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Modelling wave-particle duality of classical particles and waves

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ABSTRACT

Wave-particle duality is the fundamental phenomenon of particles and fields in quantum mechanics. In the past, the trajectory-like (particle-like) behaviour and wave-like behaviour has been considered separately. In this article, a superimposed model is derived to characterise wave-particle duality of classical particles. The superimposed model reflects an invariant mathematical structure (analogous variables and differential equations) from classical mechanics, classical field theories and quantum mechanics. Its analytical solution carries trajectory-like property (phase-independent) and wave-like property (phase-dependent) of particles that is consistent with to Schrodinger's picture. Subsequently, the presented model is applied to model duality of classical waves in electromagnetism, acoustics and elasticity. The analysis implies the existence of quantum effects of classical waves at macroscopic scale. It predicts quantum picture on energy and momentum exchange between classical particles and waves. As seen in the model, wave-particle duality reflects inherent and indispensable characteristics of classical objects.

Keywords: Wave-particle duality; Superimposed model; Phase dependence; Classical objects; Invariant structure; One-to-one map;

Introduction

Wave-particle duality is one of the most mysterious phenomena in quantum mechanics. Since the discovery of the phenomenon in double-slit experiment, this interesting topic has been studied continuously over the past century [1,2]. Experimental investigations have been extended from original double-slit experiment of photons and free electrons to relatively large physical objects. These include the testing of heavy nuclei particle (neutron), the carbon molecular (C_{60}) and recently the phthalocyanine molecules [3-7]. With the increasing the size of the testing objects from subatomic scale to large molecules, the wave-particle duality is demonstrated as one of the fundamental phenomena in quantum mechanics. To investigate wave-particle duality at macroscopic scale, recent studies have pointed out the analogous behaviour of classical objects such as small drop manifest dual behaviour under the liquid bath [8-12]. The oscillating oil droplet on the liquid surface create wave pattern that propagate though slits. These attempts provide useful method to visualise the interesting feature of wave-particle duality in simple experimental setup. The analogous between the classical droplets and quantum particles could shed light on the study of many interesting quantum process [13]. Therefore, the connections on wave-particle duality that manifested on classical objects and quantum objects may bridge the barrier in classical and quantum theories [14].

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To understand the observed behaviour, theoretical modelling of wave-particle duality has attracted attention from earlier time to the recent. Traditionally, as in some earlier textbooks, the modelling of wave-particle duality is treated as two independent frameworks to account its trajectory-like property or wave-like property [15-17]. The trajectory-like property is considered in the analysis of macroscopic classical system where its wave-like property is neglected. The wave-like property is accounted in the analysis of microscopic quantum object while its trajectory-like property is omitted. The above model is popular since it reduces the dual properties (both trajectory and wave) into single property (either trajectory or wave) that simplify the calculation. Alternatively, the pilot wave model has been investigated by different authors for modelling wave-particle duality for classical and quantum objects [18-20]. One of the important features of this model is the representation of both trajectory-like and wave-like properties in its formulation. In the reported droplet experiment, the pilot wave of droplet contains time-varying information of generated wave pattern as well as travelling trajectory. Nevertheless, pilot wave model differs from the usual Schrodinger's picture in term of the treatment of trajectory and wave [21]. The pilot wave is a real wave (surface wave of liquid bath) rather than the (statistical) probability wave in quantum mechanics.

As shown in literature, wave-particle duality has been experimentally observed from small-size quantum objects and large-size classical objects. At the same time, the topic on wave-particle duality of classical objects have not been well-studied. In this study, a superimposed model is derived to characterised wave-particle duality of classical particles and waves. This model has its root from the invariant mathematical structure of classical mechanics and classical field theories. Its analytical solution contains trivial (phase-independent) solution and non-trivial (phase-dependent) solution, respectively. The phase-independent solution manifest trajectory-like behaviour while the phase-dependent solution leads to Schrodinger's picture in quantum mechanics. In this way, we could obtain a superimposed equation of classical particles that manifest wave-particle duality. The disturbance of wave-like property under external measurement can be modelled as the phase degeneration process. Subsequently, the wave-particle duality of classical waves (electromagnetism, acoustics and elasticity) is modelled. Analytical solution of field variables with trajectory-like property and wave-like property can be shown for classical waves in electromagnetism, acoustics and elasticity. From the one-to-one transformations between dynamical variables and field variables, the quantum constraints on the classical waves can be revealed. This gives the quantum picture of energy and momentum exchange between classical particles and waves. From the model, it is shown that both the trajectory-like property and wave-like property reflect inherent and indispensable characteristics of classical objects.

Results

Superimposed model and analytical solution. A superimposed model of classical objects constitutes physical variables and differential equations that arise from the theories of classical mechanics and classical waves. The detail formulation of the superimposed model is explained in the method section. In its usual formulation, classical object (particle or waves) is uniquely characterised by its governing variable $Z(\boldsymbol{r},t)$ and dependent variables $X(\boldsymbol{r},t), \boldsymbol{Y}(\boldsymbol{r},t)$ that are subjected to the following set of differential equations,

$$\left(\nabla^2 - \frac{1}{\boldsymbol{v}^2}\partial_t^2\right)Z = 0 \quad ; \quad X = \gamma\partial_t Z \quad ; \quad \boldsymbol{Y} = \nabla Z\,, \tag{1}$$

where $\boldsymbol{v}$ denotes certain velocity and γ represents certain scalar physical quantity. It is worth to note that the above relation represents an invariant mathematical structure in theories of classical particles and waves. The detail procedures of establishing the superimposed models of classical particle and waves in the above form are presented in the method section and the obtained models are listed in Table 1. Usually, the non-trivial solution is taken the form of planewave in complex plane and the trivial solution is frequently omitted in the analysis. However, it is worth to note the trivial solution does have physical meaning just as the non-trivial solution. In this study, we consider the solution of superimposed model in the following form,

$$Z = \alpha(\bar{\boldsymbol{Y}}\boldsymbol{r} - \bar{X}t) + \beta(-i\bar{Z}\phi) \;\; ; \;\; \phi = e^{i(\boldsymbol{kr}-\omega t)}\,, \tag{2}$$

where $\bar{X}, \bar{\boldsymbol{Y}}$ denote constant magnitudes of dependent variables, $\bar{Z}$ denotes the constant magnitude of governing variable and ϕ denotes the phase function. The above solution constitutes a trivial particle which is phase-independent and a non-trivial which is phase-dependent. The parameters in Eq. (2) refers to the non-negative ($\alpha, \beta \geq 0$) weight coefficients of phase-independent and phase-dependent solutions, they satisfy the normalisation condition $\alpha + \beta = 1$.

Table 1 – Governing variables, dependent variables and differential equations of particle and fields.

Superimposed models	Governing variable	Dependent variables	Differential equations		
Basic	Z	$X, \boldsymbol{Y}$	$\left(\nabla^2 - \frac{1}{\boldsymbol{v}^2}\partial_t^2\right)Z = 0$	$X = \gamma\partial_t Z$	$\boldsymbol{Y} = \nabla Z$
Particle	S	$H, \boldsymbol{P}$	$\left(\nabla^2 - \frac{1}{\boldsymbol{v}^2}\partial_t^2\right)S = 0$	$H = -\partial_t S$	$\boldsymbol{P} = \nabla S$
Electromagnetic wave	Λ	$V, \boldsymbol{A}$	$\left(\nabla^2 - \frac{1}{\boldsymbol{c}^2}\partial_t^2\right)\Lambda = 0$	$V = -\partial_t \Lambda$	$\boldsymbol{A} = \nabla \Lambda$
Acoustic wave	χ_A	$p, \boldsymbol{v}_A$	$\left(\nabla^2 - \frac{1}{\boldsymbol{c}_A^2}\partial_t^2\right)\chi_A = 0$	$p = -\rho_A\partial_t \chi_A$	$\boldsymbol{v}_A = \nabla \chi_A$
Elastic wave (longitudinal)	χ_L	$\sigma_L, \boldsymbol{v}_L$	$\left(\nabla^2 - \frac{1}{\boldsymbol{c}_L^2}\partial_t^2\right)\chi_L = 0$	$\sigma_L = -\rho_E\partial_t \chi_L$	$\boldsymbol{v}_L = \nabla \chi_L$
Elastic wave (transverse)	χ_T	$\sigma_T, \boldsymbol{v}_T$	$\left(\nabla^2 - \frac{1}{\boldsymbol{c}_T^2}\partial_t^2\right)\chi_T = 0$	$\sigma_T = -\rho_E\partial_t \chi_T$	$\boldsymbol{v}_T = \nabla \chi_T$

Modelling wave-particle duality for single particle. To simulate wave-particle duality of particle, we apply the solution in Eq. (2) to represent the dynamical variables according to the first row in Table 1. The resultant solution of action includes a phase-independent part and a phase-independent part, that is,

$$S = \alpha(\overline{\boldsymbol{P}}\boldsymbol{r} - \overline{H}t) + \beta(-i\bar{S}\phi) \ ; \ \phi = e^{i(\boldsymbol{kr}-\omega t)} \tag{3}$$

where $\overline{H}$ and $\overline{\boldsymbol{P}}$ denote the constant magnitudes of dynamical variables, $\bar{S}$ refers to the constant magnitude of action and ϕ refers to the phase function. The trajectory-like property is represented by condition of $\alpha = 1$, in which Hamiltonian (energy) and momentum of the particle are phase-independent constants,

$$H = -\partial_t S = \overline{H} \ ; \ \boldsymbol{P} = \nabla S = \overline{\boldsymbol{P}} \, . \tag{4}$$

As the dynamical variables are independent of phase function, they describe the point-like object that can be completely determined by its trajectory at each time instant. At the same time, the wave-like property is expressed by the condition of $\beta = 1$, in which the dynamical variables are in the phase-dependent form,

$$H = -\partial_t S = i\bar{S}\partial_t\phi = \bar{S}\omega\phi \ ; \ \boldsymbol{P} = \nabla S = -i\bar{S}\nabla\phi = \bar{S}\boldsymbol{k}\phi \, , \tag{5}$$

where the magnitudes of dynamical variables are expressed by the wave parameters and phase. It is noted that these two conditions are not completely independent as they satisfy the conservation laws of energy and momentum. Consequently, the magnitudes of the dynamical variable in Eq. (4) and (5) should be identical, this leads to the following equalities between energy and momentum in term of phase-independent and phase-dependent forms,

$$\overline{H} = \bar{S}\omega \ ; \ \overline{\boldsymbol{P}} = \bar{S}\boldsymbol{k} \, . \tag{6}$$

We noted that the above equalities are formal equivalent to Planck and the de Broglie relations if the parameter $\bar{S}$ is replaced by Planck constant $\hbar$. Furthermore, the wave-like solution has clear link to the Schrodinger's picture. To see this point, we could substitute the trajectory-like and wave-like solutions in Eq. (4) and (5) into the energy-momentum relation of particle, this leads to the following superimposed equation,

$$\alpha\left(\overline{H} - \frac{\overline{\boldsymbol{P}}^2}{2m}\right) + \beta\left(i\bar{S}\partial_t - \frac{-\bar{S}^2}{2m}\nabla^2\right)\phi = 0 \, . \tag{7}$$

Under the condition $\alpha = 1$, it reflects the energy relation of classical particle in term of its trajectory-like aspect. Under the condition $\beta = 1$, it consistent with the Schrodinger' equation (free particle) by applying the substitution $\bar{S} \to \hbar$. From the above result, the superimposed solution provides the information of the particle in trajectory-like property and wave-like property in the same framework. In reality, since the observed behaviour of single particle is unique (either to be trajectory-like or wave-like), the weight coefficients of two components are mutual exclusive as constrained in the normalisation condition.

Modelling wave-particle duality for multiple particles. Result of single particle can be further generalised into the classical system contain multiple particles. For a group of particles (without mutual interaction), the overall action $S_{1\cdots n}$ of the system can be expressed in the superimposed form that manifests certain degree of trajectory-like behaviour and certain degree of wave-like behaviour,

$$S_{1\cdots n} = \sum_{n=1}^{n} S_n = \sum_{n=1}^{n} [\alpha_n(\bar{\boldsymbol{P}}_n \boldsymbol{r} - \bar{H}_n t) + \beta_n(-i\bar{S}\phi_n)] \,, \tag{8}$$

where the subscript n refers to the label of the individual particle. For particles with the same energy and momentum ($\bar{H}_n = \bar{H}, \bar{\boldsymbol{P}}_n = \bar{\boldsymbol{P}}$), the above solution can be simplified as,

$$S_{1\cdots n} = \sum_{n=1}^{n} S_n = \sum_{n=1}^{n} \alpha_n\,(\bar{\boldsymbol{P}}\boldsymbol{r} - \bar{H}t) + \sum_{n=1}^{n} \beta_n\,(-i\bar{S}\phi) \,, \tag{9}$$

where α_{all} and β_{all} refers to the summed coefficients of trajectory-like property and wave-like property, respectively. In this case, the superimposed equation of particles can be found as,

$$\sum_{n=1}^{n} \alpha_n \left(\bar{H} - \frac{\bar{\boldsymbol{P}}^2}{2m}\right) + \sum_{n=1}^{n} \beta_n \left(i\bar{S}\partial_t - \frac{-\bar{S}^2}{2m}\nabla^2\right)\phi = 0 \,. \tag{10}$$

The superimposed equation represents the trajectory-like properties and wave-like properties of the particles in the same group. Unlike the case of single particle that its behaviour is unique, the group of particles can manifest different combination of trajectory-like behaviour and wave-like behaviour depending on the weight coefficients. The weight coefficients of two components are constrained in the normalisation condition $\alpha_n + \beta_n = n$.

Double-slit experiment and which-way measurement. Double-slit experiment is the standard testing of evaluation wave-particle duality of particles. Without losing the generality, we consider the usual double-slit experiment setup of particles travelling from the ideal source and captured by the screen after crossing two identical slits. One of the usual measurement setups of measing the location information of particle is the which-way measurement. However, this measurement will cause the transition of particle from wave-like behaviour to trajectory-like behaviour. In the superimposed model, the which-way measurement can be represented as the degeneration of phase function that is localised $\phi \to 1$ within small area (the detected location). In the model, the phase degeneration will trig the following transition of dynamical variables of single particle,

$$H = \bar{S}\omega\phi \xrightarrow{\text{degen}} \bar{S}\omega = \bar{H} \;;\; \boldsymbol{P} = \bar{S}\boldsymbol{k}\phi \xrightarrow{\text{degen}} \bar{S}\boldsymbol{k} = \bar{\boldsymbol{P}} \,. \tag{11}$$

The above relation represents reduction of the (non-trivial) wave-like solution to the (trivial) trajectory-like solution or equivalently $\alpha = 1$. Although this reduction changes the formal expression of the solution, the whole reduction is within the framework of superimposed model.

As the which-way measurement is acting on multiple particles in the whole system, the action of the overall system transits as the following form,

$$S_{1\cdots n} = \sum_{n=1}^{n} S_n \xrightarrow{\text{degen}} S'_{1\cdots n} = \sum_{n=1}^{n} S'_n \tag{12}$$

and

$$S_{1\cdots n} = \sum_{n=1}^{n} \alpha_n \left(\bar{\boldsymbol{P}}\boldsymbol{r} - \bar{H}t\right) + \sum_{n=1}^{n} \beta_n \left(-i\bar{S}\phi\right) \; ; \; S'_{1\cdots n} = \sum_{n=1}^{n} \alpha'_n \left(\bar{\boldsymbol{P}}\boldsymbol{r} - \bar{H}t\right), \tag{13}$$

where the transition of particles before and after which-way measurement shows that their wave-like properties are suppressed from the analytical solution. Further, the superimposed equation of multiple particles is reduced to the normal trajectory-like form,

$$\sum_{n=1}^{n} \alpha_n \left(\bar{H} - \frac{\bar{\boldsymbol{P}}^2}{2m}\right) + \sum_{n=1}^{n} \beta_n \left(i\bar{S}\partial_t - \frac{-\bar{S}^2}{2m}\nabla^2\right)\phi \xrightarrow{\text{degen}} \sum_{n=1}^{n} \alpha'_n \left(\bar{H} - \frac{\bar{\boldsymbol{P}}^2}{2m}\right), \tag{14}$$

where the weight coefficients satisfy the following conservation relation $\alpha'_n = n$. For the particles that are not exposure to the which-way measurement, the wave-like property is retained in the analytical solution. As the number of the sampled particle is sufficient large, the accumulated pattern of double-slit experiment will be a superposition of wave interference pattern (reflect wave-like property) and Gaussian distribution pattern (reflect trajectory-like property) as illustrated in Fig. 1.

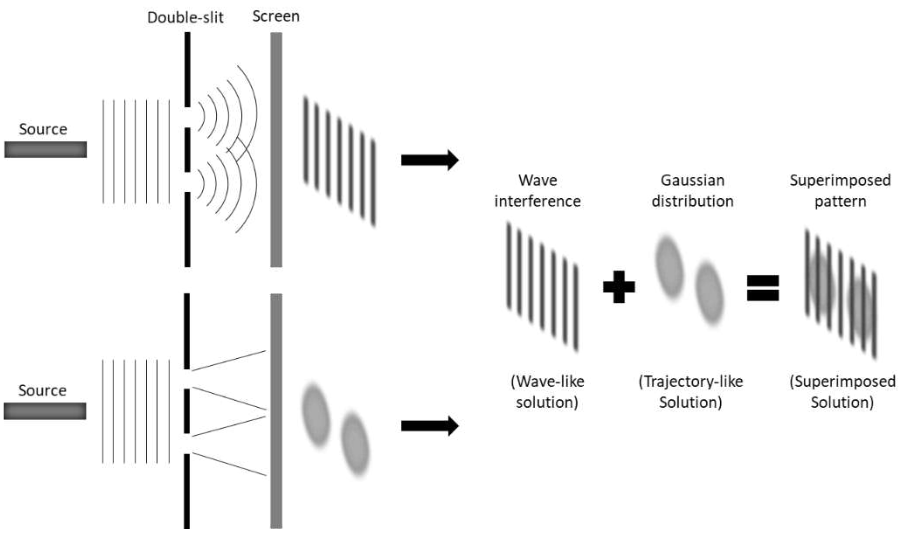

Figure 1: Superposition of Gaussian pattern and interference pattern of double-slit experiment. The upper process on the left refers to the wave-like behaviour of particles that passing through the double-slit. This leads to the wave interference pattern at the screen side. While the lower process on the left refers to the trajectory-like behaviour of particles that passing through the double-slit. This leads to the two distinct Gaussian distribution peaks at the screen side. On the right side, the overall observed pattern can be considered as the

superimposition of the wave interference pattern and Gaussian peaks that contributed from the particles in wave-like solution and trajectory-like solution, respectively.

Modelling wave-particle duality for waves. In this subsection, the superimposed model is extended from classical particle to classical waves. According to the isomorphism of superimposed models in Table 1, the analysis of classical waves will share some similarities as in the analysis of classical particle. In this section, we will borrow similar procedures to construct the superimposed solution of classical waves. As the wave-like property of the classical waves is well-known in literature, we will focus on the trajectory-like property of these classical waves that has not been fully reported. For electromagnetic wave, the following superimposed solution can be found according to the second row in Table 1,

$$\Lambda = \alpha(\bar{\boldsymbol{A}}\boldsymbol{r} - \bar{V}t) + \beta(-i\bar{\Lambda}\phi)\,, \tag{15}$$

where $\bar{V}$ and $\bar{\boldsymbol{A}}$ refer to the magnitude of field potentials, $\bar{\Lambda}$ refers to the magnitude of governing variable. To model the trajectory-like property of the electromagnetic wave, one can let $\alpha = 1$ and this gives,

$$V = -\partial_t \Lambda = \bar{V} \;\; ; \;\; \boldsymbol{A} = \nabla\Lambda = \bar{\boldsymbol{A}} \tag{16}$$

and for $\beta = 1$ it restores the usual wave-like property of the field,

$$V = -\partial_t \Lambda = \bar{\Lambda}\omega\phi \;\; ; \;\; \boldsymbol{A} = \nabla\Lambda = \bar{\Lambda}\boldsymbol{k}\phi\,. \tag{17}$$

It is worth to note that the above trajectory-like solution of electromagnetic wave has clear physical meaning. This point can be shown from the following one-to-one transformation between the field variables and the dynamical variables,

$$q\bar{V} \to \bar{H} \;\; ; \;\; q\bar{\boldsymbol{A}} \to \bar{\boldsymbol{P}}\,, \tag{18}$$

where the transformation presents the energy and momentum based on the field variables of electromagnetic waves and q refers to the electrical charge.

Moreover, the trajectory-like properties of acoustic wave and elastic wave can be also shown by following the similar procedures above (by analogy). Without going into the detail steps, the acoustic variables can be transformed to the dynamical variables through following relation,

$$m\bar{p}\rho^{-1} \to \bar{H} \;\; ; \;\; m\bar{\boldsymbol{v}}_A \to \bar{\boldsymbol{P}}\,, \tag{19}$$

where $\bar{p}$ and $\bar{\boldsymbol{v}}_A$ represent the magnitudes of acoustic pressure and particle velocity, m denotes the mass of acoustic element, ρ denotes the density of material. The elastic variables along longitudinal direction can be transformed to the dynamical variables by the following relations,

$$m\bar{\sigma}_L\rho^{-1} \to \bar{H} \ ; \ m\bar{\boldsymbol{w}}_L \to \bar{\boldsymbol{P}} \tag{20}$$

where $\bar{\sigma}_L$ and $\bar{\boldsymbol{w}}_L$ represent the magnitudes of elastic pressure and particle velocity of longitudinal elastic wave, m denotes the mass of elastic element, ρ denotes the density of material. Similarly, the elastic variables along transverse direction can be transformed to the dynamical variables by the following relations,

$$m\bar{\sigma}_T\rho^{-1} \to \bar{H} \ ; \ m\bar{\boldsymbol{w}}_T \to \bar{\boldsymbol{P}}\,, \tag{21}$$

where $\bar{\sigma}_T$ and $\bar{\boldsymbol{w}}_T$ represent the magnitudes of elastic pressure and particle velocity of transverse elastic wave. The information about the trajectory-like property and wave-like property of classical particle and waves is summarised in Table 2. It can be noted that quantum constraints on energy and momentum not only applied to classical particles but also applied to the classical waves. These constraints reflect the inherent trajectory-like properties of classical waves that have been missing in literature.

Table 2 – Trajectory-like property and wave-like property of classical particle and waves

Superimposed models	Trajectory-like $\alpha = 1$	Wave-like $\beta = 1$	Quantum relations of dynamical and field variables*	
Basic	$\bar{\boldsymbol{Y}}\boldsymbol{r} - \bar{X}t$	$-i\bar{Z}\phi$	$\bar{X} = \bar{Z}\omega$	$\bar{\boldsymbol{Y}} = \bar{Z}\boldsymbol{k}$
Particle	$\bar{\boldsymbol{P}}\boldsymbol{r} - \bar{H}t$	$-i\bar{S}\phi$	$\bar{H} = \hbar\omega$	$\bar{\boldsymbol{P}} = \hbar\boldsymbol{k}$
Electromagnetic wave	$\bar{\boldsymbol{A}}\boldsymbol{r} - \bar{V}t$	$-i\bar{\Lambda}\phi$	$q\bar{V} = \hbar\omega$	$q\bar{\boldsymbol{A}} = \hbar\boldsymbol{k}$
Acoustic wave	$\bar{\boldsymbol{v}}_A\boldsymbol{r} - \rho^{-1}\bar{p}t$	$-i\bar{\chi}_A\phi$	$m\bar{p}\rho^{-1} = \hbar\omega$	$m\bar{\boldsymbol{v}}_A = \hbar\boldsymbol{k}$
Elastic wave (longitudinal)	$\bar{\boldsymbol{v}}_L\boldsymbol{r} - \rho^{-1}\bar{\sigma}_L t$	$-i\bar{\chi}_L\phi$	$m\bar{\sigma}_L\rho^{-1} = \hbar\omega$	$m\bar{\boldsymbol{v}}_L = \hbar\boldsymbol{k}$
Elastic wave (transverse)	$\bar{\boldsymbol{v}}_T\boldsymbol{r} - \rho^{-1}\bar{\sigma}_T t$	$-i\bar{\chi}_T\phi$	$m\bar{\sigma}_T\rho^{-1} = \hbar\omega$	$m\bar{\boldsymbol{v}}_T = \hbar\boldsymbol{k}$

*We have employed the substitution relation $\bar{S} \to \hbar$ in the table.

Quantum-like interacting of classical particle and waves. The quantised energy and momentum carried by classical waves brings interesting interaction picture between particle and waves. As an example, we consider the simple scenario that particle absorbs classical waves before and after interaction. This process is governed by the following conservation relations of energy and momentum,

$$\bar{H} = \bar{H}' - \bar{H}_{EM} - \bar{H}_{AC} - \bar{H}_{EL} \ ; \ \bar{\boldsymbol{P}} = \bar{\boldsymbol{P}}' - \bar{\boldsymbol{P}}_{EM} - \bar{\boldsymbol{P}}_{AC} - \bar{\boldsymbol{P}}_{EL}\,, \tag{22}$$

where $\bar{H}_{EM}, \bar{H}_{AC}, \bar{H}_{EL}$ refers to the energy carries by the electromagnetic wave, acoustic wave, elastic wave and $\bar{\boldsymbol{P}}_{EM}, \bar{\boldsymbol{P}}_{AC}, \bar{\boldsymbol{P}}_{EL}$ refers to the momentum carries by the electromagnetic wave, acoustic wave, elastic wave. From the equalities in Eq. (6) and differential operators in Eq. (5), the above relations give the following dynamical operators of interacted particle,

$$i\hbar\partial_t\phi = (i\hbar\partial_t - \bar{H}_{EM} - \bar{H}_{AC} - \bar{H}_{EL})\phi' \,;\; -i\hbar\nabla\phi = (-i\hbar\nabla - \bar{\boldsymbol{P}}_{EM} - \bar{\boldsymbol{P}}_{AC} - \bar{\boldsymbol{P}}_{EL})\phi'. \quad (23)$$

By replacing the dynamical operators from Eq. (23) into Eq. (7), we further show the superimposed model of interacting classical particle and waves in the following form,

$$\alpha\left[\bar{H}' - \bar{H}_{EM} - \bar{H}_{AC} - \bar{H}_{EL} - \frac{1}{2m}(\bar{\boldsymbol{P}}' - \bar{\boldsymbol{P}}_{EM} - \bar{\boldsymbol{P}}_{AC} - \bar{\boldsymbol{P}}_{EL})^2\right] + \beta\left[i\hbar\partial_t - \frac{1}{2m}(-i\hbar\nabla - \bar{\boldsymbol{P}}_{EM} - \bar{\boldsymbol{P}}_{AC} - \bar{\boldsymbol{P}}_{EL})^2 + \bar{H}_{EM} + \bar{H}_{AC} + \bar{H}_{EL}\right]\phi' = 0\,, \quad (24)$$

where the first term describes the trajectory-like property while the second term describes the interaction between particle and waves in term of dynamical operators and field variables.

The above superimposed equation generalises the free superimposed equation in Eq. (7). It provides a clear quantum picture of interacting classical particle and waves. Especially, although the interaction (energy and momentum exchange) between particle (atom, electron) and electromagnetic wave (photon) is well-established, the similar results on acoustic wave and elastic wave have not been reported in literature. The above result suggests the energy and momentum exchange between classical particle and acoustic (or elastic) waves manifest similar mode as in electromagnetic wave. One the one hand, the similarity of the interaction mode reveals the opportunities of replacing electromagnetic (optical) wave by acoustic waves in air or fluid and elastic waves in solid state materials. This will contribute to many recent techniques on particle control and manipulation based on acoustic wave and elastic wave [22-26]. On the other hand, the quantum constraints on classical acoustic and elastic wave create distinct features (discretised energy and momentum) that can be beneficial for development of precision measurement techniques and devices [27, 28].

Discussion

From the results, it is shown that the trajectory-like property and wave-like property is related to the solution of dynamical variables and field variables in their phase-independent form and phase dependent form. Fig. 2 shows the relationship between the wave-particle duality of classical particles and waves from the aspect of superimposed models. Since the both phase-independent solution and phase-dependent solution are admissible, the two parts of solution reflect the dual aspects of classical particles and waves during their propagation and interaction. In the propagation, the classical particles and waves manifest phase-dependent behaviour while in the interaction they manifest phase-independent behaviour. Usually, the analysis of classical particles only considered its trajectory-like property while the analysis of classical waves only considered their wave-like property (the shaded boxes in Fig. 2). However, the present analysis on classical particle and waves revealed that both trajectory-like property and wave-like property are their inherent properties. This is due to the fact that the dynamical variables and field variables that characterise the states of classical particle and waves carrier the above properties. Consequently, the trajectory-like property and wave-like property reflect the dual aspects of inherent features of classical particle and waves.

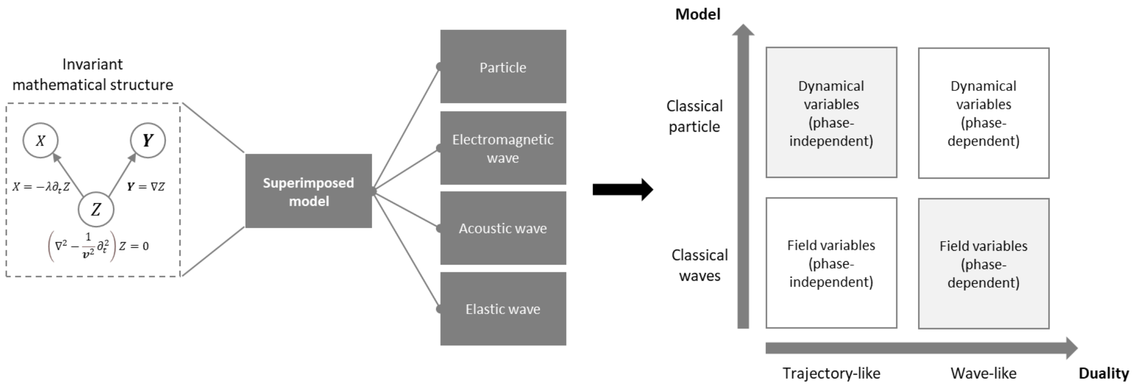

Figure 2: Relation between wave-particle duality and superimposed models. The left part of the figure denotes the superimposed model that is characterised via physical variables and differential equations in an invariant mathematical structure. The right part of the figure refers to the quadrant between models and duality behaviour. Phase-independent solutions of dynamical variables and field variables represent the trajectory-like property of classical particle and waves, respectively. Phase-dependent solutions of dynamical variables and field variables represent the wave-like property of particle and classical waves, respectively. Both trajectory-like property and wave-like property of dynamical or field variables are admissible of superimposed models of classical particle and waves.

Furthermore, inherent features of trajectory-like property and wave-like property of classical particle and waves narrow the gap between classical and quantum domains. The quantum effect can influence the macroscopic system in different ways. Quantised energy and momentum provide inherent constraints on the smallest magnitudes of the energy and momentum that carried by classical waves. However, these constraints are known for electromagnetic waves but are frequently ignored in the studies of acoustic waves and elastic waves. As the size of classical system reduced continuously from macroscopic scale to microscopic scale, the effect due to the inherent wave-like property should become influential [27]. In this trend, the governing rules of classical system transient from trajectory-like dominated behaviour to wave-like dominated behaviour. In this view, the behaviour of the classical system is not only governed by the continuum-mechanical relations as already reported in literature [29]. Its behaviour is also subjected to the quantum-mechanical relations that is worth to be considered.

In summary, the formulated superimposed models well represent the indispensable trajectory-like property and wave-like property of classical particle and waves. It is shown that how the two dual properties are emerged as the inherent features of classical particle and waves in the superimposed model. Through the analysis, it reveals the wave-like property of classical particle and trajectory-like property of classical waves. These results enhance the integrity view on the classical particle and wave in terms of their dual properties. Additionally, it is found that the employment of invariant mathematical structure (isomorphic models) can greatly facilitate the study of duality in the analogous superimposed models. It is the author's opinion that the existence of such invariance maybe not a (trivial) coincidence. Through characterising the distinct physical objects via analogous variables and equations, the isomorphism of their models may not just refer to the theoretical (mathematical) analogies but also reflect actual (physical) connections between them, such as duality.

Methods

Superimposed model of classical particle. In classical mechanics, we considered the balanced equation of energy and momentum can be represented in following form,

$$\dot{H} = \nabla H v + \frac{\partial H}{\partial \boldsymbol{P}} \dot{\boldsymbol{P}} + \partial_t H = 0 \;\; ; \;\; \dot{\boldsymbol{P}} = \nabla \boldsymbol{P} \boldsymbol{v} + \frac{\partial \boldsymbol{P}}{\partial \boldsymbol{v}} \dot{\boldsymbol{v}} + \partial_t \boldsymbol{P} = \boldsymbol{0} \, , \tag{25}$$

and action functional (Hamiltonian form) [30] is usually given as,

$$S = \int L \, dt = \int \boldsymbol{P} \, d\boldsymbol{r} - \int H \, dt \, , \tag{26}$$

where L denotes Lagrange, H denotes Hamiltonian, $\boldsymbol{P}$ denotes momentum and $\boldsymbol{v}$ refers to velocity of particle. By resolving the dynamical variables in Eq. (25) by action S in Eq. (26) and using the smoothness function condition $\partial_t \nabla S - \nabla \partial_t S = 0$, it gives the following differential equations in term of action functional [31],

$$\left(\nabla^2 - \frac{1}{\boldsymbol{v}^2} \partial_t^2 \right) S = 0 \;\; ; \;\; H = -\partial_t S \;\; ; \;\; \boldsymbol{P} = \nabla S \, . \tag{27}$$

The model of classical particle carrier the same type of mathematical structure as in the superimposed model ($\gamma = -1$). In the model of classical particle, the governing variables is the action functional, dependent variable are Hamiltonian (energy) and momentum.

Superimposed model of electromagnetic wave. In classical electrodynamics, the propagating electromagnetic wave are governed by the equations of electrical potential $\boldsymbol{V}$ and magnetic potential $\boldsymbol{A}$ (defined with respect to zero) in the following form [32],

$$\left(\nabla^2 - \frac{1}{\boldsymbol{c}^2} \partial_t^2 \right) V = 0 \;\; ; \;\; \left(\nabla^2 - \frac{1}{\boldsymbol{c}^2} \partial_t^2 \right) \boldsymbol{A} = \boldsymbol{0} \;\; ; \;\; \boldsymbol{c} = \sqrt{(u_0 \epsilon_0)^{-1}} \, , \tag{28}$$

where u_0 denotes the vacuum permeability, ϵ_0 denotes vacuum permittivity, and $\boldsymbol{c}$ denotes the velocity of the electromagnetic wave. We introduce a new scalar variable (serve as the governing variable but not necessarily to be the gauge function). By representing the equations in Eq. (28) by this variable, it leads to following set of differential equations of the electromagnetic variables [33],

$$\left(\nabla^2 - \frac{1}{\boldsymbol{c}^2} \partial_t^2 \right) \mathit{\Lambda} = 0 \;\; ; \;\; V = -\partial_t \mathit{\Lambda} \;\; ; \;\; \boldsymbol{A} = \nabla \mathit{\Lambda} \, , \tag{29}$$

where Λ refers to the governing variable of electromagnetic wave. It is seen that the model of classical electromagnetic wave carrier the same type of mathematical structure as in the superimposed model ($\gamma = -1$). In the model of classical electromagnetic wave, the dependent variables are electrical potential and magnetic potential.

Superimposed model of acoustic wave. In classical acoustics, the propagating acoustic wave are governed by the equations of acoustic pressure $\boldsymbol{p}$ and (medium) particle velocity $\boldsymbol{v_A}$ in the following form [34],

$$\left(\nabla^2 - \frac{1}{\boldsymbol{c}_A^2}\partial_t^2\right)p = 0 \quad ; \quad \left(\nabla^2 - \frac{1}{\boldsymbol{c}_A^2}\partial_t^2\right)\boldsymbol{v}_A = 0 \quad ; \quad \boldsymbol{c}_A = \sqrt{B\rho^{-1}}\,, \tag{31}$$

where B denotes the bulk modulus and ρ denotes the density of material. Here, we introduce the new governing variable and represent the model into the form like superimposed model. By employing the procedures are similar to the case of electromagnetic wave, it leads to following differential equations of the acoustic wave,

$$\left(\nabla^2 - \frac{1}{\boldsymbol{c}^2}\partial_t^2\right)\varphi = 0 \quad ; \quad p = -\rho\partial_t\varphi \quad ; \quad \boldsymbol{v}_A = \nabla\varphi\,, \tag{32}$$

where φ refers to the governing variable of acoustic wave. The model of classical particle carrier the same type of mathematical structure as in the superimposed model ($\gamma = -\rho$).

Superimposed model of elastic wave. In classical elasticity, the elastic wave is governed by Navier-Lamé equation in terms of elastic displacement [35],

$$\rho_E\partial_t^2\boldsymbol{u} = (\lambda + 2\mu)\nabla(\nabla\cdot\boldsymbol{u}) - \mu\nabla\times\nabla\times\boldsymbol{u}\,, \tag{33}$$

where $\boldsymbol{u}$ denotes the elastic deformation (displacement), ρ refers to the material density, λ and μ refer to the Lamé constants. Based on Helmholtz decomposition, the total elastic displacement can be expressed into the following two components,

$$\boldsymbol{u} = \nabla\Phi + \nabla\times\boldsymbol{\Psi} = \boldsymbol{u}_L + \boldsymbol{u}_T \tag{34}$$

where Φ denotes certain scalar function, $\boldsymbol{\Psi}$ denotes certain vector function, $\boldsymbol{u}_L$ denotes the longitudinal elastic displacement and $\boldsymbol{u}_T$ denotes transverse elastic displacement. Using the above substitution of elastic displacement, it gives the following coupled equation,

$$\{\nabla[(\lambda + 2\mu)(\nabla^2\Phi) - \rho\partial_t^2\Phi]\} + \{\nabla\times[\mu\nabla^2\boldsymbol{\Psi} - \rho\partial_t^2(\nabla\times\boldsymbol{\Psi})]\} = \boldsymbol{0}\,. \tag{35}$$

In the weak coupling condition, the above equation can be simplified into the decoupled equations in terms of the longitudinal elastic displacement and transverse elastic displacement,

$$\left(\nabla^2 - \frac{1}{\boldsymbol{c}_L^2}\partial_t^2\right)\boldsymbol{u}_L = \mathbf{0} \quad ; \quad \boldsymbol{c}_L = \sqrt{(\lambda + 2\mu)\rho^{-1}} \tag{36}$$

and

$$\left(\nabla^2 - \frac{1}{\boldsymbol{c}_T^2}\partial_t^2\right)\boldsymbol{u}_T = \mathbf{0} \quad ; \quad \boldsymbol{c}_T = \sqrt{\mu\rho^{-1}}\,, \tag{37}$$

where $\boldsymbol{c}_L$ denotes the velocity of the longitudinal elastic wave and $\boldsymbol{c}_T$ denotes the velocity of transverse elastic wave. We introduce a pair of new governing variables χ_L and χ_T of longitudinal and transverse elastic waves, respectively. The equations of elastic displacements in Eqs. (36) and (37) can be represented by the governing variables via applying spatial and temporal differential on both sides. The final obtained differential equations between governing variables, elastic stress and particle velocity can be found in the following,

$$\left(\nabla^2 - \frac{1}{\boldsymbol{c}_L^2}\partial_t^2\right)\chi_L = 0 \quad ; \quad \sigma_L = -\rho\partial_t\chi_L \quad ; \quad \boldsymbol{v_L} = \nabla\chi_L \tag{38}$$

and

$$\left(\nabla^2 - \frac{1}{\boldsymbol{c}_T^2}\partial_t^2\right)\chi_T = 0 \quad ; \quad \sigma_T = -\rho\partial_t\chi_T \quad ; \quad \boldsymbol{v_T} = \nabla\chi_T\,. \tag{39}$$

where σ_L denotes the longitudinal elastic stress, $\boldsymbol{v}_L$ denotes the longitudinal particle velocity, σ_T denotes the transverse elastic stress and $\boldsymbol{v_T}$ denotes the transverse particle velocity. The two decoupled model of classical elastic waves carrier the same type of mathematical structure as in the superimposed model ($\gamma = -\rho$).

Formulation of superimposed model. According to the derived models of classical particle and waves, these models manifest certain invariant mathematical structure that is specified by analogous physical variables and differential equations. Among these models, a scalar variable (named governing variable) is explicitly governed by the homogenous wave equation and its partial differentiations with respect to space and time defines the other pair of variables (named dependent variables). From their invariant structure, it is shown that these models form an isomorphism set in terms of their physical variables and differential equations. Therefore, the structure of the analytical solution is also formal invariant through the one-to-one transformation from one model to another. Without losing the generality, we can formulate an basic (kernel) model that carrier the invariant mathematical structure as the models in the above isomorphism set. By studying the solution structure of this model, we can connect the result into the models of classical particle and waves in the isomorphism set through one-to-one transformations.

References

[1] Bassi, Angelo, Kinjalk Lochan, Seema Satin, Tejinder P. Singh, and Hendrik Ulbricht. 2013. “Models of Wave-Function Collapse, Underlying Theories, and Experimental Tests.” Reviews of Modern Physics 85 (2): 471–527.

[2] Carnal, O., and J. Mlynek. 1991. “Young’s Double-Slit Experiment with Atoms: A Simple Atom Interferometer.” Physical Review Letters 66 (21): 2689–92.
[3] Kolesov, Roman, Bernhard Grotz, Gopalakrishnan Balasubramanian, Rainer J. Stöhr, Aurélien A. L. Nicolet, Philip R. Hemmer, Fedor Jelezko, and Jörg Wrachtrup. 2009. “Wave–particle Duality of Single Surface Plasmon Polaritons.” Nature Physics 5 (7): 470–74.
[4] Rab, Adil S., Emanuele Polino, Zhong-Xiao Man, Nguyen Ba An, Yun-Jie Xia, Nicolò Spagnolo, Rosario Lo Franco, and Fabio Sciarrino. 2017. “Entanglement of Photons in Their Dual Wave-Particle Nature.” Nature Communications 8 (1): 915–915.
[5] Arndt, Markus, Olaf Nairz, Julian Vos-Andreae, Claudia Keller, Gerbrand van der Zouw, and Anton Zeilinger. 1999. “Wave–particle Duality of C 60 Molecules.” Nature 401 (6754): 680–82.
[6] Gerlich, Stefan, Sandra Eibenberger, Mathias Tomandl, Stefan Nimmrichter, Klaus Hornberger, Paul J. Fagan, Jens Tüxen, Marcel Mayor, and Markus Arndt. 2011. “Quantum Interference of Large Organic Molecules.” Nature Communications 2 (1): 263–263.
[7] Juffmann, Thomas, Adriana Milic, Michael Müllneritsch, Peter Asenbaum, Alexander Tsukernik, Jens Tüxen, Marcel Mayor, Ori Cheshnovsky, and Markus Arndt. 2012. “Real-Time Single-Molecule Imaging of Quantum Interference.” Nature Nanotechnology 7 (5): 297–300.
[8] Couder, Y., S. Protière, E. Fort, and A. Boudaoud. 2005. “Dynamical Phenomena: Walking and Orbiting Droplets.” Nature 437 (7056): 208–208.
[9] Couder, Yves, and Emmanuel Fort. 2006. “Single-Particle Diffraction and Interference at a Macroscopic Scale.” Physical Review Letters 97 (15): 154101.
[10] Eddi, A., E. Fort, F. Moisy, and Y. Couder. 2009. “Unpredictable Tunneling of a Classical Wave-Particle Association.” Physical Review Letters 102 (24): 240401.
[11] Fort, Emmanuel, Antonin Eddi, Arezki Boudaoud, Julien Moukhtar, and Yves Couder. 2010. “Path-Memory Induced Quantization of Classical Orbits.” Proceedings of the National Academy of Sciences of the United States of America 107 (41): 17515–20.
[12] Couder, Y, and E Fort. 2012. “Probabilities and Trajectories in a Classical Wave-Particle Duality.” Journal of Physics: Conference Series 361 (1): 12001.
[13] Perrard, Stéphane, Matthieu Labousse, Marc Miskin, Emmanuel Fort, and Yves Couder. 2014. “Self-Organization into Quantized Eigenstates of a Classical Wave-Driven Particle.” Nature Communications 5 (1): 3219.
[14] Sleutel, Pascal, Erik Dietrich, Jan T. Van Der Veen, and Wouter R. Van Joolingen. 2016. “Bouncing Droplets : A Classroom Experiment to Visualize Wave-Particle Duality on the Macroscopic Level.” European Journal of Physics 37 (5): 55706.
[15] Griffiths, David Jeffrey. 1960. Introduction to Quantum Mechanics.
[16] Messiah, Albert. 1961. Quantum Mechanics.
[17] Park, David Allen. 1964. Introduction to the Quantum Theory.
[18] Bohm, D. 1952. “A Suggested Interpretation of The Quantum Theory In Terms of ‘Hidden’ Variables. I.” Physical Review.
[19] Bohm, D. 1952. “A Suggested Interpretation of The Quantum Theory In Terms of ‘Hidden’ Variables. II.” Physical Review.
[20] Bush, John W. M. 2015. “Pilot-Wave Hydrodynamics.” Annual Review of Fluid Mechanics 47 (1): 269–92.

[21] Duerr, Detlef, Sheldon Goldstein, Roderich Tumulka, and Nino Zanghi. 2009. "Bohmian Mechanics."
[22] Ashkin, A., J. M. Dziedzic, J. E. Bjorkholm, and Steven Chu. 1986. "Observation of a Single-Beam Gradient Force Optical Trap for Dielectric Particles." Optics Letters 11 (5): 288–90.
[23] Shi, Jinjie, Daniel Ahmed, Xiaole Mao, Sz-Chin Steven Lin, Aitan Lawit, and Tony Jun Huang. 2009. "Acoustic Tweezers: Patterning Cells and Microparticles Using Standing Surface Acoustic Waves (SSAW)." Lab on a Chip 9 (20): 2890–95.
[24] Anhäuser A, Wunenburger R, Brasselet E. Acoustic rotational manipulation using orbital angular momentum transfer. Physical review letters. 2012;109(3):034301.
[25] Marzo, Asier, Sue Ann Seah, Bruce W. Drinkwater, Deepak Ranjan Sahoo, Benjamin Long, and Sriram Subramanian. 2015. "Holographic Acoustic Elements for Manipulation of Levitated Objects." Nature Communications 6 (1): 8661–8661.
[26] Ahmed D, Ozcelik A, Bojanala N, Nama N, Upadhyay A, Chen Y, Hanna-Rose W, Huang TJ. Rotational manipulation of single cells and organisms using acoustic waves. Nature communications. 2016;7(1):1-1.
[27] Poot M, van der Zant HS. Mechanical systems in the quantum regime. Physics Reports. 2012;511(5):273-335.
[28] Arrangoiz-Arriola, Patricio, E Alex Wollack, Zhaoyou Wang, Marek Pechal, Wentao Jiang, Timothy P McKenna, Jeremy D Witmer, Raphaël Van Laer, and Amir H Safavi-Naeini. 2019. "Resolving the Energy Levels of a Nanomechanical Oscillator." Nature 571 (7766): 537–40.
[29] Juvé, Vincent, Aurélien Crut, Paolo Maioli, Michel Pellarin, Michel Broyer, Natalia Del Fatti, and Fabrice Vallée. 2010. "Probing Elasticity at the Nanoscale: Terahertz Acoustic Vibration of Small Metal Nanoparticles." Nano Letters 10 (5): 1853–58.
[30] Lanczos C. The variational principles of mechanics. Courier Corporation; 2012 Apr 24.
[31] Fung YC, Tong P, Chen X. Classical and computational solid mechanics. World Scientific Publishing Company; 2017.
[32] Yang C 2016 Wave-like variables of a classical particle and their connections to quantum mechanics. European Journal of Physics 38:015401.
[33] Jackson JD. Classical electrodynamics. John Wiley & Sons; 2007.
[34] Yang C 2018 Analogy between electromagnetic potentials and wave-like dynamic variables with connections to quantum theory. European Journal of Physics 39:035406.
[35] Love AE. A treatise on the mathematical theory of elasticity. Cambridge university press; 2013.

Figures

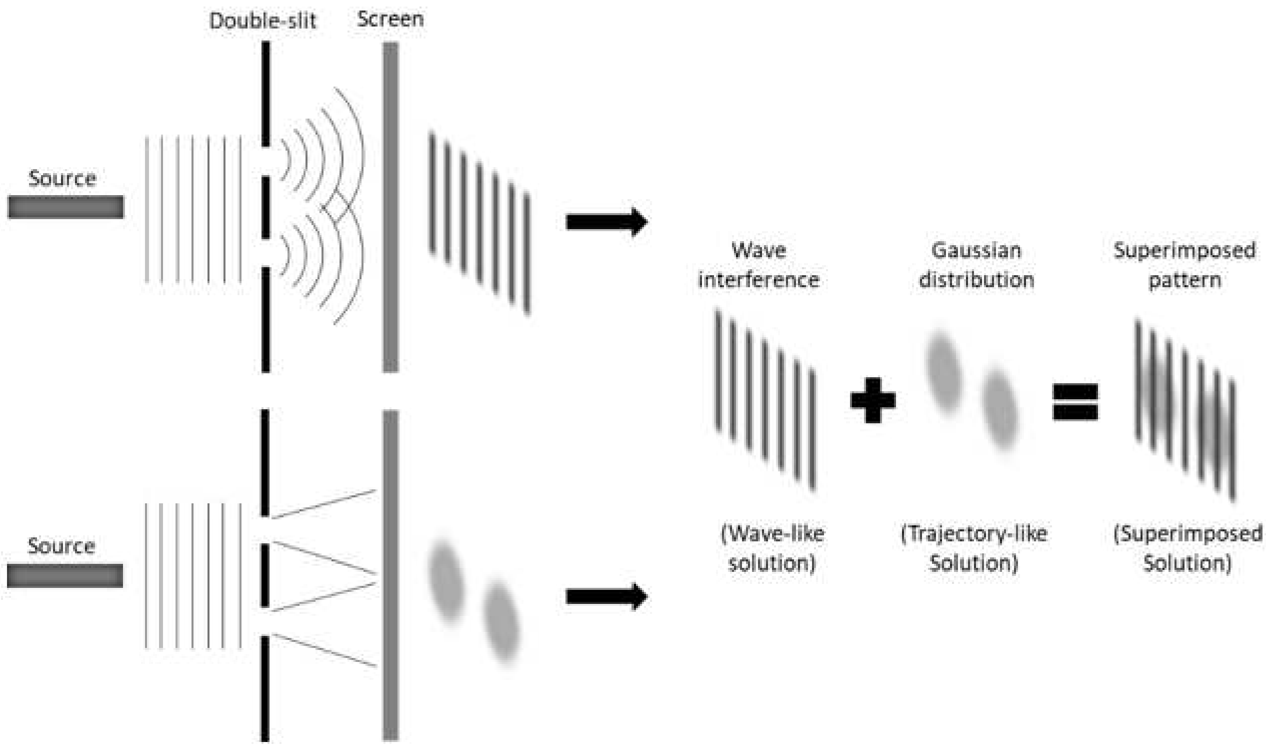

Figure 1

Superposition of Gaussian pattern and interference pattern of double-slit experiment. The upper process on the left refers to the wave-like behaviour of particles that passing through the double-slit. This leads to the wave interference pattern at the screen side. While the lower process on the left refers to the trajectory-like behaviour of particles that passing through the double-slit. This leads to the two distinct Gaussian distribution peaks at the screen side. On the right side, the overall observed pattern can be considered as the superimposition of the wave interference pattern and Gaussian peaks that contributed from the particles in wave-like solution and trajectory-like solution, respectively.

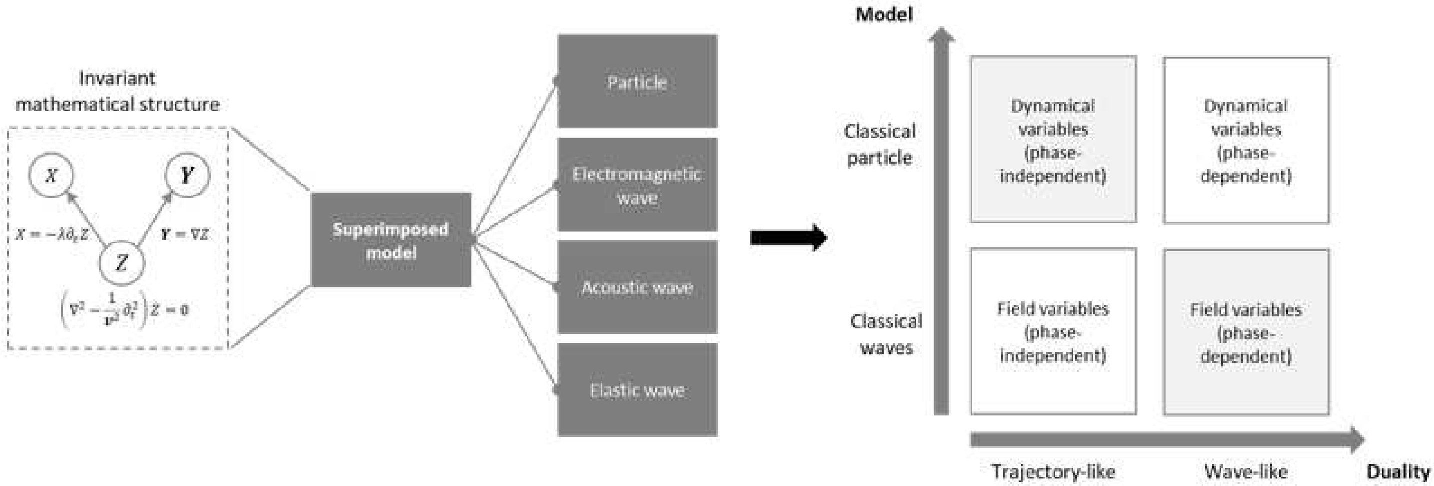

Figure 2

Relation between wave-particle duality and superimposed models. The left part of the figure denotes the superimposed model that is characterised via physical variables and differential equations in an invariant mathematical structure. The right part of the figure refers to the quadrant between models and duality behaviour. Phase-independent solutions of dynamical variables and field variables represent the trajectory-like property of classical particle and waves, respectively. Phase-dependent solutions of dynamical variables and field variables represent the wave-like property of particle and classical waves, respectively. Both trajectory-like property and wave-like property of dynamical or field variables are admissible of superimposed models of classical particle and waves.