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Unsteady squeezing flow of Cu-Al₂O₃/water hybrid nanofluid in a horizontal channel with magnetic field

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Abstract

The proficiency of hybrid nanofluid from Cu-Al₂O₃/water formation as the heat transfer coolant is numerically analyzed using the powerful and user-friendly interface bvp4c in the Matlab software. For that purpose, the Cu-Al₂O₃/water nanofluid flow between two parallel plates is examined where the lower plate can be deformed while the upper plate moves towards/away from the lower plate. Another considerable factor are the wall mass suction/injection and the magnetic field which applied on the lower plate. The reduced ordinary (similarity) differential equations are solved using the bvp4c application. The validation of this novel model is conducted by comparing a few of numerical values for the reduced case of viscous fluid. The results imply the potency of this heat transfer fluid which can enhance the heat transfer performance for both upper and lower plates approximately by 7.10% and 4.11%, respectively. An increase of squeezing parameter deteriorates the heat transfer coefficient by 4.28% (upper) and 5.35% (lower), accordingly. The rise of suction strength inflates the heat transfer at the lower plate while the presence of magnetic field shows reverse result.

Keywords: Hybrid nanofluid; Heat transfer; Horizontal channel; Magnetohydrodynamics; Squeezing flow

Nomenclature

b	constant
B	time-dependent magnetic field
B_0	constant
C_{f1}, C_{f2}	skin friction coefficients of lower and upper plates, respectively
C_p	specific heat ($\text{Jkg}^{-1}\text{K}^{-1}$)
f (subscript)	base fluid
$f(\eta)$	stream function
hnf (subscript)	hybrid nanofluid
$h(t)$	distance between two plates (m)
k	thermal conductivity ($\text{Wm}^{-1}\text{K}^{-1}$)
M	magnetic parameter
Nu_{x1}, Nu_{x2}	local Nusselt number of lower and upper plates, respectively
nf (subscript)	nanofluid
Pr	Prandtl number
t	time
T	hybrid nanofluid temperature (K)
T_1, T_2	fixed temperatures of lower and upper plates, respectively (K)
u, v	fluid velocities, respectively (ms^{-1})
u_w	velocity of the stretching lower plate (ms^{-1})
v_w	velocity of the wall mass porous lower plate (ms^{-1})
V_0	constant
V_h	velocity of the upper plate moving towards/away from lower stationary plate (ms^{-1})
S	suction parameter
Sq	unsteadiness squeezing parameter
$s1, s2$ (subscript)	Al_2O_3 and Cu nanoparticles
x, y	Cartesian coordinates (m)

Greek symbols

ϕ_1, ϕ_2	concentrations for the nanoparticles
η	similarity variable
λ	stretching parameter
μ	dynamic viscosity ($\text{kgm}^{-1}\text{s}^{-1}$)

ν	kinematic viscosity (m^2s^{-1})
θ	dimensionless fluid temperature
ρ	fluid density (kgm^{-3})
ρC_p	heat capacity ($\text{JK}^{-1}\text{m}^{-3}$)

Introduction

The squeezing flow emerged from the moving boundaries is significant in the polymer processing, lubrication equipment, molding's injection and compression including the hydrodynamical machines. The connection between the squeezing flow and the loaded bearings' performance in engines including the phenomenon of adhesion has been highlighted by Jackson¹. Stefan² used lubrication approximation to initiate his work on squeezing flow. Meanwhile, another early works considering numerical scheme on the squeezing flow were studied by Verma³ and Singh et al.⁴. Hamza⁵ inspected the squeezing flow and highlighted the impact of the suction and injection parameters. Another interesting papers reflecting the squeezing viscous flow have been debated in Ahmad et al.⁶, Shah et al.⁷, Khan et al.⁸, Magalakwe et al.⁹ and Basha¹⁰.

The addition of single nanoparticle into a base working fluid is designed to enhance the base fluid's thermal conductivity which was initiated by Choi¹¹. Meanwhile, the combination of a base fluid with suspended dissimilar nanoparticles is experimentally conducted to create hybrid nanofluid. The preparation including the thermophysical properties can be read from this papers¹²⁻¹⁹. The hybrid nanofluid's correlations by Takabi and Salehi²⁰ and Devi and Devi²¹ were extensively used in the estimation of the thermophysical properties. The advantageous of hybrid nanofluid in augmenting the heat transfer performance could be found in these references²²⁻²⁶.

Hence, this work aims to analyze the squeezing flow of hybrid nanofluid from Cu-Al₂O₃/water in the presence of magnetic field. Also, motivated by the interesting works in

Raees et al.²⁷ and Hayat et al.^{28,29}, we assume that the lower plate is permeable and stretchable with a linear velocity. The single-phase mathematical model of Cu-Al₂O₃/water is formulated and then transformed into a reduced differential equations via the similarity transformation. The bvp4c application is used for the results' generation and validated based on the available numerical values from the previous works. This study is novel and original which consider an internal hybrid nanofluid flow.

Mathematical Model

Physical assumptions and thermophysical correlations

Cu-Al₂O₃/water formation is considered to flow between two infinite parallel plates, as shown

in Fig. 1. The upper plate is placed at $y = h(t) = \sqrt{\frac{v_f(1-\alpha t)}{b}}$ from the lower plate, while the

upper plate with velocity $V_h = \frac{dh(t)}{dt} = -\frac{\alpha}{2} \sqrt{\frac{v_f}{b(1-\alpha t)}}$ is moving towards (squeezing) the

lower plate. Further assumption is the lower and upper plates are maintained at fixed temperatures T_1 and T_2 , respectively. Meanwhile, the porous lower plate is included in the physical illustration for the possible fluid suction/injection with the wall mass velocity is

denoted as $v_w = -\frac{V_0}{1-\alpha t}$; $V_0 > 0$ for suction, $V_0 < 0$ for injection and $V_0 = 0$ corresponds to an

impermeable plate. Also, the lower plate is stretchable with linear velocity $u_w = \frac{bx}{1-\alpha t}$; $t < 1/\alpha$

while the inclusion of time-dependent magnetic field is formulated with $B = \frac{B_0}{1-\alpha t}$.

Fig. 1 Physical illustration with coordinate system

Under these assumptions and using the hybrid nanofluid model proposed by Takabi and Salehi²⁰, the governing conservation equations are²⁷⁻²⁹.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial V}{\partial t} + u \frac{\partial V}{\partial x} + v \frac{\partial V}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}} \frac{\partial^2 V}{\partial y^2} - \frac{\sigma_{hnf}}{\rho_{hnf}} B^2 V, \quad (2)$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \frac{\partial^2 T}{\partial y^2}, \quad (3)$$

where $V = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$. The associate conditions at the lower and upper plates are (see Raees et al.²⁷ and Hayat et al.^{28,30})

$$\begin{aligned} u &= \lambda \frac{bx}{1-\alpha t}, \quad v = -\frac{V_0}{1-\alpha t}, \quad T = T_1 \quad \text{at} \quad y = 0 \quad (\text{lower plate}) \\ u &= 0, \quad v = \frac{dh(t)}{dt}, \quad T = T_2 \quad \text{at} \quad y = h(t) \quad (\text{upper plate}) \end{aligned} \quad (4)$$

Here u and v are the velocities along x and y directions and T is the hybrid nanofluid temperature. For the evaluation of the thermophysical properties (see Table 1), we adopt the

correlations by Takabi and Salehi²⁰ which are feasible and correct based on the experimental validation. These correlations are built based on the physical assumptions. Meanwhile, Table 2 display the the physical properties of the pure water and nanoparticles.

Table 1 Hybrid nanofluid's correlations²⁰.

Properties	Hybrid Nanofluid
Density (ρ)	$\rho_{hnf} = (1 - \phi_{hnf}) \rho_f + \phi_1 \rho_{s1} + \phi_2 \rho_{s2}$
Heat Capacity (ρC_p)	$(\rho C_p)_{hnf} = (1 - \phi_{hnf}) (\rho C_p)_f + \phi_1 (\rho C_p)_{s1} + \phi_2 (\rho C_p)_{s2}$
Dynamic Viscosity (μ)	$\frac{\mu_{hnf}}{\mu_f} = \frac{1}{(1 - \phi_{hnf})^{2.5}}$
Thermal Conductivity (k)	$\frac{k_{hnf}}{k_f} = \left[\left(\frac{\phi_1 k_1 + \phi_2 k_2}{\phi_{hnf}} \right) + 2k_f + 2(\phi_1 k_1 + \phi_2 k_2) \right] \frac{-2\phi_{hnf} k_f}{\left(\frac{\phi_1 k_1 + \phi_2 k_2}{\phi_{hnf}} \right) + 2k_f - (\phi_1 k_1 + \phi_2 k_2) + \phi_{hnf} k_f}$
Electrical Conductivity (k)	$\frac{\sigma_{hnf}}{\sigma_f} = \left[\left(\frac{\phi_1 \sigma_1 + \phi_2 \sigma_2}{\phi_{hnf}} \right) + 2\sigma_f + 2(\phi_1 \sigma_1 + \phi_2 \sigma_2) \right] \frac{-2\phi_{hnf} \sigma_f}{\left(\frac{\phi_1 \sigma_1 + \phi_2 \sigma_2}{\phi_{hnf}} \right) + 2\sigma_f - (\phi_1 \sigma_1 + \phi_2 \sigma_2) + \phi_{hnf} \sigma_f}$

Table 2 Thermophysical properties for pure water and nanoparticles³⁰.

Thermophysical Properties	H ₂ O	Nanoparticles	
		Al ₂ O ₃	Cu
ρ (kgm ⁻³)	997.1	3970	8933
C_p (Jkg ⁻¹ K ⁻¹)	4179	765	385
k (Wm ⁻¹ K ⁻¹)	0.6130	40	400

Reduced Differential Equations

According to Raees et al.²⁷ and Hayat et al.^{28, 29}, the suitable dimensionless variables are

$$\left. \begin{aligned} \psi &= \sqrt{\frac{bv_f}{1-\alpha t}} x f(\eta), \quad u = \frac{bx}{1-\alpha t} f'(\eta), \quad v = -\sqrt{\frac{bv_f}{1-\alpha t}} f(\eta), \\ \eta &= \sqrt{\frac{b}{v_f(1-\alpha t)}} y, \quad \theta(\eta) = \frac{T-T_0}{T_2-T_0} \end{aligned} \right\} \quad (5)$$

Hence, substituting (5) into Eqs. (2)-(4), the ODEs and boundary conditions are

$$\left(\frac{\mu_{hmf}/\mu_f}{\rho_{hmf}/\rho_f} \right) f^{iv} + f f''' - f f'' - \frac{Sq}{2} (3f'' + \eta f''') - \left(\frac{\sigma_{hmf}/\sigma_f}{\rho_{hmf}/\rho_f} \right) M f'' = 0, \quad (6)$$

$$\frac{1}{Pr} \frac{k_{hmf}/k_f}{(\rho C_p)_{hmf}/(\rho C_p)_f} \theta'' + f \theta' - \frac{Sq}{2} \eta \theta' = 0, \quad (7)$$

$$\begin{aligned} f'(0) &= \lambda, \quad f(0) = S, \quad \theta(0) = \delta, \\ f'(1) &= 0, \quad f(1) = \frac{Sq}{2}, \quad \theta(1) = 1. \end{aligned} \quad (8)$$

where $\lambda > 0$ refers to the stretching lower plate and $\lambda = 0$ denotes the fixed/static lower plate. Another dimensionless parameters are the unsteadiness squeeze parameter Sq , magnetic parameter M , suction parameter S , Prandtl number Pr and constant δ . These parameters are defined as²⁷⁻²⁹

$$Pr = \frac{(\rho C_p)_f}{k_f}, \quad Sq = \frac{\alpha}{b}, \quad M = \frac{\sigma_f B_0}{b \rho_f}, \quad S = \frac{V_0}{bh}, \quad \delta = \frac{T_1 - T_0}{T_2 - T_0}, \quad (9)$$

In this work, we set $\delta = 0$ which in accordance with Hayat et al.^{28, 29}. Further, we notice that Eq. (6) is compatible with the reduced momentum equation in Hayat et al.²⁸ and Hayat et al.²⁹ (if the couple stress parameter is zero) with the exclusion of the hybrid nanoparticles or $\phi_1, \phi_2 \approx 0$ (regular fluid). The reduced skin friction coefficients and local Nusselt numbers at lower and upper plates are²⁷⁻²⁹

$$\text{Lower: } Re_x^{1/2} C_{f1} = \frac{\mu_{hmf}}{\mu_f} f''(0), \quad \text{Upper: } Re_x^{1/2} C_{f2} = \frac{\mu_{hmf}}{\mu_f} f''(1), \quad (10)$$

$$\text{Lower: } \text{Re}_x^{-1/2} Nu_{x1} = -\frac{k_{hmf}}{k_f} \theta'(0), \quad \text{Re}_x^{-1/2} Nu_{x2} = -\frac{k_{hmf}}{k_f} \theta'(1), \quad (11)$$

$$\text{where } \text{Re}_x = \frac{xU_w}{\nu_f}.$$

Results and Discussion

The bvp4c solver procurable in the Matlab software successfully used as the numerical computational tool in the entire work. Equations (6) to (8) are solved by transforming it first into the language of the bvp4c code as follows:

$$f = y1, \quad f' = y2, \quad f'' = y3, \quad f''' = y4, \quad (12)$$

$$\begin{aligned} f^{iv} &= \left(\frac{\rho_{hmf}/\rho_f}{\mu_{hmf}/\mu_f} \right) \left(ff'' - ff''' + \frac{Sq}{2} (3f'' + \eta f''') + \left(\frac{\sigma_{hmf}/\sigma_f}{\rho_{hmf}/\rho_f} \right) Mf'' \right), \\ &= \left(\frac{\rho_{hmf}/\rho_f}{\mu_{hmf}/\mu_f} \right) \left(y2y3 - y1y4 + \frac{Sq}{2} (3y3 + \eta y4) + \left(\frac{\sigma_{hmf}/\sigma_f}{\rho_{hmf}/\rho_f} \right) My3 \right), \end{aligned} \quad (13)$$

$$\theta = y5, \quad \theta' = y6, \quad (14)$$

$$\begin{aligned} \theta'' &= \text{Pr} \left(\frac{(\rho C_p)_{hmf}/(\rho C_p)_f}{k_{hmf}/k_f} \right) \left(\frac{Sq}{2} \eta \theta' - f \theta' \right), \\ &= \text{Pr} \left(\frac{(\rho C_p)_{hmf}/(\rho C_p)_f}{k_{hmf}/k_f} \right) \left(\frac{Sq}{2} \eta y6 - y1y6 \right), \end{aligned} \quad (15)$$

$$\begin{aligned} f'(0) &= \lambda, \quad f(0) = S, \quad \theta(0) = \delta, \\ ya2 - \lambda, \quad ya1 - S, \quad ya5 - \delta, \\ f'(1) &= 0, \quad f(1) = \frac{Sq}{2}, \quad \theta(1) = 1, \\ yb2, \quad yb1 - \frac{Sq}{2}, \quad yb5 - 1. \end{aligned} \quad (16)$$

The results are generated and graphically presented for the distribution of the skin friction coefficients, velocity, heat transfer rates and temperature of both upper and lower plates. The

value of the Prandtl number is fixed to 6.2 which indicates the use of water at 25°C while the initial temperature at lower plate's wall is represented by $\delta = 0$ so that $\theta(0) = 0$. Another parameters are controlled within the ranges of $0 \leq Sq \leq 1$ (unsteadiness squeezing parameter), $-1.2 \leq S \leq 1.2$ (suction/injection parameter), $0 \leq \lambda \leq 2$ (stretching parameter) and $0 \leq M \leq 3$ (magnetic parameter). To further validate the precision of the present model, the numerical values are compared with Hayat et al.^{28, 29} as displayed in Tables 3 and 4 which shows identical results.

Table 5 shows variety of $\text{Re}_x^{1/2} C_{f1}$, $\text{Re}_x^{1/2} C_{f2}$, $-\text{Re}_x^{-1/2} Nu_{x1}$ and $-\text{Re}_x^{-1/2} Nu_{x2}$. With the consideration of viscous fluid, lower static plate and the exclusion of magnetic, suction/injection and squeezing parameters, the internal flow have zero frictions and equal heat transfer rates at both plates. The observation in Table 5 shows that as the stretching parameter increases from $\lambda = 0$ to $\lambda = 1$, $-\text{Re}_x^{-1/2} Nu_{x1}$ increases, but the stretching lower plate tends to increase $\text{Re}_x^{1/2} C_{f1}$. Further, we analyze for four types of fluids: viscous/water ($\phi_1, \phi_2 = 0$), Al₂O₃-water ($\phi_1 = 0.01, \phi_2 = 0$), Cu-water ($\phi_1 = 0, \phi_2 = 0.01$) and Cu-Al₂O₃/water ($\phi_1, \phi_2 = 0.01$) which reveals that the Cu-Al₂O₃/water has the highest heat transfer coefficients at both plates followed by the Cu-water, Al₂O₃-water and water. This implies the suitability of Cu-Al₂O₃/water hybrid nanofluid as the effective coolant in the engineering/technology appliances. Since the inclusion of the squeezing parameter can reduce the heat transfer rate at both plates, it is useful to know the exact parameters which can assist the heat transfer performance for this situation. In Table 6, we analyze the difference percentage of the heat transfer rate at both upper and lower plate which can be used for future assessment by other engineers or researchers. This analysis also can help in determining the preferable strength of the parameters either in the augmentation or reduction of the heat transfer in both locations.

Table 3 Comparative values of $f''(0)$ -lower plate and $f''(1)$ -upper plate when $Sq = 0$, $\lambda = 1$, $\phi_1, \phi_2 = 0$ with various S and M .

M	S	$f''(0)$		$f''(1)$	
		Present	Hayat et al. ²⁹	Present	Hayat et al. ²⁹
0	0.5	-7.4111525	-7.411153	4.7133028	4.713303
1	0.5	-7.5916177	-7.591618	4.7390165	4.739017
4	0.5	-8.1103342	-8.110334	4.8202511	4.820251
9	0.5	-8.9100956	-8.910096	4.9648698	4.964870
4	0.0	-4.5878911	-4.587891	1.8424469	1.842447
4	0.3	-6.6656620	-6.665662	3.6536948	3.653695
4	0.6	-8.8514442	-8.851444	5.3912475	5.391248
4	1.0	-11.9485843	-11.948584	7.5934262	7.593426

Table 4 Comparative values of $f''(1)$ -upper plate when $\lambda = 1$ and $\phi_1, \phi_2 \approx 0$ with various S , Sq and M .

M	Sq	S	$f''(1)$	
			Present	Hayat et al. ²⁸
0	1	0.5	1.814634	1.81463
0.25	1	0	-1.171551	-1.17155
0.25	1	0.5	1.808177	1.80818
0.25	0	0.5	4.719656	4.79166
0.25	1.5	0.5	0.283948	0.28395
0.25	1	1	4.573016	4.57302
1	1	0.5	1.789372	1.78937

Table 5 Numerical values of $\text{Re}_x^{1/2} C_{f1}$, $\text{Re}_x^{1/2} C_{f2}$, $-\text{Re}_x^{-1/2} Nu_{x1}$ and $-\text{Re}_x^{-1/2} Nu_{x2}$ with various values of the control parameters.

Sq	M	S	ϕ_1	ϕ_2	λ	$\text{Re}_x^{1/2} C_{f1}$	$\text{Re}_x^{1/2} C_{f2}$	$-\text{Re}_x^{-1/2} Nu_{x1}$	$-\text{Re}_x^{-1/2} Nu_{x2}$
0	0	0	0	0	0	0	0	1	1
0	0	0	0	0	0.5	-2.021410	0.988195	1.161853	0.898716
0	0	0	0	0	1	-4.085563	1.953179	1.336614	0.802165
0	0	0	0.01	0	1	-4.189890	2.002674	1.363747	0.831425
0	0	0	0	0.01	1	-4.194140	2.000385	1.365114	0.832586
0	0	0	0.01	0.01	1	-4.302130	2.051712	1.393870	0.863450
1	0	0	0.01	0.01	1	-1.240570	-1.220273	1.323125	0.828030
1	1	0	0.01	0.01	1	-1.322328	-1.301313	1.318921	0.831390
1	1	0.2	0.01	0.01	1	-2.688739	-0.003914	1.893656	0.668863
1	1	-0.2	0.01	0.01	1	-0.003540	-2.635965	0.887339	1.001950

Table 6 Heat transfer analysis with the addition of the control parameters.

Parameters	Develop/reduce the thermal rate at lower plate	Difference percentage of $-\text{Re}_x^{-1/2} Nu_{x1}$	Develop/reduce the thermal rate at upper plate	Difference percentage of $-\text{Re}_x^{-1/2} Nu_{x2}$
Squeezing (Sq)	Reduce	-5.35%	Reduce	-4.28%
Magnetic (M)	Reduce	-0.32%	Develop	0.40%
Suction ($S > 0$)	Develop	30.35%	Reduce	-24.30%
Injection ($S < 0$)	Reduce	-48.64%	Develop	17.02%

The exploration of pertinent parameters' impact such as squeezing, suction/injection and stretching parameters are continued with the observation on $f'(\eta)$ and $\theta(\eta)$ as displayed in Figs. 2-7. In Fig. 2, the velocity distribution enhances with the addition of unsteadiness squeezing parameter. As the squeezing parameter's magnitude increases up to $Sq=1$, the velocity distribution is depreciated at the upper plate. This is due to the squeezing effect which

originated from the upper plate. However, the temperature profile in Fig. 3 slightly reduces near to the lower plate ($\eta < 0.5$) while increases near to the upper plate ($\eta > 0.5$).

Further, the impact of suction/injection parameter on both profiles is visualized in Figs. 4 and 5. As the suction/injection parameter increases from injection to suction ($S = -1.2, -0.2, 0, 0.2, 1.2$), the velocity profile decreases which reflects the higher magnitude of suction strength can reduce the velocity distribution. Since the suction is applied through the lower permeable plate, the velocity lessens while increases near to the upper plate. The temperature profile augments at both locations (lower and upper plate). Figures 6 and 7 present the plots of velocity and temperature distribution with variety values of the stretching parameter. The velocity gradually increases when $\eta < 0.3$ while a contrary observation is obtained for $\eta > 0.3$. The temperature profile increases for both lower and upper plates.

Fig. 2 Effect of squeezing parameter on $f'(\eta)$ when $S = 0.2$, $\lambda = 0$, $M = 1$ and $\phi_1 = \phi_2 = 0.01$.

Fig. 3 Effect of squeezing parameter on $\theta(\eta)$ when $S = 0.2$, $\lambda = 0$, $Sq = M = 1$ and $\phi_1 = \phi_2 = 0.01$

Fig. 4 Effect of suction/injection parameter on $f'(\eta)$ when $\lambda = 0$, $Sq = M = 1$ and $\phi_1 = \phi_2 = 0.01$.

Fig. 5 Effect of suction/injection parameter on $\theta(\eta)$ when $\lambda = 0$, $Sq = M = 1$ and $\phi_1 = \phi_2 = 0.01$.

Fig. 6 Effect of stretching parameter on $f'(\eta)$ when $S = 0.2$, $Sq = M = 1$ and $\phi_1 = \phi_2 = 0.01$.

Fig. 7 Effect of stretching parameter on $\theta(\eta)$ when $S = 0.2$, $Sq = M = 1$ and $\phi_1 = \phi_2 = 0.01$.

Conclusion

This novel work presents a numerical study of Cu-Al₂O₃/water inside two plates (parallel lower and upper) with the appearance of magnetic field. The lower plate is permeable for the suction/injection processes and also can be stretched. Meanwhile, the upper plate can move towards the lower plate and creates the squeezing flow phenomenon. The mathematical model which suits this physical phenomenon follows the usual approximations of boundary layer flow while the bvp4c programme is fully utilized for the generation of the results. The distribution of $\text{Re}_x^{1/2} C_{f1}$, $\text{Re}_x^{1/2} C_{f2}$, $-\text{Re}_x^{-1/2} Nu_{x1}$ and $-\text{Re}_x^{-1/2} Nu_{x2}$ are examined. The heat transfer rate of the lower plate reduces with the increase of magnetic, squeezing and injection parameters. However, about 30.35% of $-\text{Re}_x^{-1/2} Nu_{x1}$ is developed with the inclusion of suction. Meanwhile, the enhancement of magnetic and injection parameters can lead to the development of upper plate's heat transfer performance. Another observation is conducted for the distribution of the velocity and temperature profiles. The addition of squeezing and

stretching parameters can increase the velocity profile whereas high suction's magnitude shows opposite trend.

Author contributions

NSK conducted the numerical computation and wrote the manuscript, IW conducted the numerical computation and validated the solutions, NMA and IP formulated the problem and provided the literature review.

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Additional information

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Figures

Figure 1

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Figure 2

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Figure 3

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Figure 4

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Figure 5
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Figure 6
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Figure 7

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