

Grouping Strategies in Numerosity Estimation between Intrinsic and Extrinsic Grouping Cues

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Abstract

The number of items in an array can be quickly and accurately estimated by dividing the array into subgroups, in a strategy termed “groupitizing.” When attempting to remember a telephone number, it is better to do so by dividing the number into several segments. Different forms of visual grouping can affect the accuracy of the enumeration of a large set of items. Previous studies have found that when groupitizing, enumeration accuracy is improved by grouping arrays using visual proximity and color similarity. Based on Gestalt theory, Palmer (1992) divided perceptual grouping into intrinsic (e.g., proximity, similarity) and extrinsic (e.g., connectedness, common region) principles. Studies have investigated groupitizing effects of intrinsic grouping. However, to the best of our knowledge, no previous study has explored groupitizing effects for extrinsic grouping cues. Therefore, this study explored whether extrinsic grouping cues differed from intrinsic grouping cues for groupitizing effects in numerosity estimation. The results showed that both extrinsic and intrinsic grouping cues improved enumeration accuracy. The extrinsic grouping cues were more accurate in terms of sensory precision of the numerosity estimation than were the intrinsic grouping cues.

Introduction

Numerosity perception is a quantitative attribute of entities that is an important dimension of nature. For example, one tree produces more fruits than another, and there are more sheep in one territory than in another. In the process of understanding and adapting to nature, humans and animals have gradually evolved in their ability to perceive numerosity¹⁻³.

Three strategies are usually used in numerosity perception: subitizing, counting, and estimation^{4,5}. For a small number of clusters (usually less than 4), humans can quickly and accurately determine the number of items in the clusters. This is called “subitizing,” which indicates that the items can be understood immediately, without thinking^{4,6,7}. With an increasing number of items (greater than 4), the time required to determine the number of items also increases correspondingly while requiring the coordination of many visual and spatial operations, as the observer determines the number of objects by counting^{8,9}. However, when the number of items in clusters is large and cannot be counted in a very short time (for example, the presentation time of the stimulus is short), numerosity perception may be inaccurate, pursuant to Weber’s Law, the approximate number system (ANS) can be relied upon¹⁰. People thus use estimation to determine the approximate number of objects¹⁰⁻¹².

Beyond counting and subitizing, recent studies have found that arrays visually divided into sub-groups can be enumerated faster and more accurately than ungrouped arrays; this is called “groupitizing”^{4,13-17}. Many studies have begun to explore the mechanisms of groupitizing. For example, Starkey and McCandliss (2014) found that groupitizing is positively correlated with the arithmetic ability of children and adults, indicating that the grouping ability may reflect the arithmetic strategies¹³. Moscoso et al. (2020) suggested that groupitizing is a process based on attention, which depends on the subitizing

system, and mathematical ability is correlated with groupitizing¹⁵. Ciccione and Dehaene (2020) indicated that in groupitizing, subjects use mental multiplication and mental addition to increase speed and accuracy of enumeration⁴. Wege et al. (2021) provided an explanation of how the numerical information needed for a mental calculation is extracted from grouped arrays, and suggested that the parallel subitizing of dots and groups in grouped arrays may represent the enumeration processes necessary for groupitizing via mental multiplication¹⁸.

The groupitizing effect must be based on visual grouping. Research into visual grouping processes and perceptual organization developed against the background of Gestalt theory approximately one-hundred years ago^{19–21}. Wertheimer (1923) proposed the main principles of perceptual grouping, which specified which regions of images constituted objects or perceptual units, such as similarity, proximity, symmetry, good continuity, connectedness, or common region²¹. Based on Gestalt theory, Palmer made an important distinction between intrinsic and extrinsic grouping principles. Like most classical Gestalt principles, intrinsic principles are based on the inherent relationships among attributes of grouped elements (e.g., color, shape, size, position). In contrast, the extrinsic principles are based on relationships among elements and other extrinsic elements that induce them to group^{22–25}.

Previous studies of groupitizing only involved intrinsic grouping cues (color similarity^{4, 14} and proximity^{4, 13–15}). To date, no research has explored numerosity estimation with extrinsic grouping cues. Thus, this study explored whether extrinsic grouping cues are different from intrinsic grouping cues in numerosity estimation. Previous studies have found that extrinsic grouping cues have advantages over intrinsic grouping cues^{22, 26–28}. Luna et al. suggested that observers respond more quickly to extrinsic than to other grouping cues^{22, 29}. Quinn and Bhatt reported that young infants (3–4 months old) are sensitive to extrinsic cues, especially common region and connectedness^{22, 29}. Therefore, we hypothesized that extrinsic grouping cues would be advantageous in numerosity estimation.

In addition, vision research has revealed that shape is crucial for object recognition^{30–33}. In the absence of other visual information, it is easy for humans to use shapes to identify objects. Human adults and children prefer to classify new objects according to their shape, given conflicting color and texture cues^{30, 33, 34}. Accordingly, in this study, a shape similarity cue was added to the intrinsic grouping cues to verify whether the shape similarity grouping cues have different effects than other intrinsic grouping cues (i.e., color similarity and proximity).

Methods

Participants

Fifty-three freshman college students (mean age = 19 years, standard deviation = 2.4, range = 18–22) with normal (or corrected-to-normal) vision, and no color blindness were selected. We replicated the experiment in three groups of participants with low, medium, or high levels of math knowledge (for a

similar approach, see Dehaene et al., 2020)⁴. At the highest level, we tested 16 science students majoring in mathematics, all of whom had scored more than 120 points on China's mathematics college entrance examination in 2020. For the medium level, we tested 18 humanities students with math scores between 60 and 90 in the college entrance examination (the mathematics component of the college entrance examination for science is more difficult than for humanities. The maximum score for mathematics is 150). We tested 19 students in the low-level group. They were sports students who had never taken university level exams in mathematics or related disciplines. The third group had much more limited familiarity with mathematics (they had not been taught mathematics for at least one year).

Materials and Procedure

Stimuli were presented using E-Prime 2.0. Participants sat in a quiet and dimly light room, 60 cm from a screen monitor (60 Hz). At the beginning of each trial, a fixation point was presented in the center of the screen and remained on the screen throughout the experiment. After 500 ms, a stimulus was displayed for 500 ms, followed by a screen with an input box. Participants estimated the number of stimuli present and entered the estimated result into the input box as quickly and accurately as possible using a numeric keypad (Fig. 1A). Response time was measured from stimulus offset to when the input box was present. Each condition was tested in separate blocks, and participants were never explicitly informed of the grouping cues.

Stimuli

All stimuli were distributed in a $6^\circ \times 6^\circ$ square grid, which consisted of 144 small squares, where each square was $0.4^\circ \times 0.4^\circ$, and the array was placed at the intersection of the grids, so that each item had 121 possible positions (Fig. 1B). We tested all numerosities between 5 and 17, so that there were 13 different numerosities. In the grouping conditions, each numerosity was divided into 2–4 subgroups, and each subgroup contained between 2 and 6 items, configurations were as following: 2, 2, 1; 3, 3; 3, 3, 1; 2, 2, 2; 3, 3, 3; 3, 3, 3, 1; 3, 3, 3, 2; 3, 3, 3, 3; 5, 5, 3; 4, 4, 3, 3; 4, 4, 4, 3; 4, 4, 4, 4; 5, 4, 4, 4. The participants were clearly informed of the range of the numerosity. Each participant completed 390 trials in total, with each numerosity presented in 30 times.

Extrinsic cues

Extrinsic cues included connectedness and common region.

Connectedness

In the connectedness conditions, the stimuli were sets of white squares ($0.4^\circ \times 0.4^\circ$) with black borders randomly distributed in the grid. The squares within subgroups were connected by a black line, with the connection at the center of the square. In the no-grouping condition, there was no black line connection, and each item was randomly distributed in the large grid (Fig. 1B).

Common region

In the common region conditions, stimuli were also sets of white squares ($0.4^\circ \times 0.4^\circ$) with black borders. The grid was divided into four quadrants, and the squares of each subgroup were randomly distributed inside the small square boxes ($2.5^\circ \times 2.5^\circ$) in the four quadrants. For example, Fig. 1B is a 3, 3, 3 group with only three subgroups, so there are only three boxes. In the no-grouping condition, there were no small square boxes, and each item was randomly distributed in a large grid (Fig. 1B).

Intrinsic cues

Intrinsic cues included color similarity, shape similarity, and proximity.

Color similarity

The color similarity conditions were the same as those used by Anobile et al. (2020)¹⁴. Individual items ($0.4^\circ \times 0.4^\circ$) could be red, blue, yellow, or green, (RGB: 255 0 0; 0 0 255; 255 255 0; 0 255 0, respectively). Colors was arranged from left to right, so that similar colors appeared in a vertical column (see Fig. 1C for a 3, 3, 3 group), where squares were first randomly arranged, then the first three squares were colored red (from the left to right), the next three blue, and the remaining three yellow (colors were randomly selected for each group). In the no-grouping condition, positions of the squares were arranged with the same logic, but the colors were randomly assigned.

Shape similarity

The shape similarity condition was similar to the color similarity condition. The only difference was that the four colors were replaced by four shapes: square, circle, triangle, and diamond; all shapes were $0.4^\circ \times 0.4^\circ$, and white with black borders (Fig. 1C).

Proximity

The proximity conditions were the same as those used by Anobile et al. (2020)¹⁴. Stimuli were arranged into four possible groups of 12 possible positions. Each group (spanning a maximum area of $4^\circ \times 2^\circ$) was located in the same quadrant and centered at 5° from the central fixation point. Each group was first randomly assigned to one quadrant (between 1 and 4); then, the individual item positions were randomly selected between one of the 12 positions in the selected quadrant. Within each quadrant, the maximum center-to-center distance between each element was 4° , and the minimum was 1° . In the no-grouping condition, each item was randomly distributed in the large grid (Fig. 1C).

Data analysis

We adopted a $2 \times 5 \times 13$ repeated-measures ANOVA that included grouping condition (grouping vs. no-grouping), grouping cue (connectedness, common region, color similarity, shape similarity, proximity), and number (5–17) as within-subjects factors. Math knowledge (high, medium, and low) was a between-subjects factor.

We excluded trials with RTs more than three standard deviations from the average reaction time. The median response times for correct answers were computed for each subject. Precision was measured by

the coefficient of variation (CV), which is a dimensionless precision index that allows cross-numerical comparison of average performance.

$$CV = \frac{i}{N_i} \quad (1)$$

N_i is the analyzed numerosity, and i is the standard deviation of the responses to numerosity i .

Data were analyzed by repeated measures ANOVA, with effect sizes reported as η^2 , using JASP and SPSS. In addition, we used Bayesian ANOVA inference for additional analysis, because quantifying evidence in favor of both difference and equality was crucial for testing our hypotheses (Wagenmakers et al., 2018). We report the Bayes factors in favor of the alternative (BF_{10}). A BF_{10} larger than 1 indicated evidence supporting the alternative hypothesis, and a BF_{10} less than 1 indicated evidence for the null hypothesis.

Statement

All coauthors agreed with the contents of manuscript. The study with human subjects was conducted in accordance with the Declaration of Helsinki. This study was approved by the School of Psychology Ethics Committee at Guizhou Normal University. All participants signed informed consent forms prior to the experiment. All methods were carried out in accordance with relevant guidelines and regulations.

Results

The results of this study found that, reaction times in the grouping condition were shorter than those in the no-grouping condition, and sensory precision was superior in the former case (see Tables 1 and 2). In addition, the extrinsic grouping cues were associated with more accurate numerosity estimation than were the intrinsic grouping cues.

Table 1
Mixed-model repeated-measures ANOVA for CV

Effect	<i>df</i>	<i>F</i>	<i>p</i>	BF incl	<i>Partial</i> <i>η</i> ²
Numerosity	12,39	96.834	<0.001 ^{***}	>100	0.968
Grouping cue	4,47	2.894	0.034 [*]	>100	0.195
Grouping condition	1,50	2.807	<0.001 ^{***}	>100	0.558
Numerosity × Grouping cue	48,3	4.798	0.11	>100	0.987
Numerosity × Grouping condition	12,39	7.395	<0.001 ^{***}	>100	0.695
Grouping cue × Grouping condition	4,47	1.495	0.219	0.000	0.113
Numerosity × Grouping cue × Grouping condition	48,3	2.197	0.285	0.000	0.972
Math knowledge	2,52	4.798	0.012 [*]	>100	0.016
Math knowledge × Numerosity	12,40	1.279	0.206	>100	0.277
Math knowledge × Grouping cue	4,48	2.807	0.008 ^{**}	0.11	0.190
Math knowledge × Grouping condition	2,50	1.496	0.234	1.50	0.056
Math knowledge × Numerosity × Grouping cue	48,4	1.009	0.552	0.000	0.924
Math knowledge × Numerosity × Grouping condition	12,40	0.697	0.840	0.000	0.173
Math knowledge × Grouping cue × Grouping condition	4,48	1.267	0.27	0.000	0.096
Math knowledge × Numerosity × Grouping condition × Grouping cue	48,4	1.034	0.533	0.000	0.925

Table 2
Mixed-model repeated-measures ANOVA for RT

Effect	<i>df</i>	<i>F</i>	<i>p</i>	BF incl	<i>Partial</i> η^2
Numerosity	12,39	289.345	<0.001 ^{***}	>100	0.989
Grouping cue	4,47	15.526	<0.001 ^{***}	>100	0.569
Grouping condition	1,50	13.001	<0.001 ^{***}	>100	0.995
Numerosity × Grouping cue	48,3	4.798	0.028 [*]	>100	0.987
Numerosity × Grouping condition	12,39	55.306	<0.001 ^{***}	>100	0.944
Grouping cue × Grouping condition	4,47	18.451	<0.001 ^{***}	>100	0.611
Numerosity × Grouping cue × Grouping condition	48,3	27.912	0.009 ^{**}	>100	0.998
Math knowledge	2,52	4.798	0.012 [*]	>100	0.016
Math knowledge × Numerosity	12,40	1.924	0.016 [*]	>100	0.366
Math knowledge × Grouping cue	4,48	2.807	0.069	>100	0.190
Math knowledge × Grouping condition	2,50	1.496	0.004 ^{**}	>100	0.056
Math knowledge × Numerosity × Grouping cue	48,4	1.795	0.190	0.000	0.956
Math knowledge × Numerosity × Grouping condition	12,40	1.434	0.088	0.000	0.301
Math knowledge × Grouping cue × Grouping condition	4,48	0.536	0.827	0.000	0.43
Math knowledge × Numerosity × Grouping condition × Grouping cue	48,4	2.221	0.112	0.000	0.964

Groupitizing and grouping of cues

As in several previous studies^{14,15}, we also investigated grouping effects on sensory precision, as indexed by the coefficient of variation (Eq. 1). CV is a classical psychophysical parameter; in numerosity perception, this parameter reflects the sensory noise associated with the estimation process: the higher the CV value, the more sensory noise, and thus the less precise the estimates. We compared the grouping effects of extrinsic grouping cues (connectedness, common region) and intrinsic grouping cues (color similarity, shape similarity, proximity) on reaction time and coefficient of variation. The ANOVA of RTs revealed a significant main effect of grouping cue $F(4, 47) = 15.526$, $p < 0.001^{***}$, $BF_{10} > 100$. The interaction between grouping cue and grouping condition was significant, $F(4, 47) = 18.451$, $p < 0.001^{***}$,

$BF_{10} > 100$. The ANOVA of CV also revealed a significant main effect of grouping cue $F(4, 47) = 2.894, p = 0.034^*$, $BF_{10} > 100$; however, its interaction with grouping condition was not significant.

For the main effect of grouping cue, the reaction time for extrinsic grouping cues was slower than that for intrinsic grouping cues, while the coefficient of variation for extrinsic grouping cues was lower than that for intrinsic grouping cues, indicating that the sensory precision for extrinsic grouping cues was greater (less sensory noise) than that for intrinsic grouping cues (Fig. 2). The interaction between grouping cue and grouping condition was not significant for CV, as shown in Fig. 3. Among all grouping cues, sensory precision in the grouping condition was more greater than in the no-grouping condition, and the grouping effect of extrinsic grouping cues (connectedness and common region) was stronger than that of intrinsic cues. The effect of the interaction on RT was significant; for extrinsic grouping cues, there was no significant difference in RT between grouping and no-grouping conditions, but a difference was present for intrinsic grouping cues. Proximity and shape similarity grouping cues had a better grouping effect (Fig. 3). The RT for extrinsic grouping cues was slower than that for intrinsic grouping cues. Moreover, we found that large numbers were underestimated for each grouping cue (Fig. 4), consistent with the results of previous studies¹⁴.

Effects of specific numerosity

Estimation in grouping conditions was better than that in no-grouping conditions in terms of RT and coefficient of variation. ANOVA of RTs revealed a significant main effect of grouping condition, $F(1, 51) = 13.001, p < 0.001^{***}$, $BF_{10} > 100$, and the interaction with numerosity was also significant, $F(12, 39) = 55.306, p < 0.001^{***}$, $BF_{10} > 100$. ANOVA of CV revealed a significant main effect of grouping condition, $F(1, 50) = 2.807, p < 0.001^{***}$, $BF_{10} > 100$, and the interaction with numerosity was also significant, $F(12, 39) = 7.395, p < 0.001^{**}$, $BF_{10} > 100$. This result shows that groupitizing was an effective estimation strategy, consistent with previous research^{4, 14, 15}.

Because the interaction between grouping condition and numerosity was significant, we next examined how RTs and coefficient of variation varied with numerosity in each condition. It can be seen from Fig. 5 that both RT and CV increased linearly with the numerosity; in the grouping condition, numbers 6, 9, 12, and 16 had faster RTs and lower coefficients of variation than adjacent numbers. In contrast, numbers 7, 11, 13, 17 had slower RTs and a higher coefficient of variation than their neighbors (Fig. 5).

Influence of math knowledge

We compared grouping effects for persons with high, medium, and low levels of math knowledge. ANOVA of RTs revealed a significant main effect of math knowledge, $F(2, 52) = 4.798, p = 0.012, BF_{10} > 100$, and its interaction with grouping condition was also significant, $F(2, 50) = 1.496, p = 0.004, BF_{10} > 100$. ANOVA of CV also revealed a significant main effect of mathematical knowledge, $F(2, 52) = 4.798, p = 0.012, BF_{10} > 100$; its interaction with grouping condition was not significant, but its interaction with grouping cue was significant, $F(4, 48) = 2.807, p = 0.008, BF_{10} > 100$.

Regarding the main effect of math knowledge, when compared with the other two groups, the subjects in the high math knowledge group had faster RTs and lower coefficients of variation. Because of the significant interaction between math knowledge and grouping condition, we performed a simple effects analysis to further test the differences in grouping condition at different levels of math knowledge (Fig. 6).

Discussion

Our results showed that when items were divided into several subgroups, this benefited estimation the number of items. Furthermore, according to Gestalt theory, perceptual grouping can be divided into extrinsic and intrinsic grouping cues²³⁻²⁵. Accordingly, this study explored whether different grouping cues had different influences on groupitizing. The results showed that although extrinsic grouping cues had longer RTs, the sensory precision was more accurate in this case, and the grouping effect was stronger.

The RT for extrinsic grouping cues was slower than that for intrinsic grouping cues, inconsistent with Luna et al. (2016), who found that extrinsic grouping cues, especially common areas, were associated with faster RTs than other grouping cues^{22, 29}. In addition, Quinn and Bhatt (2015) also found that early infants (4–6 months) were more sensitive to extrinsic grouping cues^{26, 28}. Future research should select preschool children or first-grade primary school children to explore whether children who have not studied mathematics or have no complete magnitude representation system have different groupitizing effects given different grouping cues. Although the RTs for extrinsic grouping cues were slower than for intrinsic grouping cues, the grouping effect of extrinsic grouping cues was stronger than that of intrinsic grouping cues in terms of sensory precision (Fig. 3). This may indicate that extrinsic grouping cues have the strong advantage of groupitizing, but compared with intrinsic grouping cues, the addition of connecting lines or closing lines leads to more visual interference and requires additional cognitive processing, thus leading to slower responses for extrinsic grouping cues.

In recent years, significant progress has been made in the visual science of perceptual grouping. Recent studies have focused on the temporal processes and neural basis of intrinsic and extrinsic perceptual grouping^{26, 35}. For intrinsic grouping cues, grouping by proximity was found to be related to the positive component at the occipital electrode, whose amplitude peaks 100 to 120 ms after stimulus onset. The collinearity contour integral was found to emerge 130 ms after stimulation³⁶. Grouping by similarity (shape or color) was found to appear much later, and after 300 ms from stimulus onset, the negative occipito-temporal wave was activated^{22, 35}. In contrast, the neural basis of extrinsic grouping principles has received less attention. Montoro et al. (2015) reported the neural effects associated with grouping by common regions. They found that common region grouping cues belong to the category of long-latency grouping principles, which primarily involve activity in extrastriate cortices^{22, 28}. Future research should

continue to explore the time course and neural mechanisms underlying intrinsic and extrinsic grouping cues of grouping effects.

Numerous studies³⁷⁻⁴² have found that when the dots in the array are connected by lines or placed in a closed area, the number of dots will be underestimated. This phenomenon is termed the “connectedness illusion”³⁷⁻⁴². Connectedness and closeness are similar to the extrinsic grouping cues used in the present study (connectedness and common region). Some researchers have used topological invariance to explain the underestimation of quantity caused by connectedness and closeness; they proposed that numerosity perception is influenced by topological invariance such as connectedness and closeness. Therefore, connecting and enclosing items leads to underestimation of numerosity⁴⁰. Other studies suggested that two adjacent dots are considered to be one numeral unit when connected or closed via lines^{37, 38}. Moreover, some researchers found that although the connectedness illusion can lead to underestimation, when observers are required to reach for targets quickly, reach is not affected by the connecting lines⁴².

Because many studies have found the connectedness illusion, in the present study we examined whether extrinsic grouping cues (connectedness and common region) also led to underestimation of number in grouping conditions. The numerosities 6, 9, 12, and 16 were selected for analysis because these numbers had low RTs and high precision (see Results). We calculated the mean values across all subjects of the Weber fraction for the numerosities 6, 9, 12, 16, for all grouping cues by grouping condition.

The results demonstrated that for connectedness and common region grouping cues, no significant underestimation was observed; in contrast, connectedness led to significant overestimation when compared with other grouping cues (see Fig. 7B). These results suggest that no connectedness illusion was present in our study. Possible reasons include the following: First, from the perspective of experimental tasks, previous studies mostly used a bisection task or discrimination paradigm, which ask participants to determine which of two simultaneously or sequentially presented stimuli contain more dots. In the present study, participants were asked to report the estimated values directly. Moreover, in this study, the grouping conditions and no-grouping conditions were equally distributed within each session. The participants viewed and estimated the grouping condition and estimated the no-grouping condition in each session. For the no-grouping condition, the grouped stimuli were divided into smaller subgroups, which was more conducive for the participants’ estimation, and further demonstrated that connectedness and common region³⁷ had a strong groupitizing effect. Second, from the perspective of stimuli, in previous studies, dots were usually connected by line segments or curves in pairs or connected in series (Fig. 7A). Such stimuli may add an additional variable of common fate, because no matter whether connected by line segments or curves, items were arranged together according to the trajectory of lines, and the participants could easily perceive the connected items as holistic. However, in the present study, the items in the connectedness condition were randomly distributed in the grid, and the line segments connected the center points of each item. The connected items could have encouraged the participants to perceive them as a subgroup instead of being holistic; this would not cause underestimation. Furthermore, most

previous studies involved extra lines in the unconnected condition (Fig. 7A). However, in this study, only the grouped conditions had additional connecting or enclosing lines, while the no-grouping condition had no additional lines. Regarding the mechanisms underlying the role of extrinsic grouping cues in grouping effects, different experimental paradigms could be adopted for further exploration in future research.

Interestingly, we found that RTs for grouping by shape similarity were significantly lower than those of the other groupings (Fig. 3). Studies have shown that, in the absence of other visual information, it is easy for human beings to identify objects by shape⁴³⁻⁴⁵. Adults and children prefer to categorize novel objects according to shapes, given conflicting colors and texture cues. Shape features play a more important role in inductive reasoning than do color features^{44, 45}. Shape similarity is the first strategy used in inductive reasoning in early childhood⁴⁶. Researchers presented subjects with reference stimuli of color, shape, and texture (such as square, blue, and wooden), and then presented two test stimuli with different shapes, a different color, and a different texture, so that children could judge whether the test stimulus was consistent with the reference stimulus⁴⁷. The results showed that 2–3 year old children chose shapes as the basis of induction. Future studies might select developing children as participants to explore whether the grouping effect of quantity estimation in shape similarity is different between children and adults.

Regarding math knowledge, the grouping conditions for the high math knowledge group differed significantly in reaction time, while the middle and low math knowledge groups did not exhibit such a difference, indicating that the groupitizing effect benefits the persons with high math knowledge the most. Many studies have demonstrated that an efficient ANS may be a prerequisite for the typical development of math skills^{10, 11}. Therefore, we speculate that the high math knowledge group had a more refined ANS, and that the groupitizing strategy was automatically used in quantity estimation. In the grouping condition, items were visually divided into subgroups, and since individuals with high levels of math knowledge could make better use of groupitizing strategy, RTs in the grouping condition were significantly faster than those in the no-grouping condition. For the middle and low math knowledge groups, although sensory precision was higher in the grouping condition, RTs did not differ among conditions. This may be because, in the grouping condition, when they used groupitizing strategies, they needed to employ more cognitive resources and required more time. In the no-grouping condition, they could not use any strategies; they could only make rough guesses based on their feelings. Thus, RTs were faster but accuracy was lower.

The present study found that, in the grouping condition, numbers 6, 9, 12, and 16 were associated with faster reaction times and lower coefficients of variation than adjacent numbers. This may be because those specific numerosities' configurations (6: $3, 3 = 2 \times 3 = 2$ groups of 3, 9: $3, 3, 3 = 3 \times 3 = 3$ groups of 3, 12: $4, 4, 4 = 3 \times 4 = 3$ groups of 4, 16: $4, 4, 4, 4 = 4 \times 4 = 4$ groups of 4) were divided into "equal groups," which let the subjects use mental multiplication in their estimations. This is similar to the result of Dehaene et al. in 2020⁴, who found that for 5, 7, 11, and other such prime numbers, RTs were slower than for their neighbors, and for non-prime numbers, which could be subdivided into equal numbers, RTs were faster than for their neighbors (Fig. 5).

Conclusion

The present study demonstrates that visually dividing an array into subgroups promotes quantity estimation. Moreover, for the first time, our research combined the groupitizing effect of numerosity estimation with Gestalt theory and demonstrated a difference between the groupitizing effect of extrinsic versus intrinsic grouping cues, based on Palmer et al.²³⁻²⁵. The results thus suggest that it takes longer for estimation of numerosities given extrinsic grouping cues, but the accuracy for extrinsic grouping cues is higher, due to a stronger groupitizing effect than for intrinsic grouping cues.

Limitations And Future Directions

Since the concept of “groupitizing” was proposed by Wender and Roth Kegel (2000)¹⁷ and Starkey and McCandless (2014)¹³, studies have continued to explore the effect of grouping. This study combined the grouping effect and perceptual grouping principles, thus extending the study of groupitizing and the field of perceptual grouping.

Recent studies have begun to explore the shared associative mechanisms between different perceptual features. For example, the theory of magnitude model^{48, 49} proposes that the parietal cortex of human beings processes quantitative information about space, time, and numbers together to optimize action plan and execution. It is necessary to explore the relationship between magnitude representation, space, and time. In this study, we only studied the grouping effect in space, but subsequent studies could verify the differences in the grouping effect between intrinsic and extrinsic grouping cues in the time dimension.

The participants in this study were adults. Although they were divided into high, middle, and low math knowledge groups, the difference in the math level of adults is not very prominent. Many studies have found that in the process of development and formal arithmetic learning, numerosity perception precision has been greatly improved, while in educated adults, symbolic mathematics abilities may have been stably mapped into their basic non-symbolic representation, making this connection less obvious⁴⁹⁻⁵¹. Future research could explore the differences in groupitizing effects between preschool children with a low number sense and adults, as well as between children with difficulties in math and children without math difficulties.

To date, there has been no electrophysiological or neuroscience study that explores the grouping strategies between intrinsic and extrinsic cues. Future research could be conducted from the perspective of electrophysiology to investigate the neurofunctional links between grouping strategies and intrinsic and extrinsic cues, which would delineate a possible neural hierarchical model for “groupitizing.”

Declarations

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Author contributions

All authors conceived and planned the experiments. H.Y.Y. and Y.P. developed the study concept and contributed to the study design. Testing and data collection were performed by M.M.L, J.Z.and L.H.C..The data analysis and interpretation was performed by H.Y.Y. under the supervision of Y.P. The manuscript was drafted by H.Y.Y., and Y.P. provided critical revisions. All authors approved the final version of the manuscript for submission.

Competing interests

The authors declare no competing interests.

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Figures

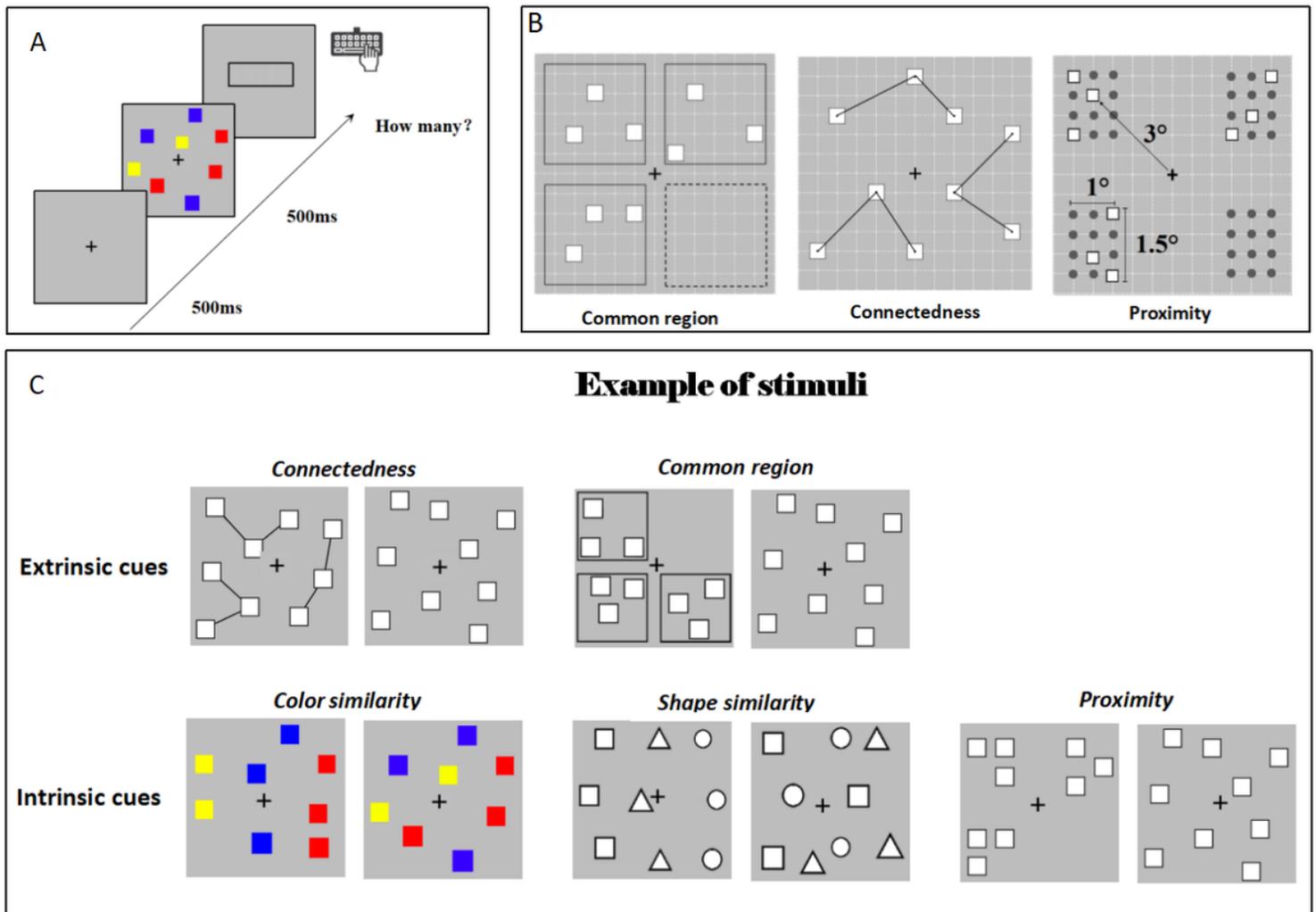


Figure 1

Illustration of the stimuli and procedure. (A) Example of the time course of the experiment. (B) Illustration of how stimulus position was defined in the grouping conditions. (C) Example of stimuli.

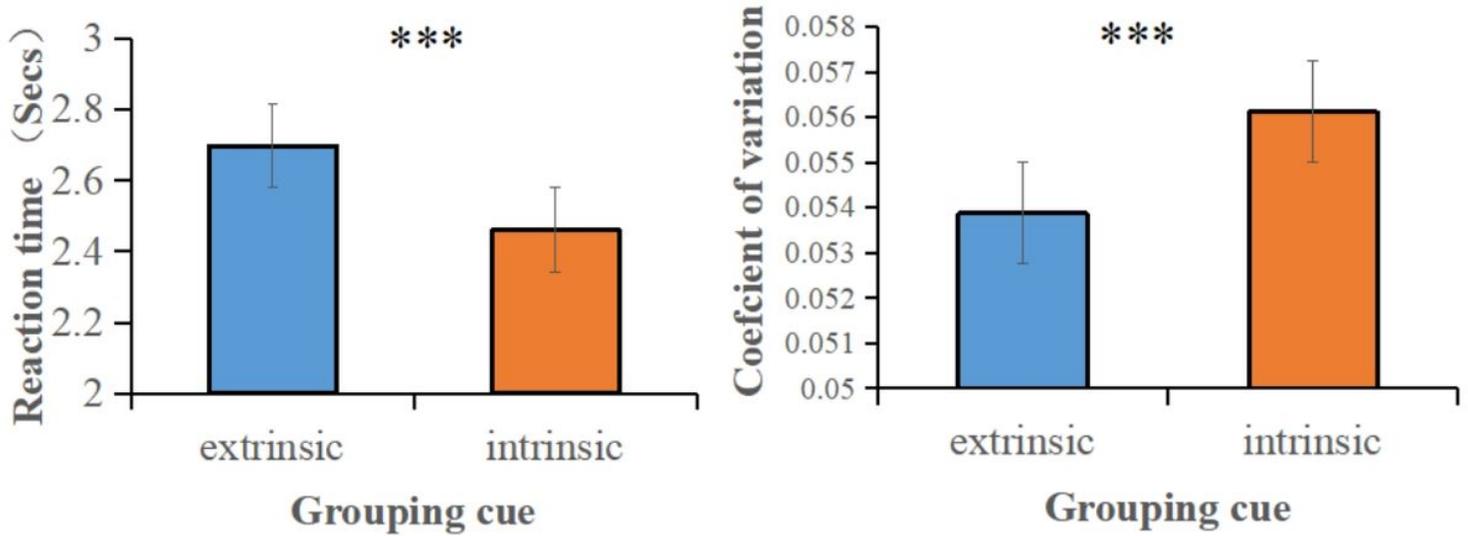


Figure 2

Results of extrinsic and intrinsic grouping cues. Reaction time and coefficient of variation (CV) for extrinsic and intrinsic grouping cues by group condition. *** $p < 0.01$, ** $p < 0.01$ * $p < 0.05$.

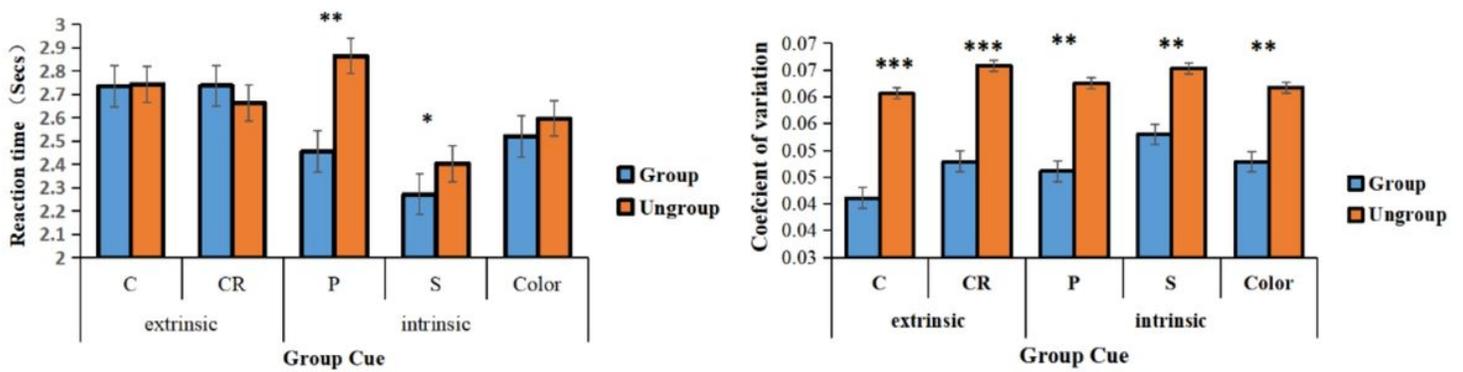


Figure 3

Grouping cue effect. Reaction time and coefficient of variation (CV) for different grouping cues by group condition. *** $p < 0.01$, ** $p < 0.01$ * $p < 0.05$.

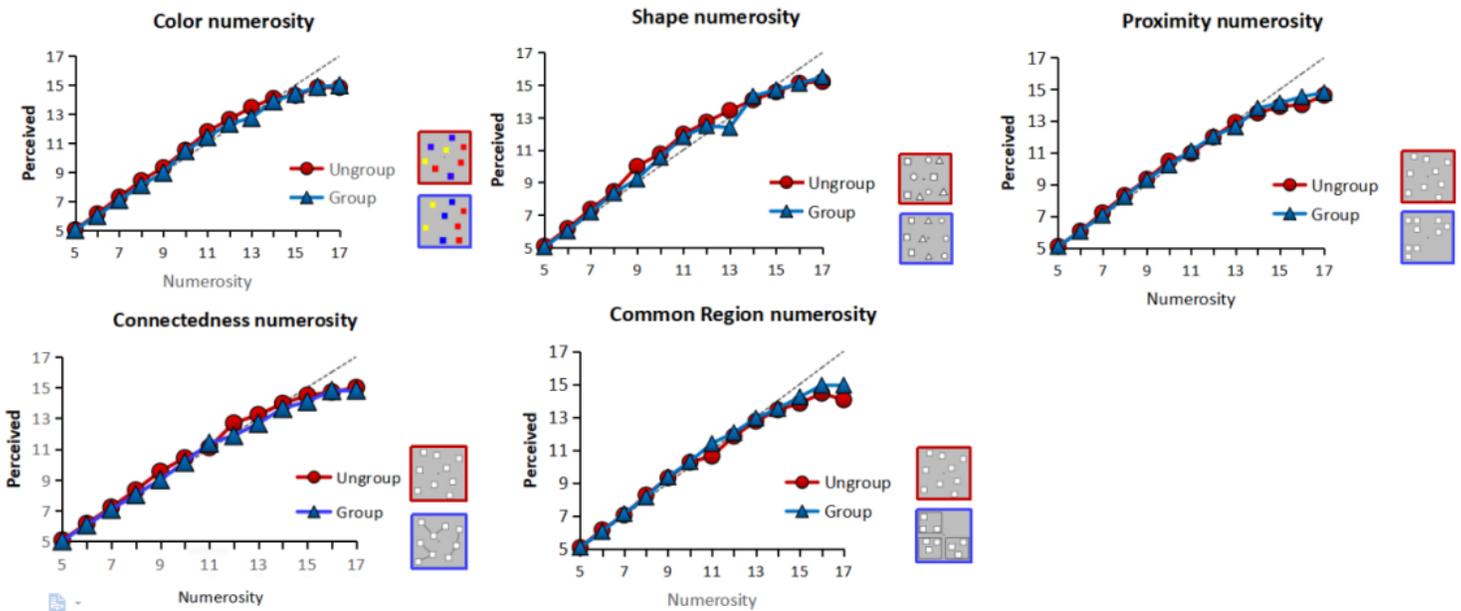


Figure 4

Perceived numerosity. Average perceived numerosity for all grouping cues and tasks, averaged across participants.

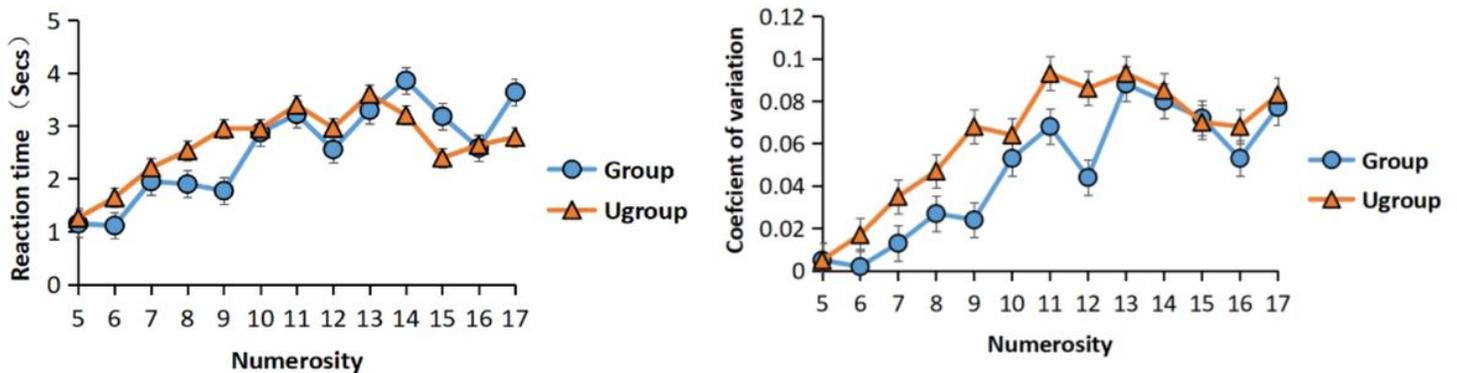


Figure 5

Specific numerosities. Reaction times and coefficient of variation (CV) for different numerosities across group conditions.

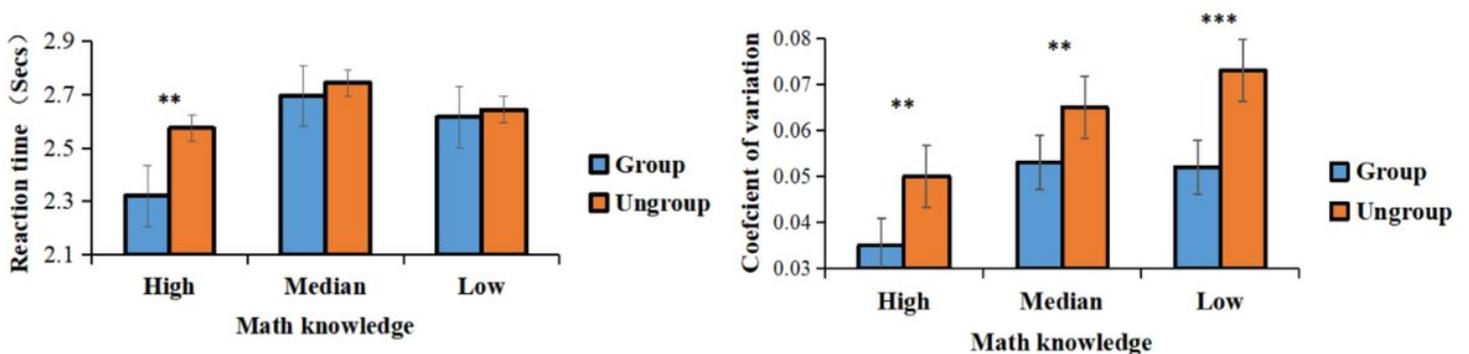


Figure 6

Results for math knowledge. Reaction time and coefficient of variation (CV) in groups with high, medium, and low levels of math knowledge by group condition. *** $p < 0.01$, ** $p < 0.01$ * $p < 0.05$.

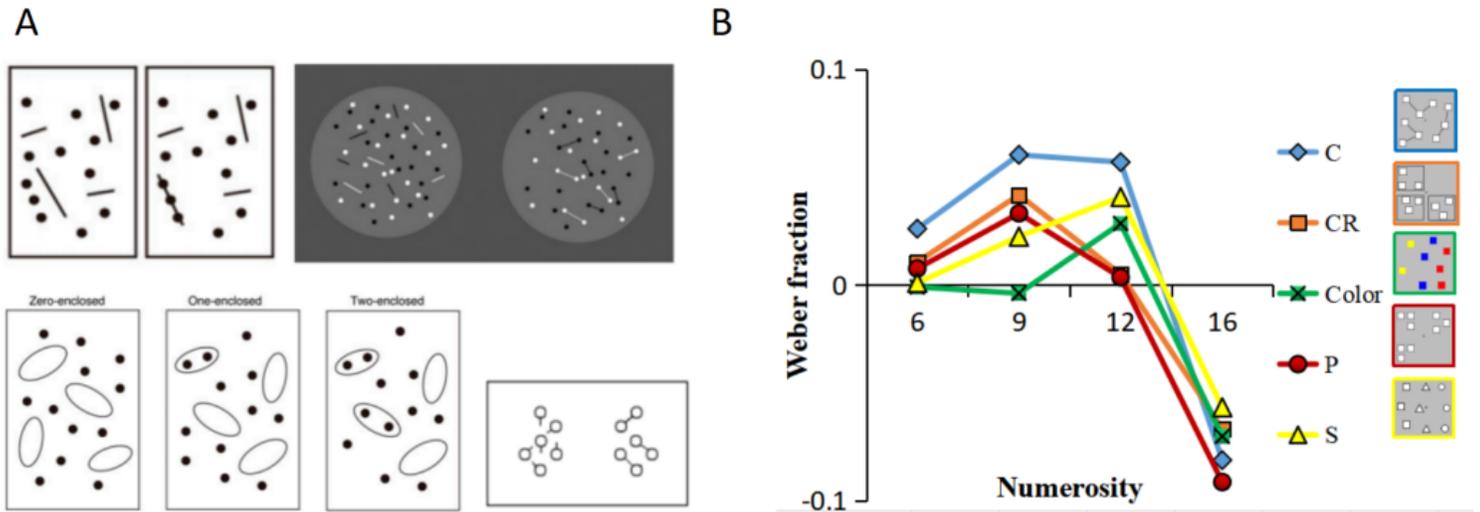


Figure 7

(A) Connectedness illusion in previous studies, and (B) Illustration of Weber fraction (a positive Weber fraction denotes overestimation; the greater the value, the greater the overestimation. Negative estimation error denotes underestimation.) for the numerosities 6, 9, 12, 16 for all grouping cues and conditions.