Fuzzy-based adaptive asymptotic tracking control design for uncertain MIMO nonlinear systems with state constraints

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Abstract This article concentrates on designing an adaptive fuzzy asymptotic tracking control strategy for uncertain multi-input and multi-output (MIMO) nonlinear systems with state constraints and unknown virtual control coefficients (UVCCs). Such design apply the fuzzy logic systems to approximate unknown dynamics, and an asymptotic controller is recursively constructed by employing a bound estimation method and some smooth functions. The predefined state constrains are guaranteed by virtue of barrier Lyapunov function. Different from the reporting achievements, the restrictive assumption about the prior knowledge of UVCCs is removed in our context. By fusing the lower bounds of UVCCs into iterative Lyapunov functions, the stability of the controlled system is guaranteed. Finally, simulation example is conducted to exhibit the validity of theoretical findings.

Keywords Asymptotic tracking control · state constraints · bound estimation method · nonlinear systems

1 Introduction

Tracking control nonlinear systems has become a hot and rapidly growing research topic. Over the past decade, many approaches and concepts have been presented in order to achieve precise control for nonlinear systems [1–5]. The backstepping control of nonlinear systems has gained tremendous attention due to its recursively character, with many representative achievements being reported [6–8]. Note that these previous work are restricted to the nonlinear systems with known nonlinear dynamics.

Since the preeminent approximation ability of fuzzy logic systems (FLSs) or neural networks (NNs), numerous intelligent fuzzy or NN control designs have been explored for uncertain nonlinear systems (UNS) with unknown dynamics [9–17]. In [9], the adaptive fuzzy control (AFC) design was presented for UNS. In [10], the fuzzy-based backstepping design scheme was fabricated for UNS by associating with high-gain filters. With the help of small-gain theorem, an AFC strategy was investigated for UNS with unmodeled dynamics [11]. In [12], the fuzzy-based prescribed performance thought was explored for UNS. Based on the inverse optimal algorithm, an adaptive fuzzy optimal thought was discovered for UNS with dynamic uncertainties [14]. In [15], the approximated-based intelligent neural thought was described for UNS. By applying the reinforcement learning algorithm [16], an adaptive NN optimized backstepping design thought was dexterously presented for UNS. In [17], the observer-based AFC scheme was firstly proposed for UNS. The above approximation-based intelligent control schemes are mainly focused on the single-input and single-output (SISO) nonlinear systems. Meanwhile, for multi-input and multi-output (MIMO) nonlinear systems, numerous fuzzy or NN control designs have been explored by virtue of backstepping technique [18–20]. In [18], the NN-based tracking design approach was presented for UNS with unknown dynamics. In [19], the fuzzy decoupling analysis strategy was fabricated for the MIMO chemical systems. By constructing auxiliary systems to address the influence of saturation character, the NN-based tracking design approach was discovered for MIMO nonlinear systems [20]. Meanwhile, some output feedback intelligent control designs were presented for MIMO systems with immeasurable states [21–24]. It is worth mentioning that the foregoing works are mainly concentrated on the nonlinear systems without state constraints.

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Due to the safe specification and performance requirement, various constraints are widespread in the practical physical systems. Therefore, it is essential to explore the effective control strategy to address this problem. Recently, the barrier Lyapunov function (BLF) has been exploited to deal with the state constraints problem. For nonlinear systems with output constraint, the BLF-based tracking design approach was discovered [25]. Based on BLF design thought, the full state constraints were subsequently investigated for UNS with parametric uncertainties [26]. Meanwhile, some intelligent fuzzy or NN control methods were proposed by fusing the BLF into backstepping design framework [27–34]. In [28], the intelligent tracking design scheme was fabricated for state-constrained UNS with faults. In [30], the BLF-based AFC method was presented for MIMO nonlinear systems with uncertainties. Associating with BLF design thought, the approximation-NN tracking control strategy was fabricated for state-constrained MIMO nonlinear systems [31]. By applying filter backstepping technique [33], a NN tracking control design was described for state-constrained MIMO nonlinear systems. Considering the MIMO chemical process with state constraints [34], the AFC algorithm was devised with the help of backstepping technique. Now that these intelligent design methods only achieve the bounded-error control effect, that is to say, the output tracking error converges into a small region, whose size can be adjusted by selecting design parameters.

Obviously, the asymptotic tracking control has better convergence feature than the bounded-error result. Since the remarkable superiority of asymptotic control, much endeavors had been made by various scientific communities [35–37]. By introducing positive integrable function to compensate the dead-zone nonlinearity [35], an adaptive asymptotic design scheme was presented for UNS. In [37], the modified backstepping asymptotic tracking method was devised for UNS with input quantization feature. The above asymptotic design thoughts suppose that the unknown virtual control coefficients (UVCCs) of nonlinear systems are constants or known functions. To surmount this restriction, by integrating the lower bounds of UVCCs into Lyapunov function construction, a fuzzy-based asymptotic controller was delicately devised for UNS with UVCCs and unknown dynamics [38]. Moreover, the design thought in [38] was extended into UNS with full state constraints and UVCCs [39]. As far as we know, how to construct an asymptotic tracking strategy for MIMO nonlinear systems with state constraints and UVCCs has not been adequately investigated, which stimulate us to study this problem.

This article will attempt to construct an adaptive fuzzy asymptotic tracking control (AFATC) scheme for uncertain MIMO nonlinear systems with state constraints and UVCCs. More specifically, the BLF is utilized to maintain the prescribed state constraints. By applying the bound estimation method and some positive integral functions, the adverse influences caused by UVCCs are surmounted. The main contributions of the AFATC design are enumerated as follows:

(1) In contrast to the published intelligent approximation methods of MIMO nonlinear systems [18–20], where only achieve bound-error control effect. By applying backstepping technique and some smooth functions, the asymptotic controller is recursively constructed.

(2) Different with the asymptotic control design [38], which is mainly focused on the SISO nonlinear systems with UVCCs. In our context, the intelligent asymptotic tracking control design is further extended to MIMO nonlinear systems. In addition, the adverse effects of UVCCs are successfully surmounted by adopting the bound estimation method.

(3) By fusing the lower bounds of UVCCs into the BLF construction in every recursively design step, the asymptotic convergence character is guaranteed and the state constraints are not violated.

2 Problem formulation and preliminaries

Consider the MIMO nonlinear systems as follows:

$$\begin{cases} \dot{x}_{i,1} = f_{i,1}\left(x_{i,1}\right) + g_{i,1}\left(x_{i,1}\right) x_{i,2} \\ \dot{x}_{i,2} = f_{i,2}\left(\bar{x}_{i,2}\right) + g_{i,2}\left(\bar{x}_{i,2}\right) x_{i,3} \\ \qquad \vdots \\ \dot{x}_{i,n_i} = f_{i,n_i}\left(x\right) + g_{i,n_i}\left(x\right) u_i \\ y_i = x_{i,1} \end{cases} \tag{1}$$

where $\bar{x}_{i,s} = [x_{i,1}, x_{i,2}, \ldots, x_{i,s}]^T \in R^s (i = 1, \ldots, m, s = 1, \ldots, n_i)$ is the state vector for the ith subsystem; $x = \left[x_1^{\mathrm{T}}, \ldots, x_i^{\mathrm{T}}, \ldots, x_m^{\mathrm{T}}\right]^{\mathrm{T}}$ with $x_i = [x_{i,1}, \ldots, x_{i,n_i}]^{\mathrm{T}}$; $u_i \in R$ and $y_i \in R$ denote the control input and system output; $f_{i,s}(\cdot)$ and $g_{i,s}(\cdot)$ denote unknown functions. All states are demanded to stay in the sets $|x_{i,s}| < k_{c_{i,s}}$ with $k_{c_{i,s}} > 0$ being constants.

This article aims to construct an AFATC strategy for system (1), such that the system output y_i can asymptotically follow the desired signal y_{di}, and the prescribed state constraints are not violated.

Assumption 1 *The desired signals $y_{di}, i = 1, \ldots, m$ and their time derivatives are continuous and bounded. Meanwhile, there are positive constants $A_{i,0}$ such that $|y_{d,i}| \leq A_{i,0} \leq k_{c_{i,1}}$.*

Assumption 2 *The signs of $g_{i,s}(\cdot)$ are known, i.e., $g_{i,s}(\cdot)$ are either positive or negative. Without losing generality, it is further supposed that $g_{i,s}(\cdot) \geq \underline{g}_{i,s} > 0$ with $\underline{g}_{i,s}$ being positive constants.*

Lemma 1 (Barbalat Lemma) *[2] For $t \geq 0$, if $\phi : R^+ \to R$ is uniformly continuous, and $\lim_{t\to\infty} \int_0^t |\phi(\tau)| d\tau$ exists and is bounded, then one has:*

$$\lim_{t\to\infty} \phi(t) = 0. \tag{2}$$

Lemma 2 *[37] For positive continuous function $\sigma(t)$ and any $x \in R$, we can get*

$$|x| \leq \frac{x^2}{\sqrt{x^2 + \sigma^2(t)}} + \sigma(t) \tag{3}$$

with $\sigma(t)$ satisfying

$$\lim_{t\to\infty} \int_{t_0}^{t} \sigma(\tau) d\tau \leq \bar{\sigma} < \infty \tag{4}$$

where $\bar{\sigma} > 0$ is a constant.

Lemma 3 *[9] Define a continuous unknown function $\mathcal{F}(\wp)$ on the compact set Ω. Therefore, for $\mathcal{C} > 0$, there is a fuzzy logic system, then*

$$\sup_{x\in\Omega} \left| \mathcal{F}(\wp) - \mathcal{P}^T \mathcal{Q}(\wp) \right| \leq \mathcal{C} \tag{5}$$

where $\mathcal{P} = [p_1, p_2, \ldots, p_l]^T$ is the ideal weight vector; $\mathcal{Q}(\wp) = [q_1(\wp), \ldots, q_l(\wp)]^T / \sum_{i=1}^{l} q_i(\wp)$ is the basis function vector and $q_i(x)$ is governed by

$$q_i(\wp) = \exp\left[\frac{-(\wp - \Xi_i)^T (\wp - \Xi_i)}{\varkappa_i^2} \right] \tag{6}$$

where $\Xi_i = [\Xi_{i1}, \Xi_{i2}, \ldots, \Xi_{in}]^T$ is the center vector and $\varkappa_i$ is the width, respectively.

3 Asymptotic controller design and analysis

3.1 Controller design

In this section, an AFATC is presented to achieve control aim. To this end, introduce the following coordinate transformation:

$$\begin{cases} \zeta_{i,1} = x_{i,1} - y_{di} \\ \zeta_{i,s} = x_{i,s} - \alpha_{i,s-1}, \quad i = 1, \ldots, m \end{cases} \tag{7}$$

where $\alpha_{i,s-1}$ are the virtual control laws to be devised later.

Step $i, 1$: By means of (1) and (7), the time derivative of $\zeta_{i,1}$ is computed as

$$\begin{aligned} \dot{\zeta}_{i,1} &= \dot{x}_{i,1} - \dot{y}_{di} \\ &= f_{i,1} + g_{i,1} x_{i,2} - \dot{y}_{di} \\ &= f_{i,1} + g_{i,1}(\zeta_{i,2} + \alpha_{i,1}) - \dot{y}_{di} \end{aligned} \tag{8}$$

where $\zeta_{i,2} = x_{i,2} - \alpha_{i,1}$.

The Lyapunov function is constructed as

$$V_{i,1} = \frac{1}{2\underline{g}_{i,1}} \log \frac{k_{b_{i,1}}^2}{k_{b_{i,1}}^2 - \zeta_{i,1}^2} + \frac{1}{2\gamma_{i,1}} \tilde{\theta}_{i,1}^2 + \frac{1}{2\lambda_{i,1}} \tilde{\beta}_{i,1}^2 \tag{9}$$

where $\gamma_{i,1} > 0$ and $\lambda_{i,1} > 0$ are constants; $\tilde{\theta}_{i,1} = \theta_{i,1} - \hat{\theta}_{i,1}$ and $\tilde{\beta}_{i,1} = \beta_{i,1} - \hat{\beta}_{i,1}$ are the estimation errors, $\hat{\theta}_{i,1}$ and $\hat{\beta}_{i,1}$ are the estimations of $\theta_{i,1}$ and $\beta_{i,1}$, whose definitions will be given in the later. Similar [31], define the set $\Xi_{\zeta_{i,1}} = \left\{ \zeta_{i,1} \in R | \, |\zeta_{i,1}| < k_{b_{i,1}} \right\}$ with $k_{b_{i,1}} = k_{c_{i,1}} - A_{i,0}$ being positive constant.

Calculating $\dot{V}_{i,1}$ along (8) yields

$$\begin{aligned}\dot{V}_{i,1} =& \frac{1}{\underline{g}_{i,1}} \kappa_{\zeta_{i,1}} \left[f_{i,1} + g_{i,1}(\zeta_{i,2} + \alpha_{i,1}) - \dot{y}_{di} \right] - \frac{1}{\gamma_{i,1}} \tilde{\theta}_{i,1} \dot{\hat{\theta}}_{i,1} - \frac{1}{\lambda_{i,1}} \tilde{\beta}_{i,1} \dot{\hat{\beta}}_{i,1} \\ =& \frac{1}{\underline{g}_{i,1}} \kappa_{\zeta_{i,1}} (g_{i,1}\zeta_{i,2} + g_{i,1}\alpha_{i,1} + \mathcal{F}_{i,1}) - \frac{1}{\gamma_{i,1}} \tilde{\theta}_{i,1} \dot{\hat{\theta}}_{i,1} - \frac{1}{\lambda_{i,1}} \tilde{\beta}_{i,1} \dot{\hat{\beta}}_{i,1}\end{aligned} \tag{10}$$

where $\kappa_{\zeta_{i,1}} = \frac{\zeta_{i,1}}{k_{b_{i,1}}^2 - \zeta_{i,1}^2}$ and $\mathcal{F}_{i,1} = f_{i,1} - \dot{y}_{di}$.

Since the functions $\mathcal{F}_{i,1}$ are unknown, which can not directly apply to control design process. Associating with Lemma 3, the FLS $\mathcal{P}_{i,1}^T \mathcal{Q}_{i,1}(\wp_{i,1})$ is utilized to approximate $\mathcal{F}_{i,1}$, that is to say

$$\mathcal{F}_{i,1} = \mathcal{P}_{i,1}^T \mathcal{Q}_{i,1}(\wp_{i,1}) + \mathcal{C}_{i,1}(\wp_{i,1}), |\mathcal{C}_{i,1}| \leq \bar{\mathcal{C}}_{i,1} \tag{11}$$

where $\wp_{i,1} = \left[x_{i,1}, \hat{\theta}_{i,1}, \hat{\beta}_{i,1}, y_{di}, \dot{y}_{di} \right]^T$.

Inserting (11) into (10) yields

$$\begin{aligned}\dot{V}_{i,1} =& \frac{g_{i,1}}{\underline{g}_{i,1}} \kappa_{\zeta_{i,1}} (\zeta_{i,2} + \alpha_{i,1}) + \frac{\kappa_{\zeta_{i,1}}}{\underline{g}_{i,1}} \mathcal{P}_{i,1}^T \mathcal{Q}_{i,1}(\wp_{i,1}) + \frac{\kappa_{\zeta_{i,1}}}{\underline{g}_{i,1}} \mathcal{C}_{i,1}(\wp_{i,1}) - \frac{1}{\gamma_{i,1}} \tilde{\theta}_{i,1} \dot{\hat{\theta}}_{i,1} - \frac{1}{\lambda_{i,1}} \tilde{\beta}_{i,1} \dot{\hat{\beta}}_{i,1} \\ \leq& \frac{g_{i,1}}{\underline{g}_{i,1}} \kappa_{\zeta_{i,1}} (\zeta_{i,2} + \alpha_{i,1}) + \frac{\|\mathcal{P}_{i,1}\|}{\underline{g}_{i,1}} |\kappa_{\zeta_{i,1}}| \, \|\mathcal{Q}_{i,1}(\wp_{i,1})\| + \frac{\bar{\mathcal{C}}_{i,1}}{\underline{g}_{i,1}} |\kappa_{\zeta_{i,1}}| - \frac{1}{\gamma_{i,1}} \tilde{\theta}_{i,1} \dot{\hat{\theta}}_{i,1} - \frac{1}{\lambda_{i,1}} \tilde{\beta}_{i,1} \dot{\hat{\beta}}_{i,1} \\ \leq& \frac{g_{i,1}}{\underline{g}_{i,1}} \kappa_{\zeta_{i,1}} (\zeta_{i,2} + \alpha_{i,1}) + \theta_{i,1} |\kappa_{\zeta_{i,1}}| \, \|\mathcal{Q}_{i,1}(\wp_{i,1})\| + \beta_{i,1} |\kappa_{\zeta_{i,1}}| - \frac{1}{\gamma_{i,1}} \tilde{\theta}_{i,1} \dot{\hat{\theta}}_{i,1} - \frac{1}{\lambda_{i,1}} \tilde{\beta}_{i,1} \dot{\hat{\beta}}_{i,1}\end{aligned} \tag{12}$$

where the definitions of $\theta_{i,1}$ and $\beta_{i,1}$ are $\theta_{i,1} = \frac{\|\mathcal{P}_{i,1}\|}{\underline{g}_{i,1}}$ and $\beta_{i,1} = \frac{\bar{\mathcal{C}}_{i,1}}{\underline{g}_{i,1}}$.

Noting that Lemma 2, for $\sigma_{i,1} > 0$, we arrive at

$$\theta_{i,1} |\kappa_{\zeta_{i,1}}| \, \|\mathcal{Q}_{i,1}(\wp_{i,1})\| \frac{\theta_{i,1} \kappa_{\zeta_{i,1}}^2 \|\mathcal{Q}_{i,1}(\wp_{i,1})\|^2}{\sqrt{\kappa_{\zeta_{i,1}}^2 \|\mathcal{Q}_{i,1}(\wp_{i,1})\|^2 + \sigma_{i,1}^2}} + \theta_{i,1} \sigma_{i,1} \tag{13}$$

$$\beta_{i,1} |\kappa_{\zeta_{i,1}}| \leq \frac{\beta_{i,1} \kappa_{\zeta_{i,1}}^2}{\sqrt{\kappa_{\zeta_{i,1}}^2 + \sigma_{i,1}^2}} + \beta_{i,1} \sigma_{i,1}. \tag{14}$$

Inserting (13) and (14) into (12) yields

$$\begin{aligned}\dot{V}_{i,1} \leq& \frac{g_{i,1}}{\underline{g}_{i,1}} \kappa_{\zeta_{i,1}} (\zeta_{i,2} + \alpha_{i,1}) + \frac{\hat{\theta}_{i,1} \kappa_{\zeta_{i,1}}^2 \|\mathcal{Q}_{i,1}(\wp_{i,1})\|^2}{\sqrt{\kappa_{\zeta_{i,1}}^2 \|\mathcal{Q}_{i,1}(\wp_{i,1})\|^2 + \sigma_{i,1}^2}} + \frac{\hat{\beta}_{i,1} \kappa_{\zeta_{i,1}}^2}{\sqrt{\kappa_{\zeta_{i,1}}^2 + \sigma_{i,1}^2}} + \sigma_{i,1}(\theta_{i,1} + \beta_{i,1}) \\ & - \frac{\tilde{\theta}_{i,1}}{\gamma_{i,1}} \left(\dot{\hat{\theta}}_{i,1} - \gamma_{i,1} \frac{\kappa_{\zeta_{i,1}}^2 \|\mathcal{Q}_{i,1}(\wp_{i,1})\|^2}{\sqrt{\kappa_{\zeta_{i,1}}^2 \|\mathcal{Q}_{i,1}(\wp_{i,1})\|^2 + \sigma_{i,1}^2}} \right) - \frac{\tilde{\beta}_{i,1}}{\lambda_{i,1}} \left(\dot{\hat{\beta}}_{i,1} - \lambda_{i,1} \frac{\kappa_{\zeta_{i,1}}^2}{\sqrt{\kappa_{\zeta_{i,1}}^2 + \sigma_{i,1}^2}} \right).\end{aligned} \tag{15}$$

The virtual control law $\alpha_{i,1}$ and adaptive laws $\dot{\hat{\theta}}_{i,1}$, $\dot{\hat{\beta}}_{i,1}$ are structured as

$$\alpha_{i,1} = - c_{i,1} \zeta_{i,1} - \frac{\kappa_{\zeta_{i,1}} \hat{\theta}_{i,1} \|\mathcal{Q}_{i,1}(\wp_{i,1})\|^2}{\sqrt{\kappa_{\zeta_{i,1}}^2 \|\mathcal{Q}_{i,1}(\wp_{i,1})\|^2 + \sigma_{i,1}^2}} - \frac{\kappa_{\zeta_{i,1}} \hat{\beta}_{i,1}}{\sqrt{\kappa_{\zeta_{i,1}}^2 + \sigma_{i,1}^2}} \tag{16}$$

$$\dot{\hat{\theta}}_{i,1} = \gamma_{i,1} \frac{\kappa_{\zeta_{i,1}}^2 \|\mathcal{Q}_{i,1}(\wp_{i,1})\|^2}{\sqrt{\kappa_{\zeta_{i,1}}^2 \|\mathcal{Q}_{i,1}(\wp_{i,1})\|^2 + \sigma_{i,1}^2}} - \gamma_{i,1} \sigma_{i,1} \hat{\theta}_{i,1} \tag{17}$$

$$\dot{\hat{\beta}}_{i,1} = \lambda_{i,1}\frac{\kappa_{\zeta_{i,1}}^2}{\sqrt{\kappa_{\zeta_{i,1}}^2+\sigma_{i,1}^2}} - \lambda_{i,1}\sigma_{i,1}\hat{\beta}_{i,1} \tag{18}$$

where $c_{i,1} > 0$, $\gamma_{i,1} > 0$ and $\lambda_{i,1} > 0$ are relevant parameters.

Applying the (16)-(18) to (15) yields

$$\dot{V}_{i,1} \leq -c_{i,1}\frac{\zeta_{i,1}^2}{k_{b_{i,1}}^2-\zeta_{i,1}^2} + \frac{g_{i,1}}{\underline{g}_{i,1}}\kappa_{\zeta_{i,1}}\zeta_{i,2} + \sigma_{i,1}\left(\theta_{i,1}+\beta_{i,1}\right) + \sigma_{i,1}\tilde{\theta}_{i,1}\hat{\theta}_{i,1} + \sigma_{i,1}\tilde{\beta}_{i,1}\hat{\beta}_{i,1}. \tag{19}$$

Step $i, s(2 \leq s \leq n_i - 1)$: From $\zeta_{i,s} = x_{i,s} - \alpha_{i,s-1}$, it follows that

$$\begin{aligned}\dot{\zeta}_{i,s} &= \dot{x}_{i,s} - \dot{\alpha}_{i,s-1}\\ &= f_{i,s} + g_{i,s}x_{i,s+1} - \dot{\alpha}_{i,s-1}\\ &= f_{i,s} + g_{i,s}\zeta_{i,s+1} + g_{i,s}\alpha_{i,s} - \dot{\alpha}_{i,s-1}\end{aligned} \tag{20}$$

where

$$\dot{\alpha}_{i,s-1} = \sum_{k=1}^{s-1}\frac{\partial\alpha_{i,s-1}}{\partial x_{i,k}}\left(f_{i,k}+g_{i,k}x_{i,k+1}\right) + \sum_{k=1}^{s-1}\frac{\partial\alpha_{i,s-1}}{\partial\hat{\theta}_{i,k}}\dot{\hat{\theta}}_{i,k} + \sum_{k=1}^{s-1}\frac{\partial\alpha_{i,s-1}}{\partial\hat{\beta}_{i,k}}\dot{\hat{\beta}}_{i,k} + \sum_{k=0}^{s-1}\frac{\partial\alpha_{i,s-1}}{\partial y_{di}^{(k)}}y_{di}^{(k+1)}. \tag{21}$$

Consider the Lyapunov function as

$$V_{i,s} = V_{i,s-1} + \frac{1}{2\underline{g}_{i,s}}\log\frac{k_{b_{i,s}}^2}{k_{b_{i,s}}^2-\zeta_{i,s}^2} + \frac{1}{2\gamma_{i,s}}\tilde{\theta}_{i,s}^2 + \frac{1}{2\lambda_{i,s}}\tilde{\beta}_{i,s}^2 \tag{22}$$

where $\gamma_{i,s} > 0$ and $\lambda_{i,s} > 0$ are constants and $|\zeta_{i,s}| < k_{b_{i,s}}$; $\tilde{\theta}_{i,s} = \theta_{i,s} - \hat{\theta}_{i,s}$ and $\tilde{\beta}_{i,s} = \beta_{i,s} - \hat{\beta}_{i,s}$ are the estimation errors, $\hat{\theta}_{i,s}$ and $\hat{\beta}_{i,s}$ are the estimations of $\theta_{i,s}$ and $\beta_{i,s}$, whose definitions will be given in the later.

The time derivative of $V_{i,s}$ is

$$\begin{aligned}\dot{V}_{i,s} = &-\sum_{k=1}^{s-1}c_{i,k}\frac{\zeta_{i,k}^2}{k_{b_{i,k}}^2-\zeta_{i,k}^2} + \sum_{k=1}^{s-1}\sigma_{i,k}\left(\theta_{i,k}+\beta_{i,k}\right) + \sum_{k=1}^{s-1}\sigma_{i,k}\tilde{\theta}_{i,k}\hat{\theta}_{i,k} + \sum_{k=1}^{s-1}\sigma_{i,k}\tilde{\beta}_{i,k}\hat{\beta}_{i,k}\\ &+ \frac{g_{i,s}}{\underline{g}_{i,s}}\kappa_{\zeta_{i,s}}(\zeta_{i,s+1}+\alpha_{i,s}) + \frac{\kappa_{\zeta_{i,s}}}{\underline{g}_{i,s}}\mathcal{F}_{i,s} - \frac{1}{\gamma_{i,s}}\tilde{\theta}_{i,s}\dot{\hat{\theta}}_{i,s} - \frac{1}{\lambda_{i,s}}\tilde{\beta}_{i,s}\dot{\hat{\beta}}_{i,s}\end{aligned} \tag{23}$$

where $\kappa_{\zeta_{i,s}} = \frac{\zeta_{i,s}}{k_{b_{i,s}}^2-\zeta_{i,s}^2}$ and $\mathcal{F}_{i,s} = f_{i,s} - \dot{\alpha}_{i,s-1} + \frac{g_{i,s-1}\underline{g}_{i,s}}{\underline{g}_{i,s-1}}\kappa_{\zeta_{i,s-1}}$.

In accordance with Lemma 3, the FLS $\mathcal{P}_{i,s}^T\mathcal{Q}_{i,s}(\wp_{i,s})$ can be utilized to approximate $\mathcal{F}_{i,s}$ as follows:

$$\mathcal{F}_{i,s} = \mathcal{P}_{i,s}^T\mathcal{Q}_{i,s}(\wp_{i,s}) + \mathcal{C}_{i,s}(\wp_{i,s}), |\mathcal{C}_{i,s}(\wp_{i,s})| \leq \bar{\mathcal{C}}_{i,s} \tag{24}$$

where $\wp_{i,s} = \left[\bar{x}_{i,s}^T, \bar{y}_{di}^T, \bar{\theta}_{i,s}^T, \bar{\beta}_{i,s}^T\right]^T$, $\bar{y}_{di} = [y_{di}, \dot{y}_{di}, \ldots, y_{di}^{(s)}]^T$, $\bar{\theta}_{i,s} = [\hat{\theta}_{i,1}, \ldots, \hat{\theta}_{i,s}]^T$, $\bar{\beta}_{i,s} = [\hat{\beta}_{i,1}, \ldots, \hat{\beta}_{i,s}]^T$.

Inserting (24) into (23) yields

$$\begin{aligned}\dot{V}_{i,s} = &-\sum_{k=1}^{s-1}c_{i,k}\frac{\zeta_{i,k}^2}{k_{b_{i,k}}^2-\zeta_{i,k}^2} + \sum_{k=1}^{s-1}\sigma_{i,k}\left(\theta_{i,k}+\beta_{i,k}\right) + \sum_{k=1}^{s-1}\sigma_{i,k}\tilde{\theta}_{i,k}\hat{\theta}_{i,k} + \sum_{k=1}^{s-1}\sigma_{i,k}\tilde{\beta}_{i,k}\hat{\beta}_{i,k}\\ &+ \frac{g_{i,s}}{\underline{g}_{i,s}}\kappa_{\zeta_{i,s}}(\zeta_{i,s+1}+\alpha_{i,s}) + \frac{\kappa_{\zeta_{i,s}}}{\underline{g}_{i,s}}\mathcal{P}_{i,s}^T\mathcal{Q}_{i,s}(\wp_{i,s}) + \frac{\kappa_{\zeta_{i,s}}}{\underline{g}_{i,s}}\mathcal{C}_{i,s}(\wp_{i,s}) - \frac{1}{\gamma_{i,s}}\tilde{\theta}_{i,s}\dot{\hat{\theta}}_{i,s} - \frac{1}{\lambda_{i,s}}\tilde{\beta}_{i,s}\dot{\hat{\beta}}_{i,s}\\ \leq &-\sum_{k=1}^{s-1}c_{i,k}\frac{\zeta_{i,k}^2}{k_{b_{i,k}}^2-\zeta_{i,k}^2} + \sum_{k=1}^{s-1}\sigma_{i,k}\left(\theta_{i,k}+\beta_{i,k}\right) + \sum_{k=1}^{s-1}\sigma_{i,k}\tilde{\theta}_{i,k}\hat{\theta}_{i,k} + \sum_{k=1}^{s-1}\sigma_{i,k}\tilde{\beta}_{i,k}\hat{\beta}_{i,k}\\ &+ \frac{g_{i,s}}{\underline{g}_{i,s}}\kappa_{\zeta_{i,s}}(\zeta_{i,s+1}+\alpha_{i,s}) + \theta_{i,s}|\kappa_{\zeta_{i,s}}|\,\|\mathcal{Q}_{i,s}(\wp_{i,s})\| + \beta_{i,s}|\kappa_{\zeta_{i,s}}| - \frac{1}{\gamma_{i,s}}\tilde{\theta}_{i,s}\dot{\hat{\theta}}_{i,s} - \frac{1}{\lambda_{i,s}}\tilde{\beta}_{i,s}\dot{\hat{\beta}}_{i,s}\end{aligned} \tag{25}$$

where $\theta_{i,s} = \frac{\|\mathcal{P}_{i,s}\|}{\underline{g}_{i,s}}$ and $\beta_{i,s} = \frac{\bar{\mathcal{C}}_{i,s}}{\underline{g}_{i,s}}$.

Associating with Lemma 2, for $\sigma_{i,s} > 0$, we have

$$\theta_{i,s}|\kappa_{\zeta_{i,s}}| \left\|\mathcal{Q}_{i,s}(\wp_{i,s})\right\| \frac{\theta_{i,s}\kappa_{\zeta_{i,s}}^2 \left\|\mathcal{Q}_{i,s}(\wp_{i,s})\right\|^2}{\sqrt{\kappa_{\zeta_{i,s}}^2 \left\|\mathcal{Q}_{i,s}(\wp_{i,s})\right\|^2 + \sigma_{i,s}^2}} + \theta_{i,s}\sigma_{i,s} \tag{26}$$

$$\beta_{i,s}|\kappa_{\zeta_{i,s}}| \leq \frac{\beta_{i,s}\kappa_{\zeta_{i,s}}^2}{\sqrt{\kappa_{\zeta_{i,s}}^2 + \sigma_{i,s}^2}} + \beta_{i,s}\sigma_{i,s}. \tag{27}$$

From (26)-(27), the (25) can be resolved as

$$\begin{aligned}\dot{V}_{i,s} \leq & -\sum_{k=1}^{s-1} c_{i,k}\frac{\zeta_{i,k}^2}{k_{b_{i,k}}^2 - \zeta_{i,k}^2} + \sum_{k=1}^{s-1}\sigma_{i,k}\left(\theta_{i,k} + \beta_{i,k}\right) + \sum_{k=1}^{s-1}\sigma_{i,k}\tilde{\theta}_{i,k}\hat{\theta}_{i,k} + \sum_{k=1}^{s-1}\sigma_{i,k}\tilde{\beta}_{i,k}\hat{\beta}_{i,k} + \frac{g_{i,s}}{\underline{g}_{i,s}}\kappa_{\zeta_{i,s}}\zeta_{i,s+1} \\ & + \frac{g_{i,s}}{\underline{g}_{i,s}}\kappa_{\zeta_{i,s}}\alpha_{i,s} + \frac{\hat{\theta}_{i,s}\kappa_{\zeta_{i,s}}^2 \left\|\mathcal{Q}_{i,s}(\wp_{i,s})\right\|^2}{\sqrt{\kappa_{\zeta_{i,s}}^2 \left\|\mathcal{Q}_{i,s}(\wp_{i,s})\right\|^2 + \sigma_{i,s}^2}} + \frac{\hat{\beta}_{i,s}\kappa_{\zeta_{i,s}}^2}{\sqrt{\kappa_{\zeta_{i,s}}^2 + \sigma_{i,s}^2}} + \sigma_{i,s}(\theta_{i,s} + \beta_{i,s}) \\ & - \frac{\tilde{\theta}_{i,s}}{\gamma_{i,s}}\left(\dot{\hat{\theta}}_{i,s} - \gamma_{i,s}\frac{\kappa_{\zeta_{i,s}}^2 \left\|\mathcal{Q}_{i,s}(\wp_{i,s})\right\|^2}{\sqrt{\kappa_{\zeta_{i,s}}^2 \left\|\mathcal{Q}_{i,s}(\wp_{i,s})\right\|^2 + \sigma_{i,s}^2}}\right) - \frac{\tilde{\beta}_{i,s}}{\lambda_{i,s}}\left(\dot{\hat{\beta}}_{i,s} - \lambda_{i,s}\frac{\kappa_{\zeta_{i,s}}^2}{\sqrt{\kappa_{\zeta_{i,s}}^2 + \sigma_{i,s}^2}}\right).\end{aligned} \tag{28}$$

The $\alpha_{i,s}$ and $\dot{\hat{\theta}}_{i,s}$, $\dot{\hat{\beta}}_{i,s}$ are formulated as

$$\alpha_{i,s} = -c_{i,s}\zeta_{i,s} - \frac{\kappa_{\zeta_{i,s}}\hat{\theta}_{i,s} \left\|\mathcal{Q}_{i,s}(\wp_{i,s})\right\|^2}{\sqrt{\kappa_{\zeta_{i,s}}^2 \left\|\mathcal{Q}_{i,s}(\wp_{i,s})\right\|^2 + \sigma_{i,s}^2}} - \frac{\kappa_{\zeta_{i,s}}\hat{\beta}_{i,s}}{\sqrt{\kappa_{\zeta_{i,s}}^2 + \sigma_{i,s}^2}} \tag{29}$$

$$\dot{\hat{\theta}}_{i,s} = \gamma_{i,s}\frac{\kappa_{\zeta_{i,s}}^2 \left\|\mathcal{Q}_{i,s}(\wp_{i,s})\right\|^2}{\sqrt{\kappa_{\zeta_{i,s}}^2 \left\|\mathcal{Q}_{i,s}(\wp_{i,s})\right\|^2 + \sigma_{i,s}^2}} - \gamma_{i,s}\sigma_{i,s}\hat{\theta}_{i,s} \tag{30}$$

$$\dot{\hat{\beta}}_{i,s} = \lambda_{i,s}\frac{\kappa_{\zeta_{i,s}}^2}{\sqrt{\kappa_{\zeta_{i,s}}^2 + \sigma_{i,s}^2}} - \lambda_{i,s}\sigma_{i,s}\hat{\beta}_{i,s} \tag{31}$$

where $c_{i,s}$, $\gamma_{i,s}$ and $\lambda_{i,s}$ are positive constants.

Substituting (29)-(31) into (28) results in

$$\dot{V}_{i,s} \leq -\sum_{k=1}^{s} c_{i,k}\frac{\zeta_{i,k}^2}{k_{b_{i,k}}^2 - \zeta_{i,k}^2} + \sum_{k=1}^{s}\sigma_{i,k}\left(\theta_{i,k} + \beta_{i,k}\right) + \sum_{k=1}^{s}\sigma_{i,k}\tilde{\theta}_{i,k}\hat{\theta}_{i,k} + \sum_{k=1}^{s}\sigma_{i,k}\tilde{\beta}_{i,k}\hat{\beta}_{i,k} + \frac{g_{i,s}}{\underline{g}_{i,s}}\kappa_{\zeta_{i,s}}\zeta_{i,s+1}. \tag{32}$$

Step i, n_i: Since $\zeta_{i,n_i} = x_{i,n_i} - \alpha_{i,n_i-1}$, there is

$$\begin{aligned}\dot{\zeta}_{i,n_i} &= \dot{x}_{i,n_i} - \dot{\alpha}_{i,n_i-1} \\ &= f_{i,n_i} + g_{i,n_i}u_i - \dot{\alpha}_{i,n_i-1}\end{aligned} \tag{33}$$

where

$$\dot{\alpha}_{i,n_i-1} = \sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial x_{i,k}}\left(f_{i,k} + g_{i,k}x_{i,k+1}\right) + \sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{\theta}_{i,k}}\dot{\hat{\theta}}_{i,k} + \sum_{k=1}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial\hat{\beta}_{i,k}}\dot{\hat{\beta}}_{i,k} + \sum_{k=0}^{n_i-1}\frac{\partial\alpha_{i,n_i-1}}{\partial y_{di}^{(k)}}y_{di}^{(k+1)}. \tag{34}$$

Consider the following Lyapunov function

$$V_{i,n_i} = V_{i,n_i-1} + \frac{1}{2\underline{g}_{i,n_i}}\log\frac{k_{b_{i,n_i}}^2}{k_{b_{i,n_i}}^2 - \zeta_{i,n_i}^2} + \frac{1}{2\gamma_{i,n_i}}\tilde{\theta}_{i,n_i}^2 + \frac{1}{2\lambda_{i,n_i}}\tilde{\beta}_{i,n_i}^2 \tag{35}$$

where $\gamma_{i,n_i} > 0$ and $\lambda_{i,n_i} > 0$ are constants; $\tilde{\theta}_{i,n_i} = \theta_{i,n_i} - \hat{\theta}_{i,n_i}$ and $\tilde{\beta}_{i,n_i} = \beta_{i,n_i} - \hat{\beta}_{i,n_i}$ are the estimation errors, $\hat{\theta}_{i,n_i}$ and $\hat{\beta}_{i,n_i}$ are the estimations of θ_{i,n_i} and β_{i,n_i}, whose definitions will be given in the later.

Differentiating V_{i,n_i} along (33) generates

$$\begin{aligned}\dot{V}_{i,n_i} =& -\sum_{k=1}^{n_i-1} c_{i,k}\frac{\zeta_{i,k}^2}{k_{b_{i,k}}^2-\zeta_{i,k}^2} + \sum_{k=1}^{n_i-1}\sigma_{i,k}\left(\theta_{i,k}+\beta_{i,k}\right) + \sum_{k=1}^{n_i-1}\sigma_{i,k}\tilde{\theta}_{i,k}\hat{\theta}_{i,k} + \sum_{k=1}^{n_i-1}\sigma_{i,k}\tilde{\beta}_{i,k}\hat{\beta}_{i,k}\\ &+ \frac{g_{i,n_i}}{\underline{g}_{i,n_i}}\kappa_{\zeta_{i,n_i}}u_i + \frac{\kappa_{\zeta_{i,n_i}}}{\underline{g}_{i,n_i}}\mathcal{F}_{i,n_i} - \frac{1}{\gamma_{i,n_i}}\tilde{\theta}_{i,n_i}\dot{\hat{\theta}}_{i,n_i} - \frac{1}{\lambda_{i,n_i}}\tilde{\beta}_{i,n_i}\dot{\hat{\beta}}_{i,n_i}\end{aligned} \tag{36}$$

where $\kappa_{\zeta_{i,n_i}} = \frac{\zeta_{i,n_i}}{k_{b_{i,n_i}}^2-\zeta_{i,n_i}^2}$ and $\mathcal{F}_{i,n_i} = f_{i,n_i} - \dot{\alpha}_{i,n_i-1} + \frac{g_{i,n_i-1}\underline{g}_{i,n_i}}{\underline{g}_{i,n_i-1}}\kappa_{\zeta_{i,n_i-1}}$.

From Lemma 3, the $\mathcal{F}_{i,n_i}$ can be reconstructed by the FLS, that is to say

$$\mathcal{F}_{i,n_i} = \mathcal{P}_{i,n_i}^T\mathcal{Q}_{i,n_i}(\wp_{i,n_i}) + \mathcal{C}_{i,n_i}(\wp_{i,n_i}), |\mathcal{C}_{i,n_i}| \le \bar{\mathcal{C}}_{i,n_i} \tag{37}$$

where $\wp_{i,n_i} = \left[\bar{x}_{i,n_i}^T, \hat{y}_{di}^T, \bar{\theta}_{i,n_i}^T, \bar{\beta}_{i,n_i}^T\right]^T$, $\hat{y}_{di} = [y_{di}, \dot{y}_{di}, \ldots, y_{di}^{(n_i)}]^T$, $\bar{\theta}_{i,n_i} = [\hat{\theta}_{i,1}, \ldots, \hat{\theta}_{i,n_i}]^T$ and $\bar{\beta}_{i,n_i} = [\hat{\beta}_{i,1}, \ldots, \hat{\beta}_{i,n_i}]^T$.

Taking (37) into (36) yields

$$\begin{aligned}\dot{V}_{i,n_i} =& -\sum_{k=1}^{n_i-1} c_{i,k}\frac{\zeta_{i,k}^2}{k_{b_{i,k}}^2-\zeta_{i,k}^2} + \sum_{k=1}^{n_i-1}\sigma_{i,k}\left(\theta_{i,k}+\beta_{i,k}\right) + \sum_{k=1}^{n_i-1}\sigma_{i,k}\tilde{\theta}_{i,k}\hat{\theta}_{i,k} + \sum_{k=1}^{n_i-1}\sigma_{i,k}\tilde{\beta}_{i,k}\hat{\beta}_{i,k}\\ &+ \frac{g_{i,n_i}}{\underline{g}_{i,n_i}}\kappa_{\zeta_{i,n_i}}u_i + \frac{\kappa_{\zeta_{i,n_i}}}{\underline{g}_{i,n_i}}\mathcal{P}_{i,n_i}^T\mathcal{Q}_{i,n_i}(\wp_{i,n_i}) + \frac{\kappa_{\zeta_{i,n_i}}}{\underline{g}_{i,n_i}}\mathcal{C}_{i,n_i}(\wp_{i,n_i}) - \frac{\tilde{\theta}_{i,n_i}\dot{\hat{\theta}}_{i,n_i}}{\gamma_{i,n_i}} - \frac{\tilde{\beta}_{i,n_i}\dot{\hat{\beta}}_{i,n_i}}{\lambda_{i,n_i}}\\ \le& -\sum_{k=1}^{n_i-1} c_{i,k}\frac{\zeta_{i,k}^2}{k_{b_{i,k}}^2-\zeta_{i,k}^2} + \sum_{k=1}^{n_i-1}\sigma_{i,k}\left(\theta_{i,k}+\beta_{i,k}\right) + \sum_{k=1}^{n_i-1}\sigma_{i,k}\tilde{\theta}_{i,k}\hat{\theta}_{i,k} + \sum_{k=1}^{n_i-1}\sigma_{i,k}\tilde{\beta}_{i,k}\hat{\beta}_{i,k}\\ &+ \frac{g_{i,n_i}}{\underline{g}_{i,n_i}}\kappa_{\zeta_{i,n_i}}u_i + \theta_{i,n_i}|\kappa_{\zeta_{i,n_i}}|\left\|\mathcal{Q}_{i,n_i}(\wp_{i,n_i})\right\| + \beta_{i,n_i}|\kappa_{\zeta_{i,n_i}}| - \frac{\tilde{\theta}_{i,n_i}\dot{\hat{\theta}}_{i,n_i}}{\gamma_{i,n_i}} - \frac{\tilde{\beta}_{i,n_i}\dot{\hat{\beta}}_{i,n_i}}{\lambda_{i,n_i}}\end{aligned} \tag{38}$$

where $\theta_{i,n_i} = \frac{\|\mathcal{P}_{i,n_i}\|}{\underline{g}_{i,n_i}}$ and $\beta_{i,n_i} = \frac{\bar{\mathcal{C}}_{i,n_i}}{\underline{g}_{i,n_i}}$.

By applying Lemma 2, for $\sigma_{i,n_i} > 0$, we can obtain

$$\theta_{i,n_i}|\kappa_{\zeta_{i,n_i}}|\left\|\mathcal{Q}_{i,n_i}(\wp_{i,n_i})\right\| \le \frac{\theta_{i,n_i}\kappa_{\zeta_{i,n_i}}^2\left\|\mathcal{Q}_{i,n_i}(\wp_{i,n_i})\right\|^2}{\sqrt{\kappa_{\zeta_{i,n_i}}^2\left\|\mathcal{Q}_{i,n_i}(\wp_{i,n_i})\right\|^2+\sigma_{i,n_i}^2}} + \theta_{i,n_i}\sigma_{i,n_i} \tag{39}$$

$$\beta_{i,n_i}|\kappa_{\zeta_{i,n_i}}| \le \frac{\beta_{i,n_i}\kappa_{\zeta_{i,n_i}}^2}{\sqrt{\kappa_{\zeta_{i,n_i}}^2+\sigma_{i,n_i}^2}} + \beta_{i,n_i}\sigma_{i,n_i}. \tag{40}$$

Inserting (39)-(40) into (38) gives

$$\begin{aligned}\dot{V}_{i,n_i} \le& -\sum_{k=1}^{n_i-1} c_{i,k}\frac{\zeta_{i,k}^2}{k_{b_{i,k}}^2-\zeta_{i,k}^2} + \sum_{k=1}^{n_i-1}\sigma_{i,k}\left(\theta_{i,k}+\beta_{i,k}\right) + \sum_{k=1}^{n_i-1}\sigma_{i,k}\tilde{\theta}_{i,k}\hat{\theta}_{i,k} + \sum_{k=1}^{n_i-1}\sigma_{i,k}\tilde{\beta}_{i,k}\hat{\beta}_{i,k}\\ &+ \frac{g_{i,n_i}}{\underline{g}_{i,n_i}}\kappa_{\zeta_{i,n_i}}u_i + \frac{\hat{\theta}_{i,n_i}\kappa_{\zeta_{i,n_i}}^2\left\|\mathcal{Q}_{i,n_i}(\wp_{i,n_i})\right\|^2}{\sqrt{\kappa_{\zeta_{i,n_i}}^2\left\|\mathcal{Q}_{i,n_i}(\wp_{i,n_i})\right\|^2+\sigma_{i,n_i}^2}} + \frac{\hat{\beta}_{i,n_i}\kappa_{\zeta_{i,n_i}}^2}{\sqrt{\kappa_{\zeta_{i,n_i}}^2+\sigma_{i,n_i}^2}}\\ &+ \sigma_{i,n_i}(\theta_{i,n_i}+\beta_{i,n_i}) - \frac{\tilde{\theta}_{i,n_i}}{\gamma_{i,n_i}}\left(\dot{\hat{\theta}}_{i,n_i} - \gamma_{i,n_i}\frac{\kappa_{\zeta_{i,n_i}}^2\left\|\mathcal{Q}_{i,n_i}(\wp_{i,n_i})\right\|^2}{\sqrt{\kappa_{\zeta_{i,n_i}}^2\left\|\mathcal{Q}_{i,n_i}(\wp_{i,n_i})\right\|^2+\sigma_{i,n_i}^2}}\right)\\ &- \frac{\tilde{\beta}_{i,n_i}}{\lambda_{i,n_i}}\left(\dot{\hat{\beta}}_{i,n_i} - \lambda_{i,n_i}\frac{\kappa_{\zeta_{i,n_i}}^2}{\sqrt{\kappa_{\zeta_{i,n_i}}^2+\sigma_{i,n_i}^2}}\right)\end{aligned} \tag{41}$$

The actual control law u_i and the adaptive laws $\dot{\hat{\theta}}_{i,n_i}$, $\dot{\hat{\beta}}_{i,n_i}$ are formulated as

$$u_i = -c_{i,n_i}\zeta_{i,n_i} - \frac{\kappa_{\zeta_{i,n_i}}\hat{\theta}_{i,n_i}\left\|\mathcal{Q}_{i,n_i}(\wp_{i,n_i})\right\|^2}{\sqrt{\kappa^2_{\zeta_{i,n_i}}\left\|\mathcal{Q}_{i,n_i}(\wp_{i,n_i})\right\|^2+\sigma^2_{i,n_i}}} - \frac{\kappa_{\zeta_{i,n_i}}\hat{\beta}_{i,n_i}}{\sqrt{\kappa^2_{\zeta_{i,n_i}}+\sigma^2_{i,n_i}}} \tag{42}$$

$$\dot{\hat{\theta}}_{i,n_i} = \gamma_{i,n_i}\frac{\kappa^2_{\zeta_{i,n_i}}\left\|\mathcal{Q}_{i,n_i}(\wp_{i,n_i})\right\|^2}{\sqrt{\kappa^2_{\zeta_{i,n_i}}\left\|\mathcal{Q}_{i,n_i}(\wp_{i,n_i})\right\|^2+\sigma^2_{i,n_i}}} - \gamma_{i,n_i}\sigma_{i,n_i}\hat{\theta}_{i,n_i} \tag{43}$$

$$\dot{\hat{\beta}}_{i,n_i} = \lambda_{i,n_i}\frac{\kappa^2_{\zeta_{i,n_i}}}{\sqrt{\kappa^2_{\zeta_{i,n_i}}+\sigma^2_{i,n_i}}} - \lambda_{i,n_i}\sigma_{i,n_i}\hat{\beta}_{i,n_i} \tag{44}$$

where c_{i,n_i}, γ_{i,n_i} and λ_{i,n_i} are positive constants.

From (42)-(44), the (41) can be resolved as

$$\dot{V}_{i,n_i} \le -\sum_{k=1}^{n_i} c_{i,k}\frac{\zeta^2_{i,k}}{k^2_{b_{i,k}}-\zeta^2_{i,k}} + \sum_{k=1}^{n_i}\sigma_{i,k}\left(\theta_{i,k}+\beta_{i,k}\right) + \sum_{k=1}^{n_i}\sigma_{i,k}\tilde{\theta}_{i,k}\hat{\theta}_{i,k} + \sum_{k=1}^{n_i}\sigma_{i,k}\tilde{\beta}_{i,k}\hat{\beta}_{i,k}. \tag{45}$$

Based on Young's inequality, there are

$$\tilde{\theta}_{i,k}\hat{\theta}_{i,k} = \tilde{\theta}_{i,k}\big(\theta_{i,k}-\tilde{\theta}_{i,k}\big) = -\tilde{\theta}^2_{i,k} + \tilde{\theta}_{i,k}\theta_{i,k} \le \frac{\theta^2_{i,k}}{4} \tag{46}$$

$$\tilde{\beta}_{i,k}\hat{\beta}_{i,k} = \tilde{\beta}_{i,k}\big(\beta_{i,k}-\tilde{\beta}_{i,k}\big) = -\tilde{\beta}^2_{i,k} + \tilde{\beta}_{i,k}\beta_{i,k} \le \frac{\beta^2_{i,k}}{4}. \tag{47}$$

then the (45) will resolved as

$$\begin{aligned}\dot{V}_{i,n_i} &\le -\sum_{k=1}^{n_i} c_{i,k}\frac{\zeta^2_{i,k}}{k^2_{b_{i,k}}-\zeta^2_{i,k}} + \sum_{k=1}^{n_i}\sigma_{i,k}\left(\theta_{i,k}+\beta_{i,k}\right) + \sum_{k=1}^{n_i}\sigma_{i,k}\left(\frac{\theta^2_{i,k}}{4}+\frac{\beta^2_{i,k}}{4}\right)\\ &\le -\sum_{k=1}^{n_i} c_{i,k}\frac{\zeta^2_{i,k}}{k^2_{b_{i,k}}-\zeta^2_{i,k}} + \sum_{k=1}^{n_i}\sigma_{i,k}\varepsilon_{i,k}\end{aligned} \tag{48}$$

where $\varepsilon_{i,k} = (\theta_{i,k}+\beta_{i,k}) + \left(\frac{\theta^2_{i,k}}{4}+\frac{\beta^2_{i,k}}{4}\right)$.

3.2 Stability analysis

Theorem 1 *Consider MIMO nonlinear systems* (1) *in presence of state constraints and UVCCs. The constructed AFATC strategy can make the output tracking error asymptotically converge to zero, and all states never transgress their prescribed regions.*

Proof: From t_0 to t, integrating (48) yields

$$\begin{aligned}V_{i,n_i}(t) \le& V_{i,n_i}(t_0) - \sum_{k=1}^{n_i} c_{i,k}\int_{t_0}^{t}\frac{\zeta^2_{i,k}}{k^2_{b_{i,k}}-\zeta^2_{i,k}}d\tau + \sum_{k=1}^{n_i}\varepsilon_{i,k}\int_{t_0}^{t}\sigma_{i,k}(\tau)d\tau\\ \le& V_{i,n_i}(t_0) + \sum_{k=1}^{n_i}\varepsilon_{i,k}\bar{\sigma}_{i,k}\end{aligned} \tag{49}$$

which shows that $\frac{\zeta^2_{i,k}}{k^2_{b_{i,k}}-\zeta^2_{i,k}}$, $\tilde{\theta}_{i,k}$ and $\tilde{\beta}_{i,k}$ are bounded. From $\tilde{\theta}_{i,k} = \theta_{i,k} - \hat{\theta}_{i,k}$ and $\tilde{\beta}_{i,k} = \beta_{i,k} - \hat{\beta}_{i,k}$, we can obtain that $\hat{\theta}_{i,k}$ and $\hat{\beta}_{i,k}$ are bounded.

Furthermore, from $\zeta_{i,1} = x_{i,1} - y_{di}$, we have $x_{i,1} = \zeta_{i,1} + y_{di}$. Associating with $|y_{d,i}| \le A_{i,0}$ and $|\zeta_{i,1}| < k_{b_{i,1}}$, we can conclude that $|x_{i,1}| \le |\zeta_{i,1}| + |y_{d,i}| < k_{b_{i,1}} + A_{i,0}$. Let $k_{b_{i,1}} = k_{c_{i,1}} - A_{i,0}$, and one has $|x_{i,1}| \le k_{c_{i,1}}$. Since $\alpha_{i,1}$ is related with the bounded signals $\zeta_{i,1}, y_{di}$ and $\hat{\theta}_{i,1}$ and $\hat{\beta}_{i,1}$, the $\alpha_{i,1}$ is bounded. Therefore, there is a constant $\bar{\alpha}_{i,1}$ such that $|\alpha_{i,1}| < \bar{\alpha}_{i,1}$, which combining with $x_{i,2} = \zeta_{i,2} + \alpha_{i,1}$ implies

$|x_{i,2}| < k_{b_{i,2}} + \bar{\alpha}_{i,1}$. Define $k_{b_{i,2}} = k_{c_{i,2}} - \bar{\alpha}_{i,1}$, and we can deduce $|x_{i,2}| < k_{c_{i,2}}$. Similarly, we can infer $|x_{i,s}| < k_{c_{i,s}}, s = 3, \ldots, n$. Therefore, the predefined state constraints are ensured.

Then, the asymptotic convergence about $\zeta_{i,s}$ will be proved. In view of (49), we can get

$$\lim_{t\to\infty} \sum_{k=1}^{n_i} c_{i,k} \int_{t_0}^{t} \frac{\zeta_{i,k}^2}{k_{b_{i,k}}^2 - \zeta_{i,k}^2} d\tau \leq V_{i,n_i}(t_0) + \sum_{k=1}^{n_i} \varepsilon_{i,k} \bar{\sigma}_{i,k} \tag{50}$$

Since all internal system signals are bounded, we can obtain that $\frac{\zeta_{i,k}^2}{k_{b_{i,k}}^2 - \zeta_{i,k}^2}$ and time derivative of $\zeta_{i,k}$ are also bounded, which shows $\frac{\zeta_{i,k}^2}{k_{b_{i,k}}^2 - \zeta_{i,k}^2}$ is uniformly continuous. Applying Barbalat Lemma, we have

$$\lim_{t\to\infty} \frac{\zeta_{i,k}^2}{k_{b_{i,k}}^2 - \zeta_{i,k}^2} = 0, k = 1, \ldots, n \tag{51}$$

which implies $\lim_{t\to\infty} \zeta_{i,k} = 0$, then the asymptotic convergence has been proved.

4 simulation studies

To elucidate the validity of our presented scheme, simulation studies are performed on the MIMO mass-spring-damper systems [22]. The schematic of is drawn in Fig.1 and the dynamic equations are

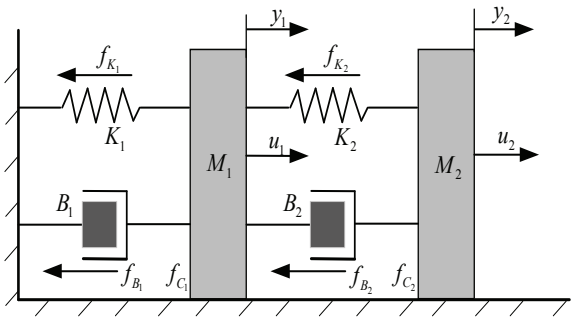

Fig. 1: Schematic of the mass-spring-damper system

$$\begin{cases} M_1\ddot{y}_1 = u_1 - f_{K_1}(x) - f_{B_1}(x) + f_{K_2}(x) + f_{B_2}(x) \\ \qquad\qquad - f_{C_1}(x) + f_{C_2}(x) + \Delta_1 \\ M_2\ddot{y}_2 = u_2 - f_{K_2}(x) - f_{B_2}(x) - f_{C_2}(x) + \Delta_2 \end{cases} \tag{52}$$

where $M_1 = 0.25$ kg, $M_2 = 0.2$ kg, $K_{10} = 1$ N/m, $K_{20} = 2$ N/m; $\Delta K_1 = 0.1, \Delta K_2 = 0.12, \Delta_1 = 0.2\sin(3t)e^{-0.2t}$ and $\Delta_2 = 0.2\cos(3t)e^{-0.1t}$. The other physical parameters can refer to Table 1.

Table 1: Description of system parameters [22]

Physical expression	Description
$f_{K_1}(x) = K_{10}y_1 + \Delta K_1 y_1^3$	Spring force
$f_{K_2}(x) = K_{20}(y_2 - y_1) + \Delta K_2(y_2 - y_1)^3$	Spring force
$f_{B_1}(x) = B_{10}\dot{y}_1 + \Delta B_1 \dot{y}_1^2$	Friction force
$f_{B_2}(x) = B_{20}(\dot{y}_2 - \dot{y}_1) + \Delta B_2(\dot{y}_2 - \dot{y}_1)^2$	Friction force
$f_{C_1} = 0.02\,\mathrm{sgn}\,(\dot{y}_1)$	Coulomb force
$f_{C_2} = 0.02\,\mathrm{sgn}\,(\dot{y}_2 - \dot{y}_1)$	Coulomb force

Define $x_{11} = y_1$, $x_{12} = \dot{y}_1$, $x_{21} = y_2$, $x_{22} = \dot{y}_2$. The (52) can be expressed as

$$\begin{cases} \dot{x}_{11} = x_{12} \\ \dot{x}_{12} = \frac{1}{M_1} u_1 + \frac{1}{M_1} \left[-f_{K_1}(x) - f_{B_1}(x) + f_{K_2}(x) + f_{B_2}(x) - f_{C_1}(x) + f_{C_2}(x) + \Delta_1 \right] \\ y_1 = x_{11} \\ \dot{x}_{21} = x_{22} \\ \dot{x}_{22} = \frac{1}{M_2} u_2 + \frac{1}{M_2} \left[-f_{K_2}(x) - f_{B_1}(x) - f_{C_2}(x) + \Delta_2 \right] \\ y_2 = x_{21} \end{cases} \tag{53}$$

From the previous analysis, the adaptive laws $\dot{\hat{\theta}}_{1,s}$, $\dot{\hat{\beta}}_{1,s}$, $\dot{\hat{\theta}}_{2,s}$ and $\dot{\hat{\beta}}_{2,s}$ with $s = 1, 2$, the virtual laws $\alpha_{1,1}$ and $\alpha_{2,1}$, the actual control laws u_1 and u_2 are governed by

$$\dot{\hat{\theta}}_{1,s} = \gamma_{1,s} \frac{\kappa_{\zeta_{1,s}}^2 \left\| \mathcal{Q}_{1,s}(\wp_{1,s}) \right\|^2}{\sqrt{\kappa_{\zeta_{1,s}}^2 \left\| \mathcal{Q}_{1,s}(\wp_{1,s}) \right\|^2 + \sigma_{1,s}^2}} - \gamma_{1,s} \sigma_{1,s} \hat{\theta}_{1,s} \tag{54}$$

$$\dot{\hat{\beta}}_{1,s} = \lambda_{1,s} \frac{\kappa_{\zeta_{1,s}}^2}{\sqrt{\kappa_{\zeta_{1,s}}^2 + \sigma_{1,s}^2}} - \lambda_{1,s} \sigma_{1,s} \hat{\beta}_{1,s} \tag{55}$$

$$\dot{\hat{\theta}}_{2,s} = \gamma_{2,s} \frac{\kappa_{\zeta_{2,s}}^2 \left\| \mathcal{Q}_{2,s}(\wp_{1,s}) \right\|^2}{\sqrt{\kappa_{\zeta_{2,s}}^2 \left\| \mathcal{Q}_{2,s}(\wp_{2,s}) \right\|^2 + \sigma_{2,s}^2}} - \gamma_{2,s} \sigma_{2,s} \hat{\theta}_{2,s} \tag{56}$$

$$\dot{\hat{\beta}}_{2,s} = \lambda_{2,s} \frac{\kappa_{\zeta_{2,s}}^2}{\sqrt{\kappa_{\zeta_{2,s}}^2 + \sigma_{2,s}^2}} - \lambda_{2,s} \sigma_{2,s} \hat{\beta}_{2,s} \tag{57}$$

$$\alpha_{1,1} = -c_{1,1}\zeta_{1,1} - \frac{\kappa_{\zeta_{1,1}} \hat{\theta}_{1,1} \left\| \mathcal{Q}_{1,1}(\wp_{1,1}) \right\|^2}{\sqrt{\kappa_{\zeta_{1,1}}^2 \left\| \mathcal{Q}_{1,1}(\wp_{1,1}) \right\|^2 + \sigma_{1,1}^2}} - \frac{\kappa_{\zeta_{1,1}} \hat{\beta}_{1,1}}{\sqrt{\kappa_{\zeta_{1,1}}^2 + \sigma_{1,1}^2}} \tag{58}$$

$$u_1 = -c_{1,2}\zeta_{1,2} - \frac{\kappa_{\zeta_{1,2}} \hat{\theta}_{1,2} \left\| \mathcal{Q}_{1,2}(\wp_{1,2}) \right\|^2}{\sqrt{\kappa_{\zeta_{1,2}}^2 \left\| \mathcal{Q}_{1,2}(\wp_{1,2}) \right\|^2 + \sigma_{1,2}^2}} - \frac{\kappa_{\zeta_{1,2}} \hat{\beta}_{1,2}}{\sqrt{\kappa_{\zeta_{1,2}}^2 + \sigma_{1,2}^2}} \tag{59}$$

$$\alpha_{2,1} = -c_{2,1}\zeta_{2,1} - \frac{\kappa_{\zeta_{2,1}} \hat{\theta}_{2,1} \left\| \mathcal{Q}_{2,1}(\wp_{2,1}) \right\|^2}{\sqrt{\kappa_{\zeta_{2,1}}^2 \left\| \mathcal{Q}_{2,1}(\wp_{2,1}) \right\|^2 + \sigma_{2,1}^2}} - \frac{\kappa_{\zeta_{2,1}} \hat{\beta}_{2,1}}{\sqrt{\kappa_{\zeta_{2,1}}^2 + \sigma_{2,1}^2}} \tag{60}$$

$$u_2 = -c_{2,2}\zeta_{2,2} - \frac{\kappa_{\zeta_{2,2}} \hat{\theta}_{2,2} \left\| \mathcal{Q}_{2,2}(\wp_{2,2}) \right\|^2}{\sqrt{\kappa_{\zeta_{2,2}}^2 \left\| \mathcal{Q}_{2,2}(\wp_{2,2}) \right\|^2 + \sigma_{2,2}^2}} - \frac{\kappa_{\zeta_{2,2}} \hat{\beta}_{2,2}}{\sqrt{\kappa_{\zeta_{2,2}}^2 + \sigma_{2,2}^2}}. \tag{61}$$

The desired signals are governed by $y_{d1} = 0.25\sin(t)$ and $y_{d2} = 0.5\cos(t)$. The constraint bounds are $k_{c_{1,1}} = 1.25$, $k_{c_{1,2}} = 3$, $k_{c_{2,1}} = 1.5$ and $k_{c_{2,2}} = 4$. Let initial conditions be $x_{1,1} = 0.1$, $x_{1,2} = 0$, $x_{2,1} = 0.6$, $x_{2,2} = 0$, $\hat{\theta}_{1,1} = 4.6$, $\hat{\theta}_{1,2} = 4.6$, $\hat{\theta}_{2,1} = 4.2$, $\hat{\theta}_{2,2} = 4.2$, $\hat{\beta}_{1,1} = 2.8$, $\hat{\beta}_{1,2} = 2.8$, $\hat{\beta}_{2,1} = 2.3$ and $\hat{\beta}_{2,2} = 2.3$. The relevant parameters of (54)-(61) are given as $c_{1,1} = 25$, $c_{1,2} = 18$, $c_{2,1} = 24$, $c_{2,2} = 16$, $\gamma_{1,1} = 0.25$, $\gamma_{1,2} = 0.25$, $\gamma_{2,1} = 0.41$, $\gamma_{2,2} = 0.41$, $\lambda_{1,1} = 0.18$, $\lambda_{1,2} = 0.18$, $\lambda_{2,1} = 0.14$, $\lambda_{2,2} = 0.14$, $\sigma_{1,1} = 6e^{-0.08t}$, $\sigma_{1,2} = 2e^{-0.1t}$, $\sigma_{2,1} = 6e^{-0.2t}$ and $\sigma_{2,2} = 4e^{-0.07t}$.

The fuzzy membership functions are set as

$$\mu_{F_{i,k}^l}(x_{i,k}) = \exp\left[-\frac{(x_{i,k} - 3 + l)^2}{4} \right], \quad i = 1, 2, k = 1, 2 \tag{62}$$

where $l = 1, \ldots, 5$, five fuzzy rules are defined on interval $[-2, 2]$.

Figs.2-3 illustrate the response of state variables $x_{1,1}$, $x_{1,2}$ and their homologous constraint bounds. Figs.4-5 shows the response of state variables $x_{2,1}$, $x_{2,2}$ and their homologous constraint bounds. The curves of adaptive laws are illustrated in Figs.6-7. The control inputs are shown Fig.8. Based on the above simulation images, it is obvious that the asymptotic control effect is achieved and the predefined state constraints are not violated.

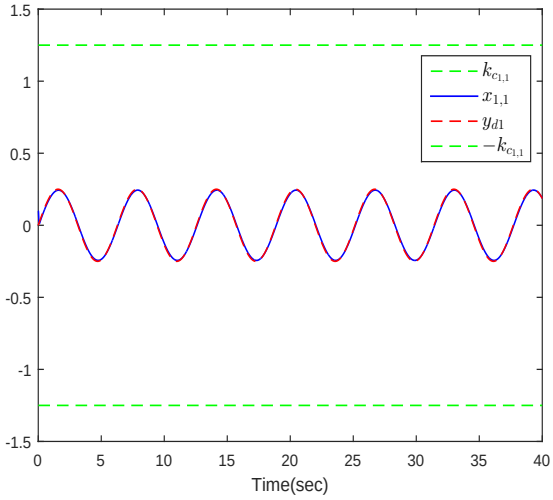

Fig. 2: Response of state variable $x_{1,1}$ and desired signal y_{d1}.

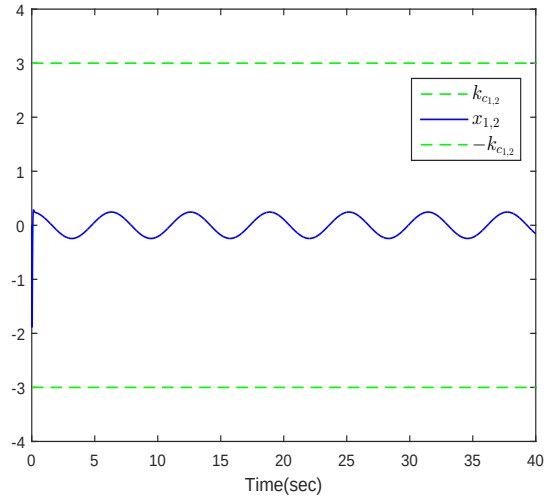

Fig. 3: Response of state variable $x_{1,2}$.

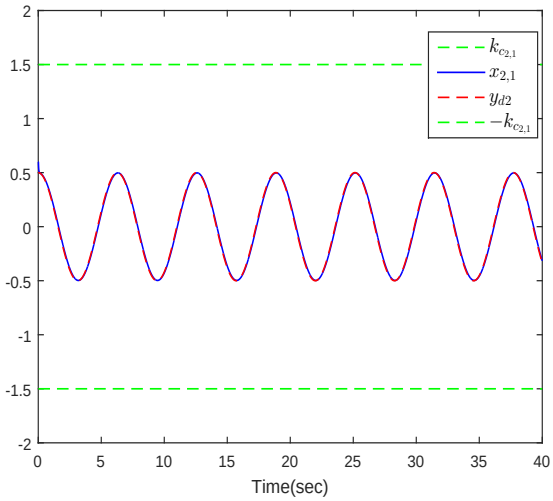

Fig. 4: Response of state variable $x_{2,1}$, desired signal y_{d2} and its constraint bounds.

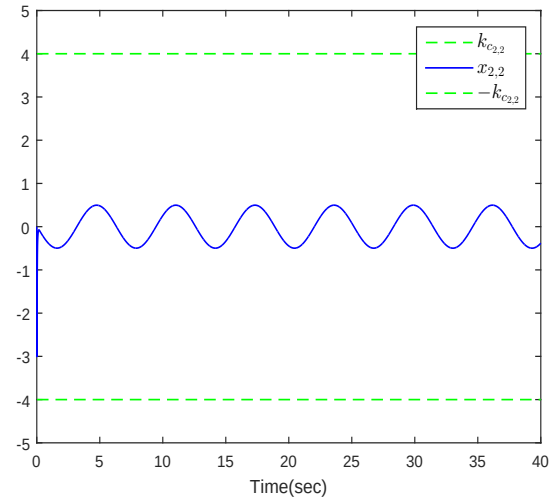

Fig. 5: Response of state variable $x_{2,2}$.

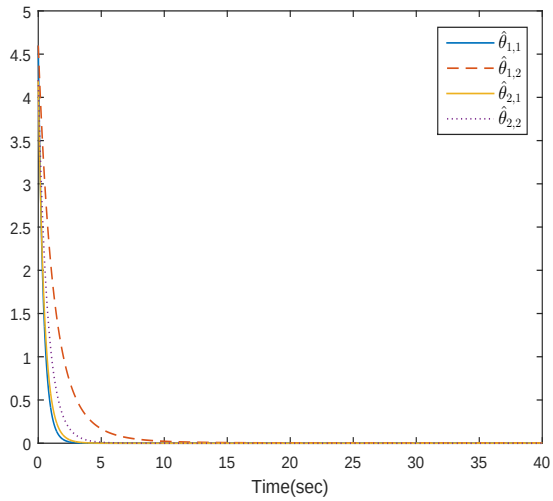

Fig. 6: Curves of the adaptive laws $\hat{\theta}_{1,1}, \hat{\theta}_{1,2}, \hat{\theta}_{2,1}, \hat{\theta}_{2,2}$.

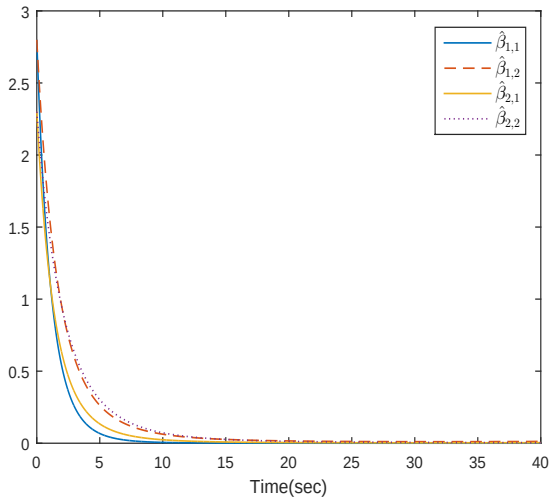

Fig. 7: Curves of the adaptive laws $\hat{\beta}_{1,1}, \hat{\beta}_{1,2}, \hat{\beta}_{2,1}, \hat{\beta}_{2,2}$.

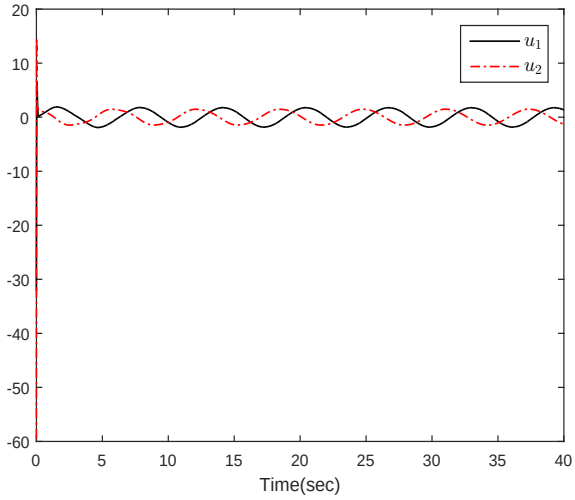

Fig. 8: Curves of the control inputs u_1 and u_2.

5 Conclusion

In this article, for MIMO nonlinear systems with state constraints, UVCCS and unknown dynamics, an AFATC strategy has been developed. More specifically, the bounded estimation scheme is explored to counteract the adverse effect deriving from UVCCs. By introducing some positive integral functions, associating with backstepping technique, the asymptotic tracking controller is recursively constructed. Such asymptotic control design not only achieves the asymptotic convergence of tracking errors, but also guarantees the prescribed state constraints. Simulation studies are conducted to elucidate the validity of the presented AFATC scheme.

Data availability statements

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

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Conflicts of interest

The authors declare that they have no conflict of interest.

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