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Naima Nossir

Chouaib Doukkali University Faculty of Sciences: Universite Chouaib Doukkali Faculte des Sciences

Latifa Dalil-Essakali

Chouaib Doukkali University Faculty of Sciences: Universite Chouaib Doukkali Faculte des Sciences

Abdelmajid Belafhal (✉ belafhal@gmail.com)

Chouaib Doukkali University <https://orcid.org/0000-0003-2735-3108>

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Behavior of the central intensity of Generalized Humbert-Gaussian beams against the atmospheric turbulence

N. Nossir, L. Dalil-Essakali, A. Belafhal*

Laboratory LPNAMME, Laser Physics Group,

Department of Physics, Faculty of Sciences, Chouaib Doukkali University,

P. B 20, 24000 El Jadida, Morocco

* Corresponding authors: belafhal@gmail.com

Abstract

Based on the generalized Huygens-Fresnel integral in the paraxial approximation and on the Rytov theory, the analytical expression of the axial intensity distribution for the Generalized Humbert-Gaussian beams (GHGBs) propagating through a turbulent atmosphere is derived in this work. The results for the special cases of GHGBs are deduced from our study and illustrated numerically. The influence of the turbulent strength and the incident beam parameters on the propagation of these beams in the atmospheric turbulence is investigated and discussed in detail.

Keywords: *Generalized Humbert-Gaussian beams; Turbulent atmosphere; Huygens-Fresnel integral; Rytov theory.*

1. Introduction

The propagation of laser beams in a turbulent atmosphere has received a lot of attention from many researchers for several years, due to their large applications in many areas of science and practical engineering including free space optical communications [1], active optical imaging system [2], and remote sensing [3]. On the other hand, the turbulent atmosphere has an important impact on the quality of the laser beam owing to the random changes in the refraction index along the path of beam propagation. This result from the small temperature variations in the atmosphere, which induce fluctuations in laser intensity.

In the literature, several works have focused on the study of the influence of the turbulent atmospheric environment on the propagation of laser beams. The propagation properties of elegant Laguerre-Gaussian beams [4], Laguerre-Gaussian Beams [5], flat-topped vortex hollow beams [6], Laguerre-Gaussian Schell beams [7], truncated Bessel-Gauss beams [8], partially coherent Bessel-Gaussian beams [9] and Hypergeometric-Gaussian beams [10] and so on, have greatly developed by the considered authors. On the other hand, in our research group Khannous et al. have interested to the propagation characteristics of some laser beams in turbulent atmosphere such as: Kummer beams [11], Hypergeometric-Gaussian beams Type II [12] and

Hollow-Gaussian beams [13]. Hennani et al. [14] have studied the axial intensity distribution of the modified quadratic Bessel-Gaussian beam in turbulent atmosphere. Kinani et al. [15] have examined the effects of the atmospheric turbulence on the propagating of Li's flat-topped beams. More recently, Ez-zariy et al. [16] have investigated the propagation of Lommel-Gaussian beams through atmospheric turbulence, Boufalah et al. [17,18] have reported a study of the propagation of Pearcey-Gaussian beam and Generalized Laguerre-Bessel-Gaussian beams in turbulent atmosphere and Yaalou et al. [19] have developed theoretically the propagation characteristics of dark and antidark-Gaussian beams in turbulent atmosphere.

We will interest in the present study to the propagation properties of the Generalized Humbert beams modulated by Gaussian envelope (GHGBs) in turbulent atmosphere, where the Generalized Humbert beams (GHBs) are new hollow beams generated recently by Belafhal and Saad [20]. These last beams can be created by illuminating Circular beams (CiBs) [21] with a spiral phase plate (SPP) passing through a paraxial ABCD optical system. We note that under certain appropriate conditions of input beam parameters, some special cases of beams derived from CiBs as Generalized Humbert beams generated from Whittaker-Gaussian beams (WGBs), Hypergeometric-Gaussian beams (HyGBs), elegant Laguerre-Gaussian (eLGBs), standard Laguerre-Gaussian beam (sLGBs), generalized Laguerre-Gaussian beam (gLGBs), and quadratic Bessel-Gaussian beam (QBGBs), respectively. This family of beams will give rise to different particular cases beams of GHGBs and possesses interesting properties. For this reason, our attention was concentrated to study the behavior of this beams family during its propagation in a turbulent atmosphere.

The coming parts of this article are structured as follows: Section 2 is devoted to perform the average intensity distribution of GHGBs propagating in atmospheric turbulence. An approximate analytical expression of the axial intensity distribution for the considered beams is expressed in Section 3. In Section 4, special cases of GHGBs are derived. The numerical simulations and discussions are treated in Section 5. A conclusion is outlined in the end of this work.

2. The average intensity distribution of GHGBs in a turbulent atmosphere

Let us consider the electric field distribution of GHGBs in the cylindrical coordinates system (r, z_0, φ) given by

$$E_{GHBs}(r, z_0, \theta) = A_l^m r^{m+l} \exp\left(\frac{ik}{2z_0} r^2\right) \exp[i(m+l)\theta] \times \Psi_1\left(m + \frac{l}{2} + 1, \beta; m+l+1; m+1; \frac{2i\alpha'}{\alpha + i\alpha'}, \frac{-(k/z_0)^2}{4(\alpha + i\alpha')} r^2\right), \quad (1)$$

where

$$A_l^m = -E_f \frac{\Gamma\left(\frac{l}{2} + m + 1\right)}{\Gamma(m+l+1)} \left(\frac{ik}{z_0}\right)^{m+l+1} \left(\frac{1}{\alpha + i\alpha'}\right)^{\frac{l}{2} + m + 1} \frac{(i\alpha')^{\frac{m}{2}}}{2^{\frac{m}{2} + l + 1}} \exp[ik(z_0 + z')] \frac{(1 + z'/q_0)^{((m+1)/2) - \beta}}{(1 + z'/q_0)^{((m+3)/2) - \beta}}. \quad (2.a)$$

with

$$\alpha = -ik \left[\frac{\omega_0^2}{4} \left(\frac{1}{z' + \tilde{q}_0} + \frac{1}{z' + q_0} \right) + \frac{1}{2z_0} \right], \quad (2.b)$$

$$\alpha' = \frac{\omega_0^2}{4k \left(\frac{1}{z' + \tilde{q}_0} - \frac{1}{z' + q_0} \right)}, \quad (2.c)$$

and

$$\beta = \frac{m+1-\mu}{2}. \quad (2.d)$$

E_f is the amplitude of the beam, ω_0 is the spot size of the fundamental Gaussian mode, μ is the radial order, $(q_0, \tilde{q}_0)$ are two complex beam parameters, $k = \frac{2\pi}{\lambda}$ represents the wave number, λ is the wavelength, l is the topological charge, m is the beam order, $\Gamma(\cdot)$ denotes the gamma function and $\Psi_1(\cdot)$ is the Humbert confluent hypergeometric function. We consider that the field is situated at a distance $z = z_0$ from the source plane, with z' is the transverse coordinate of the Circular beams.

In the following, we will interest to the Generalized Humbert beams modulated by a Gaussian envelope. The electric field expression of these beams is given by

$$E_{GHGBs}(r, z_0, \theta) = E_{GHBs}(r, z_0, \theta) \exp\left(\frac{-\eta r^2}{\omega_0^2}\right), \quad (3)$$

with η is a positive constant.

The optical system representing the propagation of the GHGBs through a turbulent atmosphere is schematized in Fig. 1. The receiver plane is positioned at propagation distance z away from the transmitter plane.

Figure 1: Schematic illustration of the GHGBs propagating in a turbulent atmosphere.

In this section, we will interest to the determination of the analytical average intensity distribution of GHGBs propagating in the turbulent atmosphere.

Based on the extended Huygens-Fresnel integral in the paraxial approximation and on the Rytov theory, the average intensity at the output plane is expressed as [22]

$$E(\rho, z, t) = \frac{-ik}{2\pi z} \exp(ikz) \int_0^a \int_0^{2\pi} E(\rho, z_0) \exp\left[\frac{ik}{2z}(\rho - \rho) + \Phi(\rho, \rho) - 2i\pi \cdot f \cdot t\right] d\rho, \quad (4)$$

where z is the propagation distance, a is the radius of the aperture, $\Phi(\rho, \rho)$ represents the random part, in the Rytov method, of the complex phase of a spherical wave propagating from the source plane to the output plane, f is the frequency and t denotes the time. Here $E(\rho, z_0)$ is the electric field of the GHGBs at the input plane.

The average intensity distribution at the z -plane can be written as

$$\begin{aligned} \langle I(\rho, z) \rangle &= \langle E(\rho, z, t) \rangle \langle E^*(\rho, z, t) \rangle \\ &= \frac{k^2}{4\pi^2 z^2} \int_0^a \int_0^{2\pi} \int_0^{2\pi} E(\rho_1, z_0) \cdot E^*(\rho_2, z_0) \exp\left[\frac{ik}{2z}(\rho_1 - \rho)^2 - \frac{ik}{2z}(\rho_2 - \rho)^2\right] \\ &\quad \times \langle \exp[\Phi(\rho_1, \rho) + \Phi^*(\rho_2, \rho)] \rangle d\rho_1 d\rho_2, \end{aligned} \quad (5)$$

where the average term in the last equation is given by [22]

$$\langle \exp[\Phi(\rho_1, \rho) + \Phi^*(\rho_2, \rho)] \rangle = \exp[-0.5D_\Phi(\rho_1 - \rho_2)] \quad (6a)$$

In these equations, asterisk $*$ denotes the complex conjugate, $\langle \rangle$ is the average, $D_\Phi(\rho_1 - \rho_2)$ is the phase structure function of the random complex in Rytov's representation given by

$$D_\Phi(\rho_1 - \rho_2) = \frac{2}{\rho_0} (\rho_1 - \rho_2)^2, \quad (6b)$$

with

$$\rho_0 = (0.545 C_n^2 k^2 z)^{-3/5}, \quad (6c)$$

is the coherence length of a spherical wave spreading in the turbulent atmosphere medium where C_n^2 is the refractive index structure constant, which characterizes the local strength of the turbulent atmosphere. By using the development of Humbert confluent hypergeometric function in terms of the Hypergeometric function ${}_2F_1(a, b; c; x)$ [23]

$$\Psi_1(a, b; c; c'; w, z) = \sum_{n=0}^{\infty} \frac{(a)_n z^n}{(c')_n n!} {}_2F_1(a+n, b; c; w), \quad (7)$$

and by introducing Eqs. (3), (6a), (6b) and (7) into Eq. (5), we can evaluate the average intensity distribution of GHGBs as

$$\begin{aligned} \langle I(\mathbf{r}, z) \rangle &= \frac{A_t^m A_l^{m*}}{\lambda^2 z^2} \sum_{q=0}^{\infty} \frac{\Gamma\left(\frac{l}{2} + m + 1 + q\right)}{q! \Gamma(m+1+q)} \left(\frac{-(k/z_0)^2}{4(\alpha + i\alpha')} \right)^q \sum_{j=0}^{\infty} \frac{\Gamma\left(\frac{l}{2} + m + 1 + j\right)}{j! \Gamma(m+1+j)} \left(\frac{-(k/z_0)^2}{4(\alpha + i\alpha')^*} \right)^j \\ &\quad \times {}_2F_1\left(\frac{l}{2} + m + 1 + q, \beta; m + l + 1; \frac{2i\alpha'}{\alpha + i\alpha'}\right) {}_2F_1\left(\frac{l}{2} + m + 1 + j, \beta; m + l + 1; \left(\frac{2i\alpha'}{\alpha + i\alpha'}\right)^*\right) \\ &\quad \times \int_0^a \int_0^a \int_0^{2\pi} \int_0^{2\pi} r_1^{m+l+2q+1} r_2^{m+l+2j+1} \exp[i(m+l)(\theta_1 - \theta_2)] \exp\left[\frac{2\cos(\theta_1 - \theta_2)r_1 r_2}{\rho_0^2}\right] \\ &\quad \times \exp\left[-\left\{\frac{ik}{2}\left(\frac{1}{z} + \frac{1}{z_0}\right) + \left(\frac{\eta}{\omega_0^2} + \frac{1}{\rho_0^2}\right)\right\}r_2^2\right] \exp\left[\left\{\frac{ik}{2}\left(\frac{1}{z} + \frac{1}{z_0}\right) - \left(\frac{\eta}{\omega_0^2} + \frac{1}{\rho_0^2}\right)\right\}r_1^2\right] \\ &\quad \times \exp\left[\frac{ik}{2z}\rho(r_1 \cos(\theta_1 - \varphi) - r_2 \cos(\theta_2 - \varphi))\right] dr_1 dr_2 d\theta_1 d\theta_2. \end{aligned} \quad (8)$$

From Eq. (8), we will deduce the on-axis average intensity distribution of GHGBs propagating in turbulent atmosphere in the following section.

3. Axial intensity distribution of truncated GHGBs in turbulent atmosphere

In this part, we will give an approximate evaluation of the axial intensity distribution of the GHGBs propagating in a turbulent atmosphere. To evaluate the integral of Eq. (8), we will introduce the hard aperture function defined as

$$H(r) = \begin{cases} 1, & |r| \leq a \\ 0, & |r| > a \end{cases}, \quad (9)$$

with a is the half-width of the rectangular function.

The approximate expression of $H(r)$ is given by expanding into a finite sum of complex Gaussian functions with finite numbers in the rectangular coordinates system [24]

$$H(r) = \sum_{k=1}^N A_k \exp\left[-\frac{B_k}{a^2} r^2\right], \quad (10)$$

where A_k and B_k designate the expansion and Gaussian coefficients, respectively. N is the number of the expansion of Gaussian function. By substituting Eq. (10) into Eq. (8) then, we put $\rho = 0$ and the average axial intensity distribution becomes

$$\begin{aligned} \langle I(0, z) \rangle &= \frac{A_l^m A_l^{m*}}{\lambda^2 z^2} \sum_{q=0}^{\infty} \frac{\Gamma\left(\frac{l}{2} + m + 1 + q\right)}{q! \Gamma(m + 1 + q)} \left(\frac{-(k/z_0)^2}{4(\alpha + i\alpha')}\right)^q \\ &\quad \times {}_2F_1\left(\frac{l}{2} + m + 1 + q, \beta; m + l + 1; \frac{2i\alpha'}{\alpha + i\alpha'}\right) \sum_{j=0}^{\infty} \frac{\Gamma\left(\frac{l}{2} + m + 1 + j\right)}{j! \Gamma(m + 1 + j)} \left(\frac{-(k/z_0)^2}{4(\alpha + i\alpha')}\right)^j \\ &\quad \times {}_2F_1\left(\frac{l}{2} + m + 1 + j, \beta; m + l + 1; \frac{2i\alpha'}{\alpha + i\alpha'}\right) \sum_{g=1}^N A_g \sum_{s=1}^N A_s \\ &\quad \times \int_0^{+\infty} \int_0^{+\infty} \int_0^{2\pi} \int_0^{2\pi} \exp\left(\frac{2r_1 r_2}{\rho_0^2} \cos(\theta_2 - \theta_1)\right) \exp(-r_1^2 \beta_1) \exp(-r_2^2 \beta_2) \exp(-i(m+l)\theta_2) \\ &\quad \times \exp(i(m+l)\theta_1) r_1^{m+l+2j+1} r_2^{m+l+2q+1} dr_1 dr_2 d\theta_1 d\theta_2, \end{aligned} \quad (11)$$

with

$$\beta_1 = \left(\frac{\eta}{\omega_0^2} + \frac{1}{\rho_0^2} + \frac{B_g}{a^2}\right) - \frac{ik}{2} \left(\frac{1}{z_0} + \frac{1}{z}\right), \quad (12a)$$

and

$$\beta_2 = \left(\frac{\eta}{\omega_0^2} + \frac{1}{\rho_0^2} + \frac{B_s}{a^2}\right) + \frac{ik}{2} \left(\frac{1}{z_0} + \frac{1}{z}\right). \quad (12b)$$

For resolution of the integrals in Eq. (11), we will apply the well-known formulas [25]

$$\int_0^{2\pi} \exp(-im\varphi_2 + x \cos(\varphi_2 - \varphi_1)) d\varphi_2 = 2\pi \exp(-im\varphi_1) I_m(x), \quad (13)$$

and

$$\begin{aligned}
& \int_0^{+\infty} x^{\delta'-1} \exp(-\mu' x) {}_mF_n(\alpha_1, \alpha_2, \dots, \alpha_m; \gamma_1, \gamma_2, \dots, \gamma_n; \lambda x) dx = \Gamma(\delta') \mu'^{-\delta'} \\
& \times {}_{m+1}F_n\left(\alpha_1, \alpha_2, \dots, \alpha_m; \delta'; \gamma_1, \gamma_2, \dots, \gamma_n; \frac{\lambda}{\mu'}\right) \\
& \left[m \leq n, \operatorname{Re} \delta' > 0; \operatorname{Re} \mu' > 0, \text{ if } m < n; \operatorname{Re} \mu' > \lambda, \text{ if } m = n \right]
\end{aligned} \tag{14}$$

where $I_m(\cdot)$ is the m-order modified Bessel function and ${}_mF_n$ is the confluent Hypergeometric function. After some algebraic calculations, we find the analytical expression of the axial intensity distribution of GHGBs as follows

$$\begin{aligned}
\langle I(z, 0) \rangle &= \frac{k^2}{4z^2} \frac{A_l^m A_l^{m*}}{\rho_0^{2m+2l}} \left[\frac{\Gamma\left(\frac{l}{2} + m + 1\right)}{m!} \right]^2 \Gamma(m+l+1) \sum_{g=1}^N A_g \sum_{s=1}^N A_s \left(\frac{1}{\beta_1 \beta_2} \right)^{m+l+1} \\
&\times \sum_{q=0}^{\infty} \frac{\left(\frac{l}{2} + m + 1\right)_q (m+l+1)_q}{(m+1)_q} \frac{\left(\frac{-(k/z_0)^2}{4(\alpha + i\alpha')\beta_1}\right)^q}{q!} {}_2F_1\left(\frac{l}{2} + m + 1 + q, \beta; m+l+1; \frac{2i\alpha'}{\alpha + i\alpha'}\right) \\
&\times \sum_{j=0}^{\infty} \frac{\left(\frac{l}{2} + m + 1\right)_j (m+l+1)_j}{(m+1)_j} \frac{\left(\frac{-(k/z_0)^2}{4(\alpha + i\alpha')^* \beta_2}\right)^j}{j!} {}_2F_1\left(\frac{l}{2} + m + 1 + j, \beta; m+l+1; \left(\frac{2i\alpha'}{\alpha + i\alpha'}\right)^*\right) \\
&\times {}_2F_1\left(m+l+1+j, m+l+1+q; m+l+1; \frac{1}{\beta_2 \beta_1 \rho_0^4}\right).
\end{aligned} \tag{15}$$

This last equation is the main result obtained in this work. To understand the GHGBs properties propagating in atmospheric turbulence, we will treat, in the next section, some particular cases by using Eq. (15).

4. Particular cases of GHGBs

We note that from Eq. (15) we can obtain the particular cases of GHGBs denoted by $GHGBs_{QBGBs}$, $GHGBs_{WGBs}$, $GHGBs_{sLGBs}$, $GHGBs_{eLGBs}$, $GHGBs_{gLGBs}$ and $GHGBs_{HyGGBs}$ and given in Table 1. Here, n is the high radial order of $eLGBs$ and $sLGBs$ beams, l is the beam parameter of the CiBs and $h \in \mathbb{R} \setminus 0$. In Table 1, d_1^+ , d_1^- , d_2^+ , d_2^- , d_3^+ and d_3^- are defined as

$$d_1^+ = \frac{2(h+i)}{2z'(h+i) + k\omega_0^2}, \tag{16.a}$$

$$d_1^- = \frac{2(h-i)}{2z'(h-i) - k\omega_0^2}, \quad (16.b)$$

$$d_2^+ = \frac{1}{q_0^* + z'} + \frac{1}{q_0 + z'}, \quad (16.c)$$

$$d_2^- = \frac{1}{q_0^* + z'} - \frac{1}{q_0 + z'}, \quad (16.d)$$

$$d_3^+ = \frac{1}{\tilde{q}_0 + z'} + \frac{1}{q_0 + z'}, \quad (16.e)$$

and

$$d_3^- = \frac{1}{\tilde{q}_0 + z'} - \frac{1}{q_0 + z'}. \quad (16.f)$$

Beams	Parameters of input beams			Parameters of output beams	
$GHGBs$	$\frac{m+1-\mu}{2}$	q_0	$\tilde{q}_0$	α	α'
$GHGBs_{sLGBs}$	$-n$	q_0	$\tilde{q}_0^*$	$-ik \left(\frac{\omega_0^2}{4} d_2^+ + \frac{1}{2z_0} \right)$	$\frac{k\omega_0^2}{4} d_2^-$
$GHGBs_{eLGBs}$	$-n$	q_0	∞	$-ik \left(\frac{\omega_0^2}{4} d_2^+ + \frac{1}{2z_0} \right)$	$-ik \left(\frac{\omega_0^2}{4} d_2^+ + \frac{1}{2z_0} \right)$
$GHGBs_{QGBs}$	$\frac{m+1}{2}$	$-\tilde{q}_0^*$	$\frac{k\omega_0^2}{2(h+i)}$	$-ik \left[\frac{\omega_0^2}{4} (d_1^+ + d_2^-) + \frac{1}{2z_0} \right]$	$\frac{k\omega_0^2}{4} (d_1^- - d_1^+)$
$GHGBs_{WGBs}$	$\frac{m+1-\mu}{2}$	$\tilde{q}_0^*$	$\frac{k\omega_0^2}{2(h+i)}$	$-ik \left[\frac{\omega_0^2}{4} (d_1^+ + d_2^-) + \frac{1}{2z_0} \right]$	$\frac{k\omega_0^2}{4} (d_1^- - d_1^+)$
$GHGBs_{gLGBs}$	$-n$	q_0	$\tilde{q}_0$	$-ik \left(\frac{\omega_0^2}{4} d_3^+ + \frac{1}{2z_0} \right)$	$\frac{k\omega_0^2}{4} d_3^-$
$GHGBs_{HyGBs}$	$\frac{-p}{2}$	q_0	0	$-ik \left[\frac{\omega_0^2}{4} \left(\frac{1}{z'} + \frac{1}{q_0 + z'} \right) + \frac{1}{2z_0} \right]$	$\frac{k\omega_0^2}{4} \left[\frac{1}{z'} - \frac{1}{q_0 + z'} \right]$

Table 1: Parameters of particular beams.

5. Numerical examples and analysis

In this section, using the analytical expression of the average intensity distribution of GHGBs propagating in atmospheric turbulence derived in Section 3, we present some numerical results of the propagation characteristics for the particular cases of GHGBs given in Table 1.

Fig. 2 shows the evolution of the distribution of the axial intensity for GHGBs versus the propagation distance z . The plots are given for three values of the beam orders: $m = 0$, $m = 1$, and $m = 2$ and for two values of the turbulent strengths $C_n^2 = 1.10^{-14} m^{-2/3}$ and $C_n^2 = 1.10^{-15} m^{-2/3}$. From the plots, it's noted that the on-axis average intensity distribution increases with the turbulent strength and decreases when the beam orders increase. Furthermore, it observed that $GHGBs_{WGBs}$ present a high intensity in comparison with other beams while $GHGBs_{eLGBs}$ are characterized by a small intensity. It is seen that the position of the maximum intensity tends toward small values of the propagation distance when C_n^2 is large which means that the propagation distance of GHGBs family becomes shorter when the atmosphere is very turbulent.

Figure 2: The average axial intensity distribution versus the propagation distance z for GHGBs for different beam orders: $m=0$, $m=1$ and $m=2$, with two values of turbulent strength: (a) $C_n^2 = 1.10^{-15} m^{-2/3}$ and (b) $C_n^2 = 1.10^{-14} m^{-2/3}$.

The others parameters are chosen as: $l = 1$, $\omega_0 = 0.5mm$, $\lambda = 1330nm$, $a = 0.15m$, $\mu = 1$ and $h = 1$.

To show the influence of the wavelength on the propagation of GHGBs, the distribution of the axial intensity is depicted versus the axial distance z in Fig. 3 for different beam orders ($m=0$, 1 and $m=2$). Several curves are plotted for two wavelength value $\lambda = 632.8nm$ and $\lambda = 1330nm$ with a fixed value of the turbulent strength $C_n^2 = 1.10^{-14} m^{-2/3}$. It can be observed that the axial intensity distribution decreases with increasing of the beam orders and the wavelength.

Figure 3: The average axial intensity distribution versus the propagation distance z for GHGBs for $m=0$, $m=1$, and $m=2$, with different beam orders and two values of wavelength: (a) $\lambda = 632.8nm$, (b) $\lambda = 1330nm$.

The other parameters are: $\omega_0 = 0.5mm$, $C_n^2 = 1.10^{-14} m^{-2/3}$, $a = 0.15m$, $\mu = 1$, $h = 1$ and $l = 1$.

It is clear that $GHGBs_{eLGBs}$ have a minimal intensity, whereas the maximum intensity appears for the $GHGBs_{WGBs}$ with all beam orders. However, we can also note that the position of the maximum intensity tends to higher values of propagation distance with large values of wavelength. We conclude that, the decrease of the intensity is faster when wavelength is small then the propagation distance of the considered beams in turbulent atmosphere is shorter when the wavelength is smaller.

Figure 4: The average axial intensity distribution versus the propagation distance z for GHGBs with different beam orders: $m=0$, $m=1$, $m=2$, and for two values of the beams waist: (a) $\omega_0 = 0.5mm$ and (b) $\omega_0 = 0.8mm$.

The other parameters are: $\lambda = 1330nm$, $C_n^2 = 1.10^{-14}m^{-2/3}$, $a = 0.5m$, $\mu = 1$, $h = 1$ and $l = 1$.

Fig. 4 represents the variation of on-axis average intensity of GHGBs with propagation distance z for $\lambda=1330nm$ and $C_n^2=1.10^{-14}m^{-2/3}$, with three beams orders. For every beam, and for a fixed beam order, the curves are plotted for two beams waist $\omega_0=0.5mm$ and $\omega_0=0.8mm$. From the plots of this figure, the increase of the beam order leads to the decrease of the average intensity. It is noted also that the axial intensity distribution keeps a similar shape profile for all beams. The $GHGBs_{WGBs}$ have a higher intensity however $GHGBs_{eLGBs}$ have the lower intensity compared to the other beams. We can deduct from Fig. 4, that the propagation distance for all beams becomes shorter when the waist beam is smaller.

6. Conclusion

In this paper, we have discussed the propagation properties of GHGBs as hollow beams in a turbulent atmospheric environment. Through this study, we developed the average axial intensity distribution of GHGBs using the extended Huygens-Fresnel integral and the Rytov theory. We can conclude from the numerical results, that the propagation distance for GHGBs in turbulent atmosphere becomes shorter when the wavelength and the waist beam are small and the atmospheric environment is highly turbulent. The present study is a generalization of some special cases deduced from our finding result concerning the GHGBs. We can note that $GHGBs_{WGBs}$ are characterized by the greatest intensity distribution in comparison with the other beams family during the propagation in turbulent atmosphere, which may be the more desirable for certain applications.

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