

Using Multi-Fractal and Joint Multi-Fractal Theories to Characterise the Spatial Variability of Soil Particle Size Distribution in an Underground Coalmine Area

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1 **Using multi-fractal and joint multi-fractal theories to**
2 **characterise the spatial variability of soil particle size**
3 **distribution in an underground coalmine area**

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15
16 **Abstract:** Underground coal mining leads to serious surface deformation, which
17 negatively affects the physical properties of soils . Soil particle size distribution (PSD)
18 is one of the most basic soil physical characteristic that influences other important
19 properties such as soil hydraulics and thermodynamics. Understanding the spatial
20 variability of the soil PSD in subsided land can provide targeted guidance for land
21 reclamation. In this study, we conducted a quantitative study on the spatial variability
22 of the soil PSD in the Pingshuo mining area on the Loess plateau, Shanxi Province in
23 China, and explored the effects of subsidence and reclamation on the soil PSD. A plot

24 experiment, including one unmined plot (UMP), one subsided plot (SUP), and one
25 reclaimed plot (RCP), was performed in Anjialing No.3 underground coal mine in the,
26 Pingshuo mining area. Four multi-fractal parameters of the soil PSD— $D(0)$, $D(1)$,
27 $\Delta\alpha(q)$, and $\Delta f(\alpha)$ —were analyzed at the three sample sites. The joint multi-fractal
28 method was carried out to analyze the spatial correlation of the soil PSD to further
29 reveal the impacts of coal mining subsidence and land reclamation on the soil PSD.
30 The multi-fractal method can reflect the local non-uniformity and heterogeneity of the
31 soil PSD, while the joint multi-fractal approach can illustrate the correlation of the
32 soil PSD between different soil depths. The range and spatial variability of the soil
33 PSD increased due to coal mining subsidence and the impact of subsidence on the
34 spatial disturbance of the surface soil PSD was greater than that of the deeper layers.
35 The spatial correlation of clay in subsided land was larger than those of unmined land
36 and reclaimed land, whereas, for silt and sand, the correlation was smaller. Land
37 reclamation decreased the spatial variability of the soil PSD, which was near that of
38 the unmined land after reclamation.

39 **Key words:** Coal mining subsidence; Soil particle size distribution; Spatial variability;
40 Multi-fractal theory; Joint multi-fractal theory.

41 **1 Introduction**

42 The coal industry, one of the most important foundations of economics, plays a key
43 role in the national economy, which will last for a long time because of the energy
44 structure of China (Yuan et al. 2018). China's coal resources are mainly distributed in

45 ecologically vulnerable areas such as the Loess Plateau area (Wang et al. 2018c), and
46 coal mining is dominated by underground mining (Zhang et al. 2019; Zhen et al.
47 2019). Open cast coal mining may result in large-scale land destruction and
48 deformation, even soil cracks (Wang et al. 2016). On account of coal mining activities
49 and human disturbance in the subsidence area, the soil structure has undergone
50 fundamental changes, which in turn, exert an influence on the soil particle size
51 distribution (PSD) and spatial variability. Only when the soil PSD, aggregate structure,
52 water, and fertilizer retention capacity are rehabilitated can the basic conditions for
53 vegetation and ecological restoration be provided (Wang et al. 2015a). Consequently,
54 understanding the spatial variability of the soil PSD in the subsided area can provide
55 effective guidance for future land reclamation work.

56 The spatial variability of the soil PSD is a key research field in current soil science
57 research. The validity and potential of application of fractal theory in soil spatial
58 variability has been verified by earlier studies (Rueth and Lennartz 2008), and it is an
59 important tool for quantifying the spatial variability of soil properties and scale
60 conversion (Caniego et al. 2005; Milne et al. 2013), which mainly includes single
61 fractals (Deng et al. 2017), multi-fractals (Manuel Miras-Avalos et al. 2016), and joint
62 multi-fractals (Biswas et al. 2012). Multi-fractals can reveal small-scale or partial
63 features of large-scale or overall soil properties, and multi-fractal singular spectra can
64 be obtained by calculating the fractal dimension of the different spatial regional scales
65 of variables (Wang et al. 2018a). In general, the wider the distribution of the
66 multi-fractal singular spectra is, the stronger the spatial variability is (Manuel

67 Miras-Avalos et al. 2016). Multi-fractal parameters can better reflect the
68 non-uniformity of the PSD compared with single fractal parameters (Guan et al. 2011;
69 Miranda et al. 2006), with high precision and sensitivity. However, the multi-fractal
70 method has a defect in its characterization of the soil PSD, calculating only the PSD
71 spatial variability of the individual soil sample and, thus, not able to indicate the
72 spatial correlation between soil properties at different layers.

73 The joint multi-fractal method can be conducted to explore the relationship between
74 two variables under the same geometric conditions (Kravchenko et al. 2000). Joint
75 multi-fractal spectrum is mostly used to analyse the correlation between two variables
76 or between variables and their influencing factors on multiple scales (Li et al. 2011;
77 Medina et al. 2017). For the convenience of analysis, the joint multi-fractal spectrum
78 can be projected onto a two-dimensional plane, and the resulting greyscale maps are
79 relatively concentrated and extended diagonally, indicating that the two variables have
80 a higher degree of correlation; conversely, if the distribution is more discrete, the
81 degree of correlation is lower and exhibits different distribution patterns (Biswas et al.
82 2012). The multi-fractal and joint multi-fractal models can be used to combine
83 different soil variables with statistical features of different scales to obtain multiple
84 singular spectra, which can directly reflect the spatial variability and scale
85 characteristics of spatial sequences, revealing the spatial variation mechanism of soil
86 characteristics more deeply (Havrylenko et al. 2016). Zeleke and Si (2006) used the
87 above two approaches to study the multi-scale relationships between soil physical
88 properties and various hydraulic parameters, indicating that these methods are more

89 accurate than geostatistical methods.

90 In summary, multi-fractal and joint multi-fractal methods are suitable approaches for
91 characterizing the spatial variability of the soil PSD; however, study on the spatial
92 variability of soil PSD in coal mining area is still in its infancy (Manuel Miras-Avalos
93 et al. 2016). Therefore, the objectives of our study were (i) to conduct descriptive
94 statistical analysis of the multi-fractal parameters of soil PSD to quantify the spatial
95 variability of soil PSD, (ii) to analyze the variation characteristics of the soil PSD
96 between different soil layers employing the joint multi-fractal method, and (iii) to
97 disclose the influence of coal mining subsidence and reclamation on soil PSD in coal
98 mining subsidence areas.

99 **2 Materials and Methods**

100 *2.1 Study area*

101 The study area is located in the Pingshuo coal mine of Shanxi Province in China, an
102 ecologically fragile area of the Loess Plateau. The geographical coordinates are 112°
103 11' to 113° 30' E, 39° 23' to 39° 37' N (Fig. 1). The geomorphology is mainly loess
104 hills and mountains, and the vegetation types are mainly farmland vegetation,
105 grassland, deciduous broad-leaved shrub and deciduous broad-leaved forest. The
106 climate of the study area is typical of temperate arid to semi-arid continental monsoon
107 climate. Winter is frigid with occasional rain, and summer is hot and rainy (Wang et al.
108 2015b). The average annual rainfall is about 450 mm, but the annual evaporation is
109 five times greater than the rainfall.

110 The specific research location was the unmined, collapsed, and restored lands of the
111 Anjialing No.3 underground coalmine in the Pingshuo mining area. One unmined plot,
112 one subsided plot, and one reclaimed plot were picked to perform the research. The
113 dimensions of each plot were 100 m × 100 m. There were 13 settlement cracks in
114 the SUP, ranging in width from 0.1 to 3.5 m. In the light of the World Reference Base
115 for Soil Resources, the soil type was Kastanozems. The subsidence area was
116 dominated by herbaceous plants.

117 The RCP and UMP were near the SUP. In June 2012, land reclamation was conducted
118 in the RCP. After stripping off 20 centimeters of topsoil, the fissures were filled with
119 local soils; afterwards, the collapsed land was reinforced into a horizontal terraces,
120 and covered with the stripped topsoils. The vegetation was naturally rehabilitated in
121 the RCP. In the RCP and UMP, the exclusive vegetation species was herbaceous
122 plants. The terrain of the unmined and restored plots was like that of the collapsed
123 plot. An overview of the three parcels is depicted in Fig. 2.

124

125 *2.2 Soil sampling and analysis*

126 In 2015, 224 soil samples were collected at 56 sampling sites at 0-20 cm, 20-40 cm,
127 40-60 cm, and 60-80 cm depths using a soil drill, and the sampling points were
128 recorded by a global positioning system (GPS) receiver. A total of 16 soil profiles
129 were excavated for each plot, and 8 additional points were placed at the subsidence
130 cracks of the SUP (Fig. 3). The samples were transported to the laboratory, air-dried
131 and treated with a grinding rod and sieved through a 2-mm mesh.

132 The PSD of the soil samples was obtained through laser diffraction employing the
133 Mastersizer-2000 particle size analyzer (Malvern Instruments, Malvern, England). It
134 was operated to record the soil particle content in the 100 segments of different
135 particle sizes obtained in the measurement interval, which are demarcated according
136 to the US Agricultural Department (USDA) soil particle classification criteria. The
137 soil PSD is classified by clay ($< 0.002\text{mm}$), silt ($0.002\text{-}0.05\text{ mm}$), and sand ($0.05\text{-}2$
138 mm). The main soil texture in the mining area is silt loam.

139

140 *2.3 Multi-fractal analysis of the soil PSD*

141 The spatial variability of the soil PSD at each sampling point were discussed by
142 multi-fractal method. The volume fraction of the soil particle size in each particle size
143 segment in measurement interval $I = [0.02, 2000]$ is obtained, and normalization is
144 performed. After the logarithmic conversion, there are 100 equidistant dimensionless
145 intervals in interval $J = [0, 5]$. There is $N(\varepsilon) = 2^k$ subintervals in each dimensionless
146 interval J , where the scale $\varepsilon = 5 \times 2^{-k}$, k takes on an integer in the range of 1-6, and
147 there is at least one observation in each subinterval. The probability density of the soil
148 particle size volume fraction spread in each sub-interval is $p_i(\varepsilon)$, and by summing up

149 the observations in all sub-intervals, $V_i = \frac{v_i}{\sum_{i=1}^{100} v_i}, i = 1, 2, \dots, 100$ can be obtained. From

150 this, the least-squares method can be used to fit and construct a distribution function
151 family:

152
$$\mu_i(q, \varepsilon) = \frac{p_i(\varepsilon)^q}{\sum_{i=1}^N p_i(\varepsilon)^q}, \quad (1)$$

153 where ε is the observation scale, $\mu_i(q, \varepsilon)$ represents the q^{th} -order probability of the i -th
 154 subinterval, q ranges from $[-10, 10]$, and the step size is 0.5. The summation

155 $\sum_{i=1}^N p_i(\varepsilon)^q$ indicates the sum of the q^{th} -order probabilities of the N sub-intervals.

156 Multi-fractal generalized dimensional spectral function $D(q)$ is widely used to
 157 characterize the unevenness of soil PSD.

158
$$D(q) = \lim_{\varepsilon \rightarrow 0} \frac{1}{q-1} \frac{\lg \left[\sum_{i=1}^{N(\varepsilon)} p_i(\varepsilon)^q \right]}{\lg \varepsilon}, \quad (2)$$

159 where $N(\varepsilon)$ denotes the sample size, and the scale is ε ; $p_i(\varepsilon)$ is the sample value of the
 160 i^{th} sub-interval; q is the order of the $p_i(\varepsilon)$ statistical moment, and different orders
 161 indicate different scanning sparse dense regions. When $q > 1$, the probability of large
 162 probability subset scanning is larger, while when $q < 1$, the probability of small
 163 probability subset scanning is larger. When $q = 0$, $D(0)$ is called the box-counting
 164 dimension, which characterizes the range of soil PSD. When $q = 1$, $D(1)$ is
 165 information entropy, which can describe the concentration degree of soil PSD.

166 The multi-fractal singularity index of the soil PSD is

167
$$\alpha(q) = \lim_{\varepsilon \rightarrow 0} \frac{\sum_{i=1}^{N(\varepsilon)} \mu_i(q, \varepsilon) \lg p_i(\varepsilon)}{\lg \varepsilon}. \quad (3)$$

168 The multi-fractal singular spectral function of soil PSD relative to $\alpha(q)$ is

169
$$f(\alpha(q)) = \lim_{\varepsilon \rightarrow 0} \frac{\sum_{i=1}^{N(\varepsilon)} \mu_i(q, \varepsilon) \lg \mu_i(q, \varepsilon)}{\lg \varepsilon}, \quad (4)$$

170 where $\alpha(q)$ is the singularity index, and the function image of $f(\alpha(q)) \sim \alpha(q)$ is the
171 multi-fractal singular spectrum. The local spatial variation of the soil particle size can
172 be determined by the spectral width $\Delta\alpha$ of the fractal structure. The larger $\Delta\alpha$ is, the
173 higher the heterogeneity is (Zhao et al. 2011). The parameter $\Delta f(\alpha)$ display the shape
174 of the multi-fractal spectral attributes, and the asymmetry of soil PSD can be
175 characterized. When $\Delta f > 0$, the asymmetrical $f(\alpha)$ - α spectra are shifted to the left,
176 and when $\Delta f < 0$, its shape is right deviation. Additionally, Δf is 0 for the
177 symmetrical spectrum. As the multifractal spectrum is shifted to the left, the effects of
178 the high values of variables on soil PSD increase. Contrariwise, the low-value
179 information exerts greater influence on the soil PSD. Therefore, the multi-fractal
180 model can efficaciously characterize the heterogeneity and complexity of the spatial
181 distribution of soil particle size (Sun et al. 2017; Du et al. 2017).

182 *2.4 Descriptive statistics of soil PSD*

183 To explore the influence of coal mining subsidence on the spatial variability of the
184 soil PSD, the one-sample Kolmogorov–Smirnov Test method was used to test the
185 multi-fractal properties (i.e., $D(0)$, $D(1)$, $\Delta\alpha$, and Δf) of the soil PSD in different soil
186 layers in three plots using IBM SPSS Statistics 20.0 software (Bowers et al. 2012;
187 Zhang et al. 2015a). Moreover, the maximum, minimum, median, mean, kurtosis,
188 skewness, and coefficient of variation (CV) of were calculated for the multi-fractal
189 properties of soil PSD. Kurtosis is used to measure the thickness of the data
190 distributed at the tail, and skewness is used to measure the skew direction and extent
191 of the distribution. The central tendency is determined by the median and mean. If

192 their values are close, the outliers will not affect the measurement of the central trend
 193 distribution (Shahbazi et al. 2013). The CV is the ratio of the standard deviation to the
 194 mean (Meng et al. 2015), and it is performed to measure the variation extent of
 195 multi-fractal characteristics (Ussiri et al. 2006). The variability of multi-fractal
 196 parameters of the soil PSD can be categorized into weak variation ($CV < 10\%$),
 197 moderate variation ($10\% \leq CV \leq 100\%$), and strong variation ($CV > 100\%$) (Aghasi
 198 et al. 2017; Hu et al. 2008).

199

200 *2.5 Joint multi-fractal analysis of the soil PSD*

201 The multi-fractal algorithm mainly studies a single variable. On the basis, the joint
 202 multi-fractal method can disclose the relationship between two variables under the
 203 same geometric conditions (Kravchenko et al. 2000). From the joint multi-fractal
 204 algorithm, research object A and the research object B on the same geometric
 205 dimension are combined to analyze, $\alpha^1(q^1, q^2)$ and $\alpha^2(q^1, q^2)$, and the corresponding
 206 $f(\alpha^1(q^1, q^2), \alpha^2(q^1, q^2))$ are important parameters in the joint multi-fractal method. The
 207 formulas are as follows:

$$208 \quad \mu_i(q^1, q^2, \delta) = p_i^1(\delta)^{q^1} p_i^2(\delta)^{q^2} / \sum_{i=1}^{N(\delta)} p_i^1(\delta)^{q^1} p_i^2(\delta)^{q^2}, \quad (5)$$

$$209 \quad \alpha^1(q^1, q^2) = -\{\log[N(\delta)]\}^{-1} \sum_{i=1}^{N(\delta)} \{\mu_i(q^1, q^2, \delta) \log[p_i^1(\delta)]\}, \quad (6)$$

$$210 \quad \alpha^2(q^1, q^2) = -\{\log[N(\delta)]\}^{-1} \sum_{i=1}^{N(\delta)} \{\mu_i(q^1, q^2, \delta) \log[p_i^2(\delta)]\}, \quad (7)$$

$$211 \quad f(\alpha^1, \alpha^2) = -\{\log[N(\delta)]\}^{-1} \sum_{i=1}^{N(\delta)} \{\mu_i(q^1, q^2, \delta) \log[\mu_i(q^1, q^2, \delta)]\}, \quad (8)$$

212 where $p_i^1(\delta)$ represents the mass probability of variable A, $p_i^2(\delta)$ represents the
213 mass probability of variable B, $\alpha^1(q^1, q^2)$ is the singularity index of study variable A,
214 and $\alpha^2(q^1, q^2)$ is the singularity index of study variable B.

215 After the values of the joint multi-fractal parameters $\alpha^1(q^1, q^2)$, $\alpha^2(q^1, q^2)$, and $f(\alpha^1, \alpha^2)$
216 are determined, the MATLAB (MathWorks, Natick, Massachusetts) is used to
217 generate a three-dimensional spatial function image composed of $\alpha^1(q^1, q^2)$, $\alpha^2(q^1, q^2)$,
218 and $f(\alpha^1, \alpha^2)$. Because of the diversification of the three-dimensional image in the
219 imaging angle of view, there are many inconveniences in the analysis process. It is
220 necessary to convert the three-dimensional space function image into a
221 two-dimensional plane function image, that is, the image is projected to $\alpha^1(q^1, q^2)$ and
222 $\alpha^2(q^1, q^2)$. On the plane of the composition, a two-dimensional greyscale image of the
223 joint multi-fractal spectral can be obtained to identify the spatial variability features
224 and their mutual relations in the fractal structure. From the concentration or dispersion
225 degree of the greyscale image, the correlation between the two research objects can be
226 judged. If the two-dimensional greyscale image exhibits a relatively concentrated
227 distribution, this indicates that the correlation between the research objects is strong.
228 The spatial distribution of variables has a certain degree of synergy. If the
229 two-dimensional greyscale shows a relatively discrete distribution, this indicates that
230 the correlation between the two research variables is relatively low, and the spatial
231 distribution law is relatively independent. The difference between the layers is large.
232 Based on the results of soil classification, the joint multi-fractal analysis was
233 conducted to analyze the soil texture of the adjacent soil layers at 0-80 cm depth (two

234 adjacent soil depths were compared with each other). Three-dimensional joint
235 multi-fractal spectra and projected 2D greyscale images are obtained.

236 **3 Results**

237 *3.1 Descriptive statistics analysis of the soil PSD*

238 The descriptive statistical characteristics of the clay content, silt content, and sand
239 content of different soil layers in three sampling plots are shown in Tables 1-3.

240 *3.1.1 Clay content*

241 The clay contents of the UMP, SUP, and RCP were 11.82%-22.30%, 4.39%-24.04%
242 and 10.05%-25.56%, respectively, and the coefficients of variation were
243 12.36%-16.37%, 21.00%-34.34% and 15.08%-19.5%, showing moderate variation.

244 The clay content in the SUP decreased, while the CV increased. The overall clay
245 content in the three samples was expressed as $UMP > RCP > SUP$, indicating that land
246 subsidence decreased the clay content, and land reclamation increased the clay
247 content.

248 *3.1.2 Silt content*

249 The silt contents in the UMP, SUP, and RCP were 40.35%-60.16%, 36.38%-63.45%,
250 and 39.55%-60.45%, respectively; and the coefficients of variation were
251 7.62%-10.56%, 6.77%-10.36% and 6.17%-8.25%, showing weak variation. All three
252 plots belong to silty clay loam according to the mean values, and the silt contents in
253 the three plots are near each other. The coefficient of variation of the SUP is slightly
254 larger than those of the other two plots. Compared to the clay content and sand

255 content, the influence of coal mining subsidence and land restoration on the soil silt
256 content are relatively low.

257 *3.1.3 Sand content*

258 The sand contents of the UMP, SUP, and RCP were 25.66%-39.79%, 31.57%-47.45%,
259 and 24.44%-39.95%, respectively, and the coefficients of variation were
260 9.96%-15.47%, 7.23%-9.62%, and 6.57%-9.28%, showing weak variation. The sand
261 content of the SUP showed a marked rise. The CV of the three plots in the same soil
262 layer showed that UMP > SUP > RCP, and land subsidence and land reclamation
263 decreased the variation in the sand content.

264 *3.2 Multi-fractal characteristics of the soil PSD*

265 Table 4, 5, 6 and 7 respectively shows the the multifractal parameters description of
266 the statistical eigenvalues of the three sampling plots. All the K-S test p values were
267 greater than 0.05, and the sample values followed the normal distribution (Hu et al.
268 2018). The median and mean values of the soil multi-fractal parameters $D(0)$, $D(1)$,
269 $\Delta\alpha(q)$, and $\Delta f(\alpha)$ were very close in the three plots , indicating that the distribution of
270 sample data was highly concentrated.

271 *3.2.1 $D(0)$*

272 The multi-fractal parameter $D(0)$ values of the UMP, SUP, and RCP were
273 0.8178-0.9501, 0.8178-0.9406, and 0.8178-0.9501, respectively. Overall, when the q
274 values are equal, the $D(0)$ of the SUP was greater than those of the UMP and RCP.

275 The multi-fractal parameter $D(0)$ has a small CV value (1.27%-5.02%), which falls in
276 the weak variation category. The CV value of the SUP is the smallest, and the CV

277 values of the RCP and UMP soil layers are near each other, indicating that the severe
278 coal mining subsidence has multi-fractal characteristics. Land subsidence was shown
279 to influence the degree of spatial variability of $D(0)$; however, the effect was
280 relatively low. After land reclamation, the soil PSD range can reach a level similar to
281 that of the unmined plot.

282 3.2.2 $D(1)$

283 The multi-fractal parameter $D(1)$ values of the UMP, SUP, and RCP were
284 0.7165-0.8710, 0.6967-0.8712, and 0.7285-0.8953, respectively. The distribution law
285 of multi-fractal parameter $D(1)$ had certain similarities with $D(0)$. When the step size
286 q is equal, the $D(1)$ of the SUP was larger than that of the UMP or RCP. Coal mining
287 subsidence increased the concentration level of the soil PSD. The change in the CV
288 values of $D(1)$ of the three plots was consistent with that of the parameter $D(0)$.
289 Compared to $D(0)$, the CV values of $D(1)$ for each plot were slightly larger than that
290 of $D(0)$. However, they were still within the weak variation range (2.40%-6.02%).

291 3.2.3 $\Delta\alpha$

292 The multi-fractal parameter $\Delta\alpha$ values of UMP, SUP, and RCP were 0.8786-1.0012,
293 0.8073-1.0614, and 0.8569-1.0033, respectively. The parameter $\Delta\alpha$ of the UMP
294 exhibited a gradual decrease with the increase in the soil depth. Mining activities
295 presented an artificial disturbance to the soil profile, and $\Delta\alpha$ exhibited fluctuations.
296 After land reclamation, the spatial variability of the surface soil layer was close to that
297 of the unmined land, and there remained fluctuations in the deep soil depths. Land
298 subsidence caused an increase in the spatial variability of the local heterogeneity of

299 the soil PSD, and land reclamation decreased the variability. However, the difference
300 in the CV values of parameter $\Delta\alpha$ among the various plots was relatively small
301 (2.55%-6.87%). The CV values in the deeper soil was higher than those in the topsoil
302 in each plot, except at the depth of 20-40 cm in the SUP.

303 3.2.4 Δf

304 The multi-fractal parameter Δf values of the UMP, SUP, and RCP were between
305 0.0235-0.4680, 0.0318-0.5259, and 0.0531-0.4940, respectively. Land subsidence
306 slightly increased the asymmetry of the soil PSD. All of the CV values of the different
307 soil depths in the three plots were within the range of moderate variation
308 (21.95%-55.79%), and the difference in the CV values of the parameter Δf among the
309 various plots was relatively small.

310 3.3 Joint multi-fractal analysis of soil PSD

311 The joint multi-fractal results of the soil PSD at the three sample sites are shown in
312 Figs. 4-12. These figures intuitively present the variability and spatial correlation of
313 soil PSD between different soil depths.

314 3.3.1 Clay content

315 The joint multi-fractal spectra and projection greyscale maps of the PSD multi-fractal
316 parameters of the clay soils in the three sample sites are reflected in Figs. 4, 5, and 6.

317 The concentration degree of dots in the clay content grey map in the UMP was the
318 highest, and the greyscale dispersion of the UMP in 0-20 cm and 20-40 cm topsoil
319 was higher than that of the SUP or RCP. The SUP had a relatively weak concentration
320 of clay content, and the distribution was more discrete. The degree of joint greyscale

321 concentration in the RCP soil layers was between that of the UMP and SUP, with
322 higher joint gray scale concentrations in 20-40 cm and 40-60 cm soils. Under natural
323 conditions, the degree of correlation between the clay content of each soil layer was
324 relatively high, and land subsidence decreased this spatial correlation while land
325 reclamation increased the spatial correlation.

326 *3.3.2 Silt content*

327 The joint multi-fractal spectra and projection greyscale maps of the PSD multi-fractal
328 parameters of the silt soils in the three sample sites are shown in Figs. 7, 8, and 9.
329 There was a certain similarity between the silt content and the clay content in the joint
330 multi-fractal greyscale pattern. The concentration of dots in the greyscale of the UMP
331 silt content was relatively high, and the degree of correlation between the middle soil
332 layers was greater than those of the upper and bottom soil layers. The concentration
333 degree of the grey map of the SUP increasingly grew with the improve of soil depth,
334 but was smaller than that of the UMP or RCP, indicating that land subsidence reduced
335 the correlation between the surface soils. The concentration of dots in the joint
336 greyscale map between the soil layers of the RCP was relatively high, and was
337 opposite to the changing trend in the concentration degree of the UMP with the
338 increase of soil depth. The concentration degree firstly decreased and then increased.
339 As a whole, the concentration degree of the greyscale maps of the silt content was
340 UMP > RCP > SUP.

341 *3.3.3 Sand content*

342 The joint multi-fractal results of sand soil PSD are presented Figs. 10, 11, and 12.

343 The distribution patterns of the joint multi-fractal greyscale image of sand content was
344 different compared to those of the clay and silt content. The correlation degree of sand
345 content of SUP was high, there was no clear law between the different soil layers, the
346 concentration degree of the greyscale of sand contents was greater than that of clay or
347 silt content. The UMP and RCP exhibited the similar multi-fractal greyscale results. In
348 UMP and RCP, the joint multi-fractal parameters of the soil layers of 0-20 cm and
349 20-40 cm, and the soil layers of 40-60 cm and 60-80 cm were all distributed in a small
350 range, while the distribution range of the joint multi-fractal parameters of the 20-40
351 cm and 40-60 cm soil layers were relatively large. The concentration degrees of the
352 UMP and RCP were less than those of the SUP, and they were less than those of clay
353 or silt in the horizontal direction. The concentration degree of sand content was
354 characterized by $SUP > UMP > RCP$, indicating that land subsidence resulted in a
355 great effect on the soil PSD of sand, leading to an increase in the correlation of the
356 soil PSD between different depths.

357 **4 Discussion**

358 *4.1 Impacts of coal mining subsidence on the spatial variability of the soil PSD*

359 Analysis of the spatial variability of the distribution of the joint multi-fractal attributes,
360 the mechanism of the impact of land subsidence on the soil PSD may be revealed. The
361 spatial correlation of the soil PSD can be significantly affected by subsidence (Wang
362 et al. 2018b; Xiao et al. 2013). In this study, the spatial correlation of the clay and silt
363 content was relatively high under natural conditions, while its counterpart in the

364 subsidence area was clearly reduced. Under natural conditions, the spatial correlation
365 of the soil sand content was relatively low, but land subsidence caused the spatial
366 correlation to increase.

367 Moreover, it was manifested that land subsidence resulted in an increase in the spatial
368 variability of the soil PSD range, concentration, heterogeneity, and asymmetry.

369 Compared to the unmined soil with the same site conditions in the study area, the
370 subsidence resulting from coal mining can give rise to serious surface deformation
371 and soil destruction, which, consequently, change soil physical properties, such as
372 particle size, bulk density, and porosity (Wang et al. 2017a). Therefore, soil erosion in
373 the subsidence area was intensified (Zhang et al. 2011). Soil erosion could give rise to
374 the breakage of some coarse particles and the deprivation of some fine particles (Jing
375 et al. 2018), changing the soil PSD (Liu et al. 2009; Zhang et al. 2010). Land
376 subsidence would lead to an increase in the range of the soil PSD, and undermine the
377 spatial auto-correlation of the soil PSD, causing the PSD to be in a disordered uneven
378 state (Zhang et al. 2015b). This change was also confirmed by the reduction in the
379 clay content and the increase in the sand content in the subsidence area in this study.
380 As a consequence, land subsidence increased the spatial variability of the soil PSD
381 range ($D(0)$), concentration ($D(1)$), heterogeneity ($\Delta\alpha$), and asymmetry (Δf).

382 *4.2 Impacts of land reclamation on the spatial variability of the soil PSD*

383 It was shown that the soil PSD and its spatial correlation between the soil depths after
384 land reclamation was close to those of unmined soils. In the RCP, the implementation
385 of land reclamation began in 2012. Soil restoration improved soil physical properties

386 (Biswas 2019; Nyamadzawo et al. 2008; Wang et al. 2008). The variability of the soil
387 PSD of the RCP approximates that of the UMP, through surface crack filling and
388 vegetation reconstruction. With increasing soil depth, the difference in the distribution
389 between the layers is reduced. Soil remediation may weaken the variability of the soil
390 physical attributes (Wang et al. 2015a; Wang et al. 2017b). In addition, land
391 reclamation exhibited a greater impact on the soil PSD at the 0-20 cm depth than that
392 at other soil layers, which also demonstrated the impact of land reclamation measures
393 on PSD in the RCP soils (Zhou et al. 2015). Therefore, land reclamation decreased the
394 spatial variability of the soil PSD range ($D(0)$), concentration ($D(1)$), heterogeneity
395 ($\Delta\alpha$), and asymmetry (Δf).

396 Affected by factors such as reclamation method, reclamation time, and surface crack
397 filling source, the soil PSD of different soil layers was inconsistent with that of the
398 unmined soil, indicating that the reclamation activity exerts a great impact on the
399 vertical distribution of the soil PSD (Wang et al. 2013). Land reclamation is an
400 artificial activity. Although it can guarantee an approximate range of soil particle size
401 to a certain extent, soils are mechanically compacted during the process because of
402 the use of large machinery (Wang et al. 2012). Therefore, land reclamation
403 technologies should be improved to reduce the influence of machinery compaction on
404 soil physical characteristics (Adeli et al. 2019). Considering the ecological restoration
405 of the mining area, the optimal time for reclamation in the subsided land should
406 receive consideration (Huang et al. 2018).

407 **5 Conclusions**

408 In this study, the spatial variability of the soil particle size distribution in an
409 underground coalmine area was investigated using multi-fractal and joint multi-fractal
410 methods, and the impacts of coal mining subsidence and land rehabilitation on the soil
411 PSD were discussed. Multi-fractal parameters ($D(0)$, $D(1)$, $\Delta\alpha(q)$, and $\Delta f(\alpha)$) can
412 reflect the spatial variability of different soil particle sizes from different aspects and
413 scales. The overall variability of the soil PSD can be characterized by the statistical
414 analysis of multi-fractal attributes, and the joint multi-fractal model can achieve
415 quantitative characterization of the soil PSD between different depths. Land
416 subsidence increased the spatial variability of the soil PSD, as well as the spatial
417 variability of the soil PSD range, concentration, heterogeneity, and asymmetry.
418 Meanwhile, land reclamation decreased the spatial variability of the soil PSD. In
419 summary, coal mining subsidence and land reclamation changed the spatial
420 correlation of the soil PSD between different depths.

421

422 **Acknowledgments**

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426

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550

Figures

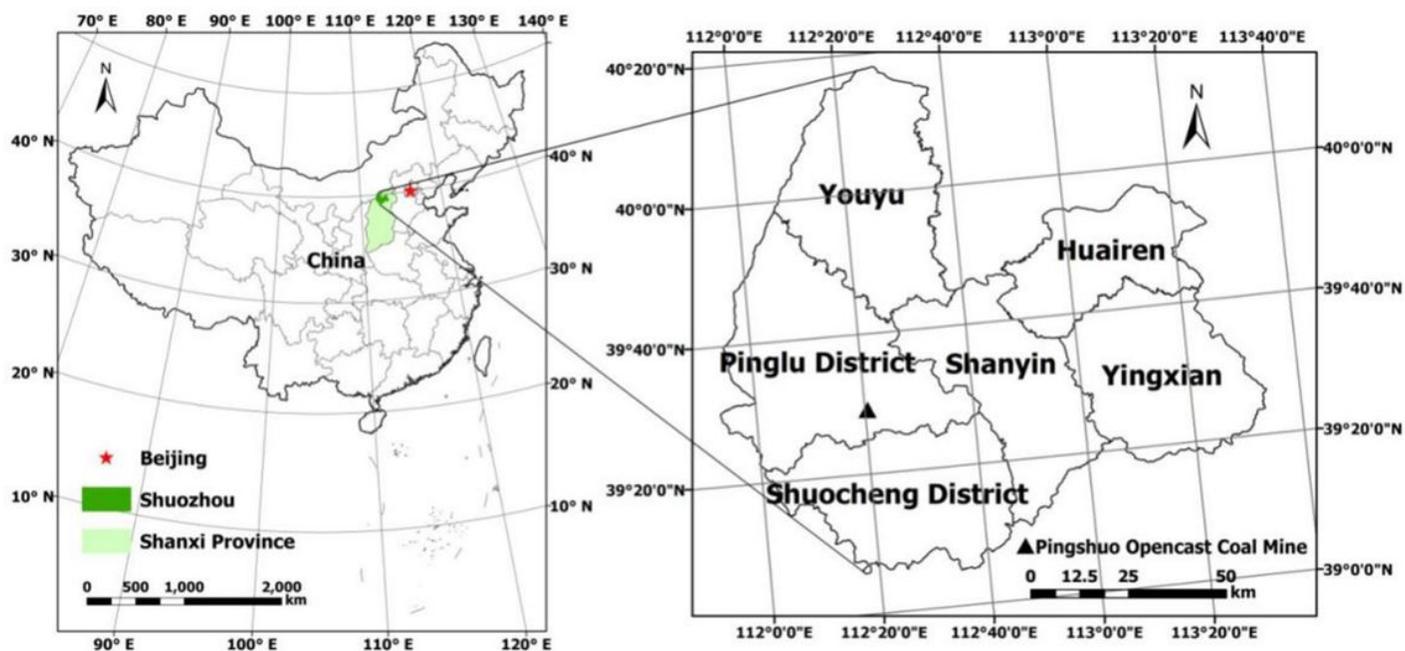


Figure 1

Schematic diagram of the location of Pingshuo mining area. Note: The designations employed and the presentation of the material on this map do not imply the expression of any opinion whatsoever on the part of Research Square concerning the legal status of any country, territory, city or area or of its authorities, or concerning the delimitation of its frontiers or boundaries. This map has been provided by the authors.



a

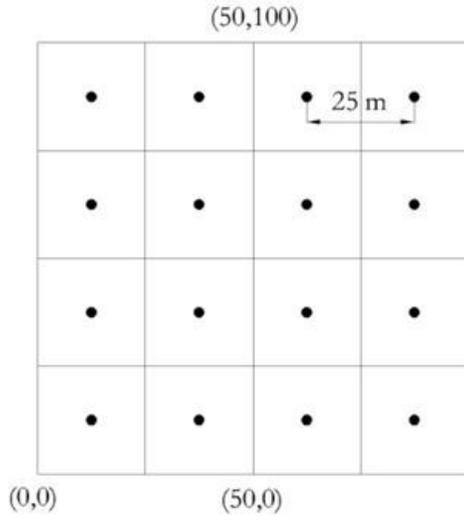
b



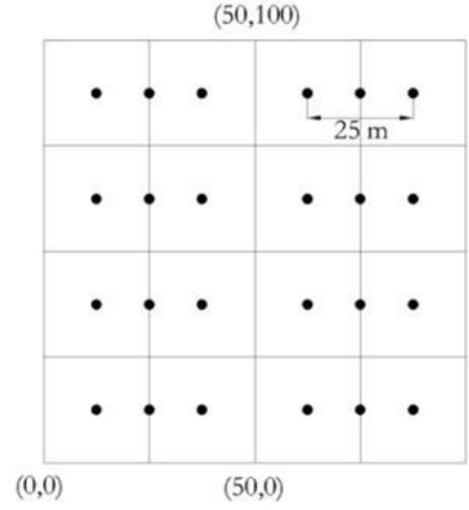
c

Figure 2

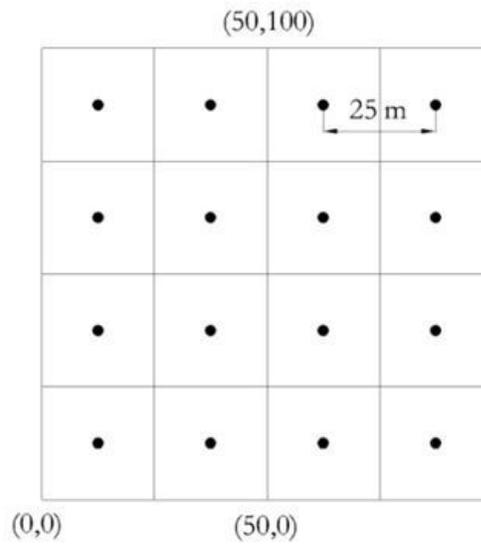
Overview of study plots (a. UMP, b. SUP and c. RCP).



a.UMP



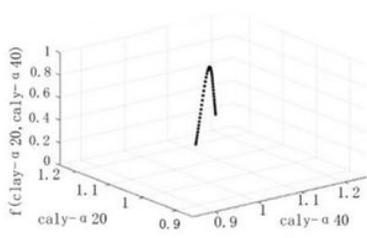
b.SUP



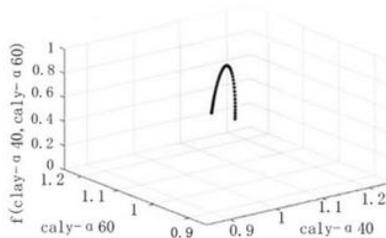
c.RCP

Figure 3

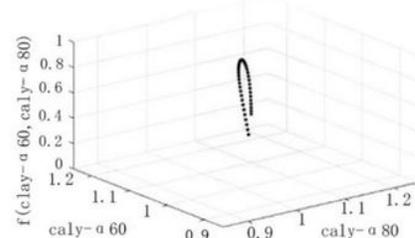
Sampling point layout in study area



0-20cm and 20-40cm

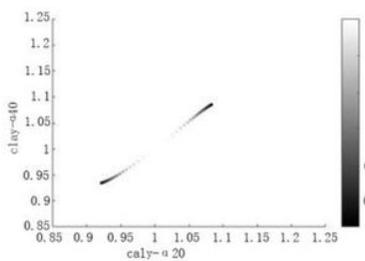


20-40cm and 40-60cm

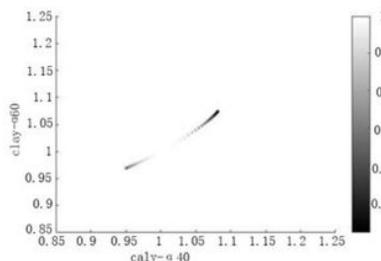


40-60cm and 60-80cm

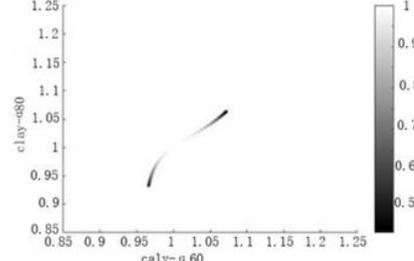
a. Joint multi-fractal spectra of clay content



0-20cm and 20-40cm



20-40cm and 40-60cm

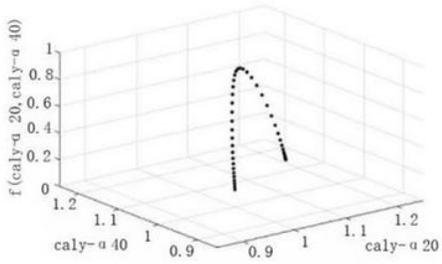


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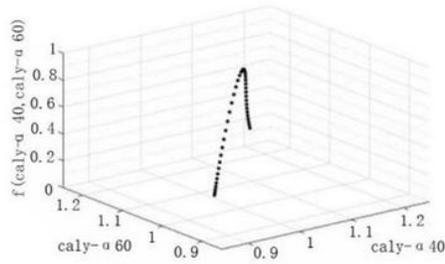
b. Joint multi-fractal spectrum projection grayscale of clay content

Figure 4

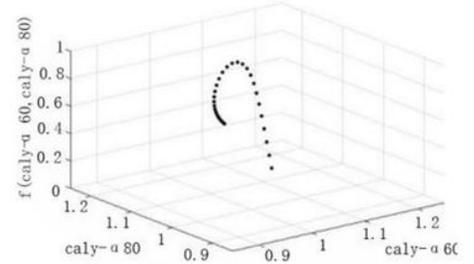
Joint multi-fractal spectra and projection grayscale of clay content in UMP.



0-20cm and 20-40cm

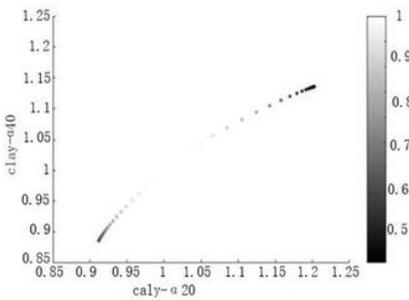


20-40cm and 40-60cm

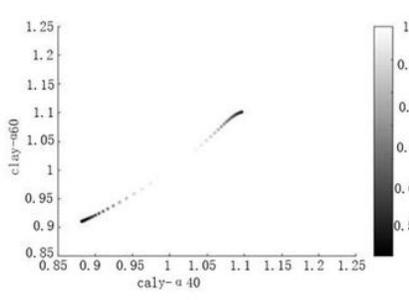


40-60cm and 60-80cm

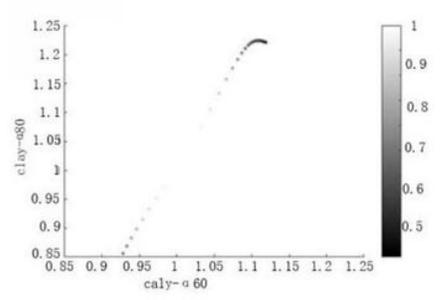
a. Joint multi-fractal spectra of clay content



0-20cm and 20-40cm



20-40cm and 40-60cm

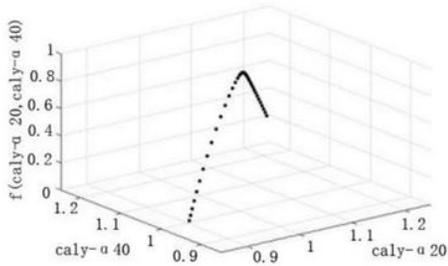


40-60cm and 60-80cm

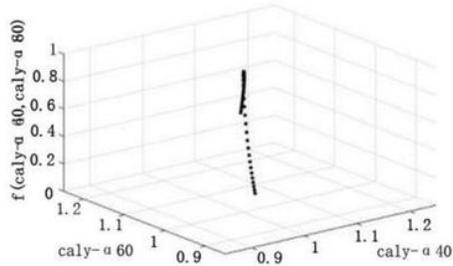
b. Joint multi-fractal spectrum projection grayscale of clay content

Figure 5

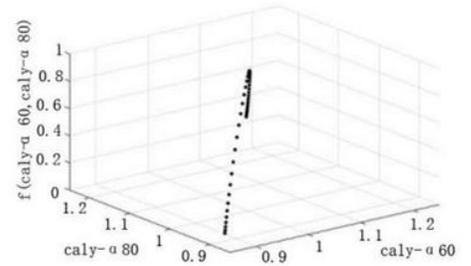
Joint multi-fractal spectra and projection grayscale of clay content in SUP.



0-20cm and 20-40cm

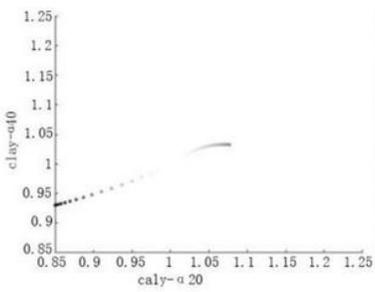


20-40cm and 40-60cm

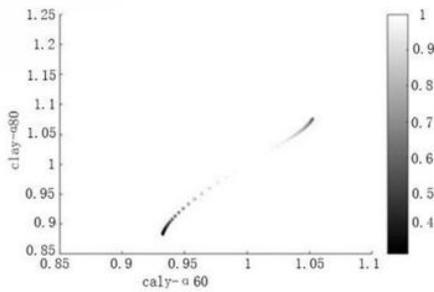


40-60cm and 60-80cm

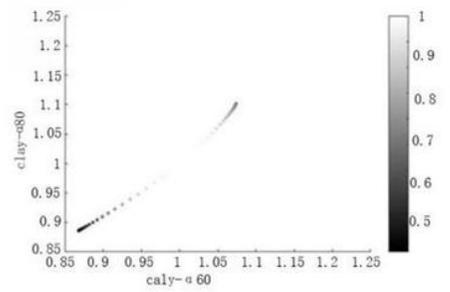
a. Joint multi-fractal spectra of clay content



0-20cm and 20-40cm



20-40cm and 40-60cm

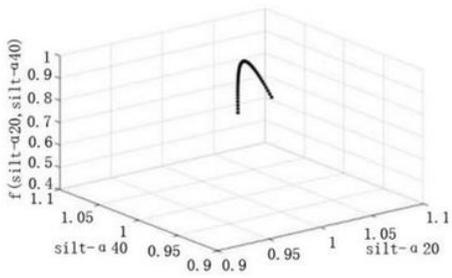


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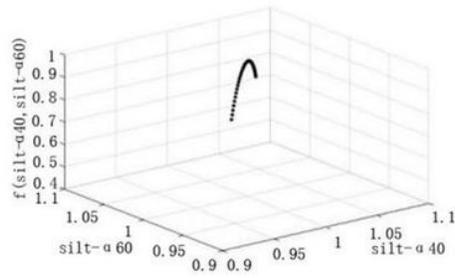
b. Joint multi-fractal spectrum projection grayscale of clay content

Figure 6

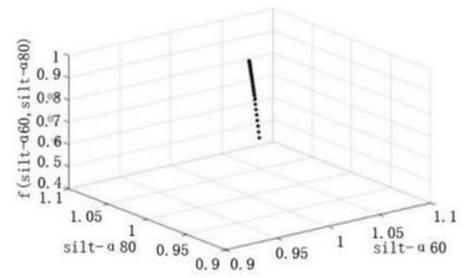
Joint multi-fractal spectra and projection grayscale of clay content in RCP.



0-20cm and 20-40cm

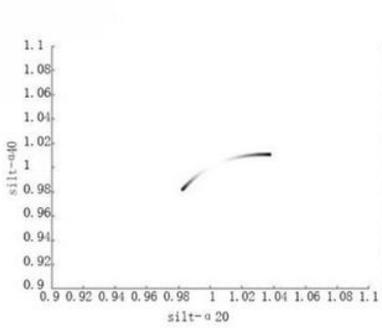


20-40cm and 40-60cm

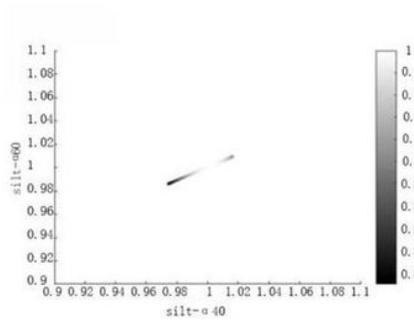


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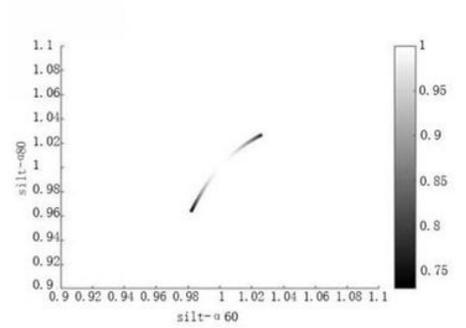
a. Joint multi-fractal spectra of silt content



0-20cm and 20-40cm



20-40cm and 40-60cm

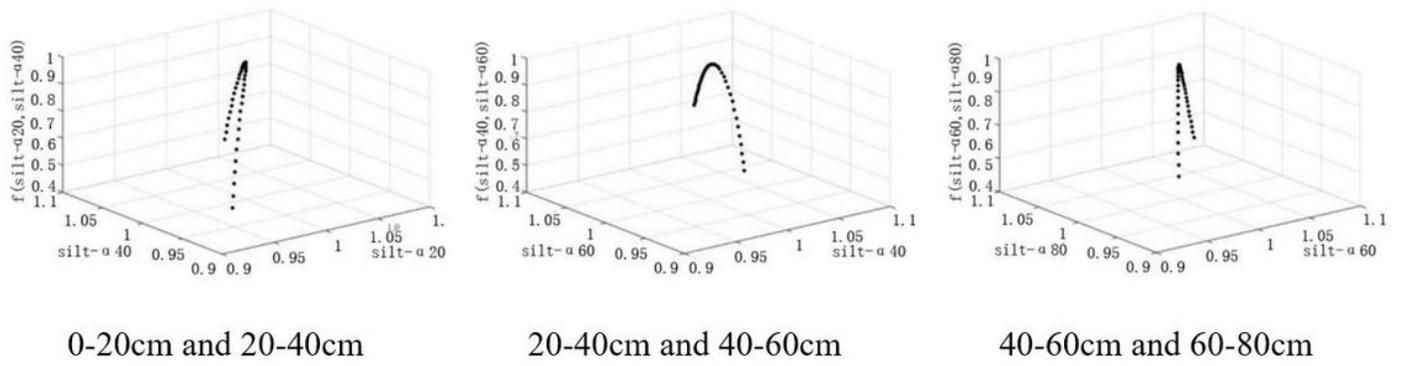


40-60cm and 60-80cm

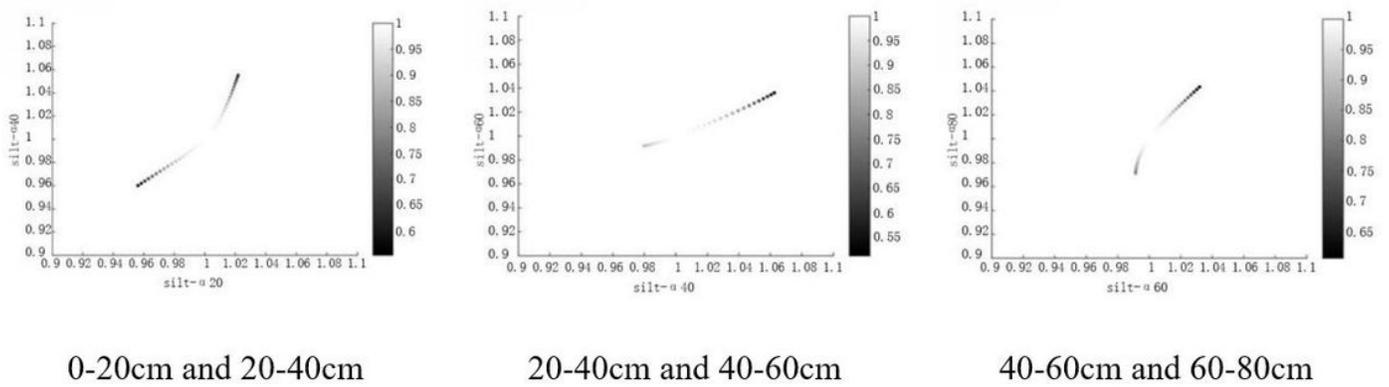
b. Joint multi-fractal spectrum projection grayscale of silt content

Figure 7

Joint multi-fractal spectra and projection grayscale of silt content in UMP.



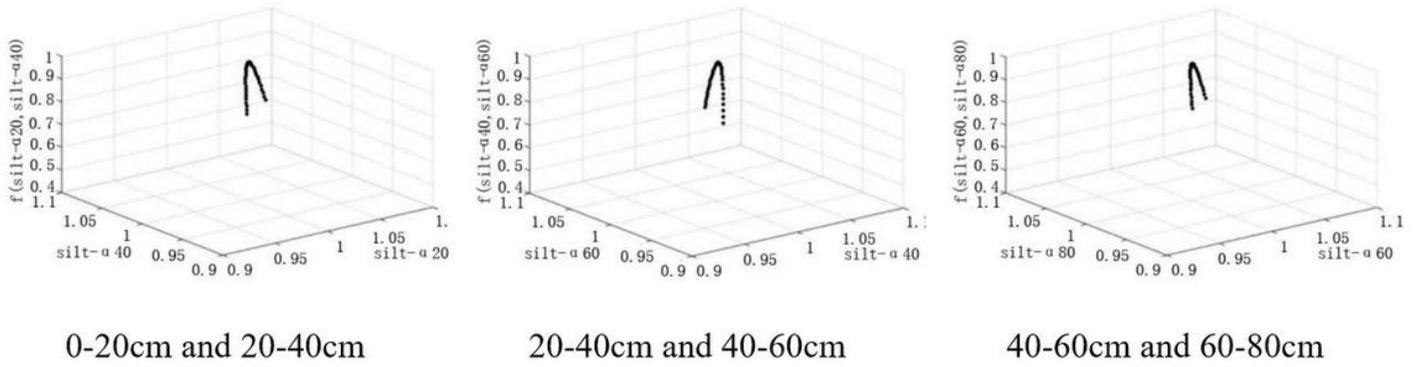
a. Joint multi-fractal spectra of silt content



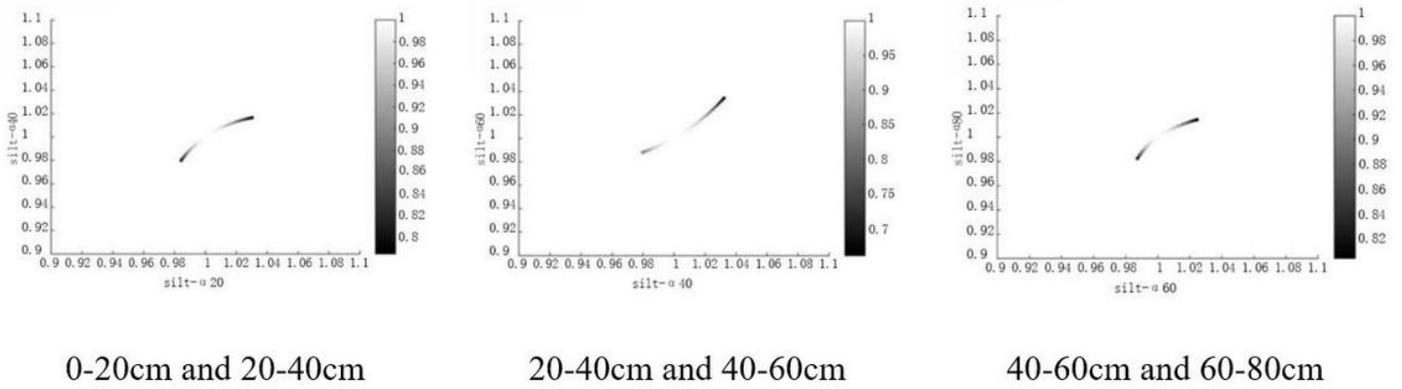
b. Joint multi-fractal spectrum projection grayscale of silt content

Figure 8

Joint multi-fractal spectra and projection grayscale of silt content in SUP.



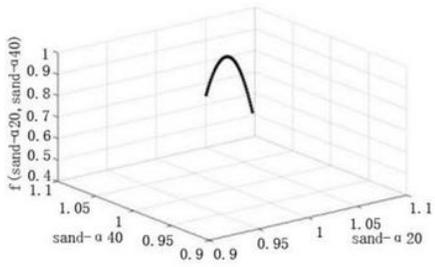
a. Joint multi-fractal spectra of silt content



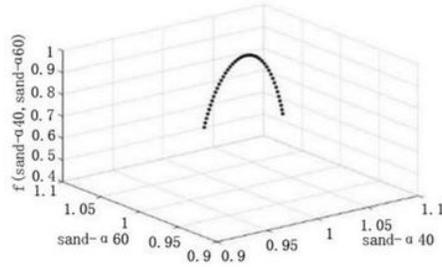
b. Joint multi-fractal spectrum projection grayscale of silt content

Figure 9

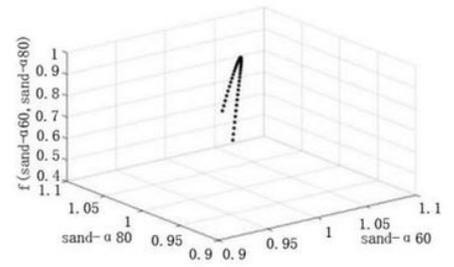
Joint multi-fractal spectra and projection grayscale of silt content in RCP.



0-20cm and 20-40cm

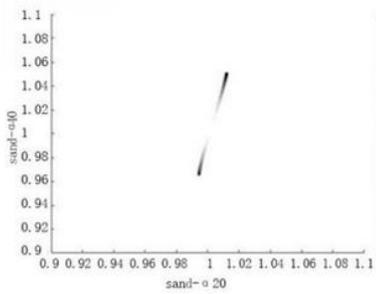


20-40cm and 40-60cm

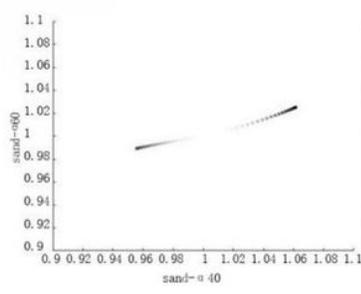


40-60cm and 60-80cm

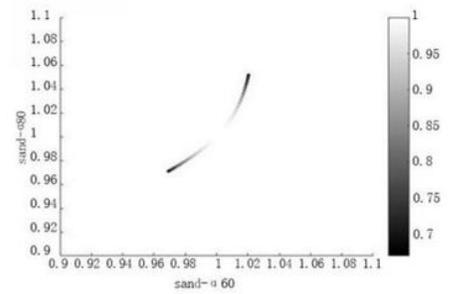
a. Joint multi-fractal spectra of sand content



0-20cm and 20-40cm



20-40cm and 40-60cm

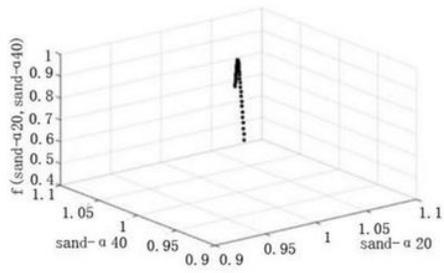


40-60cm and 60-80cm

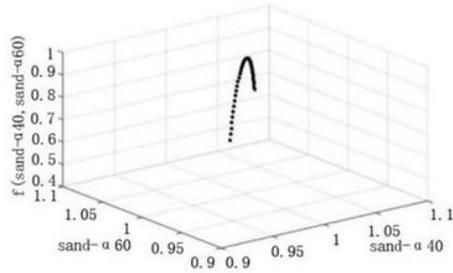
b. Joint multi-fractal spectrum projection grayscale of sand content

Figure 10

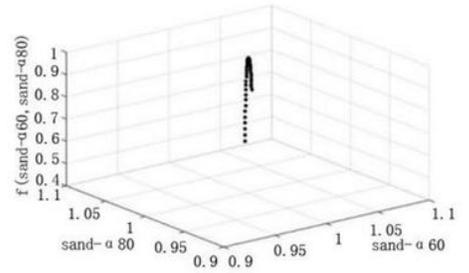
Joint multi-fractal spectra and projection grayscale of sand content in UMP.



0-20cm and 20-40cm

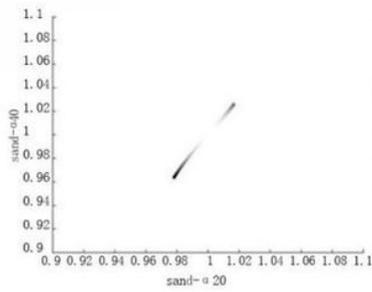


20-40cm and 40-60cm

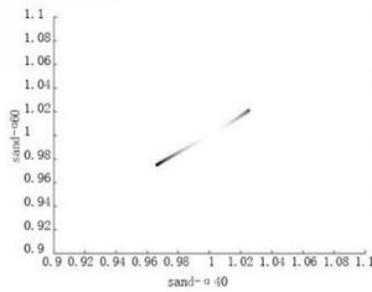


40-60cm and 60-80cm

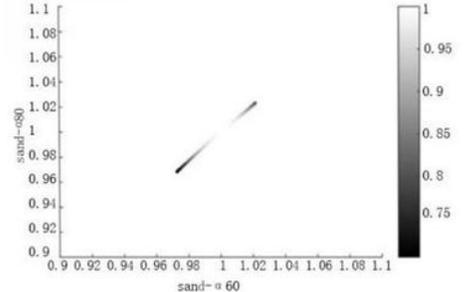
a. Joint multi-fractal spectra of sand content



0-20cm and 20-40cm



20-40cm and 40-60cm

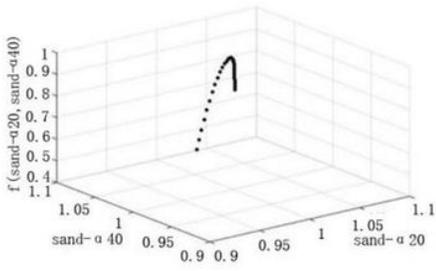


40-60cm and 60-80cm

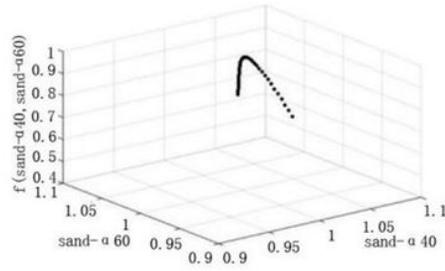
b. Joint multi-fractal spectrum projection grayscale of sand content

Figure 11

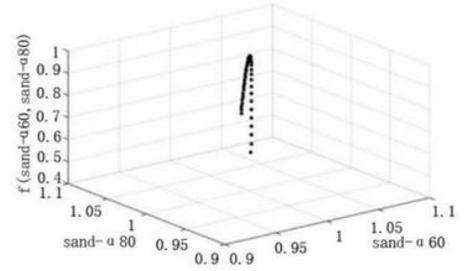
Joint multi-fractal spectra and projection grayscale of sand content in SUP.



0-20cm and 20-40cm

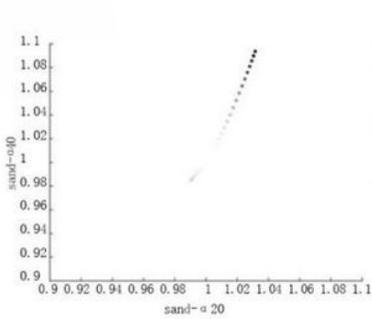


20-40cm and 40-60cm

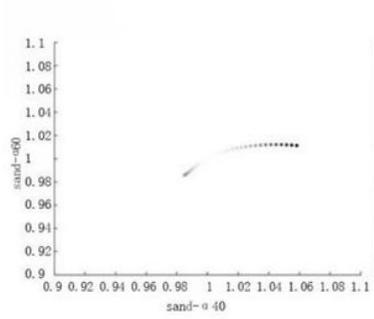


40-60cm and 60-80cm

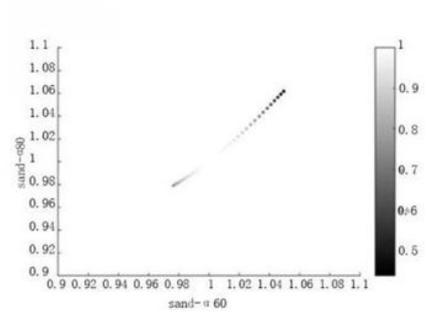
a. Joint multi-fractal spectra of sand content



0-20cm and 20-40cm



20-40cm and 40-60cm



40-60cm and 60-80cm

b. Joint multi-fractal spectrum projection grayscale of sand content

Figure 12

Joint multi-fractal spectra and projection grayscale of sand content in RCP.