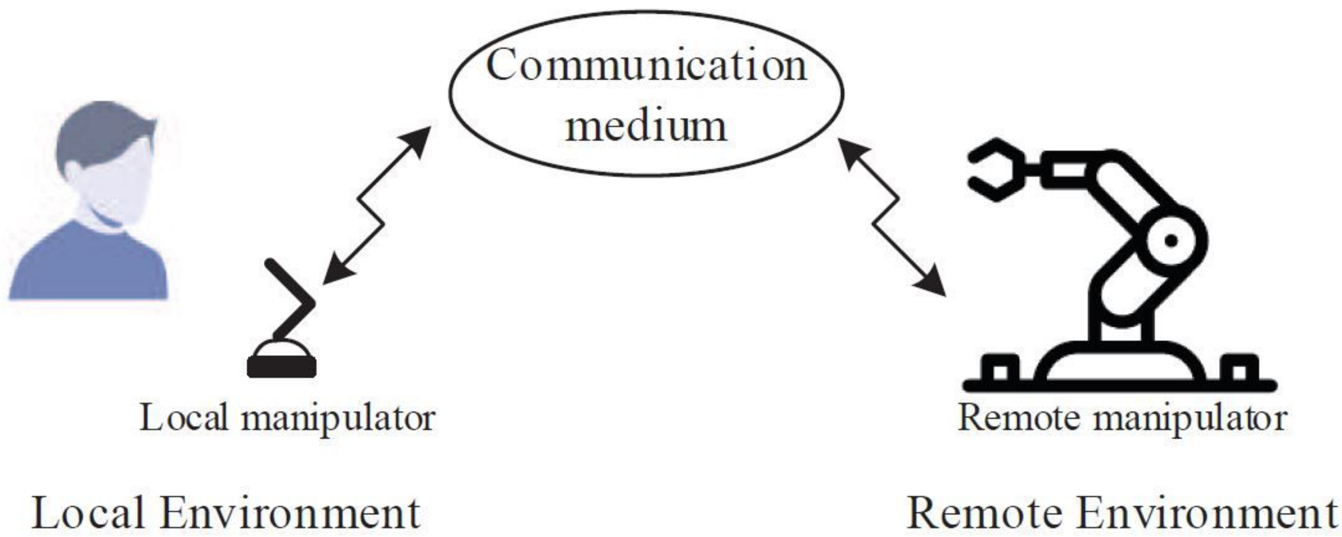


Figure 1

Bilateral teleoperation.

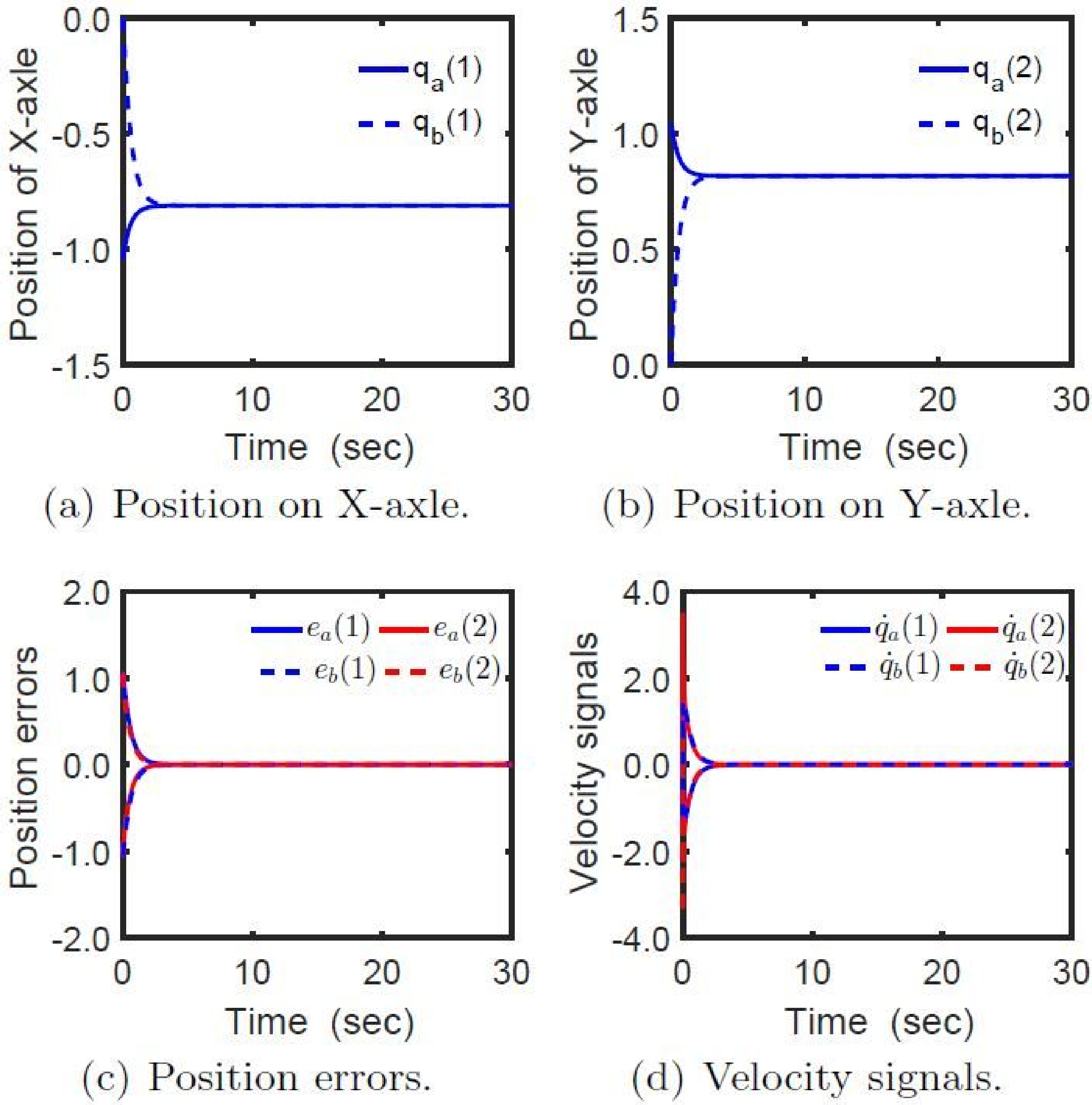


Figure 2

Performance results of telecontrol scheme in joint space.

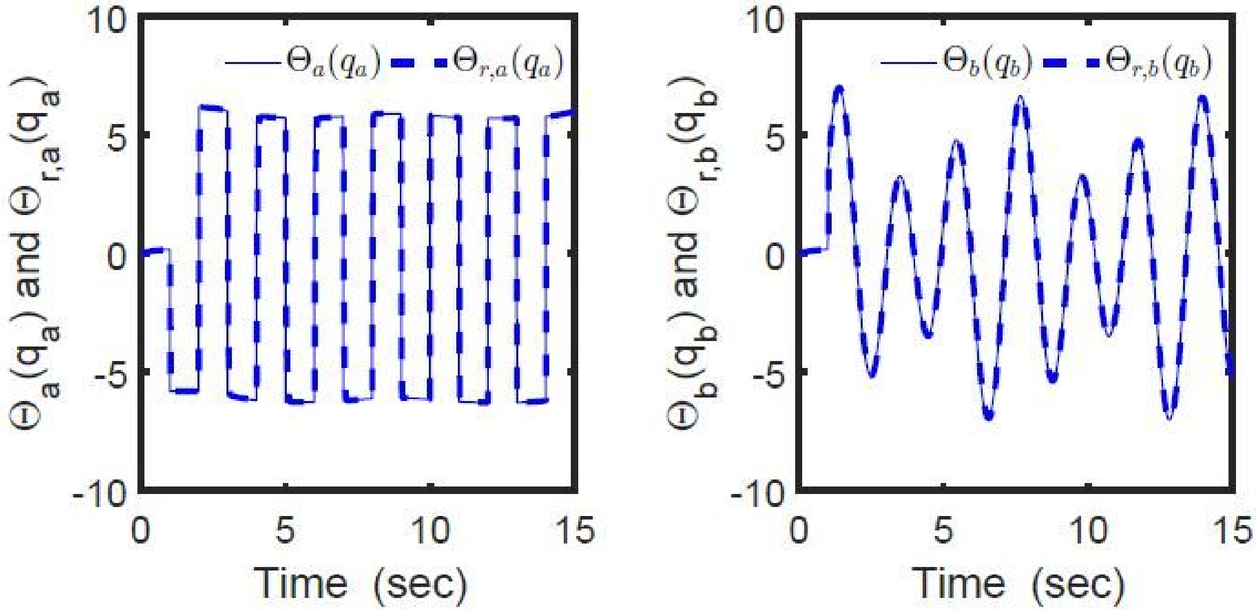


Figure 3

Performance of sliding mode observers proposed for uncertain dynamics.

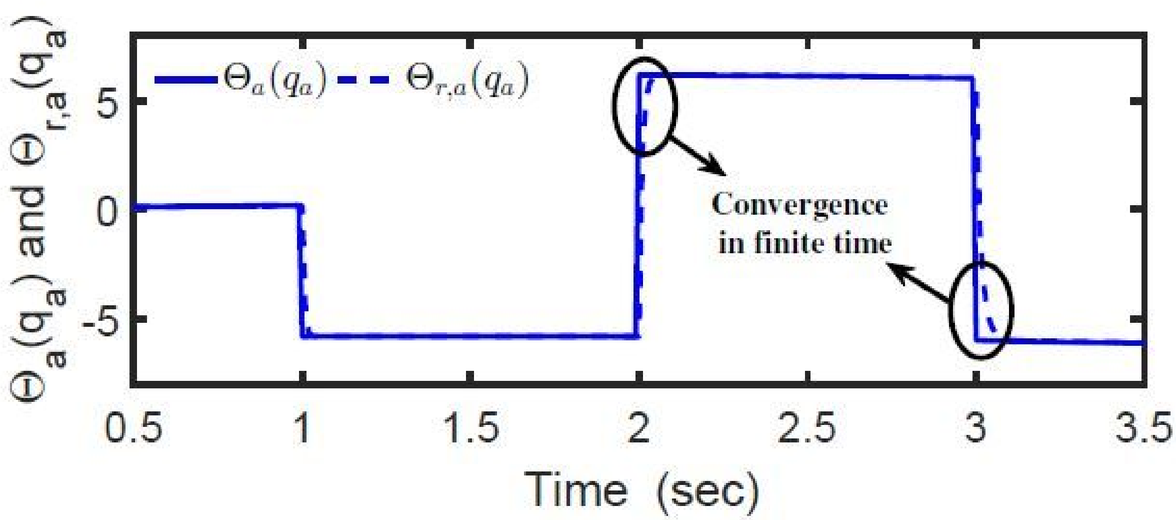


Figure 4

Detailed response of θr;a(qa).

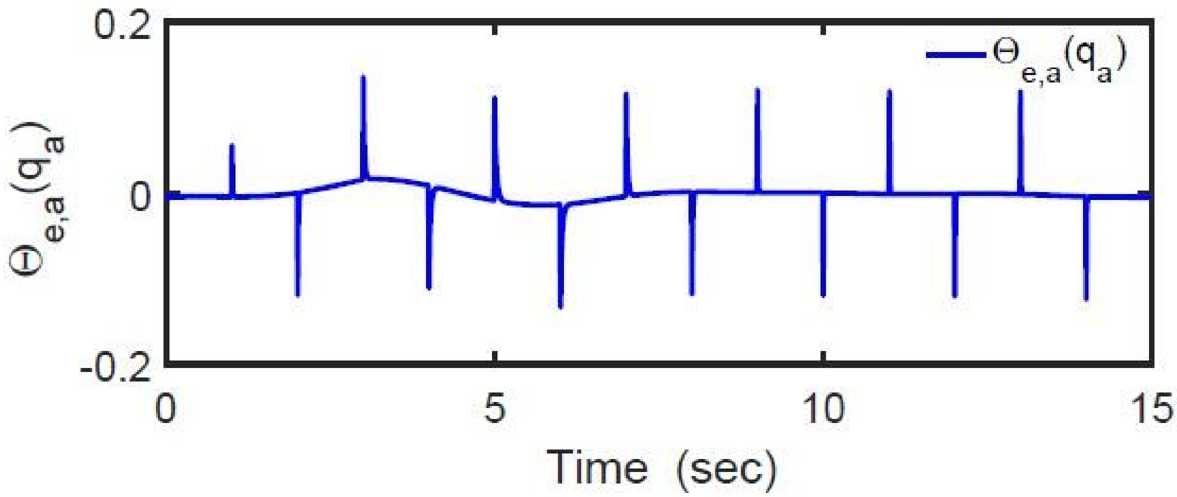
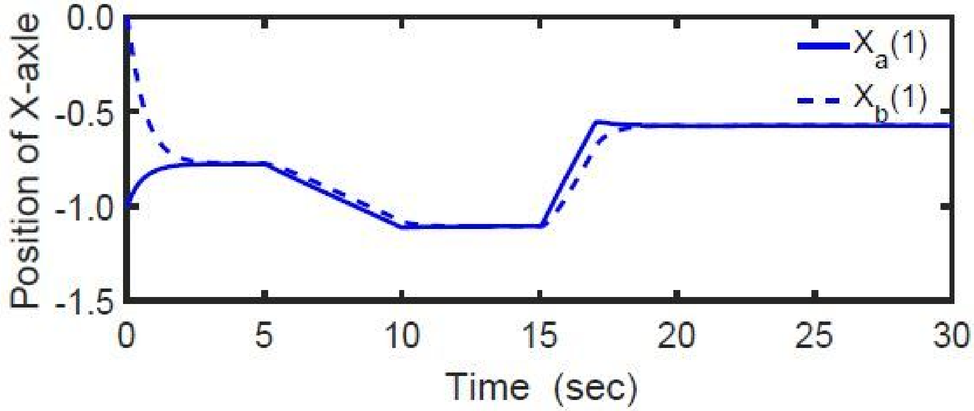
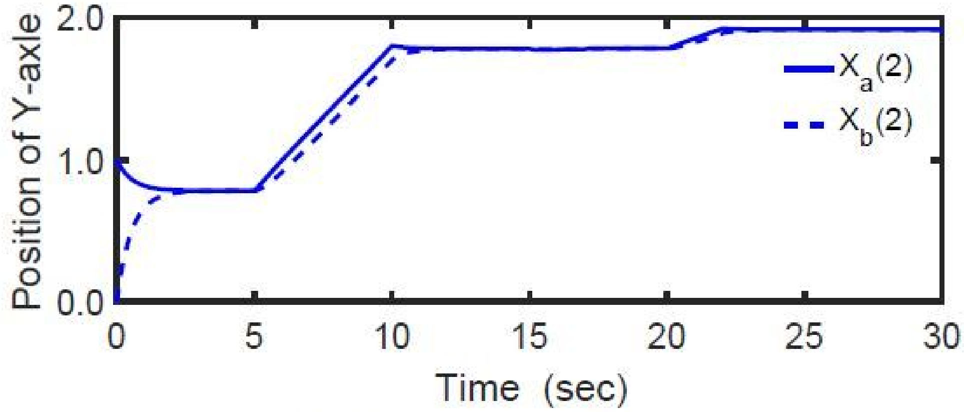


Figure 5

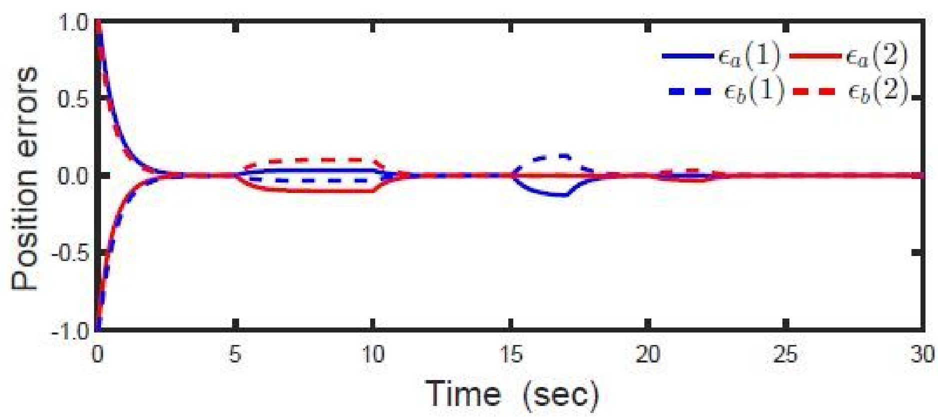
Steady response of the uncertain dynamics estimation error θe;a(qa).



(a) Position on X-axle.



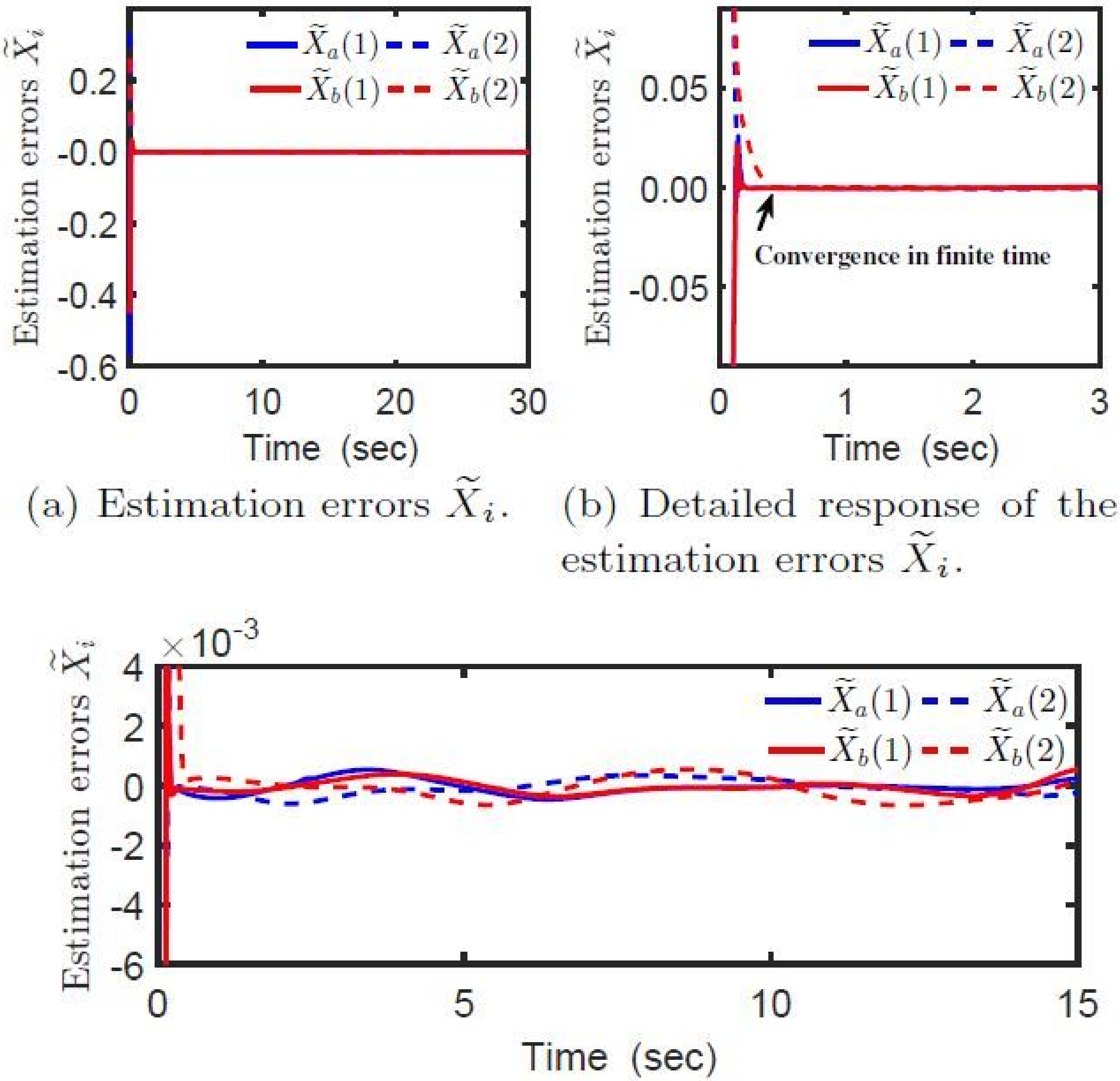
(b) Position on Y-axle.



(c) Position errors.

Figure 6

Performance results of telecontrol scheme in task space.



(a) Estimation errors $\widetilde{X}_i$.

(b) Detailed response of the estimation errors $\widetilde{X}_i$.

(c) Steady-state behavior of the estimation errors $\widetilde{X}_i$.

Figure 7

Performance of sliding mode observers proposed for uncertain kinematics.

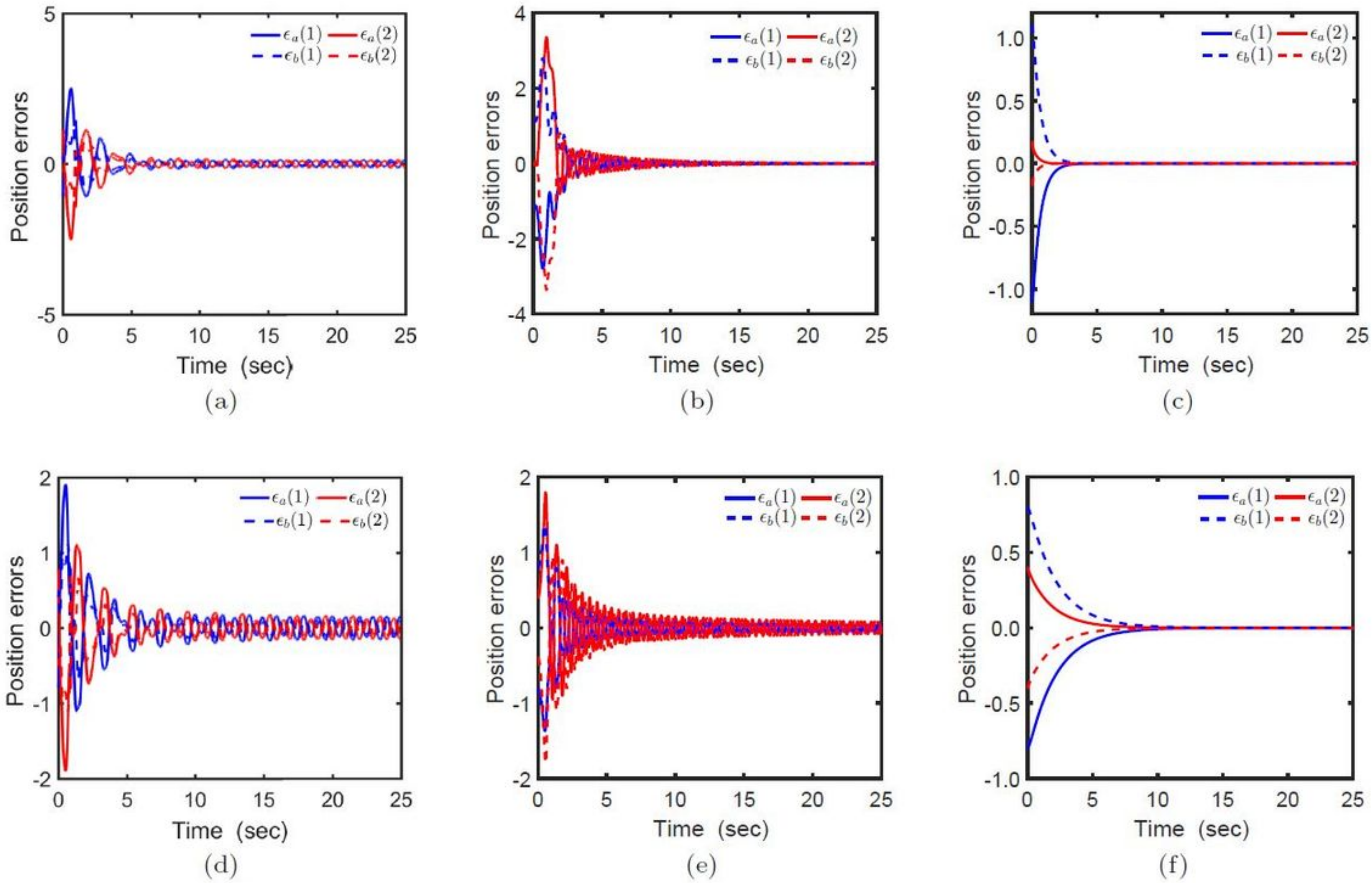


Figure 8

Simulation of position errors under two scenarios. Results of AFC scheme: (a)position errors under S1; (d)position errors under S2. Results of NAC scheme: (b)position errors under S1; (e)position errors under S2. Results of SMOBC scheme: (c)position errors under S1; (f)position errors under S2.