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Effects of turbulent atmosphere on the propagation properties of vortex Hermite-cosine-hyperbolic-Gaussian beams

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Abstract

The propagation properties of a vortex Hermite-cosh-Gaussian beam (vHChGB) in atmospheric turbulence are investigated based on the extended Huygens–Fresnel diffraction integral and Rytov method. The analytical formula for the average intensity of a vHChGB propagating in turbulent atmosphere is derived in detail. The influence of the turbulence strength on the intensity distribution under the change of beam parameters conditions is illustrated numerically and discussed. Results show the profile of the initial vHChGB remains unchanged within small propagation distance range, and at certain propagation distance a central peak intensity appears, and finally the beam evolves into Gaussian profile-like in far-field. The rising speed of the central peak intensity is faster when the turbulence strength is larger or the beam parameters such as the beam order, the vortex charge and the Gaussian waist width are smaller. With a small decentered parameter b , the beam profile changes faster as the wavelength is larger, whereas the reverse behavior occurs when b is large. The obtained results may be useful for the practical applications of vHChGB in optical communications and remote sensing.

Keywords: *Vortex Hermite-cosine-hyperbolic-Gaussian beam; Atmospheric turbulence; Rytov theory, Paraxial approximation.*

1. Introduction

In recent years, interest has increased in the studies of laser beam propagating through atmospheric turbulence due to many applications [1-9]. In particular, new types of laser excitations have been introduced and investigated to promote their applications in free space optical communication (FSO), optical microscopy, wireless communications and optical micromanipulation [10-18]. The so-called vortex hollow dark beams (vHDBs) have been extensively studied laser beams over the last few years due to their interesting properties [19-

30]. The beams have zero intensity at the center surrounded by an annular spot (or an array bright spots) due to the embedded vortex charge. Furthermore, the beams possess a spirally wave-front phase, which makes them enable to carry the orbital angular momentum. In practical applications, the vortex charge of the beam can be used as optical traps for micro-particles as well as beam spanner [31-32]. The hollow vortex Gaussian beam represents the simplest and typical model for the hollow vortex beams which has been widely investigated in many laser papers. The propagation characteristics of the hollow vortex Gaussian beam in various optical systems including turbulent media have been researched [21-23]. Recently, an extended form of the hollow vortex Gaussian beam, named as vortex cosine-hyperbolic Gaussian beam, has been introduced and its propagation in the turbulent atmosphere has been examined by our research group [33-34]. Moreover, a generalized beam expression for the hollow vortex Gaussian field has been proposed in [35], the obtained beam is called vortex-Hermite-cosh-Gaussian beam (vHChG). The vHChGB possesses three key parameters namely the beam orders, the decentered parameter b and the vortex charge number M . The beam is hollow dark array-like and can be reduced by choosing the appropriate values of the beam parameters to well-known laser beams such as the vortex-Gaussian [21], vortex Hermite-Gaussian [25] and vortex-cosh-Gaussian beams [33]. The intensity distribution pattern, the spot size and the central dark width of the vHChGB can be controlled by the appropriate choice of the beam parameters. The evolution of intensity distribution of the vHChGB upon propagating in free space has been investigated in detail in Ref. [35].

The present paper is the extension of the latest cited paper, and it therefore aimed at investigating the propagation properties of a vHChGB propagating in a turbulent atmosphere. The theoretical analysis is based on the extended Huygens–Fresnel integral and Rytov method. The evolution of a vHChGB propagating in the turbulent atmosphere, and the influences of the turbulence strength on the beam intensity distribution under different beam parameters conditions are investigated with numerical examples. The remainder of the manuscript is structured as follows: in the coming Section, we present the theoretical formulation for the propagation of a vHChGB in atmospheric turbulence, and derive a propagation equation for the average intensity distribution of the beam. In the third Section, the propagation properties of vHChGB through the turbulent atmosphere are illustrated numerically and discussed versus the turbulence strength under the change of the beam parameters. The main results are outlined the conclusion part.

2. Propagation properties of a vHChG through turbulent atmosphere

In the rectangular coordinates system, a vHChGB propagating along the z -axis in the source plane $z=0$ can be expressed in the form [35]

$$E(x_0, y_0, z=0) = H_p\left(\frac{\sqrt{2}}{\omega_0}x_0\right)H_q\left(\frac{\sqrt{2}}{\omega_0}y_0\right)\cosh\left(b\frac{x_0}{\omega_0}\right)\cosh\left(b\frac{y_0}{\omega_0}\right)e^{-\left(\frac{x_0^2+y_0^2}{\omega_0^2}\right)}(x_0+iy_0)^M, \quad (1)$$

where (x_0, y_0) are the transverse Cartesian coordinates in the source plane and ω_0 is the waist radius of the Gaussian part. b is the decentered beam parameter b (or the parameter b for short) associated to the cosh part. M is an integer which denotes the topological charge of the vortex, and (p, q) are the mode indexes associated with the Hermite polynomials $H_p(\cdot)$ and $H_q(\cdot)$ in the x - and y -directions.

When $b=0$, Eq. (1) reduces to the hollow vortex Hermite-Gaussian beam [25]. In the case $M=0$, i.e., in the absence of the vortex, Eq. (1) will describe the well-known Hermite-cosh-Gaussian beam [36-40]. For convenience and without loss generality, we will consider in the following a vHChGB with $p=q$, i.e, with xy symmetry. The profile of the intensity of the xy symmetric vHChGB at the initial plane $z=0$ has been presented in Ref. [35]. It is found that the intensity profile of vHChGB is array-like with a dark central region surrounded by a multi-spot structure. The beam shape is mirror symmetric and can have two pattern configurations depending strongly on the value of the parameter b . Indeed, for a small b ($b<1$), the beam exhibits $4p$ lobes, and the four main lobes located at the vertices of the squared beam spot. While for large b , the beam is four-petal like.

Within the frame of the paraxial approximation, the propagation of a light beam through turbulent atmosphere along the z -axis can be formulated by the extended Huygens–Fresnel diffraction integral [1]

$$E(\mathcal{P}, z) = \frac{ik}{2\pi z} \exp(-ikz) \iint E(\mathcal{P}_0, 0) \exp\left[-\frac{ik}{2z}(\mathcal{P}_0 - \mathcal{P})^2 + \psi(\mathcal{P}_0, \mathcal{P}, z)\right] d\mathcal{P}_0, \quad (2)$$

where $\mathcal{P}_0 = (x_0, y_0)$ and $\mathcal{P} = (x, y)$ are the transverse coordinates in the initial and receiver planes, respectively. z is the distance between the initial plane $z=0$ and the receiver plane. $\psi(\mathcal{P}_0, \mathcal{P}, z)$ denotes the random part for the complex phase of a spherical wave spreading from the source plane to the output plane, $k = \frac{2\pi}{\lambda}$ is the wavenumber and λ is the wavelength of radiation in vacuum. Here, the integration is performed over the whole space for the unapertured light beam. The average intensity of the vHChGB propagating through turbulent atmosphere is expressed as

$$\begin{aligned} \langle I(\mathbf{P}, z) \rangle = & \left(\frac{k}{2\pi z} \right)^2 \iiint E_0(x_{01}, y_{01}, 0) E_0^*(x_{02}, y_{02}, 0) \\ & \times \exp \left[-\frac{ik}{2z} \left\{ (x_{01} - x)^2 + (y_{01} - y)^2 - (x_{02} - x)^2 - (y_{02} - y)^2 \right\} \right] \\ & \times \langle \exp[\psi(\mathbf{P}_{01}, \mathbf{P}) + \psi^*(\mathbf{P}_{02}, \mathbf{P})] \rangle d\mathbf{P}_{01} d\mathbf{P}_{02}, \end{aligned} \quad (3)$$

where $*$ and $\langle \rangle$ denote the complex conjugation and the ensemble average over the medium statistics, respectively.

Within the Rytov method and under weak atmosphere turbulence conditions, the ensemble average term in Eq. (3) is given as [1]

$$\langle \exp[\psi(x_{01}, y_{01}, x, y) + \psi^*(x_{02}, y_{02}, x, y)] \rangle = \exp \left[-\frac{1}{\rho_0^2} \left\{ (x_{01} - x_{02})^2 + (y_{01} - y_{02})^2 \right\} \right], \quad (4)$$

where $\rho_0 = (0.545 C_n^2 k^2 z)^{-3/5}$ is the coherence length of a spherical wave propagating in the turbulent medium, with C_n^2 being the refractive index structure constant of the medium.

Substituting Eqs. (1) and (4) into Eq. (3), and recalling the following binomial formula [41]

$$(x + iy)^M = \sum_{l=0}^M C_l^M x^l (iy)^{M-l}, \quad (5a)$$

where

$$C_l^M = \frac{M!}{l!(M-l)!}, \quad (5b)$$

Eq. (3) can be rewritten as:

$$\langle I(\mathbf{P}, z) \rangle = \left(\frac{k}{2\pi z} \right)^2 \sum_{l=0}^M \sum_{n=0}^M C_l^M C_n^M \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F_n(x, y, x_{02}, y_{02}) G_l(x, y, x_{02}, y_{02}) dx_{02} dy_{02}, \quad (6a)$$

where

$$\begin{aligned} F_n(x, y, x_{02}, y_{02}) = & x_{02}^n (-iy_{02})^{M-n} H_p \left(\frac{\sqrt{2}}{\omega_0} x_{02} \right) H_p \left(\frac{\sqrt{2}}{\omega_0} y_{02} \right) \cosh \left(b \frac{x_{02}}{\omega_0} \right) \cosh \left(b \frac{y_{02}}{\omega_0} \right) \\ & \times \exp \left(-\delta^* x_{02}^2 - \frac{ikx}{z} x_{02} \right) \exp \left(-\delta^* y_{02}^2 - \frac{iky}{z} y_{02} \right) \end{aligned} \quad (6b)$$

and

$$G_l(x, y, x_{02}, y_{02}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g_l(x, y, x_{01}, y_{01}, x_{02}, y_{02}) dx_{01} dy_{01}, \quad (6c)$$

with

$$g_l(x, y, x_{01}, y_{01}, x_{02}, y_{02}) = x_{01}^l (iy_{01})^{M-l} H_p \left(\frac{\sqrt{2}}{\omega_0} x_{01} \right) \cosh \left(b \frac{x_{01}}{\omega_0} \right) \exp \left[-\delta x_{01}^2 + \left(\frac{ikx}{z} + \frac{2x_{02}}{\rho_0^2} \right) x_{01} \right] \times H_p \left(\frac{\sqrt{2}}{\omega_0} y_{01} \right) \cosh \left(b \frac{y_{01}}{\omega_0} \right) \exp \left[-\delta y_{01}^2 + \left(\frac{iky}{z} + \frac{2y_{02}}{\rho_0^2} \right) y_{01} \right], \quad (6d)$$

δ is the auxiliary parameter defined by

$$\delta = \frac{ik}{2z} + \frac{1}{\omega_0^2} + \frac{1}{\rho_0^2}. \quad (6e)$$

Using the explicit form of $\cosh(\cdot)$ and the variable separation technique to perform the double integral expression in Eq. (6c), we get

$$G_l(x, y, x_{02}, y_{02}) = (i)^{M-l} \hat{g}_l(x, x_{02}) \hat{g}_l(y, y_{02}), \quad (7a)$$

with

$$\hat{g}_l(v, v_{02}) = \frac{1}{2} \left(\hat{g}_l^+(v, v_{02}) + \hat{g}_l^-(v, v_{02}) \right), \quad v = x \text{ or } y \quad (7b)$$

where

$$\hat{g}_l^\pm(v, v_{02}) = \int_{-\infty}^{+\infty} u^l H_p \left(\frac{\sqrt{2}}{\omega_0} u \right) \exp(-\delta u^2) \exp \left[\left(\frac{ikv}{z} + \frac{2v_{02}}{\rho_0^2} \pm \frac{b}{\omega_0} \right) u \right] du. \quad (7c)$$

Now, by recalling the following integral formula [42]

$$\int_{-\infty}^{+\infty} x^l H_m(\alpha x) e^{-px^2+2qx} dx = \frac{1}{2^l} e^{\frac{q^2}{p}} \sqrt{\frac{\pi}{p}} \sum_{k=0}^{\lfloor \frac{m}{2} \rfloor} \frac{(-1)^k m!}{k!(m-2k)!} \left(\frac{\alpha}{i\sqrt{p}} \right)^{m+l-2k} H_{m+l-2k} \left(\frac{iq}{\sqrt{p}} \right), \text{ with } \text{Re } p > 0 \quad (8)$$

Eq. (7c) can be expressed as

$$\hat{g}_l^\pm(v, v_{02}) = \frac{1}{2^l} e^{\frac{q_\pm^2(v, v_{02})}{\delta}} \sqrt{\frac{\pi}{\delta}} \sum_{s=0}^{\lfloor \frac{p}{2} \rfloor} \frac{(-1)^s p!}{s!(p-2s)!} \left(\frac{\sqrt{2}}{i\omega_0 \sqrt{\delta}} \right)^{p+l-2s} H_{p+l-2s} \left(\frac{iq_\pm(v, v_{02})}{\sqrt{\delta}} \right), \quad (9a)$$

with

$$q_\pm(v, v_{02}) = \frac{ikv}{2z} + \frac{v_{02}}{\rho_0^2} \pm \frac{b}{2\omega_0}, \quad (9b)$$

in which v represents either x or y .

Substituting Eqs. 9(a-b) and (7a) into Eq. (6a) we obtain

$$\begin{aligned}
\langle I(\mathbf{r}, z) \rangle = & \frac{1}{2^{M+2}} \frac{\pi}{\delta} \left(\frac{k}{2\pi z} \right)^2 \left(\frac{\sqrt{2}}{i\omega_0 \sqrt{\delta}} \right)^M \sum_{l=0}^M \sum_{n=0}^M C_l^M(i)^{M-l} C_n^M(-i)^{M-n} \\
& \times \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} dx_{02} dy_{02} F_n(x, y, x_{02}, y_{02}) \sum_{s=0}^{\lfloor \frac{p}{2} \rfloor} \frac{(-1)^s p!}{s!(p-2s)!} \left(\frac{\sqrt{2}}{i\omega_0 \sqrt{\delta}} \right)^{p-2s} \sum_{t=0}^{\lfloor \frac{p}{2} \rfloor} \frac{(-1)^t p!}{t!(p-2t)!} \left(\frac{\sqrt{2}}{i\omega_0 \sqrt{\delta}} \right)^{p-2t} \\
& \times \left[\exp\left(\frac{q_+^2(x, x_{02})}{\delta} \right) H_{p+l-2s}\left(\frac{iq_+(x, x_{02})}{\sqrt{\delta}} \right) + \exp\left(\frac{q_-^2(x, x_{02})}{\delta} \right) H_{p+l-2s}\left(\frac{iq_-(x, x_{02})}{\sqrt{\delta}} \right) \right] \\
& \times \left[\exp\left(\frac{q_+^2(y, y_{02})}{\delta} \right) H_{p+M-l-2t}\left(\frac{iq_+(y, y_{02})}{\sqrt{\delta}} \right) + \exp\left(\frac{q_-^2(y, y_{02})}{\delta} \right) H_{p+M-l-2t}\left(\frac{iq_-(y, y_{02})}{\sqrt{\delta}} \right) \right]
\end{aligned} \tag{10}$$

The integral expression on the right-hand of Eq. (10) can be performed directly by using the separation of variable method. So, by recalling the expanding form of Hermite polynomial [41]

$$H_m(x) = \sum_{j=0}^{\lfloor \frac{m}{2} \rfloor} \frac{(-1)^j m!}{j!(m-2j)!} (2x)^{m-2j}, \tag{11}$$

and with the help again of Eq. (8), and after lengthy algebraic calculations, the average intensity of the vHChGB propagating through turbulent atmosphere can be expressed as

$$\begin{aligned}
I(x, y, z) = & \frac{1}{2^{M+4}} \frac{1}{\delta \eta} \left(\frac{k}{2z} \right)^2 \left(\frac{\sqrt{2}}{i\omega_0 \sqrt{\delta}} \right)^M \sum_{l=0}^M C_l^M(i)^{M-l} \sum_{n=0}^M C_n^M(-i)^{M-n} \\
& \times [J_{l,n}^+(x) + J_{l,n}^-(x)] [K_{M-l,M-n}^+(y) + K_{M-l,M-n}^-(y)]
\end{aligned} \tag{12}$$

where $J_{l,n}^\pm(x)$ is given by

$$\begin{aligned}
J_{l,n}^\pm(x) = & \sum_{s=0}^{\lfloor \frac{p}{2} \rfloor} \frac{(-1)^s p!}{s!(p-2s)!} \left(\frac{\sqrt{2}}{i\omega_0 \sqrt{\delta}} \right)^{p-2s} \sum_{j=0}^{\lfloor \frac{\mu}{2} \rfloor} \frac{(-1)^j \mu!}{j!(\mu-2j)!} \sum_{j_1=0}^{\mu-2j} C_{j_1}^{\mu-2j} \left(\frac{2i}{\rho_0^2 \sqrt{\delta}} \right)^{\mu-2j-j_1} \left(\frac{2i}{\sqrt{\delta}} \right)^{j_1} \frac{1}{2^N} \\
& \times \left\{ \sum_{j_2=0}^{\lfloor \frac{p}{2} \rfloor} \frac{(-1)^{j_2} p!}{j_2!(p-2j_2)!} \left(\frac{\sqrt{2}}{i\omega_0 \sqrt{\eta}} \right)^{N+p-2j_2} \left[\left(\frac{ikx}{2z} + \frac{b}{2\omega_0} \right)^{j_1} \exp(Ax^2 + B^\pm x + C^\pm) H_{N+p-2j_2} \left(\frac{i\alpha}{\sqrt{\eta}} x + \frac{i\beta^\pm}{\sqrt{\eta}} \right) \right. \right. \\
& \left. \left. + \left(\frac{ikx}{2z} - \frac{b}{2\omega_0} \right)^{j_1} \exp(Ax^2 - B^m x + C^m) H_{N+p-2j_2} \left(\frac{i\alpha}{\sqrt{\eta}} x - \frac{i\beta^m}{\sqrt{\eta}} \right) \right] \right\},
\end{aligned} \tag{13a}$$

and $K_{M-l,M-n}^\pm(y)$ is expressed as

$$\begin{aligned}
K_{M-l, M-n}^{\pm}(y) = & \sum_{t=0}^{\left[\frac{p}{2}\right]} \sum_{h=0}^{\left[\frac{d}{2}\right]} \frac{(-1)^t p!}{t!(p-2t)!} \left(\frac{\sqrt{2}}{i\omega_0 \sqrt{\delta}} \right)^{p-2t} \frac{(-1)^h d!}{h!(d-2h)!} \sum_{t_1=0}^{d-2h} C_{t_1}^{d-2h} \sum_{t_2=0}^{\left[\frac{p}{2}\right]} \left(\frac{2i}{\rho_0^2 \sqrt{\delta}} \right)^{d-2h-t_1} \left(\frac{2i}{\sqrt{\delta}} \right)^{t_1} \frac{1}{2^{N_l}} \\
& \frac{(-1)^{t_2} p!}{t_2!(p-2t_2)!} \left(\frac{\sqrt{2}}{i\omega_0 \sqrt{\eta}} \right)^{N_l+p-2t_2} \times \left[\left(\frac{iky}{2z} + \frac{b}{2\omega_0} \right)^{t_1} \exp(A y^2 + B^{\pm} y + C^{\pm}) H_{N_l+p-2t_2} \left(\frac{i\alpha}{\sqrt{\eta}} y + \frac{i\beta^{\pm}}{\sqrt{\eta}} \right) \right. \\
& \quad \left. + \left(\frac{iky}{2z} - \frac{b}{2\omega_0} \right)^{t_1} \exp(A y^2 - B^{\mu} y + C^{\mu}) H_{N_l+p-2t_2} \left(\frac{i\alpha}{\sqrt{\eta}} y - \frac{i\beta^{\mu}}{\sqrt{\eta}} \right) \right]
\end{aligned} \tag{13b}$$

with

$$\mu = p + l - 2s, \tag{13c}$$

$$d = p + M - l - 2t, \tag{13d}$$

$$N = \mu + n - 2j - j_1, \tag{13e}$$

$$N_l = d + M - n - 2h - t_1, \tag{13f}$$

$$\eta = \delta^* - \frac{1}{\delta \rho_0^4}, \tag{13g}$$

$$\alpha = \frac{ik}{2z} \left(\frac{1}{\delta \rho_0^2} - 1 \right), \tag{13h}$$

$$\beta^{\pm} = \frac{b}{2\omega_0} \left(\frac{1}{\delta \rho_0^2} \pm 1 \right), \tag{13i}$$

$$A = \frac{\alpha^2}{\eta} - \frac{k^2}{4z^2 \delta}, \tag{13j}$$

$$B^{\pm} = \frac{ikb}{2z\omega_0 \delta} + \frac{2\alpha\beta^{\pm}}{\eta}, \tag{13k}$$

and

$$C^{\pm} = \frac{b^2}{4\omega_0^2 \delta} + \frac{(\beta^{\pm})^2}{\eta}. \tag{13l}$$

Eq. (12) is the main analytical result of this paper, which provides us with a convenient way to investigate the propagation properties of a vHChGB in the turbulent atmosphere. In the particular case when $p=0$, Eq. (12) will give the propagation equation for the vChGB, and the result will be consistent with the corresponding expression given in [34].

3. Numerical examples and analysis

In the present section, based on the numerical calculation of Eq. (12), the propagation properties of a vHChGB through atmospheric turbulence are illustrated graphically as function of the turbulence strength and under different beam parameters conditions. As it is known, the profile of the incident vHChG filed can take one of the two beam shape configurations depending on the value of the parameter b as mentioned before. Hereafter, the behavior of the beam in the both configurations is examined separately. The calculation parameters are $\omega_0 = 2 \text{ cm}$, $\lambda = 1060 \text{ nm}$ and $C_n^2 = 10^{-14} \text{ m}^{-2/3}$ (otherwise, it is indicated). In addition and for the sake of clarity the numerical results related to the free space propagation case are also presented for comparison.

Fig. 1 illustrates the normalized intensity distribution of a vHChGB with $M=1$ and the beam orders, $p=1, 2$ and 3 , propagating in turbulent atmosphere. From the plots of Fig. 1, one can easily see that the perturbed beam widens gradually and its profile remains unchanged within a small distance range. Then, at a certain propagation distance the beam lobes overlap and the central hole-intensity is filled. With more increasing the propagation distance the beam evolves into Gaussian-like, and at certain propagation distance the central peak intensity reaches its maximum value. After that, the peak intensity decreases gradually with the propagation distance. It can also be seen that the beam evolution is affected strongly by the turbulent atmosphere compared to the free-space case, especially in the far-field. Furthermore, one can note the apparition of some secondary weak lobes inside the beam spot for higher beam orders ($p=2$ and 3) within the intermediate propagation distance range: $1 \text{ km} < z < 2 \text{ km}$.

Figure 1.1: The normalized average intensity distribution of a vHChGB in a turbulent atmosphere at different propagation distances z , and for $b=0.1$ and different values of p , with $\omega_0 = 0.02 \text{ m}$, $\lambda = 1060 \text{ nm}$,

$$C_n^2 = 10^{-14} \text{ m}^{-2/3}, \text{ rows (a-b) for } p=1, \text{ (c-d) for } p=2 \text{ and (e-f) for } p=3.$$

Figure 1.2: The same as Fig.1a, except $b=4$.

The effect of the topological charge M on the perturbed vHChGB is demonstrated in Fig. 2 for $p=1$ and 2. From the plots of Fig. 2, one can see that the beam lobes are slightly wider and the lobes interspace is larger when M is increased. One can also notice the elongation effect on the lobes due to the increase of M . Nevertheless, when b is large, the perturbed beam is less affected

by the change of M . In addition, one can clearly see from Fig. 2, that the rising speed of the central peak intensity becomes slower as M or p is larger.

Figure 2.1: The average intensity distribution of a vHChGB in a turbulent atmosphere for different values M , with $\lambda = 1060 \text{ nm}$, $C_n^2 = 10^{-14} \text{ m}^{-2/3}$, $\omega_0 = 0.02 \text{ m}$, $p=1$, (a,b): $M=2$ and (c,d): $M=4$.

Figure 2.2: The same as Fig. 2.1 except $p=2$.

The influence of the turbulence strength on the vHChGB spreading is demonstrated in Fig. 3, we have illustrated the normalized axial intensity distribution for different turbulence strength ($C_n^2 = 10^{-15} m^{-2/3}$, $10^{-14} m^{-2/3}$ and $5 \cdot 10^{-14} m^{-2/3}$). The plots show that if the atmosphere is more turbulent, the rising speed of the central peak intensity will become faster. In addition, one can easily note that if M is increased the beam rising speed of the central peak becomes slower, which confirms the behavior of the beam shown in Fig. 2.

Figure 3: The normalized axial intensity versus propagation distance z of a vHChGB in a turbulent atmosphere for a different values of the refractive index structure C_n^2 with $\omega_0 = 0.02 \text{ m}$, $\lambda = 1060 \text{ nm}$ and $p=1$.

Fig. 4 illustrates the effect of the waist radius ω_0 on the intensity distribution of a vHChGB in turbulent atmosphere at $z=2\text{km}$, for $M=1$ and $p=1$. It can be clearly seen that when ω_0 is larger the beam lobes becomes wider and the central hole-intensity filling is slower.

Figure 4.1: The normalized intensity of a vHChGB in a turbulent atmosphere for different values of the waist ω_0 with $M=1$, $p=1$, $z=2 \text{ km}$, $\lambda = 1060 \text{ nm}$ and $C_n^2 = 10^{-14} \text{ m}^{-2/3}$.

(a.1,b.1) $\omega_0 = 0.01 \text{ m}$, (a.2,b.2) $\omega_0 = 0.02 \text{ m}$, (a.3,b.3) $\omega_0 = 0.03 \text{ m}$ and (a.4,b.4) $\omega_0 = 0.04 \text{ m}$.

Figure 4.2: The normalized intensity in x-direction of a vHChGB in a turbulent atmosphere at different propagation distance z as follow: $M=1, p=1, C_n^2 = 10^{-14} m^{-2/3}, \lambda = 632.8 nm$, (a.1-2): $b=0.1$, (b.1-2): $b=4$, and different values of the waist ω_0 .

Figure 4.3: The normalized axial intensity versus propagation distance z of a vHChGB in a turbulent atmosphere for a different values of the waist ω_0

with $M=1, p=1, \lambda = 1060 nm$ and $C_n^2 = 10^{-14} m^{-2/3}$. (a): $b=0.1$, (b), $b=4$

The effect of the wavelength λ on the vHChGB propagating through turbulent atmosphere is depicted in Fig. 5, from which it is easily seen that the perturbed beam spreads faster as λ is

increased. Furthermore, for a beam with small b the rising speed of the central peak is slower as λ is larger, whereas the reverse behavior occurs when b is large.

Figure 5.1: The normalized intensity in x-direction of a vHChGB in a turbulent atmosphere at different propagation distance as follow: $M=1$, $p=1$, $\omega_0 = 0.02 \text{ m}$, $C_n^2 = 10^{-14} \text{ m}^{-2/3}$, (a.1-2-3): $b=0.1$, (b.1-2-3): $b=4$ and different values of the wavelength λ .

Figure 5.2: The normalized intensity of a vHChGB in a turbulent atmosphere for different values of the wavelength λ with $M=1, p=1, z=2$ km, $\omega_0 = 0.02$ m and $C_n^2 = 10^{-14} \text{ m}^{-2/3}$. Cols (a.1-b.1) $\lambda = 632.8 \text{ nm}$, (a.2-b.2) $\lambda = 1060 \text{ nm}$, (a.3-b.3) $\lambda = 1550 \text{ nm}$ and (a.4-b.4) $\lambda = 2000 \text{ nm}$.

4. Conclusion

In summary, based on the extended Huygens–Fresnel diffraction integral and the Rytov method, the propagation formula of a vHChGB in turbulent atmosphere is derived in detail. It is shown from numerical calculations that the propagation properties of the perturbed vHChGB are altered by the turbulence strength and the change of the beam parameters. The profile of the beam will change during propagation in the turbulent media from hollow array–distribution to the Gaussian-like. When the turbulence strength is larger or the beam order, vortex charge and Gaussian waist width are smaller, the speed of profile change will be faster. For a vHChGB with a small parameter b , the profile change is faster as λ is larger, whereas the reverse behavior is obtained when b is large. It is expected that this study will be useful for the applications of vHChGB in free space communication optics.

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