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Zoubir Hricha

Chouaib Doukkali University Faculty of Sciences: Universite Chouaib Doukkali Faculte des Sciences

Mohammed Lazrek

Chouaib Doukkali University Faculty of Sciences: Universite Chouaib Doukkali Faculte des Sciences

Mohammed Yaalou

Chouaib Doukkali University Faculty of Sciences: Universite Chouaib Doukkali Faculte des Sciences

Abdelmajid Belafhal (✉ belafhal@gmail.com)

Chouaib Doukkali University <https://orcid.org/0000-0003-2735-3108>

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Propagation of vortex cosine-hyperbolic-Gaussian beams in atmospheric turbulence

Z. Hricha, M. Lazrek, M. Yaalou, A. Belafhal*

Laboratory LPNAMME, Laser Physics Group,

Department of Physics, Faculty of Sciences, Chouaib Doukkali University,

P. B 20, 24000 El Jadida, Morocco

** Corresponding author E-mail: belafhal@gmail.com.*

Z. Hricha E-mail : hlichazo66@gmail.com

Abstract

In this paper, the propagation properties of a vortex cosh-Gaussian beam (vChGB) in turbulent atmosphere are investigated. Based on the extended Huygens–Fresnel diffraction integral and the Rytov method, the analytical expression for the average intensity of the vChGB propagating in the atmospheric turbulence is derived. The effects of the turbulent strength and the beam parameters on the intensity distribution and the beam spreading are illustrated numerically and analyzed in detail. It is shown that upon propagating, the incident vChGB keeps its initial hollow dark profile within a certain propagation distance, then the field loses gradually its central hole-intensity and transformed into a Gaussian-like beam for large propagation distance. The rising speed of the central peak is demonstrated to be faster when the constant strength turbulence or the wavelength are larger and the Gaussian width is smaller. The obtained results can be beneficial for applications in optical communications and remote sensing.

Keywords: *Vortex cosine-hyperbolic-Gaussian beam; Atmospheric turbulence; Huygens-Fresnel diffraction; Rytov theory.*

1. Introduction

In recent years, the propagation of laser beams in atmosphere have received a great attention from the laser researchers due to many applications such as the remote sensing, imaging, optics communications and so on [1-2]. The propagation characteristics of laser beams with various excitations in the atmospheric turbulence have been examined [3-14], and also new beam models have been discovered and studied for their applications in free space optical communication systems. Among them, one can cite the hollow vortex Gaussian beam which is a fundamental

Gaussian beam including a topological vortex charge [15]. The said beam belongs to the wide family of hollow vortex beams, i.e. the beams which can carry the orbital angular momentum. During the last few years, a great deal of attention has been paid to the propagation proprieties of hollow vortex beams in various optics medium owing to promote their applications in optical microscopy, wireless communications, micromanipulation, etc [16-26]. In particular, the propagation characteristics of the hollow vortex Gaussian beams in free space and in turbulent atmosphere have been investigated in details in Refs. [15, 27]. The extended form of the hollow vortex Gaussian beam which is called the vortex cosine-hyperbolic Gaussian beam (vChGB) has been introduced very recently in Ref. [28]. In latter paper, it is reported that if the appropriate values of beam parameters, mainly the decentered parameter b , the vChGB can reduce either to the vortex Gaussian beam or may resemble the four-petal Gaussian vortex beam. The spatial characteristics of the vChGB upon propagating in free space, through a FrFT system and in strongly nonlocal nonlinear media have been examined in detail [28-29].

The present work is aimed at investigating the propagation properties of the vChGB in the turbulent atmosphere. The formulation is based on the extended Huygens–Fresnel integral diffraction and the Rytov method. The evolution of the diffracted vChGB in turbulent atmosphere, and the influences of the beam parameters and the turbulence strength on the behavior of the beam intensity distribution are investigated in detail. The remainder of the manuscript is organized as follow: in the Second section, we present the theoretical analysis for the propagation of vChGB in turbulent atmosphere, and we derive the propagation equation of the average intensity distribution. In Section 3, numerical examples are presented to discuss the evolution of intensity of vChGB in the turbulent atmosphere as a function of the involved parameters. A conclusion is outlined in the end of the paper.

2. Propagation characteristics of a vChGB in turbulent atmosphere

In the rectangular coordinates system, a vChGB propagating along the z -axis in the source plane $z=0$ can be expressed as [28]

$$E(x_0, y_0, z=0) = \cosh\left(b \frac{x_0}{\omega_0}\right) \cosh\left(b \frac{y_0}{\omega_0}\right) e^{-\left(\frac{x_0^2 + y_0^2}{\omega_0^2}\right)} (x_0 + iy_0)^M, \quad (1)$$

where (x_0, y_0) are the Cartesian coordinates at arbitrary point in the source plane and ω_0 is the waist radius of the Gaussian part. b being a real valued parameter associated to the cosh part, it is named

as the **decentered parameter b**. M being a positive integer which denotes the topological charge of the vortex.

The field of Eq. (1) can be generated in practice by passing a collimated cosine-hyperbolic Gaussian beam (ChGB) through a spiral phase plate.

When $b=0$, Eq. (1) reduces to the hollow vortex Gaussian beam [15], while for $M=0$ (**i.e.**, in the absence of the vortex **case**), one obtains the conventional ChGB [30-34].

From the paper [28], it found that the vChGB pattern depends crucially on the parameter b . Indeed, the beam can exhibit two kinds of profiles depending on the magnitude of the **decentered parameter b**: when b is small (say $b < 1.5$), the beam has a central dark spot surrounded by a bright ring spot, whereas for large b ($b \geq 4$), the beam possesses four symmetrical bright lobes.

Within the frame of the paraxial approximation, a light beam propagating through the turbulent atmosphere along the z -axis can be formulated by the extended Huygens–Fresnel diffraction integral [35]

$$E(\mathbf{P}, z) = -\frac{ik}{2\pi z} \exp(-ikz) \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E(\mathbf{P}_0, 0) \exp\left[-\frac{ik}{2z}(\mathbf{P}_0 - \mathbf{P})^2 + \psi(\mathbf{P}_0, \mathbf{P}, z)\right] d\mathbf{P}_0, \quad (2)$$

where $\mathbf{P}_0 = (x_0, y_0)$ and $\mathbf{P} = (x, y)$ are the transverse coordinates in the source and the receiver planes, respectively. z is the distance between the initial plane $z=0$ and the receiver plane. $\psi(\mathbf{P}_0, \mathbf{P}, z)$ denotes the random part for the complex phase of a spherical wave spreading from the source plane to the output plane, $k = \frac{2\pi}{\lambda}$ is the wavenumber, and λ is the wavelength of the source radiation **in vacuum**.

The average intensity of the vChGB through turbulent atmosphere is given as

$$\begin{aligned} \langle I(\mathbf{P}, z) \rangle &= \frac{k^2}{4\pi^2 z^2} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} E_0(x_{01}, y_{01}, 0) E_0^*(x_{02}, y_{02}, 0) \\ &\quad \times \exp\left[-\frac{ik}{2z} \left\{ (x_{01} - x)^2 + (y_{01} - y)^2 - (x_{02} - x)^2 - (y_{02} - y)^2 \right\}\right] \\ &\quad \times \langle \exp[\psi(\mathbf{P}_{01}, \mathbf{P}) + \psi^*(\mathbf{P}_{02}, \mathbf{P})] \rangle d\mathbf{P}_{01} d\mathbf{P}_{02}, \end{aligned} \quad (3)$$

where $*$ and $\langle \rangle$ denote the complex conjugation and the ensemble average over the medium statistics, respectively. Within the Rytov theory, the ensemble average in Eq. (3) is given by [1]

$$\langle \exp[\psi(x_{01}, y_{01}, x, y) + \psi^*(x_{02}, y_{02}, x, y)] \rangle = \exp\left[-\frac{1}{\rho_0^2} \left\{ (x_{01} - x_{02})^2 + (y_{01} - y_{02})^2 \right\}\right], \quad (4)$$

where $\rho_0 = (0.545 C_n^2 k^2 z)^{-3/5}$ is the coherence length of a spherical wave propagating in the turbulent medium with C_n^2 is the refractive index structure constant.

Substituting Eqs. (1) and (4) into Eq. (3), and recalling the binomial formula [36]

$$(u+v)^m = \sum_{l=0}^m C_l^m u^l v^{m-l}, \quad (5a)$$

where

$$C_l^m = \frac{m!}{l!(m-l)!}, \quad (5b)$$

leads to

$$\langle I(P, z) \rangle = \left(\frac{k}{2\pi z} \right)^2 \sum_{m_1=0}^M C_{m_1}^M \sum_{m_2=0}^M C_{m_2}^M \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x_{02}, y_{02}) J(x_{02}, y_{02}) dx_{02} dy_{02}, \quad (6a)$$

where

$$f_{xy}(x_{02}, y_{02}) = x_{02}^{m_2} \left(-iy_{02} \right)^{M-m_2} \cosh \left(b \frac{x_{02}}{\omega_0} \right) \cosh \left(b \frac{y_{02}}{\omega_0} \right) \times \exp \left(-\delta^* x_{02}^2 - \frac{ikx}{z} x_{02} \right) \exp \left(-\delta^* y_{02}^2 - \frac{iky}{z} y_{02} \right), \quad (6b)$$

and

$$J_{xy}(x_{02}, y_{02}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(x_{01}, y_{01}, x_{02}, y_{02}) dx_{01} dy_{01}, \quad (6c)$$

with

$$F_{xy}(x_{01}, y_{01}, x_{02}, y_{02}) = x_{01}^{m_1} \left(iy_{01} \right)^{M-m_1} \cosh \left(b \frac{x_{01}}{\omega_0} \right) \exp \left[-\delta x_{01}^2 + \left(\frac{ikx}{z} + \frac{2x_{02}}{\rho_0^2} \right) x_{01} \right] \times \cosh \left(b \frac{y_{01}}{\omega_0} \right) \exp \left[-\delta y_{01}^2 + \left(\frac{iky}{z} + \frac{2y_{02}}{\rho_0^2} \right) y_{01} \right], \quad (6d)$$

and δ is the auxiliary parameter defined by

$$\delta = \frac{ik}{2z} + \frac{1}{\omega_0^2} + \frac{1}{\rho_0^2}. \quad (6e)$$

Using the definition of cosh function and the separation of variable method to perform the double integration in Eq. (6c), one gets

$$J_{xy}(x_{02}, y_{02}) = (i)^{M-m_1} F_m(x_{02}, x) F_m(y_{02}, y), \quad (7a)$$

with

$$F_m(s_{02}, s) = \frac{1}{2} \left(F_m^+(s_{02}, s) + F_m^-(s_{02}, s) \right), \quad s = x \text{ or } y \quad (7b)$$

where

$$F_m^\pm(s_{02}, s) = \int_{-\infty}^{+\infty} u^m \exp(-\delta_1 u^2) \exp\left[\left(\frac{iks}{z} + \frac{2s_{02}}{\rho_0^2} \pm \frac{b}{\omega_0}\right)u\right] du, \quad (7c)$$

Now, by using the integral formula [37-38]

$$\int_{-\infty}^{+\infty} x^n e^{-px^2+2qx} dx = e^{\frac{q^2}{p}} \sqrt{\frac{\pi}{p}} \left(\frac{1}{2i\sqrt{p}}\right)^n H_n\left(\frac{iq}{\sqrt{p}}\right), \quad (8)$$

where $H_n(\cdot)$ is the Hermite polynomial of n^{th} -order, Eq. 7(c) reads

$$F_m^\pm(s_{02}, x) = e^{\frac{q_\pm(s_{02}, s)^2}{\delta_1}} \sqrt{\frac{\pi}{\delta_1}} \left(\frac{1}{2i\sqrt{\delta_1}}\right)^{m_1} H_{m_1}\left(\frac{iq_\pm(s_{02}, s)}{\sqrt{\delta_1}}\right), \quad (9a)$$

where

$$q_\pm(s_{02}, s) = \frac{iks}{2z} + \frac{s_{02}}{\rho_0^2} \pm \frac{b}{2\omega_0}, \quad (9b)$$

in which s represents either x or y .

The substitution of Eqs. 9(a-b) and (7a) into Eq. (6a) leads to

$$\begin{aligned} \langle I(P, z) \rangle = & \frac{1}{4} \frac{\pi}{\delta} \left(\frac{k}{2\pi z}\right)^2 \left(\frac{1}{2i\sqrt{\delta}}\right)^M \sum_{m_1=0}^M C_{m_1}^M (i)^{M-m_1} \sum_{m_2=0}^M C_{m_2}^M \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} f(x_{02}, y_{02}, x, y) \\ & \times \left\{ \left[\exp\left(\frac{q_+(x_{02}, x)^2}{\delta_1}\right) H_{m_1}\left(\frac{iq_+(x_{02}, x)}{\sqrt{\delta_1}}\right) + \exp\left(\frac{q_-(x_{02}, x)^2}{\delta_1}\right) H_{m_1}\left(\frac{iq_-(x_{02}, x)}{\sqrt{\delta_1}}\right) \right] \right. \\ & \left. + \left[\exp\left(\frac{q_+(y_{02}, y)^2}{\delta_1}\right) H_{M-m_1}\left(\frac{iq_+(y_{02}, y)}{\sqrt{\delta_1}}\right) + \exp\left(\frac{q_-(y_{02}, y)^2}{\delta_1}\right) H_{M-m_1}\left(\frac{iq_-(y_{02}, y)}{\sqrt{\delta_1}}\right) \right] \right\} dx_{02} dy_{02}. \end{aligned} \quad (10)$$

The integral expression on the right-hand of the last equation can be performed straightforwardly as following: By using the separation of variable method, and the expanding form of Hermite polynomial [38],

$$H_m(x) = \sum_{j=0}^{\lfloor m/2 \rfloor} \frac{(-1)^j m!}{j!(m-2j)!} (2x)^{m-2j}, \quad (11)$$

and with the help a second time of Eq. (8), and after lengthy algebraic calculations, the average intensity of the diffracted vChGB in the turbulence atmosphere is expressed as

$$\begin{aligned} I(x, y, z) = & \left(\frac{k}{2z}\right)^2 \frac{1}{16\delta_1 \eta} \left(\frac{1}{2i\sqrt{\delta_1}}\right)^M \sum_{m_1=0}^M C_{m_1}^M (i)^{M-m_1} \sum_{m_2=0}^M C_{m_2}^M (-i)^{M-m_2} \\ & \times \left[G_{m_1 m_2}^+(x, z) + G_{m_1 m_2}^-(x, z) \right] \left[H_{m_1 m_2}^+(y, z) + H_{m_1 m_2}^-(y, z) \right] \end{aligned} \quad (12)$$

where

$$G_{m_1 m_2}^{\pm}(x, z) = \sum_{s_1=0}^{\left[\frac{m_1}{2}\right]} \frac{(-1)^{s_1} m_1!}{s_1! (m_1 - 2s_1)!} \left(\frac{2i}{\sqrt{\delta}}\right)^{m_1 - 2s_1} \sum_{j_1=0}^{m_1 - 2s_1} C_{j_1}^{m_1 - 2s_1} \left(\frac{1}{2i\sqrt{\eta}}\right)^{m_2 + j_1} \left(\frac{1}{\rho_0^2}\right)^{j_1} \\ \times \left\{ \left[\left(\left(\frac{ikx}{2z} + \frac{b}{2\omega_0} \right) \right)^{m_1 - 2s_1 - j_1} \exp(Ax^2 + B_1^{\pm}x + C_1^{\pm}) H_{m_2 + j_2} \left(\frac{i\alpha}{\sqrt{\eta}}x + \frac{i\beta^{\pm}}{\sqrt{\eta}} \right) \right] \right. \\ \left. + \left[\left(\frac{ikx}{2z} - \frac{b}{2\omega_0} \right) \right]^{m_1 - 2s_1 - j_1} \exp(Ax^2 + B_2^{\mu}x + C_2^{\mu}) H_{m_2 + j_2} \left(\frac{i\alpha}{\sqrt{\eta}}x - \frac{i\beta^{\mu}}{\sqrt{\eta}} \right) \right] \right\}, \quad (13a)$$

$$H_{m_1 m_2}^{\pm}(y, z) = \sum_{s_2=0}^{\left[\frac{M-m_1}{2}\right]} \frac{(-1)^{s_2} (M-m_1)!}{s_2! (M-m_1 - 2s_2)!} \left(\frac{2i}{\sqrt{\delta}}\right)^{M-m_1 - 2s_2} \sum_{j_2=0}^{M-m_1 - 2s_2} C_{j_2}^{M-m_1 - 2s_2} \left(\frac{1}{2i\sqrt{\eta}}\right)^{M-m_2 + j_2} \left(\frac{1}{\rho_0^2}\right)^{j_2} \\ \times \left\{ \left[\left(\left(\frac{iky}{2z} + \frac{b}{2\omega_0} \right) \right)^{M-m_1 - 2s_2 - j_2} \exp(Ay^2 + B_1^{\pm}y + C_1^{\pm}) H_{M-m_2 + j_2} \left(\frac{i\alpha}{\sqrt{\eta}}y + \frac{i\beta^{\pm}}{\sqrt{\eta}} \right) \right] \right. \\ \left. + \left[\left(\frac{iky}{2z} - \frac{b}{2\omega_0} \right) \right]^{M-m_1 - 2s_2 - j_2} \exp(Ay^2 + B_2^{\mu}y + C_2^{\mu}) H_{M-m_2 + j_2} \left(\frac{i\alpha}{\sqrt{\eta}}y - \frac{i\beta^{\mu}}{\sqrt{\eta}} \right) \right] \right\}, \quad (13b)$$

with

$$\eta = \delta^* - \frac{1}{\delta \rho_0^4}, \text{ and } \delta^* \text{ denotes the complex conjugate of } \delta, \quad (13c)$$

$$\alpha = \frac{ik}{2z} \left(\frac{1}{\delta \rho_0^2} - 1 \right), \quad (13d)$$

$$\beta^{\pm} = \frac{b}{2\omega_0} \left(\frac{1}{\delta \rho_0^2} \pm 1 \right), \quad (13e)$$

$$A = \frac{\alpha^2}{\eta} - \frac{k^2}{4z^2\delta}, \quad (13f)$$

$$B_1^{\pm} = \frac{ikb}{2z\omega_0\delta} + \frac{2\alpha\beta^{\pm}}{\eta}, \quad (13g)$$

$$B_2^{\mu} = - \left(\frac{ikb}{2z\omega_0\delta} + \frac{2\alpha\beta^{\mu}}{\eta} \right), \quad (13h)$$

$$C_1^{\pm} = \frac{b^2}{4\omega_0^2\delta} + \frac{(\beta^{\pm})^2}{\eta}, \quad (13i)$$

and

$$C_2^\mu = \frac{b^2}{4\omega_0^2\delta} + \frac{(\beta^\mu)^2}{\eta}. \quad (13j)$$

Eq. (12) is the main analytical result of this work, and it will be convenient for analyzing the evolution of the intensity of vChGB **propagating** through turbulent atmosphere.

In the limit case $C_n^2 = 0$, i.e., in the absence of the **atmospheric** turbulence, Eq. (12) reduces to the propagation equation of vChGB in free-space, the obtained result is consistent with Eq. (8a) of Ref. [28].

The **on-axis** intensity can be deduced by putting $x=y=0$ in Eq. (12), so we get

$$I(0,0,z) = \left(\frac{k}{2z}\right)^2 \frac{1}{16\delta\eta} \left(\frac{1}{2i\sqrt{\delta}}\right)^M \sum_{m_1=0}^M C_{m_1}^M (i)^{M-m_1} \sum_{m_2=0}^M C_{m_2}^M (-i)^{M-m_2} \times [G_{m_1m_2}^+(0,z) + G_{m_1m_2}^-(0,z)] [H_{m_1m_2}^+(0,z) + H_{m_1m_2}^-(0,z)] \quad (14a)$$

where

$$G_{m_1m_2}^\pm(0,z) = \sum_{s_1=0}^{\left[\frac{m_1}{2}\right]} \frac{(-1)^{s_1} m_1!}{s_1!(m_1-2s_1)!} \left(\frac{2i}{\sqrt{\delta}}\right)^{m_1-2s_1} \sum_{j_1=0}^{m_1-2s_1} C_{j_1}^{m_1-2s_1} \left(\frac{1}{2i\sqrt{\eta}}\right)^{m_2+j_1} \left(\frac{1}{\rho_0^2}\right)^{j_1} \quad (14b)$$

$$\times \left\{ \left[\left(\frac{b}{2\omega_0}\right)^{m_1-2s_1-j_1} \exp(C_1) H_{m_2+j_2} \left(\frac{i\beta^\pm}{\sqrt{\eta}}\right) + \left(\frac{b}{2\omega_0}\right)^{m_1-2s_1-j_1} \exp(C_2) H_{m_2+j_2} \left(-\frac{i\beta^\mu}{\sqrt{\eta}}\right) \right] \right\},$$

$$H_{m_1m_2}^\pm(0,z) = \sum_{s_2=0}^{\left[\frac{M-m_1}{2}\right]} \frac{(-1)^{s_2} (M-m_1)!}{s_2!(M-m_1-2s_2)!} \left(\frac{2i}{\sqrt{\delta}}\right)^{M-m_1-2s_2} \sum_{j_2=0}^{M-m_1-2s_2} C_{j_2}^{M-m_1-2s_2} \left(\frac{1}{2i\sqrt{\eta}}\right)^{M-m_2+j_2} \left(\frac{1}{\rho_0^2}\right)^{j_2} \quad (14c)$$

$$\times \left\{ \left[\left(\frac{b}{2\omega_0}\right)^{M-m_1-2s_2-j_2} \exp(C_1) H_{M-m_2+j_2} \left(\frac{i\beta^\pm}{\sqrt{\eta}}\right) + \left(\frac{b}{2\omega_0}\right)^{M-m_1-2s_2-j_2} \exp(C_2) H_{M-m_2+j_2} \left(-\frac{i\beta^\mu}{\sqrt{\eta}}\right) \right] \right\}.$$

It is worth noting that in the two limiting cases $b=0$ and $M=0$, the **above formulation** gives the propagation equation and the intensity distribution for the vortex Gaussian and ChGB beams (respectively) in turbulent atmosphere [7].

3. Numerical results and analyses

The propagation characteristics of vChGB propagating in atmospheric turbulence are numerically investigated based on the main formula given by Eq. (12). Since the initial shape of the vChGB is dependent on the **value** of the **decentered parameter b**, therefore, in the following,

the two configurations of the beam, i.e., the beam with small and large values of b will be separately examined.

In Figs. (1)-(2), we illustrated the normalized intensity distribution of vChGB in turbulent atmosphere at different propagation distance z ($z=0.1$ km, 1 km, 2 km and 5 km). The calculation parameters are set as $\omega_0 = 2$ cm, $M=1$, $\lambda= 1060$ nm and $C_n^2=10^{-14} \text{ m}^{-2/3}$. In addition, the same characteristics of the beam in free-space are presented for the sake of comparison. From the illustrated plots, it is seen that the vChGB keeps its initial profile, i.e., a ringed pattern with hole-intensity at the center, until a certain propagation distance, and then the beam gradually changes profile and the central hole-intensity is filled upon propagation at large propagation distance (see Figs. 1-a3 and Fig. 2-a4). From the plots (b1-b4), it can be seen that the evolution of vChGB in atmospheric turbulence is different from that in free-space; in free-space the hole-intensity persists even in far-field. Furthermore, one can note that the widening of the beam in turbulent atmosphere is stronger compared to the free-space case.

Figure 1: The normalized intensity of a vChGB in a turbulent atmosphere for different values of the propagation distance z with $\omega_0 = 0.02$ m, topological charge $M=1$, $b=0.1$

$$\lambda = 1060 \text{ nm} \text{ and } C_n^2 = 10^{-14} \text{ m}^{-2/3}.$$

Figure 2: The normalized intensity of a vChGB in a turbulent atmosphere for different values of the propagation distance z with $\omega_0 = 0.02 \text{ m}$, topological charge $M=1$, $b=4$, $\lambda = 1060 \text{ nm}$ and $C_n^2 = 10^{-14} \text{ m}^{-2/3}$.

Figs. 3(a)-(b) display the intensity distribution pattern and the on-axis intensity (respectively) of vChGB in turbulent atmosphere for different values of the topological charge M ($M= 0, 1, 2$ and 3). It is clearly seen that the rising speed of peak intensity at the center (for both beam configurations) **is faster as M increases**.

Figure 3a: The average intensity of a vChGB in a turbulent atmosphere for different values of the topological charge with $\lambda = 1060 \text{ nm}$, $C_n^2 = 10^{-14} \text{ m}^{-2/3}$ and $\omega_0 = 0.02 \text{ m}$.

Figure 3b: The normalized axial intensity of a vChGB in a turbulent atmosphere versus propagation distance versus the topological charge M , for $b=0.1$ and $b=4$, with $\lambda = 1060 \text{ nm}$, $C_n^2 = 10^{-14} \text{ m}^{-2/3}$ and $\omega_0 = 0.02 \text{ m}$.

In order to analysis the effect of the turbulence strength on the beam propagation, we have illustrated in Fig. 4 the normalized intensity distribution for different values of turbulence strength C_n^2 ($C_n^2 = 10^{-16} \text{ m}^{-2/3}$, $10^{-15} \text{ m}^{-2/3}$ and $5 \cdot 10^{-15} \text{ m}^{-2/3}$). The plots show that the rising speed of the central peak is faster when the turbulence strength is stronger for both beam configurations. In addition, one can note the increase of the beam widening with increasing the turbulence strength. C_n^2 .

Figure 4: The normalized intensity of a vChGB in a turbulent atmosphere for a different values of the refractive index structure C_n^2 with $\omega_0 = 0.02 \text{ m}$, $z=2 \text{ km}$ and $\lambda = 1060 \text{ nm}$.

Fig. 5 presents the effect of the waist size ω_0 on the **evolution of the intensity distribution** of vChGB in turbulent atmosphere at the propagation distance $z=2\text{km}$, for $M=1$. From the illustrations in Fig. 5, one can clearly see that the **beam widens faster and the rising speed of the central peak is slower when ω_0 increases.**

Figure 5: The normalized intensity of a vChGB in a turbulent atmosphere for different values of the waist ω_0 with

$$M=1, z=2 \text{ km}, \lambda = 1060 \text{ nm and } C_n^2 = 10^{-14} \text{ m}^{-2/3}.$$

(a.1,b.1) $\omega_0 = 0.02 \text{ m}$, (a.2,b.2) $\omega_0 = 0.05 \text{ m}$, (a.3,b.3) $\omega_0 = 0.1 \text{ m}$ and (a.4,b.4) $\omega_0 = 0.2 \text{ m}$.

The influence of the wavelength λ on the evolution intensity distribution is depicted in Fig. 6, from which it is readily seen that the perturbed beam widens gradually as λ increases. Furthermore, the central hole-intensity **is filled faster when λ is smaller in the small b case, whereas the opposite evolution occurs with large b case.**

Figure 6: The normalized intensity of a vChGB in a turbulent atmosphere for different values of the wavelength λ with $M=1$, $z=2$ km, $\omega_0 = 0.02$ m and $C_n^2 = 10^{-14} \text{ m}^{-2/3}$.
(a.1,b.1) $\lambda = 632.8 \text{ nm}$, (a.2,b.2) $\lambda = 1060 \text{ nm}$, (a.3,b.3) $\lambda = 1550 \text{ nm}$ and (a.4,b.4) $\lambda = 2000 \text{ nm}$.

4. Conclusion

The propagation characteristics of a vChGB propagating in turbulent atmosphere are investigated in detail. The analytical expression of the average intensity of the diffracted vChGB in turbulent atmosphere is **derived within** the framework of the Huygens–Fresnel diffraction and the Rytov method. Numerical examples illustrating the effects of the turbulence strength, the beam parameters and the wavelength on the beam propagation are performed. It is found that the incident vChGB keeps its initial profile within a certain propagation distance, and then loses gradually its central hole-intensity and transformed into a Gaussian-like beam. The rising speed of central peak is faster for larger strength turbulence, higher vortex charge and smaller Gaussian waist **size**. The obtained results can be beneficial for applications **of vChG** in free space optical communication systems.

References

- [1] L. C. Andrews, R. L. Philips, “Laser beam propagation through Random media”. SPIE Press, Washington (1998).
- [2] F. Wang, X. Liu, Y. Cai, “Propagation of partially coherent beam in turbulent atmosphere: a review”. Prog. Electromag. Res. 150, 123-143 (2015).
- [3] Y. Baykal, “Correlation and structure functions of Hermite-sinusoidal-Gaussian beams in the turbulent atmosphere”. J. Opt. Am. A : Opt. Imag. Sci. Vis.21, 1290-1299 (2004).
- [4] Y. Cai, S. He, “Propagation of various dark hollow beams in a turbulent atmosphere”. Opt. Express 14, 1353-1367 (2006).
- [5] Y. Cai, “Propagation of various flat-topped beams in a turbulent atmosphere”. J. Opt. A.: Pure Appl. Opt. 8, 537-545 (2006).
- [6] R.J. Noriega-Manez, J.C. Gutiérrez-Vega, “Rytov theory for Helmholtz-Gauss beams in turbulent atmosphere”. Opt. Express15, 16328-16341 (2007).
- [7] X. Chu, Y. Ni, G. Zhou, “Propagation of cosh-Gaussian beams diffracted by a circular aperture in turbulent atmosphere”, Appl. Phys. B, 87, 547-552 (2007).
- [8] J. Cang, Y. Zhang, “Axial intensity distribution of truncated Bessel–Gauss beams in a turbulent atmosphere”. Optik 121, 239-245 (2010).
- [9] F. Wang, Y. Cai, H.T. Eyyboglu, Y. Baykal, “Average Intensity and spreading of partially coherent standard and elegant Laguerre-Gaussian beams in turbulent atmosphere”. Prog. In Electromag. Res. PIER 103, 33-56 (2010).
- [10] F. Khannous, M. Boustimi, H. Nebdi, A. Belafhal, “Theoretical investigation on the Hollow Gaussian beams propagating in atmospheric turbulent”. Chin. J. of Phys. 54, 194-20 (2016).
- [11] F. Boufalah, L. Dalil-Essakali, L. Ez-zariy, A. Belafhal, “Introduction of generalized Bessel-Laguerre-Gaussian beams and its central intensity traveling a turbulent atmosphere”. Opt. Quant. Electron. 50, 305-325 (2018).
- [12] F. Saad, E. M. El Halba, A. Belafhal, “A theoretical study of the on-axis average intensity of Generalized spiraling Bessel beams in a turbulent atmosphere”. Opt. Quant. Electron. 49, 94-106 (2018).
- [13] M. Yaalou, E. M. El Halba, Z. Hricha, A. Belafhal, “Propagation characteristics of Dark and Antidark Gaussian beams in a turbulent atmosphere”. Opt. Quant. Electr. 51, 255-266 (2019).
- [14] Z. Hricha, M. Yaalou, A. Belafhal, “Intensity characteristics of double–half inverse Gaussian hollow beams through turbulent atmosphere”. Opt. Quant. Electron. 52, 201-207 (2020).

- [15] G.Q. Zhou, Y. Cai, C.Q. Dai, “Hollow vortex Gaussian beams”. *Sci. Chin.* 56 (5), 896-903 (2013).
- [16] L. Allen, M.W. Begersbergen, R.J.C. Spreeuw, J.P. Woerdman, “Orbital angular omentum of light and the transformation of Laguerre-Gaussian laser modes”. *Phys. Rev. A.* 45, 8185- 8189 (1992).
- [17] T. Kuga, Y. Torii, N. Shiokawa, T. Hirano, Y. Shimizu, H. Sasada. “Novel optical trap of atoms with a doughnut beam”. *Phys. Rev. Lett.* 78, 4713–4716 (1997).
- [18] L. Paterson, M. P. MacDonald, J. Arlt, W. Sibbett, P.E. Bryant, K. Dholakia, “Controlled rotation of optically trapped microscopic particles”. *Sci.* 292 (5518), 912-914 (2001).
- [19] S.A. Ponomarenko, “A class of partially coherent beams carrying optical vortices”, *J. Opt. Am.A*, 18, 150-156 (2001).
- [20] Y. Cai, X. Lu, and Q. Lin. “Hollow Gaussian beam and its propagation”. *Opt. Lett.* 28, 1084-1086 (2003).
- [21] A.I. Bishop, T. A. Nieminen, N. R. Heckenberg, H. Rubinsztein, “Optical microrhology using rotating laser-trapped particles”. *Phys. Rev. Lett.* 92 (19), 198104- 198107 (2004).
- [22] Z. Wang, Q. Lin, and Y. Wang, “Control of atomic rotation by elliptical hollow beam carrying zero angular momentum”. *Opt. Commun* 240, 357–362 (2004).
- [23] V.P. Lukin, P.A. Konyaev, V. A; Sennikov, “Beam Spreading of vortex beams propagating in turbulent atmosphere”. *Applied Optics*, 51 (10), 84-87 (2012).
- [24] J. Wang, J.Y. Yang, I.M. Fazal, N. Ahmed, Y. Yan, H. Huang, Y. Ren, Y. Yue, S. Dolinar, M. Tur, A. E. Willner, “Terabit free-space data transmission employing orbital angular momentum multiplexing”. *Nat. Phot.* 16 (7), 488-412 (2012).
- [25] X. Zhu, G. Wu, B. Lu, “Propagation of elegant vortex Hermite-Gaussian beams in turbulent atmosphere”. *Proc. SPIE* 10158, optical communication, optical fiber sensors, and Optical memories for Big data storage, 101580F-6 (2016).
- [26] H. Rubinsztein-Dunlop et al., “Roadmap on structured light”. *J. Opt.* 19(1), 013001-1-01300-51 (2017).
- [27] Q.X. Mei, Z. W. Yue, R.R. Zhong, “Intensity distribution properties of Gaussian vortex beam propagation in atmospheric turbulence”. *Cin. Phys. B* 24 (4), 044201-5 (2015).
- [28] Z. Hricha, M. Yaalou, A. Belafhal, “Introduction of a new vortex cosine-hyperbolic-Gaussian beam and the study of its propagation properties in Fractional Fourier Transform optical system”. *Opt. Quant. Electron* 52, 296-302 (2020).

- [29] Z. Hricha, M. Yaalou, A. Belafhal “Propagation properties of vortex cosine-hyperbolic-Gaussian beams in strongly nonlocal nonlinear media”. *JQSRT*, 265 (6) :10755A (2021).
 - [30] L.W. Casperson, A. A. Tovar, “Hermite-Sinusoidal-Gaussian beams in complex optical systems”. *J. Opt. Am. A* 15, 954-961 (1998).
 - [31] A.A. Tovar, L.W. Casperson, “Production and propagation of Hermite-sinusoidal-Gaussian laser beams”. *J. Opt. Am. A* 15, 2425-2432 (1998).
 - [32] B. Lu, B. Zhang, “propagation properties of cosh-Gaussian beams”. *Opt. Comm.* 164 (4-5), 165-170 (1999).
 - [33] A. Belafhal, M. Ibnchaikh, “Propagation properties of Hermite-cosh-Gaussian laser”. *Opt. Commun.* 186, 269-276 (2000).
 - [34] Z. Hricha, A. Belafhal, “Focusing properties of focal shift in hyperbolic-cosine-Gaussian beams”. *Opt. Commun.* 253, 242-249 (2005).
 - [35] M. Born, E. Wolf, “Principles of Optics”. Seventh (expanded) ed, Cambridge University Press, Cambridge, UK (1999).
 - [36] M. Abramowitz, I.A. Stegun (Eds.), “Handbook of Mathematical Functions”. Nat. Bureau of Standards Washington, DC (1964).
- Belafhal
- [37] A. Belafhal, Z. Hricha, L. Dalil-Essakali, T. Usman, “A note on some integrals involving Hermite polynomials and their applications”. *Adv. Math. Mod. and appl.* 5 (3), 313-319 (2020).
 - [38] I.S. Gradshteyn, I. M. Ryzhik, “Tables of Integrals, Series, and Product”. Fifth ed., Academic Press, New York (1994).

Figures

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